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Entrepreneurship, Knowledge, and the Industrial Revolution

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Abstract
This paper constructs a two-sector unified growth model. Learning-by-doing in agriculture eventually allows the preindustrial economy to leave its Malthusian trap. But entrepreneurs in the manufacturing sector do not attempt invention if not much is known about natural phenomena. This delays the industrial revolution. Since entrepreneurs identify new useful knowledge at all times in a serendipitous way, the industrial revolution is an inevitable outcome. The paper characterizes the equilibrium path from ancient times to the infinite future. According with this characterization, the model economy successfully captures the qualitative aspects of England's unified growth experience.

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Keywords Unified growth theory; useful knowledge; industrial enlightenment; demographic transition; endogenous technological change

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1 Introduction

The literature following Galor and Weil's (2000) unified growth theory (UGT) studies the very long-run patterns of economic growth and development within dynamic general equilibrium environments. A unified growth model features Malthusian stagnation and modern growth equilibria. It also accounts for factors that trigger and govern an endogenously occurring and gradual transition from stagnation to growth.[1]

An industrial revolution is the turning point in an economy's transition from stagnation to growth. For this reason, the UGT literature is a promising research program for understanding

- why the first Industrial Revolution in Britain started *when* it did, and
- *which factors* had kept today's developed societies and others in a quasi-trap of poverty for several millennia.

Some recent papers focusing on endogenous technological change—e.g., Desmet and Parente (2012), O'Rourke et al. (2013), Strulik et al. (2013), Peretto (2013), and Strulik (2014)—provide new insights into the causes of the first Industrial Revolution. From an empirical perspective, Madsen et al. (2010) show that British economic growth in the last couple of centuries is a story of productivity growth resulting from inventions. Complementing this result, Clark (2014) argues that only models endogenizing technological progress would be successful in explaining England's transition from Malthusian stagnation to modern growth.

This paper constructs a unified growth model in which *entrepreneurs*' role in the accumulation of *useful knowledge* explains the timing and the inevitability of an industrial revolution. An entrepreneur in the model behaves very much like Schumpeter's (1934) "entrepreneur-inventor." Economic growth is due to new inventions created by entrepreneurs. The industrial revolution is an endogenously occurring switch from an equilibrium regime of zero inventive effort to one of positive inventive effort.

[1] Early contributions to the UGT literature include those of Kögel and Prskawetz (2001), Jones (2001), Hansen and Prescott (2002), Lucas (2002), Tamura (2002), Hazan and Berdugo (2002), Boldrin and Jones (2002), Lagerlöf (2003), and Doepke (2004). Galor (2005, 2010) provides two illuminating surveys of the literature.

1.1 Premises

Three premises specify the role of entrepreneurs in economic growth and the accumulation of useful knowledge:

First, as in Hellwig and Irmen (2001) and Grossmann (2009), entrepreneurs establish and manage the firms producing and selling a consumption good. The sector they operate in is perfectly competitive. Managing their own firms allows entrepreneurs to appropriate positive profits under perfect competition. Thus, *entrepreneurial invention*—allocating some of the time endowment to invention and decreasing the time spent on routine management—may be an optimal strategy.

Second, building on Mokyr's (2002) theory, the model differentiates *inventions* from *useful knowledge*. If the stock of useful knowledge is smaller, being successful in achieving a given number of inventions is less likely for a given level of effort directed to invention. This, as in O'Rourke et al. (2013), endogenizes the productivity term of a standard invention technology exhibiting constant-returns-to-scale (CRS) with respect to its rival labor input.

Finally, the stock of useful knowledge expands in time through *collective discovery*. Entrepreneurs identify new useful knowledge about natural phenomena underlying the production processes in a serendipitous way. Besides, they share new useful knowledge with one another in their common environment. This formalizes, albeit imperfectly, what Mokyr (2002) calls *industrial enlightenment*, i.e., the creation and diffusion of useful knowledge among British/European entrepreneurs and capitalists.

1.2 Motivation

Meisenzahl and Mokyr's (2012) prosopographical evidence on 759 British inventors born between 1660 and 1830 motivate the emphasis on entrepreneurial invention.[2] Among 598 inventors with a known business ownership status, only 88 inventors—around 15% of all—are non-managers, and 467 of them—around 78% of all—are *business owners*. Given the latter statistic, understanding the role of

[2] Prosopography is a type of historical investigation focusing on common characteristics of a group of people.

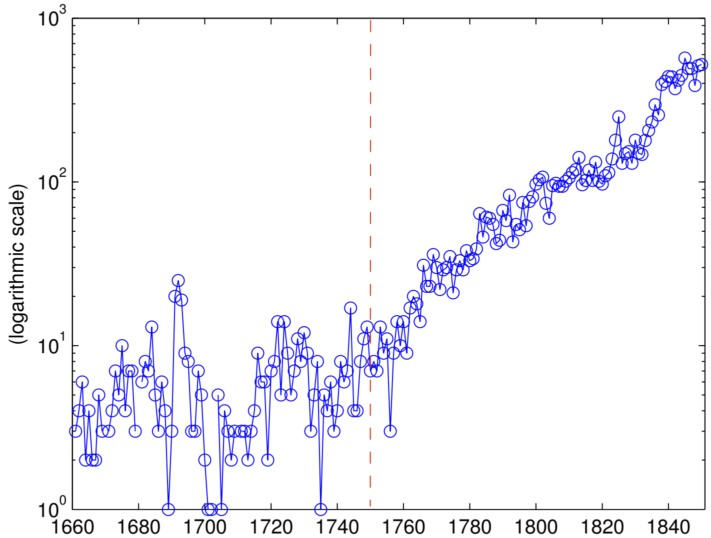

Figure 1: Patented Process Innovations in England, 1661–1850

Data Source: Sullivan (1989)

inventors incentivized with profit motive during the first Industrial Revolution may be of prime importance.

Sullivan (1989) motivates the focus on useful knowledge. In England, the number of patented inventions starts increasing around 1750 without any change in patent regulations and the propensity to patent. Figure 1 pictures Sullivan's (1989) data on patented process innovations. The stock of useful knowledge reaching to a critical level around 1750 might be the trigger of this trend break.

1.3 Main Results

The model of this paper incorporates the three premises introduced above with Strulik and Weisdorf's (2008) simple unified growth model—a two-sector model of learning-by-doing and endogenous fertility with a closed-form solution. The main results are the following:

1. The stock of useful knowledge has an endogenously determined threshold level: Entrepreneurs choose to not to invest in inventive activities if the existing stock of useful knowledge is not larger than this threshold level.

2. An industrial revolution is inevitable: Collective discovery makes the stock of useful knowledge to continuously grow in preindustrial times and to cross its threshold level in finite time.

3. The preindustrial economy faces a "trade-off" for the timing of its industrial revolution: Two determinants of fertility—the initial level of productivity in the manufacturing sector and the preference parameter increasing the (marginal) utility from reproduction—affect the growth rate of useful knowledge through the mass of entrepreneurs. The effects of these exogenous givens on the timing of the industrial revolution, however, are ambiguous. A higher level of fertility implies a larger *mass* of population but a smaller *share* of entrepreneurs. For this reason, the mass of entrepreneurs might increase or decrease.

1.4 Contribution

Thinking a bit seriously about the productivity term of an otherwise standard invention technology and bringing the "entrepreneur-inventor" back to the scene of economic development have important implications: They allow us to understand

- why invention may not be optimal for a very long episode of history, and
- why an industrial revolution is inevitable.

The main message is twofold: First, since inventors are business owners, the industrial revolution starts only when the stock of useful knowledge is large enough to make invention optimal. Second, the pace at which the economy approaches its industrial revolution depends on the supply of entrepreneurship. The historical narratives of the Industrial Revolution strongly acknowledge these notions, but existing unified growth models, to the best of my knowledge, do not pay attention to this dual role of entrepreneurs.

In contrast with Strulik and Weisdorf's (2008) model, the model of this paper tells a story of Schumpeterian innovation: It makes explicit the incentives that govern technological progress in the long run. But building upon Strulik and Weisdorf's (2008) model with *demographic transition* and *structural transformation* enriches our understanding in another respect: Fertility choice and the dynamics of the agricultural sector affect the timing of the industrial revolution. This is not a trivial point as the timing results are in complete accordance with those obtained by Desmet and Parente (2012) and Peretto (2013).

Invention is subject to uncertainty, and this leads to ex post heterogeneity across firms in the manufacturing sector. Hence, the model provides a richer understanding of modern industrial structure than most unified growth models do. Specifically, it explains why more innovative firms are larger on average, and why the size distribution of innovative firms is skewed. These are two well-known regularities recently reiterated by Akcigit and Kerr (2010) and Klette and Kortum (2004), respectively.

Finally, this paper complements some recent papers—e.g., O'Rourke et al. (2013) and Strulik (2014)—that formalize Mokyr's (2002) theory of useful knowledge. In this theory, the purposeful activation of invention technologies centrally depends on the creation and diffusion of useful knowledge in preindustrial times.

1.5 Outline

Section 2 briefly reviews the related literature. Section 3 introduces the model economy, and Section 4 defines and analyzes static, dynamic, and asymptotic equilibria. Section 5 characterizes the model economy's equilibrium path in the very long run. Section 6 discusses the implications of the model, and Section 7 concludes. Appendix A presents the proofs of all lemmas and propositions.

2 The Related Literature

At least since Schumpeter (1934), “entrepreneur-inventor” is the leading actor in the narratives of the Industrial Revolution. Solo (1951), Baumol (1990), Murphy et al. (1991), and Mokyr (2010) argue that entrepreneurial invention was the prime cause of technological progress during the first Industrial Revolution—long before the rise of modern R & D lab. Peretto (1998) emphasizes the distinction between entrepreneurial invention and corporate R & D activities in a second generation Schumpeterian model. Michelacci (2003) studies the functioning of entrepreneurial skills in bringing inventions to markets. From an evolutionary perspective, Doepke and Zilibotti (2008) and Galor and Michalopoulos (2012) show why entrepreneurial traits do matter for economic development in the very long run. This paper, differently than these papers do, incorporates both occupational choice and entrepreneurial invention with the UGT framework. Two-occupation structure is similar to, and even simpler than, those of Lucas (1978), Murphy et al. (1991), and Michelacci (2003). The formulation of entrepreneurial invention under perfect competition is similar to those of Hellwig and Irmen (2001), Grossmann (2009), and Haruyama (2009).

