Comment on "Perfecting Imperfect Competition" by Goetz Seißer

The paper by Seißer proposes a profit tax, $\tau(q)$, equal to the Lerner index as a means of improving economic efficiency. Since the Lerner index, (price-marginal cost)/price, is generally decreasing in output,[1] firms will have an incentive to increase output to reduce their profits tax and this can reduce the deadweight burden of monopoly. As a regulatory devise a profits tax has the advantage of requiring rather limited intervention and preserving the profit motive. There is a further advantage that if firms were induced to produce at the competitive equilibrium no tax would actually be imposed or collected.

The proposed tax has some similarities to the suggestion made by Alan Walters, economic advisor to Mrs Thatcher 1981–4, for an "output related profit levy". Walters suggested the levy as a means to regulate British Telecommunications after privatization. The output related profit levy was examined in the Littlechild report to the Secretary of State for Industry [2] along with other regulatory schemes and was rejected in favor of price-cap regulation.

A detailed analysis of regulation through an output related profits tax is provided by Glaister [1]. Glaister shows how to construct a profits tax to induce a Ramsey optimum. Although his focus is on the multi-product firm with independent products, it is useful to consider the single product firm so as to compare his results with those of Seißer's paper. Let $\rho(q) = (1-\tau(q))$ be the retained profit rate and let $\eta(q) = q\rho'(q)/\rho(q)$ denote the elasticity of profit retention. Let $\Pi(q) = R(q) - C(q)$ denote pre-tax profits where $R(q)$ is total revenue and $C(q)$ is total cost. The first-order condition for a maximizing after tax profits $\rho(q)\Pi(q)$ is

$$\rho(q)\left(MR(q) - MC(q)\right) + \rho'(q)\Pi(q) = 0$$

[1] A sufficient condition is that marginal cost is non-decreasing.

where $MR(q)$ is marginal revenue and $MC(q)$ is marginal cost. Since $MR(q) = p(q)(1 + (1/\epsilon(q)))$ this first-order condition can be rewritten as

$$\frac{p(q) - MC(q)}{p(q)} = \frac{1}{\epsilon(q)} \left(1 - \eta(q)\epsilon(q)\pi(q)\right) \tag{1}$$

where $\epsilon(q)$ is the elasticity of demand written as a function of output q and $\pi(q) = \Pi(q)/R(q)$ is the pre-tax profit margin. The left-hand-side of (1) is the Lerner index $\mathcal{L}(q)$. At the Ramsey optimal solution the Lerner index is equated to the Ramsey index $\mathcal{R}(q)$ given by

$$\mathcal{R}(q) = \frac{1}{\epsilon(q)} \left(1 - \frac{1}{1+s}\right) \tag{2}$$

where s is the shadow price on the profit constraint. Following [1] and comparing (1) and (2) it can be seen that choosing the elasticity of profit retention such that

$$\eta(q) = \frac{1}{(1+s)} \frac{1}{\pi(q)} \frac{1}{\epsilon(q)} \tag{3}$$

can achieve the Ramsey optimal outcome. This has some intuitive features. The elasticity of profit retention is inversely related to the profit margin: if the profit margin is small more of extra profits can be retained. It is inversely related to elasticity as if the elasticity is high the divergence from the optimal solution will be small without taxation. It is inversely related to the shadow price on the profit constraint: if s is high then the regulator does not want to encourage output expansion very much and the slope of the profit retention function will be low.

As an example of the single product case consider a linear inverse demand function $p(q) = 1 - q$ with quadratic cost function $C(q) = cq^2$ for some

$c > 0$. The Ramsey optimal solution has[2]

$$q^{\mathcal{R}} = \frac{1+s}{(1+2c)+2(1+c)s}.$$

As $s \to 0$, $q^{\mathcal{R}} \to 1/(1+2c)$ the perfectly competitive output and as $s \to \infty$, $q^{\mathcal{R}} \to 1/(2(1+c))$ which is the monopoly output. In this example the elasticity of demand is $\epsilon(q) = (1-q)/q$ and the profit margin is given by $\pi(q) = (1-(1+c)q)/(1-q)$. Profits are positive provided $q < 1/(1+c)$. Using equation (3) gives

$$\frac{\eta(q)}{q} = \frac{\rho'(q)}{\rho(q)} = \frac{1}{(1+s)}\frac{1}{(1-(1+c)q)}.$$

Solving this differential equation for $\rho(q)$ setting $\rho(\hat{q}) = 1$ for some output $\hat{q} < 1/(1+c)$ gives

$$\rho(q) = \left(\frac{1-(1+c)\hat{q}}{1-(1+c)q}\right)^{\frac{1}{(1+c)(1+s)}}.$$

This is defined for $q < 1/(1+c)$, that is for output levels for which the firm makes positive profits. Firms maximizing $\rho(q)\Pi(q)$ will choose the Ramsey-optimal output level $q^{\mathcal{R}}$.[3] It is perhaps natural to chose $\hat{q}$ to be the Ramsey solution $q^{\mathcal{R}}$ so that all profits are retained and no tax is actually imposed at the optimal solution.[4]

Seisser's proposed tax is $\tau(q) = L(q)$. In the example this gives the elasticity of profit retention $\eta(q) = 1/(1-q)$. This is independent of c and s. Since $\pi(q)$ varies with c we can see from equation (3) that the solution to

[2]The superscript $\mathcal{R}$ denotes it is the Ramsey optimum quantity.

