

# **Yes, We Should Discount the Far-Distant Future at Its Lowest Possible Rate: A Resolution of the Weitzman–Gollier Puzzle**

*Mark C. Freeman*

*Bradford University School of Management, UK*

**Abstract** In this paper the author proves that the Expected Net Future Value (ENFV) criterion can lead a risk neutral social planner to reject projects that increase expected utility. By contrast, the Expected Net Present Value (ENPV) rule correctly identifies the economic value of the project. While the ENFV increases with uncertainty over future interest rates, the expected utility decreases because of the planner’s desire to smooth consumption across time. This paper therefore shows that Weitzman (1998) is “right” and that, within his economy, the far-distant future should be discounted at its lowest possible rate.

**JEL** D61, E43, G12, G31, Q51

**Keywords** Discount rates; term structure; capital budgeting; interest rate uncertainty; environmental planning

**Correspondence** Mark C. Freeman, Bradford University School of Management, Emm Lane, Bradford, West Yorkshire, BD9 4JL, United Kingdom,  
Email: [M.C.Freeman@Bradford.ac.uk](mailto:M.C.Freeman@Bradford.ac.uk)

## Introduction

In this paper I show that the Expected Net Future Value (ENFV) investment criterion is not necessarily consistent with expected utility maximisation. If it is chosen as the capital budgeting technique by risk neutral social planners then this can lead to them rejecting projects that increase expected utility. By contrast, the Expected Net Present Value (ENPV) criterion is consistent with expected utility maximisation and therefore this should be the preferred technique. As a consequence, when future interest rates are persistent and unknown, the term structure of social discount rates should be downward sloping. This is consistent with recommendations contained within the UK Treasury Guidance on Appraisal and Evaluation in Central Government (the “Green Book”). The French government also requires that public institutions apply lower discount rates once the cash flow maturity becomes more than thirty years (Gollier (2009b)). Furthermore, the findings in this paper support the analysis contained within the Stern Review (Stern (2007)), where a baseline social discount rate of around 1.4% is used to evaluate the future costs of climate change even though many authors have noted that this is substantially below shorter-term market rates of return (for example, Nordhaus (2007), Weitzman (2007) and Dasgupta (2008)).<sup>1</sup>

The theoretical justification for using a downward sloping term structure of social discount rates is given in Weitzman (1998) and Weitzman (2001) in an economy where policy makers are risk neutral but cannot perfectly forecast future interest rates. He shows that, through a Jensen’s inequality effect, cost of capital uncertainty increases the ENPV of future cash flows. When interest rates are highly persistent, this leads to a sharply declining schedule of social discount rates. Calibrations of the term structure for more realistic interest rate processes have been constructed by Newell and Pizer (2003), Newell and Pizer (2004), Guo et al. (2006), Groom et al. (2007) and Gollier et al. (2008). They demonstrate that this effect is of economic significance for far horizon projects and generates important policy implications for the evaluation of long term environmental and energy projects. Tackling climate change, for example, becomes a more urgent

---

<sup>1</sup> The relationship between the long-term discount rates recommended in the Green Book and those used in the Stern Review is discussed in the UK Treasury’s supplementary guidance on intergenerational wealth transfers (Lowe (2008)).

priority while using nuclear power as a means to reduce greenhouse gas emissions is relatively less attractive as an option given the future costs of decommissioning.

As a response to this, Gollier (2004) has extended a paradox that can be traced back to Pazner and Razin (1975). He proves that the recommendation that low discount rates should be applied at far horizons is highly sensitive to the chosen investment appraisal technique. If policy makers were to put money aside today to deal with environmental problems in the future, then again the effect of Jensen's inequality means that the expected future value of this saving increases with cost of capital uncertainty. As a consequence, this implies that policy makers, using an ENFV criterion, should give lower priority today to climate change abatement programmes. This apparent sensitivity of the optimal policy decision concerning long term initiatives to the method of capital appraisal is sometimes referred to as the Weitzman-Gollier puzzle.

In this paper, I resolve this paradox. Previous interesting proposed resolutions have been presented by Hepburn and Groom (2007), Gollier (2009a), Gollier (2009b) and Buchholz and Schumacher (2009). This paper makes a number of important extensions to this debate. In particular, I show that within the setting of Weitzman's (1998) paper, a fixed future cost becomes unambiguously more unattractive as interest rate uncertainty increases. As a consequence, it is the Weitzman ENPV that is the correct evaluation criterion and not the Gollier ENFV criterion. This contrasts with Gollier (2004): "Clearly, Weitzman and I cannot be both right. In fact, to tell the truth, I believe that we are both wrong" (p.88), Hepburn and Groom (2007): "Our conclusion, perhaps surprisingly, is that Weitzman and Gollier are both right" (p.107), Gollier (2009a): "This demonstrates that, as suggested by Hepburn and Groom (2007), both Weitzman (1998) and Gollier (2004) are right" (p.6), Gollier (2009b): "In a sense, contrary to our conclusion in Gollier (2004), both Weitzman (1998) and Gollier (2004) are right..." (p.8) and Buchholz and Schumacher (2009): "Much more is in favor of Gollier's approach because he puts the risk to the right place, i.e. to the future period" (p.4).

This paper also makes methodological improvements on previous explanations for the puzzle. In contrast to Gollier (2009a), Gollier (2009b) and Buchholz and Schumacher (2009), the social planner remains risk neutral within the economy of this paper. This is consistent with the original paradox and shows that there is no requirement to call on risk aversion to resolve the problem. Further,

in contrast to Hepburn and Groom (2007), there is no need to introduce arbitrary evaluation dates to reconcile the different approaches and show that policy makers are correct to use declining schedules of social discount rates.