This paper builds upon Mokyr’s (2002) conceptual framework. In his theory, useful knowledge and inventions refer to different things. Useful knowledge is propositional and does not have direct technological applications. Laws and principles answering “What?” questions about natural phenomena constitute the stock of useful knowledge. Inventions, in contrast, are prescriptive blueprints or recipes that answer “How?” questions. Landes (1969), Rosenberg (1974), Nelson (1982), and Easterlin (1995) also emphasize this distinction, i.e., the usefulness of some sort of foundational knowledge for inventive activity. According to Weitzman (1998: 345), knowledge accumulation has distinct *recombination* and *productivity* aspects; inventions resulting in productivity increases are recombinations of useful ideas. With reference to the *knowledge capital*, Lucas (2002: 12) asks "[w]hat can be gained by disaggregating into two or more knowledge-related variables." The model studied in this paper answers this question: Entrepreneurial invention may not be optimal if the productivity of inventive effort depends on the stock of useful knowledge. Haruyama (2009) also builds upon this distinction in an endogenous growth model, and O’Rourke et al. (2013) incorporate it with the formal analysis of unified growth. Also related is Strulik's (2014) emphasis on existing vs. accessible knowledge, and the access cost in Strulik (2014) is decreasing in time as suggested by Mokyr (2002). Entrepreneurs' dual role, however, remains unexplored in a unified model with population growth and structural transformation.

Landes (2006: 6) describes collective discovery as "the seventeenth-century European mania for tinkering and improving." Bekar and Lipsey (2004) further argue that the diffusion of Newtonian mechanics among British industrialists was the prime cause of the first Industrial Revolution. Jacob (1997) makes a similar argument on the diffusion of scientific culture with an emphasis on British success. Kelly (2005) develops a network model analyzing this type of collective learning in relation with the Industrial Revolution. Lucas (2009) also emphasizes collective learning in a model differentiating propositional knowledge from productivity. O'Rourke et al. (2013) and Milionis and Klasing (2011) link the accumulation of propositional knowledge to human capital accumulation within environments similar to that of Galor and Weil (2000). Howitt and Mayer-Foulkes (2005) assume that the skill level of entrepreneurs is proportional to average productivity in the production of intermediate inputs. This paper's formulation of industrial enlightenment accentuates the *mass* of entrepreneurs: More entrepreneurs in the manufacturing sector create and disseminate more useful knowledge given the quality of creating and disseminating it. The mass of urban population, as argued by Crafts and Mills (2009), is more convincing as a driver of knowledge growth than the mass of the entire population.

3 The Model Economy

This section constructs the model economy. The first four subsections introduce the demographic structure, endowments, preferences, and technologies, i.e., the model environment. Then follow occupations and the market structures. After formally defining the decision problems, the section concludes with the market clearing conditions.

Time in the model, denoted by t, is discrete and diverges to the infinite future: $t \in \{0, 1, ...\}$. Following the UGT literature, the economy is closed, and there is no physical capital accumulation. The produced goods of the model are food and the manufactured good, and the primary inputs are land and labor.

3.1 The Demographic Structure

Consider two overlapping generations. Individuals who are *adults* in period t give birth to *children* at the beginning of period t. The surviving children become adults at the beginning of period $t+1$.

For simplicity, reproduction is asexual, and $n_t \in \mathbb{R}_{++}$ denotes the number of surviving children a generic adult in period t optimally chooses, i.e., net fertility per adult. n_t, not being an integer number, represents *average* net fertility among period-t adults.

Denote by $N_t \in \mathbb{R}_{++}$ the adult population in period t. Since there exists a common level of net fertility for all adults in equilibrium, N_{t+1} simply reads

$$N_{t+1} = n_t N_t, \tag{1}$$

and $N_0 > 0$ is exogenously given.

3.2 Endowments

Normalize the length of a period to unity. A period-t adult has then a unit time endowment. This is the only source of homogeneous labor force in this economy. Children, not having a time endowment, remain idle until they become adults next period.

Land is a production factor of the agricultural technology. The total land endowment of the economy is fixed. As it is common in the UGT literature, there do not exist property rights over land, and the ownership structure is immaterial from an analytical point of view.

3.3 Preferences

An adult in period t derives lifetime utility from her consumption C_t of the manufactured good and her net fertility n_t. As in Strulik and Weisdorf (2008) and de la

Croix and Licandro (2013), the utility function representing these preferences is quasi-linear and satisfies

$$U(C_t, n_t) \equiv C_t + \phi \ln(n_t) \quad \phi > 0 \tag{2}$$

with a boundary restriction in the form of

$$n_t \geq 1. \tag{3}$$

$U(C_t, n_t)$ takes its non-homothetic form for two reasons: First, linearity in C_t eliminates the direct income effect on fertility; this leads to a fertility decline at the advanced stages of economic development. Second, adapting risk neutral preferences with respect to C_t simplifies the decision problem of entrepreneurs under uncertainty.

Equation (3) represents the parental preference for *reproductive success*. This is to transmit genes to the next generations.[3] The restriction in (3) is the simplest way of introducing reproductive success in a model of fertility choice, and it begets a desired property:[4] The baseline level of net fertility is equal to unity as in Jones (2001), and this implies a stabilizing *level* of population in the very long run.

3.4 Technologies

This subsection introduces the model economy's technologies without reference to the ownership and the market structures.

Reproduction

The only input of the reproduction technology is food, i.e., child rearing requires only the food intake by children. For simplicity, adults do not consume food, and each surviving child requires one unit of food.[5] Then, the budgetary cost of having n_t surviving children is equal to n_t units of food.

[3] All surviving children become fecund at the beginning of their adulthood by construction.

[4] de la Croix and Licandro (2013) and Strulik and Weisdorf (2014) incorporate reproductive success, respectively, with continuous and discrete time environments in which parents simultaneously choose the number of children they have and the survival probability of their children.

[5] This assumption, as in Strulik and Weisdorf (2008), simplifies the model to allow for a closed-form solution.

Agriculture

Consider any farm f operating in the agricultural sector. The Cobb-Douglas technology of production for farm f is

$$Y_{ft} = (L_f X_{ft})^{1-\eta} H_{ft}^{\eta} \quad \eta \in (0,1). \tag{4}$$

Here, Y_{ft} denotes output, L_f denotes land input, X_{ft} denotes productivity, and H_{ft} denotes the flow of worker hours.

The productivity X_{ft} of farm f changes in time owing to learning-by-doing at the farm level. The technology of learning-by-doing is simply

$$X_{ft+1} = X_{ft} + \psi Y_{ft} \quad \psi > 0. \tag{5}$$

Manufacturing

Consider any firm i operating in the manufacturing sector. The Cobb-Douglas technology of production for firm i is

$$y_{it} = (X_{it} h_{wit})^{\lambda} h_{mit}^{1-\lambda} \quad \lambda \in (0,1). \tag{6}$$

Here, y_{it} denotes output, X_{it} denotes productivity associated with a unit of worker hour h_{wit}, and h_{mit} denotes the flow of manager hours.

Recall that the labor endowment of individuals is homogeneous. For this reason, only the nature of tasks associated with production differentiates worker and manager hours in (6): Workers produce the good in its finished form with their eye-hand coordination, and managers tell workers *what* to do and *how* to do it.

Productivity growth in the manufacturing sector is due to inventions. An invention project requires research hours as its only rival input. It then generates a stochastic number of inventions, and each invention increases a baseline level of productivity by some fixed factor. As in Desmet and Parente (2012), this baseline level of productivity for firm i in period t, denoted by $\bar{X}_t$, is the average operating productivity attained by the firms of the previous generation. Formally, we have

$$\bar{X}_t \equiv E_{t-1}^{-1} \int_0^{E_{t-1}} X_{it-1} \mathrm{d}i \tag{7}$$

where $E_{t-1} > 0$ denotes the mass of firms operating in the manufacturing sector in period $t-1$.

Define, now, firm i's operating productivity X_{it} as in

$$X_{it} \equiv \sigma^{z_{it}} \bar{X}_t \tag{8}$$

where $\sigma > 1$ is the step-size of inventions, and $z_{it} \in \{0, 1, ...\}$ is the stochastic number of inventions satisfying

$$z_{it} \sim \text{Poisson}\,(a_{it})\,. \tag{9}$$

Here, $a_{it} \in \mathbb{R}_+$ denotes the arrival rate of inventions for firm i and is tied to the research effort via

$$a_{it} = \theta f\,(K_t)\,h_{rit} \quad \theta > 0 \tag{10}$$

where h_{rit} denotes the flow of hours allocated to research by firm i. This invention technology features CRS with respect to its rival input h_{rit}.

The novelty here is the term $\theta f\,(K_t)$, i.e., research productivity per unit of inventive effort. $K_t \in \mathbb{R}_+$ denotes the stock of useful knowledge, and $f : \mathbb{R}_+ \to [0,1]$ is a function representing the role of K_t. A discussion of three restrictions on $f\,(K_t)$ is in order.

First, suppose that the expected number of inventions is higher if a larger stock of useful knowledge is available. A plausible way to restrict $f\,(K_t)$ accordingly is to assume that $f(K_t)$ is strictly increasing.

The second restriction is the following: Useful knowledge not only is useful for invention but also is the essential input of an invention project:

$$f\,(0) = 0. \tag{11}$$

In Mokyr's (2002: 13–14) words, "[t]he likelihood that a laptop computer would be developed in a society with no knowledge of computer science, advanced electronics, materials science, and whatever else is involved is nil."

Finally, there exists an upper limit of $\theta f\,(K_t)$ for $K_t \to +\infty$ as in Weitzman (1998). Even if every single thing about natural phenomena underlying the production processes is known, i.e., $K_t \to +\infty$, the arrival rate a_{it} is bounded above.

