Reply to comments on: A Note on Aoki-Yoshikawa Model

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Abstract

This is a collective reply to the seven comments/assessments we received so-far on the manuscript: "A Note on Aoki-Yoshikawa Model". Based on the comments and on the present reply, we shall resubmit a revised version of the manuscript.

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I. REPLY TO: ANONYMOUS - ASSESSMENT, JANUARY 16, 2009 - 10:21

A. Comment

The present entry considers a dynamic version of the multi-sectoral, demand-driven macro model proposed by Aoki and Yoshikawa (AY). While AY derived an equilibrium distribution of factors across sectors from combinatorial principles Eanalogous to Boltzmann's solution for the statistical equilibrium of n particles under conservation of energy Ethe authors derive alternative equilibrium distributions under certain assumptions on the dynamics of factor movements between sectors. With a parameterized family of transition probabilities, a generalized Polya distribution is obtained. Overall, I found this a nice and concise contribution to the new statistical approach to macroeconomics. As a grain of salt, it should nevertheless be mentioned that the new results in this note all hinge on the relatively strong assumption that all productivities are multiples of the lowest productivity. It seems, however, that this restriction is unnecessary from a certain perspective. If one would allow for deviations from the aggregate equality (5) (i.e. demand equal to supply) because of frictions arising from structural change, a similar outcome might be obtained for the admissible factor movements with arbitrary specification of productivities. The authors might wish to elaborate on this idea.

B. Our reply

The referee is right in writing that our results depend on a finitary model, where we assume that productivities are multiples of the lowest productivity. Such a method can be traced back to Boltzmann's discretization of the μ-space in order to apply combinatorics to the continuous motions of molecules [1]. Boltzmann's work shows that this assumption is not limiting at all, as the continuous limit can be recovered, and a degeneracy can be attributed to every energy level, so that one is free to assume any density of states (that is the weight of each energy level). This issue is discussed in section II, where we show that, in this way, binary moves can satisfy the two bounds (1) and (5). Incidentally, notice that thes bounds can be satisfied also by $2m$-ary moves, where m is an integer. A more realistic model would for sure include deviations from equation (5), but we were interested in exploring and understanding a fully conservative version of the AYM. We already discussed

possible deviations from this case at the end of section III, and what we can do is explicitly write that it is possible to relax the conservative dynamics, allow for non conservative moves and recover the equilibrium $Y = D$ only at the end of a period. However, the dynamics of such a model would be similar to a transformation in a thermodynamic system. An increase in demand makes possible to occupy higher productivity levels of the system, and it is analogous to heating a physical system. An increase in the number of workers is analogous to adding particles to a physical system. Both operations lead to a variation of Entropy connected to the intensive variables *temperature* (such as in AYM) and *chemical potential* whose counterpart in AYM is yet to be specified. On all these aspects we are working; however, we will show that the statistical approach to Economics cannot be reduced to simple combinatorics.

II. REPLY TO: MISHAEL MILAKOVIC - READER COMMENT, JANUARY 27, 2009 - 08:47

A. Comment

[...] Although the paper is generally well-written, I have two minor issues that I would like to see addressed in a (basically effortless) revision. First, since the book of AY contains a plethora of models, the title is at best unfortunate (and, also, reference should be made to the appropriate chapter in the book, I believe Chap. 3). How about "A Note on the AY Sectoral Productivity Model" or something to that effect? Second, I am generally wary of allegories like "...demand has the meaning of energy" (p. 3 after Eq. (9)) since they often miss crucial differences between physics and economics. The passage should be rephrased to make clear that this is merely a formal analogy to a well-understood problem in physics, without any economic content or implications. The concept of energy is the prime example for such a conundrum, since in economics most relevant quantities are rarely "conserved". Hence subsequent passages that emphasize the conservation of demand are most unfortunate from this perspective. Finally, and somewhat unfairly, I cannot resist but to point out that neither AY nor the authors of this note subject the model to empirical scrutiny. After all, the supposed migration of workers between sectors in situations of exogenously given aggregate demand will help us very little in understanding how effective demand is shaped over time,

or why and how productivity differences across sectors arise in the first place.

B. Our reply

The three suggestions by Mishael Milakovic are very useful. The title will be changed in the resubmitted version to *A Dynamic Probabilistic Version of Aoki-Yoshikawa Sectoral Productivity Model.* We will rephrase our text in order to stress that the analogy between energy and demand is merely formal (see also the discussion above). Finally, a short discussion will be included on how to empirically validate or falsify this class of models. These models have some unrealistic features. For instance the migration of workers from one productive sector to another may take time due to the need of learning a new job (even if this is not always the case). Moreover, one would like to add unemployed workers who should belong to a category of zero productivity, etc.. However, with appropriate information, one could measure the number of workers who move across different sectors and see if this dynamics is approximately described by equation (22). In any case we think that conservative systems have not yet been fully exploited in Economics, and, at this stage, we prefer to introduce "moving constraints" to make this model richer, rather than introducing a more realistic model where no analytical calculations are possible.

III. REPLY TO: VICTOR YAKOVENKO - READER COMMENT, JANUARY 29, 2009 - 08:43

A. Comment

[...] I agree with the complaints of other commentators about the uninformative title of the paper. First, the book by AY discusses many things, so the title should specify which AY model is being studied. Second, it is not an informative way to title a paper "A note on ...". Tell the reader right away in the title what is the subject or the result of a paper.

B. Our reply

Yes, we agree too and we will choose a more informative title, based on the suggestion of Mishael Milakovic.