The paper proceeds as follows. Section 1 briefly describes the paradox and the resolution proposed by Hepburn and Groom (2007). Section 2 develops the new proposed resolution, shows that the ENPV and ENFV criteria can be reconciled and demonstrates why the ENPV method is correct. Section 3 explains further why the ENFV method cannot be used to evaluate projects when interest rates are stochastic. Section 4 concludes.

## 1 The Puzzle

The Weitzman-Gollier puzzle arises in the following, highly stylised, economy. At time  $-\delta$ , the future short-term interest rate  $\tilde{r}$  is unknown but lies in the range  $[r_{\min}, r_{\max}]$ . The true interest rate,  $\tilde{r} = r$ , will be revealed in the next instant, time 0, and then never change again.

For Expected Net Present Values (ENPV), a risk-neutral social planner contemplates spending  $p$  at time 0 to avoid a fixed cost,  $D_T$ , that will otherwise arise with certainty at time  $T$ . As all uncertainty is resolved at time 0, the planner values the proposal at this time using a discounted cash flow technique with the cost of capital equal to the risk-free rate;  $NPV_0 = -p + D_T e^{-rT}$ . One instant earlier, at time  $-\delta$ , the expectation of this NPV is  $E[NPV_0] = -p + D_T E[e^{-\tilde{r}T}]$ . As the social planner is risk neutral, *ceteris paribus*, this represents the economic value of the project at time  $-\delta$ ;  $ENPV = E[NPV_0]$  and:

$$ENPV = -p + D_T E[e^{-\tilde{r}T}] \quad (1)$$

The project is accepted (rejected) if the  $ENPV > 0 (< 0)$ . The  $T$ -period discount rate,  $r_d(T)$ , is defined by:

$$e^{-r_d(T)T} = E[e^{-\tilde{r}T}] \quad (2)$$

Under the Expected Net Future Value (ENFV) criterion,  $p$  has already been put aside to deal with a potential threat. The social planner is considering taking this

money away from the preventative measure and investing it in a rolling portfolio of Treasury bills instead. The proceeds of this investment strategy will be used to deal with the threat, which results in a certain cost  $D_T$ , when it arises. The incremental cash flows from this change of strategy are zero at time 0 and  $pe^{rT} - D_T$  at time  $T$ . At time  $-\delta$ , the project is accepted (rejected) provided that  $ENFV > 0$  ( $< 0$ ) where:

$$ENFV = pE \left[ e^{\tilde{r}T} \right] - D_T \quad (3)$$

The  $T$ -period compound rate,  $r_c(T)$ , is defined by:

$$e^{r_c(T)T} = E \left[ e^{\tilde{r}T} \right] \quad (4)$$

The paradox arises from differences between  $r_d(T)$  and  $r_c(T)$ . As Gollier (2004) explains, (2) and (4) can be interpreted as exercises in exponential utility.  $r_d(T)$  is the certainty equivalent of  $\tilde{r}$  when a pseudo-investor has a constant coefficient of absolute risk aversion  $T$ . As  $T$  gets larger, so the risk aversion of the pseudo-agent increases and  $r_d(T)$  decreases. In the limit, as  $T \rightarrow \infty$ , the pseudo-agent becomes infinitely risk averse and  $r_d(T) \rightarrow r_{\min}$ . By contrast,  $r_c(T)$  is the certainty equivalent of  $\tilde{r}$  when a pseudo-investor has a coefficient of absolute risk aversion  $-T$ . The pseudo-investor now becomes increasingly risk seeking with growing  $T$  and  $r_c(T) \rightarrow r_{\max}$  in the limit. Equivalently, for fixed  $T$ ,  $r_d(T)$  decreases and  $r_c(T)$  increases if the uncertainty surrounding  $r$  rises in a mean-preserving way.

Weitzman (1998) and Weitzman (2001) use this argument in relation to  $r_d(T)$  to contend that low discount rates should be applied to far-horizon costs, raising their perceived net present value. Newell and Pizer (2003), Newell and Pizer (2004), Guo et al. (2006), Groom et al. (2007) and Gollier et al. (2008) calibrate interest rate models to show that this effect can be of major economic significance. These recommendations currently influence both British and French governments' advice on social discounting.

Gollier (2004), by contrast, uses the argument in relation to  $r_c(T)$  to contend that, if we start saving today to deal with threats in the future, then interest rate uncertainty increases our expected future wealth to deal with the problem when it arises. Equation (1) suggests that, with rising interest rate uncertainty, we

should place more money today into preventing future environmental costs while (3) suggests that we should simultaneously take money away from similar existing projects and invest in financial assets instead. This is the puzzle.

This paradox had previously been recognised by Pazner and Razin (1975). It is also a restatement of a well-known result of Cox et al. (1981) that the local expectations hypothesis of the term structure of interest rates is inconsistent with the returns-to-maturity expectations hypothesis. If  $B_{-\delta T}$  is the time  $-\delta$  price of the default risk-free zero coupon bond with face value of \$1 and maturity at time  $T$ , then Cox et al. (1981) show that:

$$\begin{aligned} B_{-\delta T} &= E[e^{-\tilde{r}T}] & \text{i.i.w. } B_{-\delta T}^{-1} &= E[e^{\tilde{r}T}] \\ \implies 0 &= E[e^{-\tilde{r}T}] - B_{-\delta T} & \text{i.i.w. } 0 &= B_{-\delta T} E[e^{\tilde{r}T}] - 1 \end{aligned} \quad (5)$$

where “i.i.w” reads as “is inconsistent with”. The right-hand sides of these two equations are respectively the ENPV and ENFV criteria (1) and (3) with  $p = B_{-\delta T}$  and  $D_T = 1$ .

