A report on Graciela Chichilnisky: Avoiding Extinction Equal Treatment of the Present and the Future

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Consider an economy which uses trees as a necessary input to production or consumption. The dynamics of tree reproduction are as follows. If n out of $2n$ subsequent generations cut the forest at a maximal rate, the species become extinct after the $2n$'th generation, in which case there is zero utility at every period from then on. A typical utility stream representing this strategy is of the form $u^n = (0, \ldots, 0; 1, \ldots, 1; 0, 0, \ldots)$ with the first (resp. last) 1 at the $n+1$ (resp. $2n$)'th place, in which generations $n+1, \ldots, 2n$ cut at a full capacity and exhaust the forest. Note that when the consumption of the forest is delayed (i.e. n becomes larger), the forest slightly expands and more generations can benefit. Alternatively, generations can invest in the forest and only cut at an equilibrium rate which allows the forest to survive. This strategy results in the utility stream $u^\infty = (1/5, \ldots, 1/5, \ldots)$ in which each generation reaches the same utility level.

Let us evaluate the different policies by means of the normalized[1] discounting rule

$$u = (u_1, u_2, \ldots) \longmapsto d_\beta(u) = (1-\beta) \times \sum_{k=1}^{\infty} \beta^{k-1} u_k.$$

We obtain $d_\beta(u^n) = \beta^n - \beta^{2n}$ and $d_\beta(u^\infty) = 1/5$. For each β in the open interval $(0,1)$ there exists an n such that $\beta^n - \beta^{2n} \geq 1/4$. On the other hand, if we focus on the long term future of these policies, then we obtain $\lim_{t\to\infty} u_t^n = 0$ and $\lim_{t\to\infty} u_t^\infty = 1/5$.

Conclusion: optimization with respect to a discounting rule leads to the elimination of this forest. In order to avoid the elimination of this forest, some weight should be given to the limiting value. Ferejohn and Page (1978, p274) summarize as follows:

> *... the search for a fair rate of discount is a vain one. Instead of searching for the right number, i.e. 'the' social rate of discount, we must look to broader principles of social choice to incorporate ideas of intertemporal equity. Once found, these principles might be used as side conditions in a discounting procedure to rule*

[1] An evaluation F of infinite utility streams is said to be normalized if $F(r, r, \ldots, r, \ldots) = r$ for each r in $\mathbb{R}$. Due to this normalization the discounted sum is premultiplied with $(1-\beta)$.

out gross inequities that can arise with discounting, even with a low discount rate.

If we judge the long term future as important, then we should use the right tools to evaluate a long run policy. This paper provides such a tool.

Graciela Chichilnisky presents a welfare criterion that avoids dictatorship of the present (i.e. discounting the future) and dictatorship of the future (i.e. being insensitive for effects in the short run). This welfare criterion is used to order the set l_∞ of bounded infinitely long utility streams. For each generation t, the coordinate u_t of such an infinite utility stream $u = (u_1, u_2, \ldots)$ in l_∞ is interpreted as the utility of generation t obtained from its consumption. The utilities u_t are assumed to be bounded. The welfare criterion W is defined as follows:

$$W : l_\infty \longrightarrow \mathbb{R} : u = (u_1, u_2, \ldots) \longmapsto \sum \mu_t u_t + \Phi(u),$$

where $\mu_t > 0$ for each t and $\sum \mu_t < \infty$, and where $\Phi : l_\infty \to \mathbb{R}$ is an increasing purely finitely additive map. The first term $\sum \mu_t u_t$ is sensitive in each generation: if the utility stream v dominates u—i.e. the utility streams u and v are different and for each t we have $v_t \geq u_t$—then $\sum \mu_t v_t > \sum \mu_t u_t$. The discounting rule d_β with $0 < \beta < 1$ is a particular case. The second term $\Phi(u)$ is insensitive for finite sets of generations: if u and v are equal up to a finite number of coordinates, then $\Phi(u) = \Phi(v)$. The map Φ only satisfies a weak form of sensitivity: if v dominates u and if there exist a T in $\mathbb{N}_0$ and an $\varepsilon > 0$ such that $v_t \geq u_t + \varepsilon$ for each $t > T$, then $\Phi(v) > \Phi(u)$. The value $\Phi(u)$ depends on the limit points in the infinite stream u. Furthermore, the criterion W is continuous with respect to the sup norm and is independent, i.e. $W(u+v) = W(u)+W(v)$ for each pair u and v of infinite utility streams. This criterion is said to be sustainable (Chichilnisky, 1996).

The evaluation of the constant utility stream $(1, 1, \ldots, 1, \ldots)$ reads

$$W(1, 1, \ldots, 1, \ldots) \;=\; \underbrace{\sum \mu_t}_{P>0} \;+\; \underbrace{\Phi(1, 1, \ldots, 1, \ldots)}_{F>0}.$$

The fraction $P/(P+F)$ can be interpreted as the weight allocated to the present, and the fraction $F/(P+F)$ as the weight of the future in the sustainable evaluation criterion W.