[3]Appropriate second-order conditions are met, at least in this example.

[4]Here $\rho(q)$ is increasing so there would be a subsidy to profits for $q > q^{\mathcal{R}}$. There would be a subsidy at the optimum for any $\hat{q} < q^{\mathcal{R}}$.

the after-tax profit maximization problem will not in general yield the Ramsey optimal solution even for $s = 0$. It can be shown that the solution to the problem of choosing q to maximize $(1 - L(q))\,\Pi(q)$ is given by

$$q^{\mathcal{S}} = \frac{4}{4 + 3c + \sqrt{c}\sqrt{8 + 9c}}$$

where $q^{\mathcal{S}}$ denotes the solution with the Seißer tax.[5] An interesting special case is $c = \frac{1}{2}$. In this case $q^{\mathcal{S}} = \frac{1}{2}$ which is the competitive solution. Moreover, since price equals marginal cost at the competitive solution, $\tau(\frac{1}{2}) = 0$ and no tax is levied at the optimum.[6] In the case of $c = \frac{1}{2}$ equation (3) is satisfied at the solution $q^{\mathcal{S}} = \frac{1}{2}$ but not for other values of q. Thus we can conclude from the example that Glaister's tax will implement the Ramsey optimum whereas Seißer's tax, although it has many desirable properties, will implement the Ramsey or competitive optimum exactly only for certain parameter values.

It is perhaps interesting to speculate whether the output related profit tax has a longer history. Glaister [1] quoting Littlechild [2] suggests that prior Alan Walter's suggestion there had been very little analysis of an output related profit tax. As Littlechild and Glaister note economist's have previously speculated about an output subsidy. For example Joan Robinson [3] suggested that an unit output subsidy equal to the difference in marginal cost and marginal revenue at the competitive solution with a lump-sum tax to

[5] Again second order conditions are satisfied in this example.

[6] Seißer is careful to set the tax rate to zero for output equal or greater than the competitive solution. For $c = 1$, which is the case considered in the supplementary materials, the solution given in the above equation is $q^{\mathcal{S}} = 4/(7 + \sqrt{17})$ which is greater the competitive outcome where $q = 1/3$. Seißer's solution gives the correct competitive outcome by correctly treating the appropriate corner solution. For other parameter values there will be an interior solution. For example, if $c = 1/9$ then $q^{\mathcal{S}} = 3/4$ which is less than the competitive output where $q = 9/11$ but equal to the Ramsey optimum if $s = 1/8$.

claw back the subsidy.[7] I know of no analysis of an output related taxes prior to [1] but regulation by sliding scales has a history dating back to the 19th century so perhaps other readers can suggest further historical precedents for an output related profit tax.

It is fair to say that [3] and [1] treat the output related tax or subsidy as an interesting theoretical analysis but dismiss its practical importance. Robinson [3] says that "there is not likely to be much scope for applying it in actual cases" and Glaister [1] says that while the "scheme has the advantage of limiting detailed intervention and preserving the profit motive ... unrealistic assumptions are necessary and implementation would require a great deal of information" and that this "limits its practical value".

As Seißer recognizes his tax suffers from similar problems. To implement it requires knowledge of demand and cost functions. The fact that information about these functions may be incomplete or information asymmetric between the firm and regulator has been the focus of much of the regulatory literature. Moreover, there may be further difficulties to applying a profits tax to firms with multiple dependent products and defining profits in a way which minimizes the opportunities for misrepresentation so as to achieve the regulatory aims. The challenge is to know if it is possible to design an output related profits tax that can overcome some of these difficulties and be better or more cost effective than other regulatory schemes.

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[7] She ascribes the suggestion of a corrective subsidy to Austin Robinson in an answer given in an examination.

References

[1] Glaister, S., (1987): "Regulation Through Output Related Profits Tax" *Journal of Industrial Economics*, 35(3), 281-296. http://www.jstor.org/stable/2098635

[2] Littlechild, S., (1983): "Regulation of British Telecommunications' Profitability," (Department of Industry, London).

[3] Robinson, J., (1933): "The Economics of Imperfect Competition," Macmillan & Co, London.