Hepburn and Groom (2007) propose a resolution. Their certainty equivalent discount rate,  $r_{ca}(T, \tau)$  depends on both the horizon of the threat and an evaluation date,  $\tau$ , and is defined by:

$$e^{-(T-\tau)r_{ca}(T,\tau)} = E[e^{-\tilde{r}(T-\tau)}] \quad (6)$$

This measure nests  $r_d(T)$  when  $\tau = 0$  and  $r_c(T)$  when  $\tau = 2T$ . Now the coefficient of absolute risk aversion of the pseudo-investor is  $T - \tau$ . The appropriate cost of capital is decreasing in  $T$ , as with ENPV, but increasing in  $\tau$ , as with ENFV. This analysis, though, provides no insights into the appropriate evaluation date and thus cannot objectively judge between the ENPV and ENFV criteria. I propose an alternate resolution to the paradox that overcomes these limitations.

## 2 Resolving the Puzzle

In this section, I take a utility based approach to show that when agents are risk neutral (i) increased interest rate uncertainty unambiguously makes a fixed future cost more unattractive, (ii) that the ENFV and ENPV strategies have the same

expected utility and (iii) that the ENPV investment criterion is consistent with expected utility maximisation while the ENFV criterion is not.

The analysis in this section extends utility based work on the Weitzman-Gollier puzzle by Gollier (2009a), Gollier (2009b) and Buchholz and Schumacher (2009). In particular, Gollier (2009a) shows the consistency of ENPV and ENFV when evaluated when using expected utility; a result that is also demonstrated here. Gollier (2009b) proves that the ENPV criterion reflects the socially efficient discount rate when the social planner has logarithmic utility, which is a more restrictive version of the results presented here. This section extends previous analysis by incorporating explicit risk neutrality into a Lucas (1978) endowment (“pure exchange”) economy.

The original Weitzman-Gollier puzzle is set in the context of a risk-neutral social planner. However, Gollier (2009a), Gollier (2009b) and Buchholz and Schumacher (2009) all assume that the utility function is time-separable. As a consequence, curvature in the utility function captures both the agent’s desire to smooth consumption across time (the Elasticity of Intertemporal Substitution; EIS) and across states at any given time (risk aversion). As the third anonymous referee to the working paper version of Gollier (2009b) asks “Would these results hold in a more general model where the representative agent smooths consumption over time, but is not necessarily risk averse at any given point in time? The Weitzman-Gollier puzzle was raised in a risk-free setting, and answering this question would give us more traction on whether it is optimality or risk aversion or both that is required to resolve it.”<sup>2</sup> In this section, the EIS is separated from the coefficient of risk aversion, allowing for concavity in the utility function while retaining risk neutrality, which directly addresses the referee’s question.

This paper also extends Gollier (2009b) by adapting the economic setting. Gollier (2009b) takes a highly stylised production economy where (i) there exists a risk-free production asset, (ii) there is constant marginal productivity of capital and (iii) there are no transactions costs from investing in production. At time  $-\delta$ , the social planner does not know the productivity of capital, which in turn must equal the risk-free rate as the production asset is risk-free. Once this parameter is revealed at time 0, the planner can, without friction, optimise current and fu-

---

<sup>2</sup> <http://www.economics-ejournal.org/economics/discussionpapers/2009-7>

ture consumption by investing in the production asset to maximise intertemporal welfare. In this section, I follow Lucas (1978) and assume that the consumption good cannot be stored or saved and so aggregate endowment is fully consumed at the time it becomes available. It is assumed that time 0 endowment is known by the planner at time  $-\delta$ , but that the growth rate of the future endowment path is not. Once information about future endowment growth is revealed at time 0, the risk-free rate adjusts to ensure that financial markets remain in equilibrium. This endowment economy has been extensively applied in financial economics, particularly in the literature on the equity premium puzzle following Mehra and Prescott (1985). A discussion of the relative merits of modelling with linear production and endowment economies is given by Cochrane (2005).

An advantage of using an endowment economy here is that it fixes time zero consumption. By contrast, in Gollier's (2009b) production framework, time zero consumption will adjust to the revealed productivity of capital unless the social planner has logarithmic utility. This means that while, in Gollier's (2009b) set-up the ENPV criterion is only consistent with the socially efficient discount rate when the coefficient of relative risk aversion is exactly equal to one, this consistency always holds here.