This is because a unit of h_{rit} must have finite capacity to process knowledge. One simple way to impose such a limit is to assume that

$$\lim_{K_t \to +\infty} f(K_t) = 1. \tag{12}$$

Equation (12) basically implies that $K_t \to +\infty$ as the ultimate enlightenment eliminates what causes a unit of inventive effort to be less productive than its full potential of θ. With $K_t \to +\infty$, firm i has access to knowledge formed with practically everything about natural phenomena. The firm then generates an expected number $a_{it} = \theta h_{rit}$ of inventions with constant (maximum) productivity θ as it is usual in endogenous growth theory.[6]

Collective Discovery

The process of collective discovery governs how K_t changes in time. As entrepreneurs own and manage the firms operating in the manufacturing sector, they discover new pieces of useful knowledge during their lifetime. Moreover, they not only create new knowledge in this individual and serendipitous way but also share what they create with one another in their common environment, e.g., in coffeehouses. This network effect is the most consistent with the public good characterization of useful knowledge.

The simplest way to formalize collective discovery is a linear knowledge production function in the form of

$$K_{t+1} = K_t + \omega E_t \quad \omega > 0 \tag{13}$$

where E_t is the mass of firms operating in the manufacturing sector in period t. As we shall see in a moment, each manufacturing firm is owned by a single

[6] Note that Strulik (2014) models the access cost to existing knowledge for firms with an increasing function described by (11) and (12): Knowledge diffusion accelerates if the stock of capital expands, knowledge does not diffuse if there is no capital, and knowledge diffusion is perfect if the stock of capital converges to positive infinity. The two postulations, however, are on entirely different notions as $f(K_t)$ described above governs the research productivity and K_t is collectively known by all period-t individuals. Extending the present framework with the access cost as in Strulik (2014) is left for future research.

entrepreneur in equilibrium. Thus, the total mass E_t of all entrepreneurs is, in fact, the time input of collective discovery.

The parameter $\omega > 0$ in (13) represents the quality of environment for creating and sharing useful knowledge. This constant then represents geographical, cultural, and social determinants of collective discovery.[7]

3.5 Occupations and the Market Structures

Two occupational groups in this economy are *entrepreneurs* and *workers*, and the occupation is chosen optimally. An entrepreneur establishes a firm producing and selling the manufactured good under perfectly competitive conditions. A worker, on the other hand, inelastically supplies her labor endowment to these firms and to the farms producing food. Thus, three things are traded in this economy: First, all adults consume the manufactured good produced and sold by the firms; this is the numéraire. Second, a worker hour is traded at the wage $W_t > 0$ in a perfectly competitive labor market. In this market, firms run by entrepreneurs and farms producing food are the buyers. Finally, food is traded at the price $P_t > 0$ in a perfectly competitive food market. Here, all adults are, again, buyers.

An entrepreneur has an incentive to engage in inventive activities since she manages her own firm; the flow of profit to an entrepreneur is strictly positive under all circumstances. Instead of allocating all her labor endowment to routine management, then, an entrepreneur may choose to allocate some of it to invention for an increased market share and profit. Firm i operating in period t is shut down when entrepreneur i dies at the end of period t. Entrepreneurs of the next generation then establish new firms at the beginning of period $t+1$.

There exists a continuum $[0,1]$ of identical and perfectly competitive farms. Workers in the agricultural sector own each of these farms with equal ownership shares as in Desmet and Parente (2012). Given the production technology (4) and the normalization of $L_f = 1$ for all $f \in [0,1]$, this implies the following: A worker in the agricultural sector earns price times her average product $P_t \times \left(X_{ft}^{1-\eta} H_{ft}^{\eta} / H_{ft} \right)$ if H_{ft} units of worker hours are allocated to production. Moreover, the perfect

[7] Note that the main results of this paper are not sensitive to the linearity with respect to E_t. The qualitative nature of the results does not change if $K_{t+1} - K_t$ is an increasing function of E_t.

mobility of workers between agriculture and manufacturing dictates that

$$P_t\left(\frac{X_{ft}^{1-\eta}H_{ft}^{\eta}}{H_{ft}}\right)=W_t. \tag{14}$$

3.6 Decision Problems

There are three decision problems. These are the problem of the representative worker, the problem of the representative entrepreneur, and the problem of occupational choice for all individuals. The solution to the problem of occupational choice builds upon the solutions of the first and the second problems: In equilibrium, an individual is indifferent between becoming a worker and becoming an entrepreneur.

Workers

Let C_{wt} and n_{wt} respectively denote consumption and net fertility for the worker. Since the worker earns only the wage income W_t, the worker's deterministic problem, given (2), is to maximize

$$U_{wt}\equiv C_{wt}+\phi\ln(n_{wt}) \tag{15}$$

subject to the budget constraint

$$C_{wt}+P_tn_{wt}=W_t \tag{16}$$

and the boundary constraint $n_{wt}\geq 1$. Equivalently, the problem is

$$\max_{n_{wt}\geq 1} W_t-P_tn_{wt}+\phi\ln(n_{wt}). \tag{17}$$

Entrepreneurs

With a slight abuse of notation, let entrepreneur i be the representative entrepreneur. This entrepreneur's income is equal to the flow Π_{it} of profit, and Π_{it} is stochastic because of the uncertainty in invention. Equation (2) and entrepreneur i's budget constraint

$$C_{it}+P_tn_{it}=\Pi_{it} \tag{18}$$

imply the expected utility of entrepreneur i as in

$$\mathbf{E}[U_{it}] \equiv \mathbf{E}[\Pi_{it}] - P_t n_{it} + \phi \ln(n_{it}) \tag{19}$$

where $n_{it} \geq 1$ is her net fertility, and $\mathbf{E}[\bullet]$ is the expectation operator.

Entrepreneur i maximizes $\mathbf{E}[U_{it}]$ by choosing an optimal level of n_{it} and by maximizing $\mathbf{E}[\Pi_{it}]$. To achieve the latter, operating with the production technology (6) and the invention technology (10), she chooses an optimal level h_{wit} of the demand for worker hours and allocates her time between management and invention under the resource constraint

$$h_{mit} + h_{rit} = 1. \tag{20}$$

Let entrepreneur i choose h_{wit} contingent on (X_{it}, h_{mit}, W_t).[8] Using (6), define the profit function as in

$$\Pi_{it} \equiv \Pi\left(h_{wit}, X_{it}, h_{mit}, W_t\right) \equiv \left(X_{it} h_{wit}\right)^{\lambda} h_{mit}^{1-\lambda} - W_t h_{wit}.$$

The problem of maximizing $\Pi\left(h_{wit}, X_{it}, h_{mit}, W_t\right)$ by choosing $h_{wit} \geq 0$ has a unique interior solution satisfying

$$h_{wit} = \frac{\lambda^{\frac{1}{1-\lambda}} X_{it}^{\frac{\lambda}{1-\lambda}} h_{mit}}{W_t^{\frac{1}{1-\lambda}}}. \tag{21}$$

At this solution, we have

$$\Pi_{it} = (1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}} \left(\frac{X_{it}}{W_t}\right)^{\frac{\lambda}{1-\lambda}} h_{mit}. \tag{22}$$

Using (8), (9), (10), (20), and (22), define now the expected profit $\mathbf{E}[\Pi_{it}]$ of entrepreneur i as in

$$\mathbf{E}[\Pi_{it}] \equiv \sum_{z=0}^{+\infty} \left[\frac{a_{it}^z \exp\left(-a_{it}\right)}{z!}\right] \left[(1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}} \left(\frac{\sigma^z \bar{X}_t}{W_t}\right)^{\frac{\lambda}{1-\lambda}} \left(1 - \frac{a_{it}}{\theta f(K_t)}\right)\right]. \tag{23}$$

[8] This is not an uncommon assumption in the quality ladder framework adapted here—see, e.g., Aghion and Howitt (2009). Forcing the entrepreneur to choose h_{wit} and h_{rit} simultaneously results in a "second-best" result for the entrepreneur; she cannot perfectly insure herself against too low or too high levels of h_{wit} relative to X_{it}. The main results of the paper, however, are not altered.

The term in the first brackets on the right-hand side denotes the Poisson probability of generating z inventions given a_{it}, and the term in the second brackets is the level of optimal profit when the entrepreneur generates z inventions given a_{it}.

Lemma 1. *The expected profit in (23) can be rewritten as*

$$\mathbf{E}[\Pi_{it}] = \exp(\Sigma a_{it})\Lambda\left(\frac{\bar{X}_t}{W_t}\right)^{\Gamma}\left(1-\frac{a_{it}}{\theta f(K_t)}\right) \tag{24}$$

where the parameters

$$\Gamma \equiv \frac{\lambda}{1-\lambda} > 0, \quad \Lambda \equiv (1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}} > 0, \quad \textit{and} \quad \Sigma \equiv \sigma^{\frac{\lambda}{1-\lambda}} - 1 > 0$$

are defined for notational ease.

Equation (24) identifies the return to and the cost of inventive activity. $\mathbf{E}[\Pi_{it}]$ increases with $\exp(\Sigma a_{it})$ that depends, through Σ, on the step-size σ of invention and the elasticity λ of output with respect to X_{it}. The last term in the last parentheses, $a_{it}/[\theta f(K_t)]$, is the time cost of inventing with an expected number a_{it} of inventions. Clearly, (24) specifies the *deterministic* level of profit for $a_{it} = 0$. In this case, the entire labor endowment is spent on management, i.e., $h_{mit} = 1$. z is then equal to zero, and X_{it} is equal to $\bar{X}_t$.

