A Comment: A Note on Aoki-Yoshikawa Model

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1. The backstage of the paper and the AYM framework of econophysics

It has become recognized among scholars/researchers that we were faced with a severe epistemological/methodological conflict also in the field of economics. One camp is the methodological individualism derived from Lionel Robbins in 1930's. This idea decisively depends on the so-called "individualistic/microscopic benefits only base" to which all the behavioural decisions exclusively are referred. On the contrary, the recent progress of complexity thinking is revealing much greater irrelevance of methodological individualism. For instance, it is easy to verify that greater intensive interactions may be incompatible with the methodological individualism. As J. Holland (1995) and others who adopted the multi-agent based approach and also the complex adaptive system approach already argued in the case of animative beings including single-cells, the autotelic spending of energy is much lesser than the spending of it for interaction with others for their subsistence. We a fortiori have a strong reason to leave from the methodological individualism.

Mainzer (2007b) demonstrated philosophically as follows:[1] If we reconsider “a free act of human self-determination in a stream of nonlinearity and randomness in history,” our classical philosophical arguments could be revived. It is easy to realize that individuals never rely upon their own a priori preferences only. On the other hand, we cannot neglect “chances” independently of “individual abilities” (including learning abilities) and “circumstances” which we must be forced to accept. Things like novelties emerging either from nature or society could never happen without intermediation of chances. When we deal with our equilibrium game, equilibrium does not imply a kind of self-filling prophecy on the game result for participating agents. Their expectations must often be circumvented. Given interactive chances, we may never depend on the individualistically rational principles of traditional economics. The dominance of these principles should rather be regarded as harmful for understanding the heterogeneous interaction.

Now we move to another important idea. Immanuel Kant distinguished contingencies as being more empirical, more logical, and more intelligible. An empirical contingency which depends on a certain cause is meant as the “Event”. Sun irradiation is not a necessary condition for heating a stone. While a stone necessarily falls on the basis of gravity to the earth at any moment. Seemingly we have opposite observations. A concept may fail to catch all the events around it. A property (attribute or type) that does not follow from the definition of a concept (category) is called logically “Chance”. (Mainzer, 2007a, p. 408).

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[1] In the remaining part of this section, the reviewer utilize several parts of Aruka(2009).

This is the reason why we should employ the idea of type distribution with exchangeable agents, because it is difficult to identify the attributes or types by themselves with reference to events at hand. These considerations may suggest the promotion of the stochastic modelling approach with exchangeable agents. In this context, thus, we may be allowed to introduce such an idea as configurations $x = (x_1, x_2, \ldots, x_n)$, with $x_i \in \{1,\ldots, g\}$. [2]It may also be useful to introduce the number of distinct configurations belonging to a given occupation vector: $W(x|n) = n!/ \prod_{i=1}^{g} n_i!$

2. The features of the paper

The AYM, i.e.,Aoki and Yoshikawa Model (Aoki and Yoshikawa 2006) is a result of Prof. Masanao Aoki's seminal works in cooperation with Hiroshi Yoshikawa. The description by the authors of the paper is elegant, and the focus to be discussed is also concisely summed up. The AYM consists of two fundamental equations on total factors (population) of production n and effective demand D. Population is distributed among the different sectors g_i with their different productivities a_i giving $Y_i=a_i n_i$. It then both holds $\sum_i^{g} n_i = n$ (1) and $\sum_i^{g} a_i n_i = D$ (6); In particular, the authors illustrated so carefully the historical argument by Ludwig Boltzmann as to reveal the essential features of the AYM. The authors are successful to demonstrate that Equation (10) [$\pi(n)=CW(x|n) =C\, n!/ \prod_{i=1}^{g} n_i!$] holds true, that is the equiprobability of all the configurations x compatible with the constraints (1) and (6); This is typical in classical statistical mechanics, where the uniform x-distribution is the only one compatible with the underlying deterministic dynamics, by the following clear argument:

> The problem of AYM coincides with a well-known problem in Statistical Physics, namely, finding the statistical equilibrium allocation of n particles into g energy levels ε_i so that the number of particles is conserved: $\sum_i n_i = n$ (8) and the total energy E is conserved: $\sum_i \varepsilon_i n_i = E$ (9) ; One can immediately see that the levels of productivity a_i correspond to energy levels, whereas the demand D has the meaning of total energy E. …[Thus,] occupation vectors that maximize $\pi(n)$ must minimize $\prod_{i=1}^{g} n_i!$ subject to the two constraints (8) and (9). …Boltzmann noticed that, when statistical equilibrium is reached, the probability $\pi(n)$ of an accessible occupation state is proportional to $W(x|n)$; this means that $\pi(n)=CW(x|n) =C\, n!/ \prod_{i=1}^{g} n_i!$ (10); [The explicit solution of the bounded extremum problem could give an equilibrium state n* under the assumption that $a_i = ia$ with $i = 1, 2, \ldots, g$,[3] if we had some conditions to make the sums of (1) and (6) expansible in the infinite series.] Hence the marginal probability that a worker is in sector i, given n* follows the exponential distribution. (p.3)

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[2] Here $x_i = j$ means that the i-th worker is active in sector j.

[3] This mena that productivities are multiples of the lowest productivity a_1.

The authors also in turn employ the Master Equation approach to elucidate the micro-macro link of the relationship discussed in the above. It is noticed that the same way is often found in Aoki's serial works as he loves to identify his method with the jump Markovian process. The authors insightfully applies the AYM (2006) to be successful to prove the special characteristics of the AYM. Here the authors prefer to call unary moves of agent, instead of jumps. As the authors also demonstrated, this approach suggested the usefulness to apply the generalized Polya urn process, in particular, the Ehrenfest-Brillouin Model to the AYM.

In an economic model, we must be subject to the conservation of effective demands under unary moves of transitions. The adopted assumption of a_i = ia can always guarantee the conserve demand, as the authors showed(p.6) This considerably simplifies the argument to find an invariant distribution of the binary chain. The authors also specified the transition probability of i-->l and j-->m in the following manner:

$$P(n_{ij}^{lm}|n) = A_{ij}^{lm}(n)n_i n_j(1 + cn_l)(1 + cn_m) \quad (22)$$

Her $A_{ij}^{lm}(n)$ is a suitable normalization factor; c is a model parameter. Given these settings, the authors have characterized the property of the AYM by the criteria of some particular c values.
It is sure that in general the worker will not choose the arrival sector independently from its occupation before the move. But due to the restriction of conserving demand, the AYM equilibrium is confined to the special value of c=0. This means just the case that workers' configurations are uniformly distributed. The other equilibria corresponds to c=1 and c=－1. The cases c=0, 1 or －1 in view of equation (22) may correspond to the cases that the payoffs of transitions in the urn modeling are 0, positive, or negative. These cases coincide with all the states n compatible with the constraints. According to the authors, such constrained domain may simply be useful in Physics. In this sense, the AYM is only special.

3. Concluding remarks

The authors expositions to sate their results are quite professional and skilful. The connections of the AYM with Physics seem to be brightly revealed by this paper. This paper must be an important contribution not only to the AYM but also the similar kinds of stochastic modelling. So this paper surely gives a very useful guide to future modelling in this direction. The reviewer suggests a possible application to a dyadic pair interactive game often in the evolutionary games. In the latter case, it may sometimes be advantageous in a constrained domain.

Whereas we must notice that we have the new subsequent contributions by Aoki and Yoshikawa (2007, 2008) where innovations are dealt with the Ewens formulas of population genetics. In these

cases, we can argue the emergence of genuine mutants as well as non-convergence[4] of the economics system.

My notes:

P.6 line 6: n_{ij}^{nm} The super suffix "n" should be replaced "l."

p.9 line 1: an exclusion principle. Is the term familiar?

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[4] Precisely speaking, non-convergence implies *non-self averaging*.