IV. REPLY TO: ANONYMOUS - ASSESSMENT, JANUARY 29, 2009 - 21:05

A. *Comment*

The generalization of the AY model using Markovian dynamics is signicant, innovative and useful contribution. Nice the analogy between energy level and productivity. Anyway there strong and limited assumption (some of Aoki-Yoshikawa and some of Scalas and Garibaldi), for instance: the produc- tion depend only a one factor: labour (=population), why the productivity (formula 3) cannot be equal?, the binary move, the total amount of workers is constant n. [...] Finally, some minor issues.

1. *A more specific title.*
2. *Different Keywords.*
3. *page 2: Why define Y, D, GDP if everything is the same.*
4. *Use different letter C of formula (10) and c of formula (22).*

B. Our reply

In the discussion section, we will include a short discussion of empirical validation (see also the reply to Milakovic) and this will also contain a discussion of the intrinsic limits of this model. Requests in points 1 and 2 can be easily satisfied. As for point 3, GDP is just an acronym and defines Y, whereas D is the aggregate demand. $Y = D$ is not an identity but an equality, meaning that this is not a tautology, but a statement that the aggregate demand and the aggregate product have the same value. D is given (and fixed) exogenously. In a future work, we would like to let it change, both reversibly and by means of discrete shocks.

V. REPLY TO: ANONYMOUS - READER COMMENT, FEBRUARY 03, 2009 - 08:34

A. Comment

[...] To maximise the potential readership, it might be helpful to enlighten these readers on a few issues. For instance, why, in physics, are the values $c = 0, \pm 1$ *the only values appearing in physical systems (p. 9, l. 11)? What is it that makes economic systems so different from systems usually studied in statistical physics? Is it the fact that Liouvillefs theorem does not hold, for instance? [...]*

Some minor issues

1. *In the top line on page 8, replace "is said reversible" by "is called reversible".*

2. *Page 9, line 9: "Polya" should be "Pólya".*

B. Our reply

There are many differences between physical systems and economic systems, even if an "atomistic" point of view is used in modelling the latter. For instance the typical number of interacting agents in Economics is much less than the typical number of interacting particles in a fluid. This means that the so-called thermodynamic limit may not be relevant for economic systems also when a statistical equilibrium is reached. It is an empirical fact that, in Physics, the only relevant cases of equation (33) are $c = 1$ for bosons, $c = -1$ for fermions and $c = 0$ for "classical particles". Bosons are particles that can occupy a state without limits; it is the case of photons, the quanta of the electromagnetic radiation. This property was used by Planck in 1900 to derive the right equation for the black-body radiation [2]. Fermions are particles subject to Pauli's exclusion principle, so that no two identical particles can occupy a given state at a time [3]. Electrons are instances of fermions. In both cases the labels (names) of particles are not interesting and what is important is the occupation number of each state. In the classical case, it is also interesting to consider the labels (names) of the particles. In Economics, equation (33), or, better, equation (22) can be fitted from empirical data (see also our reply to Milakovic) and there is no *a priori* restriction on the value of c.

VI. REPLY TO: YUJI ARUKA - READER COMMENT, FEBRUARY 12, 2009 - 08:34

A. Comment

We directly refer to the link
`http://www.economics-ejournal.org/economics/discussionpapers/2008-38`.

B. Our reply

The insightful comment of Prof. Aruka would deserve much more space in order to discuss all the points raised. Here, we would like to stress that it is possible to reconcile the point of view of methodological individualism with an "emergentist" approach where the role of interactions between and among agents is not neglected. This has been done e.g. by philosopher Mario Bunge and readers of *Economics* can reead a summary of Bunge's ideas in his book *Philosophy in Crisis: The Need for Reconstruction* [4].

VII. REPLY TO: ANONYMOUS - READER COMMENT, FEBRUARY 16, 2009 - 08:38

A. Comment

Report on:
A Note on the Aoki-Yoshikawa Model
by Masanao Aoki and Hiroshi Yoshikawa

The contribution of Scalas and Garibaldi to the original model of Aoki and Yoshikawa (AY hereafter) consists essentially in two main aspects. On one hand they extend the exponential equilibrium distribution of AY model to the more general Polya distribution, and, on the other hand, they introduce a dynamical dimension in the model, via the Markovian transition rates from eq. (22). The time domain aspect gives a new essential contribution to the original model, especially if someone want to confront its descriptive power with real data. In order to conserve the aggregate demand, the authors have to introduce a rather artificial condition

on the allowed movements of workers across the different sectors, which in principle would require a very strong coordination among workers. Which is the mechanism responsible for this highly coordinated system?

All in all, the paper is suitable for publication as a relevant contribution in the field of distributional approach to Macroeconomics, perfectly in line with the spirit of the original contribution of Aoki and Yoshikawa.

B. Our reply

One could consider the hypothesis that moving agents (those who have been fired, or who decide to move) are reallocated by some Authority who takes into account the exact amount of productivity needed in order to fulfill demand. Indeed, the system is less coordinated than it seems. Binary moves preserving D may be replaced by suitable multiple moves, where many workers leave a sector, observe what places are available, and eventually change sector. If these moves are taken seriously, then during the accommodation process $Y \neq D$. Yet, this is still an instance of strong coordination as at the end of the accommodation process, one recovers $Y = D$. In other words, accommodation takes place between period t and period $t + 1$. This process may recall Walras' auctioneer mechanism, but it has the advantage that it can be effectively realized, at least in the fictitious world of Monte Carlo simulations!

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[1] A. Bach, Arch. Ex. Sci. **41**, 1, (1990).

[2] M. Planck, Annalen der Physik **4**, 553, (1901).

[3] W. Pauli, Z. Physik, **31**, 373, (1925).

[4] M. Bunge, *Philosophy in Crisis: The Need for Reconstruction*, Prometheus Books, Amherst, NY, 2001.