Equations (19) and (24) imply entrepreneur i's problem of maximizing $\mathbf{E}[U_{it}]$ as in

$$\max_{n_{it}\geq 1, a_{it}\in[0,a_t^{\max}]} \exp(\Sigma a_{it})\Lambda\left(\frac{\bar{X}_t}{W_t}\right)^{\Gamma}\left(1-\frac{a_{it}}{\theta f(K_t)}\right) - P_t n_{it} + \phi \ln(n_{it}) \tag{25}$$

where $a_t^{\max} \equiv \theta f(K_t)$ is the upper bound of a_{it} associated with $h_{rit} = 1$. Since all entrepreneurs face the same set $\{\Sigma, \Lambda, \bar{X}_t, W_t, \Gamma, \theta, K_t, P_t, \phi\}$ of givens, any solution to (25) implies a unique $\mathbf{E}[U_{it}]$.

Occupational Choice

Given (15) and (19), occupational choice is optimal if we simply impose

$$\mathbf{E}[U_{it}] = U_{wt}. \tag{26}$$

This dictates that all period-t adults are indifferent between becoming an entrepreneur and becoming a worker at the beginning of period t.

3.7 Market Clearing Conditions

This subsection closes the model through the market clearing conditions. In the next three equations, the left-hand sides denote the quantity supplied, and the right-hand sides denote the quantity demanded.

First, the food market clears via

$$Y_{ft} = (N_t - E_t)n_{wt} + \int_0^{E_t} n_{it}\mathrm{d}i. \tag{27}$$

Second, the market for the manufactured good clears via

$$\int_0^{E_t} y_{it}\mathrm{d}i = (N_t - E_t)C_{wt} + \int_0^{E_t} C_{it}\mathrm{d}i. \tag{28}$$

Finally, the market for worker hours, determining the equilibrium mass $E_t \in (0, N_t)$ of entrepreneurs in a residual way, clears via

$$N_t - E_t = H_{ft} + \int_0^{E_t} h_{wit}\mathrm{d}i. \tag{29}$$

4 Static, Dynamic, and Asymptotic Equilibria

This section defines and analyzes the model economy's equilibria. The main purpose is to establish the analytical foundations of the model economy's equilibrium path from some initial period to the infinite future.

4.1 Static General Equilibrium (SGE)

Definition 1. *A SGE of the model economy, for any $t \in \{0, 1, \ldots\}$, is a collection*

$$\{n_{wt}, C_{wt}, H_{ft}, Y_{ft}, E_t, \{n_{it}, a_{it}, h_{rit}, h_{mit}, z_{it}, X_{it}, h_{wit}, y_{it}, \Pi_{it}, C_{it}\}_{i\in[0,E_t]}\}$$

of quantities and a pair $\{P_t, W_t\}$ of relative prices such that, given the state vector $(N_t, \bar{X}_t, X_{ft}, K_t)$,

- *n_{wt} solves the worker's problem (17),*

- (n_{it}, a_{it}) *solves the entrepreneur's problem (25),*
- *all adults are indifferent between becoming a worker and becoming an entrepreneur through (26),*
- *the food market and the market for worker hours clear, respectively, via (27) and (29),*[9] *and*
- *(4), (6), (8), (9), (10), (14), (16), (18), (20), (21), and (22) are satisfied.*

Proposition 1. *There exists a unique SGE with* $n_{it} = n_{wt} = n_t \geq 1$ *and* $a_{it} = a_t \geq 0$. *Furthermore, depending on the given state vector* $(N_t, \bar{X}_t, X_{ft}, K_t)$, *this unique SGE features either*

- *Regime 1:* $n_t = 1$ *and* $a_t = 0$ *or*
- *Regime 2:* $n_t > 1$ *and* $a_t = 0$ *or*
- *Regime 3:* $n_t > 1$ *and* $a_t > 0$ *or*
- *Regime 4:* $n_t = 1$ *and* $a_t > 0$.

The unique SGE might feature these equilibrium regimes depending on $(N_t, \bar{X}_t, X_{ft}, K_t)$, and it is constructive to discuss these regimes under two separate headings:

1. the equilibrium in manufacturing with regard to $a_t \geq 0$, and
2. the equilibrium in agriculture with regard to $n_t \geq 1$.

Invention and the Manufacturing Sector

The industrial revolution *in the model* is defined as the endogenously occurring switch from the equilibrium regime of $a_t = 0$ to one of $a_t > 0$. The main result originating from the proof of Proposition 1 is the following:

[9] Note that the market clearing condition (28) for the manufactured good is satisfied via Walras' Law in general equilibrium, and it is not an equilibrium-defining equation.

Corollary 1. *The arrival rate a_t of inventive activity is characterized by a unique threshold such that*

$$a_t = \begin{cases} 0 & \text{if } K_t \leq \hat{K} \\ \theta f(K_t) - \left(\sigma^{\frac{\lambda}{1-\lambda}} - 1\right)^{-1} & \text{otherwise} \end{cases} \tag{30}$$

where the threshold level $\hat{K}$ is defined as in

$$\hat{K} \equiv f^{-1}\left[\theta^{-1}\left(\sigma^{\frac{\lambda}{1-\lambda}} - 1\right)^{-1}\right] > 0. \tag{31}$$

Corollary 1 directly follows from the solution (A.2) of the entrepreneur's problem. Notice from (24) that the return to and the cost of invention are not additively separable: The marginal cost of increasing the expected number of inventions from zero to an infinitesimally small amount is a strictly positive number that may well exceed its marginal return. This follows because the entrepreneur has to decrease her management input to increase her inventive effort, and this is not insignificant since the entrepreneur's role as manager-inventor is essential in implying this non-separability.

Naturally, then, the stock K_t of useful knowledge determines, through the time cost of inventive effort, whether invention is optimal given $(\sigma, \lambda, \theta)$. Given K_t, on the other hand, higher values of σ and λ increase the return to inventive effort, and a higher value of θ decreases the time cost of it.

Real wage W_t at the unique SGE, along with other things, determines the level of economic development in this economy. Specifically, it affects the size of the agricultural sector and the optimal level of net fertility. Thus, it is useful to underline how W_t is tied to $\bar{X}_t$.

Corollary 2. *The mapping from productivity to wage in manufacturing is*

$$W_t = \delta(a_t, K_t)(1-\lambda)^{1-\lambda}\lambda^{\lambda}\bar{X}_t^{\lambda} \tag{32}$$

where $\delta(a_t, K_t) \geq 1$ is an auxiliary function defined as in

$$\delta(a_t, K_t) \equiv \exp\left[(1-\lambda)\left(\sigma^{\frac{\lambda}{1-\lambda}} - 1\right)a_t\right]\left(1 - \frac{a_t}{\theta f(K_t)}\right)^{1-\lambda}. \tag{33}$$

When inventive activity is not optimal ($a_t = 0$), we have $\delta(0,\bullet) = 1$ implying $W_t = (1-\lambda)^{1-\lambda}\lambda^{\lambda}\bar{X}_t^{\lambda}$. This simply corresponds to the unit price of a worker hour that would prevail in a competitive model of occupational choice with Cobb-Douglas technology and *without* entrepreneurial invention.

When inventive activity is optimal ($a_t > 0$), management input is related with the invention technology through the optimal use of entrepreneurs' time. W_t in competitive equilibrium embeds this effect via $\delta(\bullet,\bullet)$ function. An important result for the equilibrium regime of $a_t > 0$ is the following:

Corollary 3. *Ex ante symmetry across entrepreneurs translates into ex post heterogeneity: For any arrival rate $a_t > 0$, the ex ante probability of generating z inventions is equal to the ex post fraction of entrepreneurs with z inventions under (Borel's version of) the law of large numbers. Thus, the unique cross-section distribution of any element of the set $\{X_{it}, h_{wit}, y_{it}, \Pi_{it}, C_{it}\}_{i\in[0,E_t]}$ is the Poisson distribution with the parameter $a_t > 0$.*

Corollary 3 emphasizes that the stochastic nature of inventive activity creates winners and losers among ex ante symmetric entrepreneurs as in Galor and Michalopoulos (2012). Consequently, more innovative entrepreneurs/firms attain higher productivities, higher firm sizes, and higher market shares.

The final result to be noted regarding the manufacturing sector is on the equilibrium supply of entrepreneurship. The mass E_t of entrepreneurs is central to the equilibrium path of the model economy. This is because the growth rate of K_t is increasing in E_t. The next corollary highlights the equilibrium solution of E_t:

Corollary 4. *At the unique SGE, E_t satisfies*

$$E_t = (1-\lambda)\left(1 - \frac{H_{ft}}{N_t}\right)N_t \tag{34}$$

where H_{ft}/N_t denotes the labor share of the agricultural sector.

E_t satisfies (34) regardless of $a_t \geq 0$ and $n_t \geq 1$. Here, $(1-\lambda)$ is the Cobb-Douglas exponent of manager hours in (6); the fraction $(1-\lambda)$ of all adult individuals occupying the manufacturing sector, i.e., $N_t - H_{ft}$, become entrepreneurs. E_t then negatively depends on the size of the agricultural sector as expected.

Fertility and the Agricultural Sector

Corollary 5. *Given $W_t = \delta(a_t, K_t)(1-\lambda)^{1-\lambda}\lambda^\lambda \bar{X}_t^\lambda$, net fertility ($n_t$) satisfies*

$$n_t = \begin{cases} \left(\frac{\phi}{W_t}\right)^\eta \left(\frac{X_{ft}}{N_t}\right)^{1-\eta} & \text{if } X_{ft} > \frac{N_t}{\left(\frac{\phi}{W_t}\right)^{\frac{\eta}{1-\eta}}} \\ 1 & \text{otherwise} \end{cases}, \tag{35}$$

and the labor share of the agricultural sector (H_{ft}/N_t) reads

$$\frac{H_{ft}}{N_t} = \begin{cases} \frac{\phi}{W_t} & \text{if } X_{ft} > \frac{N_t}{\left(\frac{\phi}{W_t}\right)^{\frac{\eta}{1-\eta}}} \\ \left(\frac{X_{ft}}{N_t}\right)^{-\frac{1-\eta}{\eta}} & \text{otherwise} \end{cases}. \tag{36}$$

Corollary 5 follows from (A.1) and the solutions of P_t and H_{ft} given (A.4) and (A.5). Almost exactly as in Strulik and Weisdorf (2008), n_t changes positively with X_{ft} and negatively with N_t and $\bar{X}_t$. These follow from the equilibrium value of the price P_t of food; P_t responds negatively to agricultural productivity X_{ft} and positively to adult population N_t and manufacturing productivity $\bar{X}_t$ (see the proof of Proposition 1). Also as in Strulik and Weisdorf (2008), n_t converges to unity when N_t and $\bar{X}_t$ get sufficiently high relative to X_{ft} given (η, ϕ). Differently from Strulik and Weisdorf (2008), however, the regime of $n_t = 1$ prevails mainly because of the reproductive success constraint $n_t \geq 1$. The constraint, for large enough P_t, forces adults to have the minimum number of surviving children for the maximization of lifetime utility.