## 2.1 The Economic Framework

Assume that there is an immortal social planner who consumes  $\tilde{c}_t$  of the single consumption good at time  $t$ . This social planner has an intertemporal welfare function at time  $\zeta$ ,  $\Psi_\zeta$ , similar to that described by Gollier (2002):

$$\Psi_\zeta = \sum_{t=\zeta}^{\infty} e^{-\rho(t-\zeta)} u(m_t) \quad (7)$$

where  $\rho$  is the constant time preference factor of utility.  $u(\cdot)$  is monotonic increasing, strictly concave and whose first derivative has a well-defined inverse function. This function captures the agent's desire to smooth consumption across time.  $m_t$  is the certainty equivalent of consumption at time  $t$  and is defined by:

$$v(m_t) = E_\zeta[v(\tilde{c}_t)] \quad (8)$$

$v(\cdot)$  captures the agent's aversion to instantaneous risk. When  $v(\cdot) \equiv u(\cdot)$ , the utility function becomes time separable and the utility framework is the same as Gollier (2009a), Gollier (2009b) and Buchholz and Schumacher (2009). Here, though, to ensure that the social planner remains risk neutral, it is assumed that  $v''(\cdot) = 0$ . By Jensen's inequality,  $E_{\zeta}[v(\tilde{c}_t)] = v(E_{\zeta}[\tilde{c}_t])$  and thus  $m_t = E_{\zeta}[\tilde{c}_t]$ . The required curvature in the utility function is explicitly generated through  $u(\cdot)$  rather than  $v(\cdot)$ , thus demonstrating that the puzzle arises from the desire to smooth consumption across time rather than across states.

In a pure exchange economy, the social planner will receive an exogenous endowment stream  $y_t$  for all  $t \in [0, 1, \dots]$ . Because the consumption good cannot be stored or saved,  $c_t = y_t$  for all  $t$ . At time  $-\delta$ ,  $y_0$  is known perfectly and it is also known at this time that future income will be generated by either one or the other of the following two processes (the agent does not need to know which one):

$$y_t = \begin{cases} u'^{-1} [e^{\rho - \tilde{r}} u'(y_{t-1})] + \omega_t \\ \eta_t u'^{-1} [e^{\rho - \tilde{r}} u'(y_{t-1})] \end{cases} \quad (9)$$

where  $\omega_t, \eta_t$  are additive and multiplicative noise terms  $E_{t-1}[\omega_t] = 0, E_{t-1}[\eta_t] = 1$  and  $\tilde{r}$  is a constant whose value is not known at time  $-\delta$ . At time 0 the value of  $\tilde{r}$  is revealed as  $r$  and then never changes. The functional restrictions on  $y_t$  given in (9), which ensure that the interest rate will take the constant value  $r$  after time 0, are discussed in more detail in the appendix. In particular, it is shown that, when  $u(\cdot)$  takes power form, then identically and independently normally distributed logarithmic endowment growth satisfies (9). The endowment processes specified in (9) do not require that all stochasticity in the system is removed at time  $t = 0$  as  $\eta_t, \omega_t$  remain unknown until time  $t$ .

Let  $B_{0T}, R(T)$  denote respectively the time 0 price and yield of a default risk-free zero-coupon bond that matures at time  $T$  with a face value of \$1. Financial markets are in equilibrium if the welfare of the social planner is unchanged when reducing consumption at time 0 by  $B_{0T}$  in exchange for an additional \$1 at time  $T$

$$u(c_0 - B_{0T}) + e^{-\rho T} u(E_0[c_T] + 1) = u(c_0) + e^{-\rho T} u(E_0[c_T])$$

Taking a first order Taylor's series expansion:

$$e^{-R(T)T} = B_{0T} = e^{-\rho T} u'(E_0[c_T]) / u'(c_0) \quad (10)$$

As interest rates are non-stochastic after time 0, the local expectations hypothesis can be invoked, so that  $R(T) = r$  for all  $T$ : Gollier (2009a) presents a similar result for time-separable utility. As interest rates are driven by the marginal utility of future expected consumption, high payouts from a strategy of investing in a rolling portfolio of Treasury bills are expected to occur at times when the consumption good gives low additional utility. By concentrating on wealth alone, the ENFV approach fails to reflect this.

## 2.2 Welfare Changes using ENPV and ENFV

Providing that the potential future threat is sufficiently small in relation to  $E_0[c_T]$ , the change in welfare at time 0,  $\Delta\Psi_0$ , from undertaking an ENFV strategy that results in an incremental future cash flow of  $pe^{rT} - D_T$  is given by a first order Taylor's series expansion:

$$\Delta\Psi_0 = e^{-\rho T} (pe^{rT} - D_T) u'(E_0[c_T]) \quad (11)$$

Substituting from (10):

$$\begin{aligned} \Delta\Psi_0 &= (pe^{rT} - D_T) e^{-rT} u'(c_0) \\ &= (p - D_T e^{-rT}) u'(c_0) \end{aligned} \quad (12)$$

The expected change in welfare one instant earlier is:

$$\begin{aligned} E_{-\delta}[\Delta\Psi_0] &= \left( p - D_T E \left[ e^{-\tilde{r}T} \right] \right) u'(c_0) \\ &= \left( p - D_T e^{-r_d(T)T} \right) u'(c_0) \end{aligned} \quad (13)$$

As  $r_d(T)$  can be interpreted as the certainty equivalent of  $\tilde{r}$  for an investor with coefficient of absolute risk aversion  $T$ , so  $r_d(T)$  decreases as interest rate uncertainty increases, all else being equal. Therefore, while the ENFV measure increases as interest rates become more uncertain, the associated expected change in utility decreases. This demonstrates the weakness of the ENFV approach, the limitations of which are discussed in more detail in the next section.