The main result (Theorem 3) shows that maximizing $W(u) = \sum \mu_t u_t + \Phi(u)$ over a feasible set Ω of infinite utility streams boils down to the constraint optimization problem of maximizing the countably additive part $\sum \mu_t u_t$ over Ω restricted to $\lim_{t\to\infty} u_t \geq K$. Furthermore, the weight of the future in W is related to the Frechet derivative of W computed at a path that satisfies the constraint and, hence, depends on K.

Graciela Chichilnisky definitely exposes important insights on the issue of the evaluation of long term policies!

Related literature

Part of the literature on ordering infinite utility streams starts from two different basic principles: anonymity and Pareto. The Pareto principle asks for sensitivity and excludes dictatorship of the future. Anonymity imposes indifference between a utility stream and its (finite) permutations and excludes the existence of a dictatorial generation. Svensson (1980), Basu and Mitra (2007), and Bossert, Sprumont, and Suzumura (2007) all present incomplete criteria that combine Pareto and finite anonymity. In order to obtain a 'complete' Pareto and anonymous relation the use of non-constructive mathematics —ultrafilters, Szpilrajn's lemma, the Axiom of Choice—is unavoidable. This statement was conjectured by Fleurbaey and Michel (2003) and confirmed by Lauwers (2006) and by Zame (2007). The sustainable welfare function proposed by Chichilnisky satisfies the Pareto principle and violates anonymity.

On the other hand, Ng (2003, 2005) proposes to leave the framework of infinite utility streams and argues that the finiteness of our universe solves the paradox of having to compare different infinite values in optimal growth/conservation theories:

> *... rational choice requires weighting all welfare values by the respective probabilities of realization. As the risk of non-survival of mankind is strictly positive for all time periods and as the probability of non-survival is cumulative, the probability weights operate like discount factors, though justified on a morally justifiable and completely different ground. ... the effective discount rate on future welfare values (distinct from monetary values) justified on this ground is likely to be less than 0.1 per annum.* (Ng, 2005, from the abstract)

Let me refer to Matheny (2007) for an excellent introduction to and review of the literature on low-probability-high-consequence-risks.

Remarks

Page 13, Proof of Theorem 3. The statement that the set $\tilde{Z}$ of all ultrafilters on the integers is formally equivalent to $\{1, 2, \ldots, N\}$ for some integer N should be better/further explained.

Page 14, Proposition 2. As a matter of fact the existence of the Φ-part in the welfare criterion already implies the existence of a non-constructible object. As explained at the start of Section 5, the extension of the uniform distribution within a finite set towards an infinite set of successive generations is at the origin of the problem. In the limit each single generation is given zero weight. This is known as the uniform distributions puzzle (e.g. Kadane and O'Hagan, 1995). Lauwers (2007) shows that the existence of a purely finitely additive measure on $\mathbb{N}_0$ implies the existence of a non-constructible object. More precisely, the following "sequence of implications" is obtained.

The Axiom of Choice implies the existence of free ultrafilters; the existence of free ultrafilters implies the Hahn Banach theorem; the Hahn Banach theorem implies the existence of purely finitely additive measures on $\mathbb{N}_0$; the existence of a purely finitely additive

measure on $\mathbb{N}_0$ implies the existence of a non-Ramsey set. From Mathias (1977) we know that the existence of non-Ramsey sets does not follow from the Zermelo-Fraenkel axioms of set theory (i.e. without the Axiom of Choice).

Page 18, middle. The $\liminf$, i.e. the operator on l_∞ that selects the infimum of the set of accumulation points, is not purely finitely additive. Indeed, consider the sequences

$$u_{\text{odd}} = (1, 0, 1, 0, 1, 0, ...) \qquad \text{and} \qquad u_{\text{even}} = (0, 1, 0, 1, 0, 1, ...).$$

We have, $\liminf(u_{\text{odd}}) = \liminf(u_{\text{even}}) = 0$. Furthermore, $u_{\text{odd}} + u_{\text{even}} = (1, 1, 1, 1, ...)$ and $\liminf(u_{\text{odd}} + u_{\text{even}}) = 1$. In combination with the previous remark, one is invited to consider the sustainable criterion

$$W : l_\infty \longrightarrow \mathbb{R} : u = (u_1, u_2, \ldots) \longmapsto \sum \mu_t u_t + \lambda_0 \liminf u + \lambda_1 \limsup u,$$

where $\limsup$ selects the supremum of the set of accumulation points. This map is well defined without recourse to non-constructive mathematics and satisfies both Chichilnisky-axioms. See Chambers (2009) for a characterization of $\liminf$ and $\limsup$.

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Typos

A few words/names/formulas are misspelled:

Van Liedekerke, Bernoulli, Katholieke Universiteit

p1 Abstract. identi.ed should be identified

p10(15) footnote 22(25). 111 should be 1,1,1

p12 middle. Mox should be Max