The labor share H_{ft}/N_t of the agricultural sector is a decreasing function of W_t (and, hence, of $\bar{X}_t$) for the regime of $n_t > 1$. This is the pull effect of the manufacturing sector. In the regime of $n_t = 1$, the role of W_t vanishes completely. The labor share of agriculture, in this regime, depends negatively on X_{ft} because higher productivity releases labor out of agriculture and positively on $N_t (= N_{t+1} = \bar{N})$ because food remains essential for reproduction where $\bar{N}$ is some fixed level of adult population.

For completeness, it is necessary to state the conditions required for both sectors to operate in equilibrium, i.e., $H_{ft}/N_t < 1$. In the regime of $n_t > 1$, $\phi < (1-\lambda)^{1-\lambda}\lambda^\lambda \bar{X}_0^\lambda$ for $t = 0$ is sufficient to imply $\phi/W_t < 1$ for all t since $\delta(a_t, K_t)$

is bounded below by unity and $\bar{X}_t$ is non-decreasing. In the regime of $n_t = 1$ where $H_{ft}/N_t = (X_{ft}/N_t)^{-(1-\eta)/\eta}$, the sufficient condition simply reads $X_{ft} > N_t (= N_{t+1} = \bar{N})$ since X_{ft} increases without bound given (5).

4.2 Dynamic General Equilibrium (DGE)

To define a DGE, how the vector $(N_t, \bar{X}_t, X_{ft}, K_t)$ of state variables evolves from t to $t+1$ should be specified. The laws of motion for N_t, X_{ft}, and K_t are respectively (1), (5), and (13). To derive the law of motion for $\bar{X}_t$, iterate (7) to obtain

$$\bar{X}_{t+1} = E_t^{-1} \int_0^{E_t} X_{it} \mathrm{d}i.$$

Substituting X_{it} with $\sigma^{z_{it}} \bar{X}_t$ and noting, once again, that the *ex post* fraction of entrepreneurs with z inventions is equal to the *ex ante* probability of generating z inventions result in

$$\bar{X}_{t+1} = \bar{X}_t \sum_{z=0}^{+\infty} \left(\frac{a_t^z \exp(-a_t)}{z!} \right) \sigma^z.$$

This law of motion reduces into the following after some arrangements as in the proof of Lemma 1:

$$\bar{X}_{t+1} = \bar{X}_t \exp\left[(\sigma - 1) a_t\right]. \tag{37}$$

Thus, as in Aghion and Howitt (1992) and others, the growth rate of (average) productivity in the innovating sector is explained by the step-size σ and the arrival rate a_t of inventions.

Definition 2. *Given the vector $(N_0, \bar{X}_0, X_{f0}, K_0) \in \mathbb{R}^4_{++}$ of initial values, a DGE for the entire history from $t = 0$ to $t \to +\infty$ is a sequence of SGE that satisfies the laws of motion (1), (5), (13), and (37) together with the sequences $\{N_t, \bar{X}_t, X_{ft}, K_t\}_{t=1}^{+\infty}$.*

Proposition 2. *There exists a unique DGE.*

4.3 Global Dynamics and the Asymptotic Equilibrium

Since the model economy's dynamical system cannot be transformed into an autonomous dynamical system of (normalized) state variables, the analysis of the asymptotic equilibrium builds upon a conditional dynamical system as in Galor and Weil (2000) and others. This subsection constructs this conditional dynamical system, defines an asymptotic equilibrium of the model economy, and shows that the unique asymptotic equilibrium is globally stable.

To ease the exposition below, define two endogenous state variables, x_{ft} and v_t, as in

$$x_{ft} \equiv \frac{X_{ft}}{N_t} \quad \text{and} \quad v_t \equiv \left(\frac{\phi}{W_t}\right)^{\frac{\eta}{1-\eta}},$$

and let $G_{Wt} \equiv W_{t+1}/W_t \geq 1$ denote the gross growth rate of W_t. Since $\bar{X}_{ft}$ is the prime determinant of the growth of living standards in this economy in the very long run, the rest of the analysis focuses on G_{Wt} as the main indicator of economic growth.[10]

With the new notation introduced, (35) implies

$$n_t = \begin{cases} (x_{ft}v_t)^{1-\eta} & \text{if } (x_{ft}v_t)^{1-\eta} > 1 \\ 1 & \text{otherwise} \end{cases} \tag{38}$$

given (x_{ft}, v_t), and Table 1 summarizes the equilibrium regimes the unique SGE at t might feature given (x_{ft}, v_t, K_t).

Lemma 2. *G_{Wt} is increasing in t for $a_t > 0$ with*

$$\lim_{t\to+\infty} G_{Wt} = G_W^\star \equiv \exp\left[\lambda(\sigma-1)\left(\theta - (\sigma^{\frac{\lambda}{1-\lambda}} - 1)^{-1}\right)\right] > 1. \tag{39}$$

[10] Real GDP per worker, with $t = 0$ being the base period, is defined as in

$$y_t \equiv N_t^{-1}\left(P_0 Y_{ft} + \int_0^{E_t} y_{it}\mathrm{d}i\right)$$

where the integral term denotes the total volume of output in the manufacturing sector. Because (i) this total volume is proportional to E_t given the cross-section Poisson distribution and (ii) Y_{ft}/N_t declines in the long run as we see below, the secular growth of y_t originates from the growth of $\bar{X}_t$.

Table 1: SGE Regimes

Regime	Conditions	Invention	Fertility
1	$K_t \leq \hat{K}$ & $v_t \leq 1/x_{ft}$	$a_t = 0$	$n_t = 1$
2	$K_t \leq \hat{K}$ & $v_t > 1/x_{ft}$	$a_t = 0$	$n_t > 1$
3	$K_t > \hat{K}$ & $v_t > 1/x_{ft}$	$a_t > 0$	$n_t > 1$
4	$K_t > \hat{K}$ & $v_t \leq 1/x_{ft}$	$a_t > 0$	$n_t = 1$

Lemma 3. *Given $G_{Wt} \geq 1$, there exists a conditional dynamical system of (x_{ft}, v_t) satisfying*

$$\frac{x_{ft+1}}{x_{ft}} = \begin{cases} \frac{1}{x_{ft}^{1-\eta} v_t^{1-\eta}} + \frac{\psi}{x_{ft}} & \text{if } n_t > 1 \\ 1 + \frac{\psi}{x_{ft}} & \text{if } n_t = 1 \end{cases}, \text{ and} \tag{40}$$

$$\frac{v_{t+1}}{v_t} = G_{Wt}^{-\frac{\eta}{1-\eta}}. \tag{41}$$

Figure 2 pictures the global dynamics of x_{ft} and v_t on (x_{ft}, v_t) plane for $G_{Wt} = 1$ and $G_{Wt} > 1$. For $G_{Wt} = 1$, we simply have $v_{t+1} = v_t$ for all t, and, for $G_{Wt} > 1$, Lemma 2 implies that v_t is decreasing toward zero as W_t grows without bound.

Define, now, the $N^\star$ locus using (38) as in

$$N^\star \equiv \left\{ (x_{ft}, v_t) : n_t = (x_{ft} v_t)^{1-\eta} = 1 \text{ or, equivalently, } v_t = \frac{1}{x_{ft}} \right\}$$

where we have $n_t > 1$ above and $n_t = 1$ below and over the $N^\star$ locus. Thus, the $N^\star$ locus divides the plane into Regimes 1 and 2 for $G_{Wt} = 1$ and into Regimes 3 and 4 for $G_{Wt} > 1$.

Next, the $x_f x_f$ locus, for $n_t > 1$, is defined as in

$$x_f x_f \equiv \left\{ (x_{ft}, v_t) : x_{ft+1} = x_{ft} \text{ or, equivalently, } v_t = \left(\frac{x_{ft}^{\eta}}{x_{ft} - \psi} \right)^{\frac{1}{1-\eta}} \right\}$$

given (40), where x_{ft} is decreasing above and increasing below the $x_f x_f$ locus. For $n_t = 1$, on the other hand, there does not exist a pair (x_{ft}, v_t) that implies $x_{ft+1} = x_{ft}$, and x_{ft} is increasing for any (x_{ft}, v_t).