Turn next to the ENPV strategy of spending an amount  $p$  at time 0 to save  $D_T$  at time  $T$ . In this case, the change in welfare is:

$$\begin{aligned} \Delta\Psi_0 &= -pu'(c_0) + D_T e^{-\rho T} u'(E_0[c_T]) \\ \implies E_{-\delta}[\Delta\Psi_0] &= (D_T e^{-r_d(T)T} - p)u'(c_0) \end{aligned} \quad (14)$$

and it follows immediately that rising interest rate uncertainty decreases the incentive to disinvest by exactly the same amount as it increases the incentive to invest in new similar projects. The equivalence between the ENPV and ENFV strategies when evaluated using expected utility is unsurprising. The ENFV approach apparently reveals value by investing in financial markets. However, in an efficient capital market, trading in financial assets always has a zero net effect on expected utility. Therefore the initiative is either attractive or not, irrespective of how it is funded. Gollier (2009a) presents a similar result for a risk averse planner by showing that the ENPV, ENFV and Ramsey discount rates are equivalent when the relationship between interest rates and marginal utility is explicitly modelled. Here, this result is extended to a risk-neutral endowment economy. This allows for the main result of this paper:

**Theorem 1.** The ENPV criterion correctly identifies the attractiveness of social initiatives. The ENFV criterion, by contrast, can lead a risk-neutral social planner to reject projects that increase expected utility. ■

**Proof.** The first statement follows immediately from (14). A project increases expected utility if and only if  $p < D_T e^{-r_d(T)T}$  as the ENPV criterion states. In addition, (14) shows that the current economic value of the initiative is  $D_T e^{-r_d(T)T} - p$ , which is exactly the same as the ENPV. For the second statement, consider a project where  $D_T = p \exp[(r_c(T) + r_d(T))T/2]$ . We can express  $r_c(T) = r_d(T) + 2\varepsilon/T$  for some  $\varepsilon > 0$  when interest rates are stochastic, so  $D_T = p \exp[r_c(T)T - \varepsilon] = p \exp[r_d(T)T + \varepsilon]$ . Then, from (14) the expected change in welfare from this project is:

$$\begin{aligned} E_{-\delta}[\Delta\Psi_0] &= (p e^{r_d(T)T + \varepsilon} e^{-r_d(T)T} - p)u'(c_0) \\ &= p[e^\varepsilon - 1]u'(c_0) \end{aligned} \quad (15)$$

which is greater than zero. Therefore, the social planner should accept this initiative. However, from Gollier (2004), if she uses the ENFV criterion, a project is accepted if and only if  $D_T - pe^{r_c(T)T} > 0$ . In this case, according to the ENFV criterion:

$$\begin{aligned} D_T - pe^{r_c(T)T} &= pe^{r_c(T)T-\varepsilon} - pe^{r_c(T)T} \\ &= p[e^{-\varepsilon} - 1]e^{r_c(T)T} \end{aligned} \quad (16)$$

and this is negative, leading the social planner to reject a project that increases expected utility. ■

It is straightforward to verify that the ENFV measure “falsely” rejects projects whenever  $p_T e^{r_d(T)T} < D_T < p_T e^{r_c(T)T}$ . By multiplying both sides of the first inequality by  $e^{-r_d(T)T}$ , the project passes the ENPV hurdle, which, from Theorem 1, is consistent with expected utility maximisation. However, from the second inequality, the project would be rejected under ENFV.

### 2.3 Comparison with Gollier (2009b)

It is important to contrast this result with Proposition 2 in Gollier (2009b). He argues that the ENPV criterion is consistent with welfare maximisation if and only if utility is logarithmic. In the analysis presented here, this result is always true. To understand this distinction, return to the expectations taken in equations 13 and 14. In both cases, if  $c_0$  is unknown at time  $-\delta$ , these become:

$$E_{-\delta}[\Delta\Psi_0] = -pE_{-\delta}[u'(c_0)] + D_T \left\{ E_{-\delta}[u'(c_0)e^{-\tilde{r}T}] \right\} \quad (17)$$

Therefore, in both cases, expected welfare increases if and only if:

$$D_T \left\{ e^{-r_d T} + \frac{Cov_{-\delta}[u'(c_0), e^{-\tilde{r}T}]}{E_{-\delta}[u'(c_0)]} \right\} - p > 0 \quad (18)$$

For the ENPV criterion to be consistent with welfare maximisation, it is necessary that  $c_0$  is independent of  $\tilde{r}$ . In the endowment economy presented here, this

holds with certainty as it has been assumed that  $y_0$ , and hence  $c_0$ , is known by the social planner at time  $-\delta$ . In Gollier's (2009b) production economy,  $c_0$  can adapt to the revealed value of  $r$  and therefore independence is no longer assured. In general, only when the coefficient of relative risk aversion is equal to one will  $Cov_{-\delta} [u'(c_0), e^{-rT}] = 0$ .

Which of these frameworks is more realistic? A justification for the complete inelasticity of investment presented in this endowment economy is that, in equilibrium, borrowing must always equal lending. It is, therefore, not possible for aggregate savings to adjust through the risk-free asset whatever the revealed interest rate. Gollier (2009b) counters this problem by having a risk-free production asset whose supply is perfectly elastic. As Cochrane (2005) observes "Which of these possibilities is correct? Well, neither, of course" (p.39). The true relationship between  $c_0$  and  $r$  must either be derived from sophisticated general equilibrium models or detailed empirical analysis. This work lies beyond the scope both of this paper and Gollier (2009b).

### 3 A Re-Examination of the ENFV Criterion

The previous section extends Proposition 2 of Gollier (2009b) into a risk-neutral endowment economy. It is shown that the ENPV criterion is always consistent with welfare maximisation while the ENFV criterion rejects some welfare increasing projects. In this section, I critique the ENFV criterion in a different way that, to my knowledge, has not previously been recognized in the literature on the Weitzman-Gollier puzzle.

