Referee Report on
"Non-Self-Averaging in Macroeconomic Models: A Criticism of Modern Micro-founded Macroeconomics"
by Masanao Aoki and Hiroshi Yoshikawa

This paper applies the concept of "non-self-averaging" from statistical mechanics to address the issue whether economic models based on the assumption of a "representative agent" yield predictions that are adequately captured by the sample means of realizations of economic variables of interest.

More precisely, "self-averaging" is the property of a (physical or here economic) system - with potentially very many decision makers - that may be completely disordered. Such a system is called "self-averaging", if its properties can be described by taking averages over a sufficiently large sample.

Take, for example, an economy with very many, heterogenous agents. These agents may interact. Suppose that that happens in a random way. The latter assumption may make sense from the perspective of someone studying such a system. Thus, such a system might be called random and it can be taken to represent a disordered (economic) system. The questions arises if we can say something about the properties of such a system. Suppose you are interested in particular realizations of economic activities (like, potentially millions of, demand and supply decisions of individual agents etc.) captured by a matrix X of these realizations. One way to elicit information about such an economic system X is to have a description in terms of the sample average of X, called $E(X)$. The latter concept would require to take averages over the realizations of X. The economic system is indeed described well by the sample *average* of X, if no dispersion in the realizations matter. This is given when the realizations are characterized by featuring a variance, $Var(X) \geq 0$, and, importantly, when the coefficient of variation, i.e. the ratio of the variance of X to the mean of X, converges in distribution to zero for large enough realizations of X. If this convergence result holds, the (economic) system is called "self-averaging". It would entail that sample *averages* of economic variables of interest would indeed describe the (possibly very disordered) economic system adequately with respect to the essential properties of interest (e.g. consumption, investment, or labor market outcomes) of such a system.

If the coefficient of variation of X does *not* converge in distribution to zero, then the (economic) system is said to be *non-self-averaging*. This means that it would be misleading to analyze only averages when trying to find out about the the essential properties of interest of such a system. In this case the (evolution of the) distribution of the variables of interest would matter. Dispersion in the (economic) variables of interest would be of concern for the description of the system in this case.

The latter concept is at the center stage of the arguments made in this paper. It takes at its starting point a technical concept to criticize the common methodology in macroeconomics that wishes to provide micro-foundations for macroeconomic variables of interest.

General Comments

The paper addresses a methodological concept, namely that of the "representative agent", and reminds us of some problematic features inherent in standard macroeconomic modelling. One important point addressed in the paper is that, for instance, in endogenous growth models the often use (one-parameter) Poisson-Drichlet distributions which capture the (stochastic) arrival rate of innovations may not be the same for a local grocery store and a multinational corporation. To capture this difference the authors introduce a two-parameter Poisson-Drichlet distribution. With this the authors then show in a very simple growth set-up that an economy characterized by a two-parameter Poisson-Drichlet distribution would be non-self-averaging. Thus, in this case it would be misleading to analyze such an economy by only focussing on the sample means of, for example, GDP.

The authors then go on that the property of non-self-averaging might be a generic problem. For this they analyze triangular urn models which may capture properties of very large class of macroeconomic models.

It is not easy to asses the authors' contribution. First of all, the paper is not written in a style that is amenable to the readership of a general or even special interest economics journal. Up to page 8 the arguments are presented in a reasonable clear way but from then on (especially, p. 8 to 14), the paper is often full of math that is not explained well. For instance, the derivation leading up to proposition 1 should be explained in a lot more intuitive way. Furthermore, the paper would gain in clarity if all the arguments leading to the second proposition, using urn models, would be accompanied by far more intuition.

All in all, I share many of the points raised in the critique of the paper provided by John Seater, which is available on the journal's website under "comments and questions". He criticizes the paper from the perspective of standard macroeconomic modelling.

In what is to follow I will, however, try to complement his critique by focussing on aspects of the paper that I think may deserve further attention.

1. The concept of "non-self-averaging" may indeed be new in economics. I think it may be worthwhile to explore its implications in more detail. For instance, the first part of the paper with the two parameter PD distribution may actually serve as a tool to be used in a more standard and more fully developed model about innovations of different sectors in an economy.

2. The authors criticize the concept of a "representative agent". But they do not relate to the literature that deals with this issue. For instance, an early critique of the "representative agent" is provided by Kirman (1992). Among other things he provides an example that a "representative agent" may actually do/choose the opposite of what individual agents would choose. Furthermore, Hartley (1997) provides an account and a discussion of the concept and the origin of the "representative agent". Their points are (explicitly) being recognized in (some) standard macro textbooks as e.g. Turnovsky (1995), ch. 9. For instance, he notes the following, see p. 275, "*Any model employed as widely as the representative agent model begins to take on a life of its own and to be accepted almost as an axiom. It is therefore useful to remind ourselves periodically of its limitations. ... It [the representative agent framework] should be viewed as a step in the continuing development and understanding of macroeconomic theory,... Over time, models become superseded, and indeed the extension to heterogeneous agents seems like a promising avenue for future research.*"

   It is a bit unfortunate that in the present paper these and other contributions on this issue are not mentioned.

3. The paper criticizes Lucas's 1972 article. But in later work models with many agents were explicitly presented that cannot easily be summarized under the standard heading of a "representative agent" economy. See, for example, Lucas and Stokey (1984).

4. One of the results of criticizing the "representative agent" approach has been to develop models with (locally) interacting agents. In this sense the concept of "non-self-averaging" may be useful in that it may help to establish whether it is so wrong to work with "averages" of economic variables. Thus, when arguing that we do not know what the approximation error precisely is, when working with standard macro models, and that simplicity may have much to recommend the use of the standard modelling approach, it may also be argued that the property of "non-self-averaging" may provide some sort of "measure" for the approximation error.

   Thus, showing to what (possibly) constant value the coefficient of variation converges to may provide a measure to a situation where the coefficient of variation converges to zero and the model would be self-averaging.

   In summary, I think the paper needs a very major overhaul. It contains interesting thoughts that, however, need to be presented in a far more intuitive way. I believe it would be no loss whatsoever, if the paper gained a lot in pages, as long as the underlying economic intuition and its added-value are brought forward in a more convincing way.

Specific Comments

1. On page 6 the two-parameter Poisson-Drichlet (PD) distribution is introduced. What exactly is the economic interpretation of α in $p_i = \frac{n_i - \alpha}{n + \theta}$? As α makes the two-parameter PD distribution different from the one-parameter one, i.e. when $\alpha = 0$, an economic interpretation of α (a central parameter in this part of the paper) appears to be called for.

2. Footnote 4: What precisely is the *economic* intuition for the implication that with $\alpha > 0$ the probability that a new sector emerges is higher?

References

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