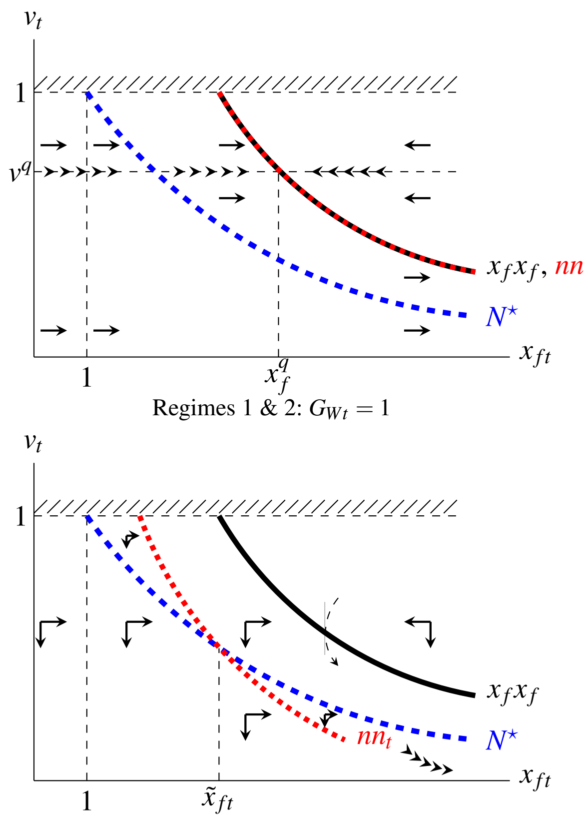

Figure 2: (x_{ft}, v_t) dynamics for $G_{Wt} = 1$ (top) and $G_{Wt} > 1$ (bottom)

Finally, using (38), (40), and (41), the nn locus—below which n_t is increasing and above which n_t is decreasing—is defined as in

$$nn \equiv \left\{ (x_{ft}, v_t) : n_{t+1} = n_t \text{ or, equivalently, } v_t = \left(\frac{x_{ft}^{\eta}}{G_{Wt}^{\frac{\eta}{1-\eta}} x_{ft} - \psi} \right)^{\frac{1}{1-\eta}} \right\}$$

and overlaps with the $x_f x_f$ locus for $G_{Wt} = 1$. For $G_{Wt} > 1$, Lemma 2 implies that the *nn* (or nn_t) locus moves toward the origin as G_{Wt} increases.

The $N^\star$ locus resides below the $x_f x_f$ locus for any $\psi > 0$; these two loci do not intersect for $x_{ft} > 1$ since we have $\psi > 0$. Also notice that the $x_f x_f$ and the *nn* loci either do not intersect (for $G_{Wt} > 1$) or overlap for any (x_{ft}, v_t) (for $G_{Wt} = 1$). On the other hand, the $N^\star$ locus and the *nn* locus intersect at a unique point $\tilde{x}_{ft}$ such that

$$\tilde{x}_{ft} = \frac{\psi}{G_{Wt}^{\frac{\eta}{1-\eta}} - 1} \tag{42}$$

given $G_{Wt} > 1$. Accordingly, the *nn* locus resides below the $N^\star$ locus for any $x_{ft} > \tilde{x}_{ft}$.

Proposition 3. *Given any initial value $\bar{X}_0$ of $\bar{X}_t$, there exists a unique quasi-steady-state equilibrium (x_f^q, v^q) in Regime 2 such that*

$$v^q = v_0 = \left[\frac{\phi}{(1-\lambda)^{1-\lambda}\lambda^\lambda \bar{X}_0^\lambda}\right]^{\frac{\eta}{1-\eta}} \quad \textit{and} \tag{43}$$

$$x_f^q = \arg\operatorname*{solve}_x \left\{ v^q - \left(\frac{x^\eta}{x-\psi}\right)^{\frac{1}{1-\eta}} = 0 \right\}. \tag{44}$$

The quasi-steady-state (x_f^q, v^q) in Regime 2 is an equilibrium state. The economy may converge to this in finite time if the unique SGE features $a_t = 0$. This quasi-steady-state is a balanced growth equilibrium with adult population N_t and agricultural productivity X_{ft} growing at the same gross growth rate

$$n_t = n^q = \left(x_f^q v^q\right)^{1-\eta} > 1.$$

At this quasi-steady-state equilibrium, the growth of X_{ft} allows the economy to sustain a growing N_t with a constant P_t.

Proposition 4. *Suppose that $K_0 \leq \hat{K}$, i.e., the economy is either in Regime 1 or in Regime 2 initially. Then, there exists a period $\hat{t} > 0$ such that $a_{\hat{t}-1} = 0$ and $a_{\hat{t}} > 0$: If the economy starts its evolution with invention being not optimal, an industrial revolution inevitably starts at some future period.*

That K_t and W_t are increasing for all t after the industrial revolution motivates the following definition of the asymptotic equilibrium:

Definition 3. *An asymptotic equilibrium of the model economy, for $t \to +\infty$, is the (unique) limiting SGE of the model economy in Regime 4 with*

$$n_t = n^\star = 1 \quad \textit{for } t \geq t^\star \quad \textit{and} \quad a_t \to a^\star = \theta - \left(\sigma^{\frac{\lambda}{1-\lambda}} - 1\right)^{-1} > 0.$$

At this asymptotic equilibrium, the labor share of the agricultural sector declines toward zero for $t \to +\infty$ as implied by (36) and (40), and adult population N_t is stabilized at some $N^\star > 0$ for $t \geq t^\star + 1$ given $n_{t^\star} = 1$. As the growth of K_t implies $f(K_t) \to 1$ through (12), a_t converges to $a^\star$. Thus, the asymptotic equilibrium is the one with the perpetual growth of $\bar{X}_t$—and, therefore, of W_t and y_t.

Proposition 5. *The unique asymptotic equilibrium of the model economy is (asymptotically) globally stable. That is, for any $(N_0, \bar{X}_0, X_{f0}, K_0) \in \mathbb{R}^4_{++}$, the model economy's SGE converges to the asymptotic equilibrium for sufficiently large t.*

5 The Equilibrium Path from Stagnation to Growth

The economy's unique asymptotic equilibrium is globally stable, and an industrial revolution inevitably starts at some finite t as stated in Proposition 4. The economy, however, may transit directly from Regime 1 to Regime 4 without a demographic transition. On the other hand, if the model economy starts its evolution at $t = 0$ in Regime 1 *and* if $\hat{t}$ is sufficiently large, then the model economy's unique DGE features an equilibrium path from stagnation to growth *with* an industrial revolution *and* a demographic transition. Specifically, under these two assumptions, the model economy exhibits regime transitions in the order of their numbering.

Proposition 6. *Suppose that (i) $v_0 \leq 1/x_{f0}$ and $K_0 \leq \hat{K}$, i.e., the economy is in Regime 1 initially, and that (ii) $\hat{t}$ is sufficiently large, i.e., the industrial revolution is to start sufficiently late. Then, the economy enters Regime 2 at some $\check{t} > 0$, then enters Regime 3 at some $\hat{t} > \check{t}$, and eventually enters Regime 4 at some $t^\star > \hat{t}$, where $\check{t}$, $\hat{t}$, and $t^\star$ are all endogenously determined.*

Remark 1. *Both assumptions of Proposition 6 are in complete accordance with historical evidence. The first one requires the preindustrial economy to have a sufficiently low level X_{f0} of agricultural productivity, given N_0 and $\bar{X}_0$. In this case, historically low values of N_0 and $\bar{X}_0$ imply a low level of upper bound for X_{f0} for Regime 1 to prevail, i.e.,*

$$v_0 \leq \frac{1}{x_{f0}} \Leftrightarrow X_{f0} \leq N_0 \left(\frac{(1-\lambda)^{1-\lambda} \lambda^{\lambda} \bar{X}_0^{\lambda}}{\phi} \right)^{\frac{\eta}{1-\eta}} .$$

The economy without a sufficiently advanced agricultural sector, then, cannot support fertility above replacement. Moreover, since x_{f0} is itself sufficiently low, the labor share of the agricultural sector is at its historical maximum at $t = 0$.

The second assumption is consistent with the fact that the first Industrial Revolution in Britain started around 50,000 years later than the rise of modern human populations, i.e., populations that share cultural universals such as language, art, religion, and toolmaking. The takeoff to modern growth through an industrial revolution is a very recent phenomenon if one takes a very long-run perspective.

The convergence to Regime 4 through Regimes 2 and 3 may be of two types. If the growth rate of x_{ft} in Regime 2 is large enough given the growth rate of K_t, then the economy enters Regime 3 after completing the convergence to the quasi-steady-state in Regime 2. In this scenario, fertility remains stable at its historical maximum before the industrial revolution. This shape of the equilibrium path is arguably consistent with the 20th century experience of today's least developed economies. These economies have remained agrarian but sustained an increasing level of population with a high (and a stable) level of fertility.

The second possibility is the one where the growth rate of x_{ft} in Regime 2 is not large enough given the growth rate of K_t. Then, the industrial revolution starts before the economy converges to its quasi-steady-state. In this case, the fertility decline starts at a level of fertility that is less than its quasi-steady-state level. This scenario is consistent with the unified growth experience of early industrialized countries such as England and France. For these countries, we do not observe very long episodes of fertility remaining stable at a very high level.

Figure 3 pictures the economy's transition from Malthusian stagnation to modern growth for the second case—the industrial revolution starting before the

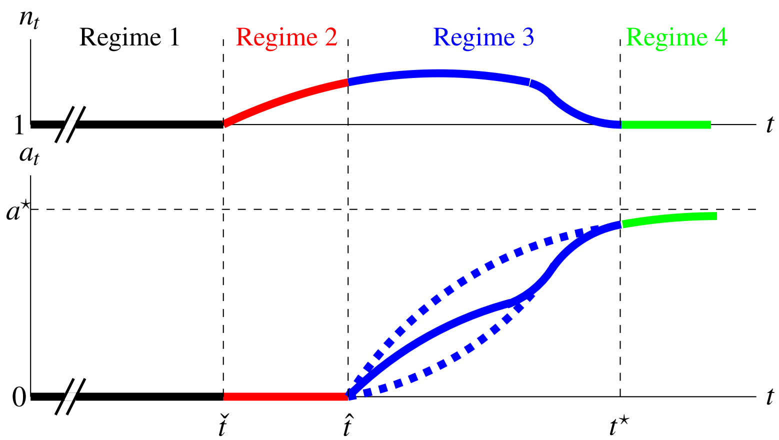

Figure 3: The Equilibrium Path from Stagnation to Growth

Note: This figure pictures the equilibrium path under the assumption that the industrial revolution starts before the economy converges to the quasi-steady-state equilibrium. The dashed lines of the bottom panel show alternative paths of a_t in Regime 3 depending on different parameterizations of $f(K_t)$.

convergence to the quasi-steady-state is completed. A narrative of this transition is in order.

At the very beginning of history, productivity and population are at historically lowest levels. Conversely, the labor share of the agricultural sector is at its historical maximum to sustain a stable population level under poverty. Since useful knowledge about natural phenomena underlying the manufacturing technology is limited, entrepreneurs allocate their entire labor endowment to routine management. This implies stagnating productivity in manufacturing. Agricultural productivity, however, grows in time through learning-by-doing and results in a slowly declining labor share of agriculture.