Gollier (2004) justifies the ENFV criterion by stating that (using the notation of this paper) "This is equivalent to requiring that the future payoff  $D_T$  be larger than the expected payoff of investing the money on the capital market" (p.87). Here, I argue that  $pe^{r_c(T)T}$  cannot be properly interpreted as the expected value of investing  $p$  in the risk-free asset until time  $T$ .

The intuitive motivation for the ENFV presented in Section 1 of Gollier (2004) is that we ask a group of experts who give us different forecasts of the true interest rate. This is in the spirit of the way in which Weitzman (2001) motivates gamma discounting. If there are  $N$  experts, each giving a forecast  $x_i$  for  $i \in \{1, \dots, N\}$ , then

Gollier gives the compounding rate as

$$e^{r_c(T)T} = \frac{1}{N} \sum_{i=1}^N e^{x_i T} \quad (19)$$

What Gollier (2004) does not model is the relationship between the individual forecasts and the realisation of the interest rate  $r$ . If we assume that the individual forecasters are unbiased, then  $x_i = r + \varpi_i$  where  $E[\varpi_i] = 0$  for all  $i$ . If we ask a sufficient number of independent experts and assume that  $Var(\varpi_i) = \sigma_\varpi^2$  for all  $i$ , then, by the Central Limit Theorem:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \sim N(r, \sigma_\varpi^2/N) \quad (20)$$

From the finite form of Jensen's inequality and the convexity of the exponential function, it follows that:

$$e^{\bar{x}T} < e^{r_c(T)T} \quad (21)$$

Also, from (20):

$$E[e^{\bar{x}T}] = e^{rT + 0.5\sigma_\varpi^2 T^2/N} \quad (22)$$

The realised future value of each dollar invested over a time interval  $T$  is  $e^{rT}$  and, from (22) and (21):

$$e^{rT} < E[e^{\bar{x}T}] < E[e^{r_c(T)T}] \quad (23)$$

This proves that  $r_c(T)$  is an upward biased measure of the expected future value.<sup>3</sup>

To illustrate this, apply the gamma discounting framework of Weitzman (2001) to the ENFV criterion. In this setting, it is argued that the individual responses,  $x_i$ , can be accurately modelled as being approximately distributed as

---

<sup>3</sup> A related discussion to this that involves the unbiased estimate of the future value of a portfolio of equities is given in Blume (1974), Indro and Lee (1997), Jacquier et al. (2003) and Jacquier et al. (2005).

the realisations of a random variable  $x$ , whose probability density function  $f(x)$  is defined by:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad (24)$$

$$\alpha = \frac{\mu^2}{\sigma^2}, \quad \beta = \frac{\mu}{\sigma^2}$$

$\mu, \sigma^2$  are the average estimate of the risk-free rate and the variance of the estimates from the  $N$  experts respectively. The ENFV of each \$1 invested at time 0 until time  $T$  in this case is given by:<sup>4</sup>

$$e^{r_c(T)T} = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty x^{\alpha-1} e^{-(\beta-T)x} dx \quad (25)$$

As  $T \rightarrow \beta$  from below,  $r_c(T) \rightarrow \infty$ , with the integral being undefined for  $T > \beta$ . Letting  $\mu = 4\%$  and  $\sigma = 3\%$ , which are very similar to the values estimated in Weitzman (2001), the ENFV criterion is undefined for  $T > 44.4$ ; under half a century.

## 4 Conclusion

In this paper I have shown that the ENPV criterion gives decisions that are consistent with expected utility maximisation. If social planners use the ENFV criterion, by contrast, then they may erroneously reject some viable initiatives. The ENFV criterion fails because, while trading in financial assets appears to increase expected future wealth, it does not capture changes in expected utility. Further, the apparent increase in wealth is in itself a mirage; the future value of a portfolio is only determined by the true underlying asset generation process rather than by investors knowledge about the process. This has led Jacquier et al. (2005) and other to argue that the term structure of compounding rates should be downward sloping.

---

<sup>4</sup> This equation is analogous to equation 9 in Weitzman (2001), where the focus is on the ENPV.

By coming down so unambiguously on Weitzman's side in this debate, this paper differs from previous resolutions of the puzzle which either conclude that both measures are correct (Hepburn and Groom (2007); Gollier (2009a), Gollier (2009b)), that both are wrong (Gollier (2004)) or that Gollier's approach is to be preferred (Buchholz and Schumacher (2009)). This paper also makes important methodological improvements on previous work in this area. In particular, it is shown that the puzzle can be resolved within a risk neutral endowment economy without the need to introduce arbitrary evaluation dates. It is the social planner's desire to smooth consumption across time, rather than across states, that lies at the heart of this paradox.

**Acknowledgement:** *The author is grateful for suggestions made by seminar participants at the Universities of Durham and Warwick. He particularly thanks Ben Groom and the anonymous referees for their detailed comments. All errors, sadly, remain the author's own.*

## Appendix

In this appendix, I briefly elaborate on the functional restrictions placed on the endowment process  $y_t$  as given in (9). First, I prove that these functional forms for  $y_t$  ensure that the interest rate is fixed at  $r$  once this value is revealed at time 0. Then, using a standard form for  $u(\cdot)$ , I show that plausible endowment processes satisfy the restrictions. Finally, I compare interest rates in the risk neutral case ( $v''(\cdot) = 0$ ) with the time-separable case ( $v(\cdot) \equiv u(\cdot)$ ) for one specific example.

To first establish that the interest rate will be fixed at  $r$  after time zero, notice that under either of the restrictions in (9) for  $t \geq 1$ ,  $E_{t-1}[y_t] = u'^{-1}[e^{\rho-r}u'(y_{t-1})]$ . As  $c_t = y_t$  for all  $t$ , so:

$$e^{-\rho} \frac{u'(E_{t-1}[c_t])}{u'(c_{t-1})} = e^{-r} \quad (26)$$

The left hand side of this expression is, by analogy with (10), the price of a single-period risk-free asset at time  $t - 1$ ,  $B_{t-1t}$ . This gives  $r$  a clear interpretation as the

risk-free interest rate, even though it is economically motivated through the growth rate of the endowment process. It then follows immediately that, by fixing the value of  $r$  after time zero, the risk-free rate will also be constant. The revelation of  $r$  at time zero does not remove all stochasticity from the system, though, as  $\omega_t, \eta_t$  still remain unknown until time  $t$ . These noise terms do not influence the analysis as the risk-free rate is driven by the expectation of  $c_t$  at time  $t - 1$ , rather than the realisation of this variable. This contrasts with the production function in Gollier (2009a), where all stochasticity is removed from the system at time 0 by the revelation of the risk-free rate of return to capital.

To illustrate with an example, suppose that  $u(c_t)$  takes power form;  $u'(c_t) = c_t^{-\gamma}$  for some constant  $\gamma$ . In this case, a permissible income process is:

$$\ln \left( \frac{y_t}{y_{t-1}} \right) = \frac{r - \rho}{\gamma} + e_t \quad (27)$$

where  $e_t \sim N(-0.5\sigma_e^2, \sigma_e^2)$ . This corresponds to an identically and independently normally distributed logarithmic endowment growth process.

Under the income process specified in (27), the risk-free rate would also be fixed if the utility function was time separable with  $v(\cdot) \equiv u(\cdot)$ . In this case, the risk-free rate,  $r_{\text{sep}}$ , is given by

$$e^{-r_{\text{sep}}} = e^{-\rho} E_{t-1} \left[ \left( \frac{y_t}{y_{t-1}} \right)^{-\gamma} \right] = e^{-r+0.5\gamma(\gamma+1)\sigma_e^2} \quad (28)$$

This is then lower than in the time non-separable case. When  $v(\cdot) \equiv u(\cdot)$ , the coefficient  $\gamma$  also captures the prudence of the social planner, whose aversion to uncertain consumption at time  $t$  reduces the risk-free rate through the precautionary savings motive.

## References

Blume, M. E. (1974). Unbiased Estimators of Long-Run Expected Rates of Returns. *Journal of the American Statistical Association*, 69(347): 634–663. URL <http://www.jstor.org/pss/2285993>.

- Buchholz, W., and Schumacher, J. (2009). Discounting the Long Distant Future: Simple Explanation for the Weitzman-Gollier Puzzle. Discussion paper 2357, CESifo. URL [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1187622](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1187622).
- Cochrane, J. H. (2005). *Asset Pricing*. Princeton: Princeton University Press, revised edition. URL <http://press.princeton.edu/titles/7836.html>.
- Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1981). A Re-Examination of Traditional Hypotheses about the Term Structure of Interest Rates. *Journal of Finance*, 36(4): 769–799. URL <http://www.jstor.org/stable/2327547>.
- Dasgupta, P. (2008). Discounting Climate Change. *Journal of Risk and Uncertainty*, 37(2–3): 141–169. URL <http://www.springerlink.com/content/633517qw4j526470>.
- Gollier, C. (2002). Discounting an Uncertain Future. *Journal of Public Economics*, 85(2): 149–166. URL <http://www.sciencedirect.com/science/article/B6V76-463KSVW-1/2/c1e77d94f931a39eed4a9a2822178783>.
- Gollier, C. (2004). Maximizing the Expected Net Future Value as an Alternative Strategy to Gamma Discounting. *Finance Research Letters*, 1(2): 85–89. URL <http://www.sciencedirect.com/science/article/B7CPP-4CBC4CX-1/2/28ed8b8bb0063a1b937c883d4a97e66e>.
- Gollier, C. (2009a). Expected Net Present Value, Expected Net Future Value, and the Ramsey Rule. Discussion paper 2643, CESifo. URL [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1400649](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1400649).
- Gollier, C. (2009b). Should We Discount the Far-Distant Future at its Lowest Possible Rate? *Economics: The Open-Access, Open-Assessment E-Journal*, 3(2009-25). URL <http://www.economics-ejournal.org/economics/journalarticles/2009-25>.
- Gollier, C., Koundouri, P., and Pantelidis, T. (2008). Declining Discount Rates: Economic Justifications and Implications for Long-Run Policy. *Economic Policy*, 23(56): 757–795. URL <http://www.ingentaconnect.com/content/bpl/ecop/2008/00000023/00000056/art00005>.