The growth in agricultural productivity eventually allows the economy to sustain a level of fertility above replacement; population starts growing. The ongoing growth of agricultural productivity implies an increasing rate of population growth in this regime. The economy starts converging to the quasi-steady-state.

The labor share of the agricultural sector is now constant because of stagnating productivity in the manufacturing sector.

An industrial revolution starts when a sufficiently larger stock of useful knowledge becomes available; entrepreneurs start allocating resources to invention. The growth rate of the stock of useful knowledge increases with a declining labor share of agriculture in Regime 1 and with an expanding population in Regime 2. It is now increasing with both of these factors. In this regime, more time allocated to invention by each generation means faster growth of manufacturing productivity. Besides, an increasing share of entrepreneurs' time allocated to invention implies a more skewed cross-section distribution of productivity and firm size in the manufacturing sector. Population and manufacturing productivity get sufficiently high some time after the industrial revolution starts. This leads to a sufficiently high price for the food, and net fertility starts declining. In the meantime, the growth of manufacturing productivity may accelerate at some period before it decelerates and converges to its asymptotic equilibrium. This is because the declining labor share of agriculture and increasing population further stimulate the growth of the stock of useful knowledge.

When it is too costly to have more than one child, the economy enters the final stage of its demographic transition. Population level and fertility are stabilized while the decline of the agricultural sector continues. Since the total mass of entrepreneurs is also stabilized, the growth rate of the stock of useful knowledge starts decreasing. At the very end of history, humanity has access to every bit of knowledge about natural phenomena, and investing into new technology for higher prosperity continues.

To conclude, Proposition 6 shows us the following: *If the model is located at the correct initial position in historical time*, it successfully replicates the very long-run equilibrium path of an early industrialized economy such as England. After several thousand years in a Malthusian trap, population in England starts increasing in mid-1600s, at the period $\check{t}$, and the first Industrial Revolution starts at around 1750, at the period $\hat{t}$. According with the most recent population projections of the United Nations (2012), population in the United Kingdom will stabilize at around 77 million at the end of this century, at the period $t^{\star}+1$.

6 Discussion

A special generation of entrepreneurs directs resources into risky inventive activities unlike those of past generations. These entrepreneurs are special because the stock of useful knowledge they have access to is large enough to signal a higher level of expected profit. They benefit from standing on the shoulders of dead entrepreneurs who collectively created all of this useful knowledge in a serendipitous way.

The invention threshold in the model leads to a kinked time-series of labor productivity in manufacturing. This, in turn, implies a kinked time-series of real wage that exhibits exponential growth starting with the industrial revolution. The Industrial Revolution in history is an invention revolution in the model. After this invention revolution, exerting inventive effort for an increasing profit continues to be optimal.

6.1 The Industrial Revolution: Break or Continuity?

Whether the first Industrial Revolution, roughly covering the period from 1760 to 1830, was a break from the past or a continuity with it remains controversial among some economic historians. Crafts and Harley's (1992) *gradualist* view suggests that, in per capita terms, there was little economic growth in England until the early 19th century. Crafts and Harley (1992) also argue that the scope of fast technological progress was limited with the textile sector before the diffusion of the steam technology. An industrial revolution as a structural break characterized by very slow growth in per capita terms, however, is not controversial. As Pereira (2004) documents, several variables of interest—including total industrial output and population—exhibit endogenously determined upward trend breaks during the first Industrial Revolution in Britain. Complementing these results, Mokyr (2004) and others suggest that the fast expansion of English population did keep output per capita at a low level during the first Industrial Revolution.

The model of this paper, as a unified model, captures such a relationship between population and technology. The model predicts that an industrial revolution may start while population growth is accelerating. Besides, productivity gains would be modest during the early stages of the industrial revolution. These naturally imply that the acceleration in the pace of economic growth would be slow during the first Industrial Revolution.

6.2 The Timing of the Industrial Revolution

Desmet and Parente (2012) and Peretto (2013) ask a specific *timing question*: Which factors do affect the period at which the industrial revolution starts? For the model of this paper, the answer would be a solution of $\hat{t}$ as a function of the model's exogenous givens. These givens are the values of structural parameters and the initial values of endogenous state variables.

A closed-form solution of $\hat{t}$ is not available because the model is not simple enough. Some concrete answers for the timing question, however, can be obtained by inspecting the economy's evolution in Regimes 1 and 2.

The timing effects are of two types: First, we have the *threshold effects* through which the fixed threshold $\hat{K} > 0$ of K_t determines *how far away the industrial revolution is* given K_0. Second, the gross growth rate $G_{Kt} \equiv K_{t+1}/K_t$ determines *how fast the economy moves toward its invention threshold* through the *growth effects*.

The exogenous component $\theta > 0$ of research productivity, the step-size $\sigma > 1$ of inventions, and the labor exponent $\lambda \in (0,1)$ of manufacturing production determine $\hat{K}$ as in (31). Given the partial derivatives

$$\frac{\partial \hat{K}}{\partial \theta} < 0, \quad \frac{\partial \hat{K}}{\partial \sigma} < 0, \quad \text{and} \quad \frac{\partial \hat{K}}{\partial \lambda} < 0,$$

the higher values of these parameters have hastening threshold effects. Clearly, a higher initial value K_0 of K_t also implies a lower value of $\hat{t}$.

More complicated is the analysis of growth effects. Here, the preliminary task is to express G_{Kt} in Regimes 1 and 2 as in

$$\begin{aligned} G_{Kt}^{\text{Regime 1}} &= 1+\omega(1-\lambda)\left(1-x_{ft}^{-\frac{1-\eta}{\eta}}\right)\left(\frac{N_0}{K_t}\right) \\ G_{Kt}^{\text{Regime 2}} &= 1+\omega(1-\lambda)\left(1-\frac{\phi}{\lambda^{\lambda}(1-\lambda)^{1-\lambda}\bar{X}_0^{\lambda}}\right)\left(\frac{N_t}{K_t}\right) \end{aligned}$$

using (13), (34), and (36).

Representing the quality of the process of collective discovery, $\omega > 0$ unambiguously increases G_{Kt} for any t and in both regimes. Hence, it unambiguously decreases $\hat{t}$ and hastens the industrial revolution. A higher initial value N_0 of adult population also implies an unambiguously higher value for G_{Kt} in both regimes. For λ, the growth effect is ambiguous because we have $\partial G_{Kt}^{\text{Regime 2}}/\partial\lambda \gtrless 0$.

A faster growth of x_{ft} with a higher value of $\psi > 0$ increases the share of entrepreneurs in adult population and implies a higher level of $G_{Kt}^{\text{Regime 1}}$. Besides, since faster growth of x_{ft} increases n_t without affecting the labor share of the agricultural sector in Regime 2, we have $\partial G_{Kt}^{\text{Regime 2}}/\partial\psi > 0$. Thus, a faster growth of agricultural productivity unambiguously hastens the industrial revolution.

The preference parameter ϕ and the initial value $\bar{X}_0$ of manufacturing productivity affect both n_t and H_{ft}/N_t in Regime 2. For this reason, these have ambiguous growth effects. To see this, notice that we have

$$\begin{aligned} n_t^{\text{Regime 2}} &= \left(\frac{\phi}{\lambda^\lambda(1-\lambda)^{1-\lambda}\bar{X}_0^\lambda}\right)^\eta x_{ft}^{1-\eta} \quad \text{and} \\ \left(\frac{H_{ft}}{N_t}\right)^{\text{Regime 2}} &= \frac{\phi}{\lambda^\lambda(1-\lambda)^{1-\lambda}\bar{X}_0^\lambda} \end{aligned}$$

in Regime 2. A higher level of ϕ and a lower level of $\bar{X}_0$ increase the labor share of agriculture—for the economy needs to sustain faster population growth with a higher level of agricultural production. Hence, these changes imply a lower share of entrepreneurs contributing to collective discovery; $G_{Kt}^{\text{Regime 2}}$ is decreasing with ϕ and increasing with $\bar{X}_0$. A higher level of ϕ and a lower level of $\bar{X}_0$, through a higher level of n_t, also imply a larger population N_t for all t in Regime 2, and this leads to a higher value of $G_{Kt}^{\text{Regime 2}}$. In other words, the economy faces a “trade-off” for the timing of the industrial revolution as faster population growth, through higher ϕ or lower $\bar{X}_0$, implies both a lower labor share of the manufacturing sector and a larger mass of adult population.

These results on threshold and growth effects perfectly overlap with those obtained by Desmet and Parente (2012) and Peretto (2013). In Desmet and Parente’s (2012) model, the parameters most closely related with institutional quality and policies are the ones representing the cost of innovative activities. Higher values of these parameters imply a delayed industrial revolution similarly to the unambiguous hastening effect of higher θ. Desmet and Parente (2012) also document the hastening timing effect of improving infrastructure. A higher value of ω unambiguously increasing G_{Kt} in both regimes represents this effect. Regarding the (exogenous and fixed) growth of agricultural productivity before the industrial revolution, Desmet and Parente (2012) find an unambiguous effect where a faster growth of agricultural productivity hastens the industrial revolution. Peretto's (2013, Prop.s 6-7) analytical results on the timing of the industrial revolution also show that

- population growth has an ambiguous effect under all scenarios,
- a higher initial level of population implies a sooner industrial revolution, and
- a higher level of fixed operating cost of firms delays the takeoff.

6.3 England vs. China

Comparing preindustrial economies may be problematic, according to Pomeranz (2000), if the selection is based on contemporaneous borders of nations and ignores the differences in geographical size, population, and internal diversity. The model of this paper, however, has interesting implications for the question of "Why England, but not China?"

As noted earlier, ω represents the quality of the environment in which entrepreneurs create and disseminate useful knowledge. The preindustrial England, here, has the advantage of being a small country in terms of its geographical size. Also advantages of the preindustrial England, as noted by Mokyr (2002) and others, are business owners' gentlemanly behavior and technological motivation, and the efficiency of social networks and informal institutions.