- Groom, B., Koundouri, P., Panopoulou, E., and Pantelidis, T. (2007). Discounting the Distant Future: How Much does Model Selection Affect the Certainty Equivalent Rate? *Journal of Applied Econometrics*, 22(3): 641–656. URL <http://www3.interscience.wiley.com/journal/114212041/abstract?CRETRY=1&SRETRY=0>.
- Guo, J., Hepburn, C. J., Tol, R. S., and Anthoff, D. (2006). Discounting and the Social Cost of Carbon: a Closer Look at Uncertainty. *Environmental Science & Policy*, 9(3): 205–216. URL [http://www.economics.ox.ac.uk/members/cameron.hepburn/Guo,%20Hepburn,%20Tol,%20Anthoff%20\(2006,%20ESP\)%20Discounting%20and%20the%20social%20cost%20of%20carbon.pdf](http://www.economics.ox.ac.uk/members/cameron.hepburn/Guo,%20Hepburn,%20Tol,%20Anthoff%20(2006,%20ESP)%20Discounting%20and%20the%20social%20cost%20of%20carbon.pdf).
- Hepburn, C., and Groom, B. (2007). Gamma Discounting and Expected Net Future Value. *Journal of Environmental Economics and Management*, 53(1): 99–109. URL [http://economics.ouls.ox.ac.uk/12723/1/Hepburn%2C%2520Groom%2520\(2006%2C%2520JEEM\)%2520Gamma%2520discounting%2520and%2520ENFV.pdf](http://economics.ouls.ox.ac.uk/12723/1/Hepburn%2C%2520Groom%2520(2006%2C%2520JEEM)%2520Gamma%2520discounting%2520and%2520ENFV.pdf).
- Indro, D. C., and Lee, W. Y. (1997). Biases in Arithmetic and Geometric Averages as Estimates of Long-Run Expected Returns and Risk Premia. *Financial Management*, 26(Winter): 81–90. URL <http://www.jstor.org/pss/3666130>.
- Jacquier, E., Kane, A., and Marcus, A. J. (2003). Geometric or Arithmetic Mean: A Reconsideration. *Financial Analysts Journal*, 59(Nov/Dec): 46–53. URL <http://www.jstor.org/pss/4480527>.
- Jacquier, E., Kane, A., and Marcus, A. J. (2005). Optimal Estimation of the Risk Premium for the Long Run and Asset Allocation: A Case of Compounded Estimation Risk. *Journal of Financial Econometrics*, 3(1): 37–55. URL <http://jfec.oxfordjournals.org/cgi/content/abstract/3/1/37>.
- Lowe, J. (2008). Intergenerational Wealth Transfers and Social Discounting: Supplementary Green Book Guidance. Discussion paper, HM Treasury. URL [http://www.hm-treasury.gov.uk/d/4\(5\).pdf](http://www.hm-treasury.gov.uk/d/4(5).pdf).

- Lucas, R. E. (1978). Asset Prices in an Exchange Economy. *Econometrica*, 46(6): 1429–1446. URL <http://www.jstor.org/stable/1913837>.
- Mehra, R., and Prescott, E. C. (1985). The Equity Premium: A Puzzle. *Journal of Monetary Economics*, 15(2): 145–161.
- Newell, R. G., and Pizer, W. A. (2003). Discounting the Distant Future: How Much do Uncertain Rates Increase Valuations? *Journal of Environmental Economics and Management*, 46(1): 52–71. URL <http://www.ingentaconnect.com/content/els/00950696/2003/00000046/00000001/art00031>.
- Newell, R. G., and Pizer, W. A. (2004). Uncertain Discount Rates in Climate Policy Analysis. *Energy Policy*, 32(4): 519–529. URL <http://www.sciencedirect.com/science/article/B6V2W-4985V6J-5/2/f85673d1a762210e9d03c72a35dc9f24>.
- Nordhaus, W. D. (2007). A Review of the Stern Review on the Economics of Climate Change. *Journal of Economic Literature*, 45(September): 686–702. URL <http://www.atypon-link.com/AEAP/doi/abs/10.1257/jel.45.3.686>.
- Pazner, E. A., and Razin, A. (1975). On Expected Present Value vs. Expected Future Value. *Journal of Finance*, 30(3): 875–877. URL <http://www.jstor.org/stable/pdfplus/2326865.pdf>.
- Stern, N. (2007). *The Stern Review on the Economics of Climate Change*. Cambridge: Cambridge University Press. URL [http://webarchive.nationalarchives.gov.uk/+http://www.hm-treasury.gov.uk/independent\\_reviews/stern\\_review\\_economics\\_climate\\_change/stern\\_review\\_report.cfm](http://webarchive.nationalarchives.gov.uk/+http://www.hm-treasury.gov.uk/independent_reviews/stern_review_economics_climate_change/stern_review_report.cfm).
- Weitzman, M. L. (1998). Why the Far-Distant Future Should be Discounted at its Lowest Possible Rate. *Journal of Environmental Economics and Management*, 36(3): 201–208. URL [http://ws1.ad.economics.harvard.edu/faculty/weitzman/files/why\\_far-distant\\_future.pdf](http://ws1.ad.economics.harvard.edu/faculty/weitzman/files/why_far-distant_future.pdf).
- Weitzman, M. L. (2001). Gamma Discounting. *American Economic Review*, 91(1): 260–271. URL <http://www.jstor.org/stable/2677908>.

Weitzman, M. L. (2007). A Review of the Stern Review on the Economics of Climate Change. *Journal of Economic Literature*, 45(September): 703–724.  
URL <http://www.atypon-link.com/AEAP/doi/abs/10.1257/jel.45.3.703>.

Please note:

You are most sincerely encouraged to participate in the open assessment of this article. You can do so by either recommending the article or by posting your comments.

Please go to:

[www.economics-ejournal.org/economics/journalarticles/2010-13](http://www.economics-ejournal.org/economics/journalarticles/2010-13)

The Editor