Next, if we assume that λ is not radically different for England and China in preindustrial times, the prime determinant of the share of entrepreneurs in adult population would be the labor share of the agricultural sector. Available data show that England, in preindustrial times, had a higher rate of urbanization in comparison with China—see, e.g., Voigtländer and Voth (2006).

More generally, any rival use of time endowment is important in determining the supply, or the *lack*, of entrepreneurship. The labor shares of occupations not contributing to collective discovery would have delaying growth effects for the timing of the industrial revolution. One such occupation—regarding which England had arguably an advantage compared with China—is state bureaucracy. A larger state bureaucracy would imply, *ceteris paribus*, a lower level of G_{Kt} because the mass of entrepreneurs would be smaller. Here, the conjecture is twofold: First, as noted by Mokyr (1998), the preindustrial England benefits from avoiding a large professional bureaucracy. Second, as emphasized by Landes (2006), China's large and ineffective bureaucracy limits its potential. In the terminology of Rodríguez-Pose (1999), the preindustrial England is an *innovation prone* society whereas the preindustrial China is an *innovation averse* one.

6.4 Serendipitous Inventions

There is no technological progress in manufacturing *before* the industrial revolution in the model. Clark's (2010) data, however, reveal that real wage in England has an increasing trend after mid-1600s. Besides, a minuscule rate of growth in real wage before the Industrial Revolution is consistent with Sullivan's (1989) patent data pictured in Figure 1. The question is, then, whether the model can be extended to account for a haphazard type of technological progress.

The simplest extension is to allow for *serendipitous inventions* to exogenously increase the baseline productivity $\bar{X}_t$. Serendipitous inventions can be thought of occurring exogenously without altering the optimal behavior of entrepreneurs. The law of motion for $\bar{X}_t$ can simply be extended as in

$$\bar{X}_{t+1} = \bar{X}_t \exp[(\sigma - 1)(a_t + a_s)]$$

where $a_s > 0$ represents the arrival rate of serendipitous inventions. Clearly, whenever $a_t = 0$, the gross growth rate of $\bar{X}_t$ reduces into $\exp[(\sigma - 1)a_s]$.

6.5 Adult Longevity

The simplest way of capturing the role of adult longevity is to assume, as in Hazan and Zoabi (2006), that all period-t adults live a *fraction* $\ell_t \in [0,1]$ of period t. For simplicity, ℓ_t is exogenous, common across period-t adults, and known by period-t adults with certainty.

This extension generalizes the main results in two respects: First, the threshold value $\hat{K}$ now depends on ℓ_t because a longer life, as an endowment, implies a higher level of profit for entrepreneurs. Then, we have

$$\hat{K}_t \equiv f^{-1}\left[\theta^{-1}\left(\sigma^{\frac{\lambda}{1-\lambda}}-1\right)^{-1}\ell_t^{-1}\right] > 0$$

with $\partial\hat{K}_t/\partial\ell_t < 0$.

Second, since the total lifetime of entrepreneurs now is equal to $E_t\ell_t$, the process of collective discovery in (13) reads

$$K_{t+1} = K_t + \omega E_t \ell_t$$

with $\partial G_{Kt}/\partial\ell_t > 0$. Since a higher level of ℓ_t decreases $\hat{K}_t$ and increases G_{Kt}, adult longevity unambiguously hastens the industrial revolution by implying a lower value of $\hat{t}$.

6.6 Mortality Shocks and the Loss of Useful Knowledge

Whereas (13) implies $K_{t+1} > K_t$ for any t, mortality shocks in an extended model would lead to the loss of useful knowledge before the industrial revolution. Since knowledge resides, at least partially, in the minds of people, a mortality shock such as the Black Death affects the growth of K_t as in Bar and Leukhina (2010).[11]

Suppose, then, that m_t is some random measure of mortality for adult individuals, presumably with a deterministic component, such that

- a fraction $1-s(m_t) \in [0,1]$ of entrepreneurs E_t dies at some interim point between t and $t+1$ (with $s' < 0$) before participating to collective discovery but after completing reproduction, and that

[11] Strictly speaking, the type of knowledge being lost because of mortality shocks in Bar and Leukhina (2010) is prescriptive knowledge in the terminology of Mokyr (2002) and corresponds to $\bar{X}_t$ of the model of this paper.

- a fraction $d(m_t) \in [0,1]$ of useful knowledge K_t depreciates, again, at some interim point between t and $t+1$ (with $d' > 0$).

G_{Kt} in Regimes 1 and 2, then, reads

$$G_{Kt} = 1 + \omega(1-\lambda)\left(1 - \frac{H_{ft}}{N_t}\right)\left(\frac{N_t}{K_t}\right)s(m_t) - d(m_t)$$

where $G_{Kt} > 1$ requires

$$\frac{d(m_t)}{s(m_t)} < \omega(1-\lambda)\left(1 - \frac{H_{ft}}{N_t}\right)\left(\frac{N_t}{K_t}\right).$$

Thus, for K_t to grow before the industrial revolution, the deterministic component of m_t should be decreasing to imply a sufficiently lower value of $d(m_t)/s(m_t)$ as in the case of England. In general, one can conclude that sizable mortality shocks—e.g., the Black Death—unambiguously delay the industrial revolution because of the loss of useful knowledge.

6.7 The Fishing-Out Effect

Productivity growth in the manufacturing sector is not subject to the so-called *fishing-out* effect: The (unit) productivity of labor directed to inventive activity does not decrease with the level of baseline productivity $\bar{X}_t$. No matter how high $\bar{X}_t$ is, the arrival rate a_{it} of inventions is fixed for given levels of K_t and h_{rit}. The endless expansion of K_t makes inventive effort more productive in time as postulated, but the only limit realistically imposed is on the *level* of $f(K_t)$ for $K_t \to +\infty$.

The fishing-out effect can be incorporated by extending $f(K_t)$ into $f(K_t, \bar{X}_t)$ with $\partial f(K_t, \bar{X}_t)/\partial \bar{X}_t < 0$. This extension would have two implications: First, although the threshold $\hat{K}_t$ would now be increasing in $\bar{X}_t$, the economy starting in Regime 1 or Regime 2 at which $a_t = 0$ eventually enters Regime 3 with an industrial revolution. This follows because we have $\bar{X}_t = \bar{X}_0$ in Regimes 1 and 2. There is, therefore, *technological lock-in* as a higher level of $\bar{X}_0 > 0$ delays the industrial revolution. The industrial revolution, however, is still an inevitable outcome of collective discovery.

Second, the shape of the equilibrium path would be affected by the evolution of $(K_t, \hat{K}_t, \bar{X}_t)$. The growth of $\bar{X}_t$ after the date $\hat{t}$ of the industrial evolution may eventually imply $K_{t_1} \leq \hat{K}_{t_1}$ at some $t_1 > \hat{t}$. This would stop the wheel of invention and fix $\bar{X}_t$ at $\bar{X}_{t_1}$. Then, it would take some number of periods for the growth of K_t to catch up with now fixed $\hat{K}_{t_1}$, and the arrival rate a_t would become positive again at some $t_2 > t_1$. There might then exist a "punctuated" equilibrium of $\bar{X}_t$ over alternating episodes of $a_t = 0$ and $a_t > 0$ depending on the parameterization of $f(K_t, \bar{X}_t)$. However, the shape of the equilibrium path, once smoothed, would not be dramatically different because a_t would be periodically positive. Or, provided that a limit-cycle of a_t can be constructed analytically, its period would be at least one.[12]

7 Concluding Remarks

The turning point of the transition from Malthusian stagnation to modern growth was the Industrial Revolution. It was a structural break; technological progress was no longer simply due to serendipitous inventions. Schumpeter's (1934) "entrepreneur-inventor" entered the scene for higher market share and profit, and the world was not the same when Thomas Edison opened the first corporate R & D lab in 1876.

This paper studies a stylized view of the Industrial Revolution. Entrepreneurs, according with this view, alter the Malthusian fate of the preindustrial economy. They manage the firms using the production processes, and they create and disseminate useful knowledge. The mass of entrepreneurs is initially small because both the level of population and the rate of urbanization are at very low levels. Useful knowledge then expands very slowly in preindustrial times, and economic

[12] There may exist parameterizations of $f(K_t, \bar{X}_t)$ leading $\hat{K}_t$ to converge to a constant for some large $\bar{X}_t$. In such a case, the model tells a more involved story of technological progress in the long run: There exist episodes before the industrial revolution in which $\bar{X}_t$ grows and then stabilizes. The industrial revolution, in such a setting, starts at the period where the fishing-out effect of $\bar{X}_t$ is no longer binding with $K_t > \hat{K}_t$ for large enough t. Mokyr (2002, Ch. 3), in fact, proposes such an interpretation: The waves of invention in preindustrial times are subject to diminishing returns; not enough is known about natural phenomena underlying the production techniques given the achieved sophistication represented by $\bar{X}_t$.

history records no such thing as an industrial revolution for a very long episode. An industrial revolution, however, is an inevitable outcome since entrepreneurs keep creating and disseminating useful knowledge.

The timing of the industrial revolution depends on several structural parameters and initial values. The results are largely consistent with those obtained by other scholars. A faster growth of agricultural productivity, a lower cost of innovative activity, and a social network more conducive to knowledge sharing imply a sooner industrial revolution. A faster growth of population, however, has ambiguous effects.

This paper analyzes the transition from Malthusian stagnation to modern growth within a simple framework. The model economy abstracts from

- the demand-side determinants of inventive activity,
- the importance of patents for the industrial revolution,
- the role of professional scientists in the *second* Industrial Revolution,
- the effect of useful knowledge on the decline of mortality, and
- how *the enlightenment through useful knowledge* affects the rise of democracy and formal education.

These are intriguing issues for future research.

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Appendix A Proofs

Proof of Lemma 1:

Rearrange (23) as in

$$\mathbf{E}[\Pi_{it}] = \exp(-a_{it})(1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}}\left(\frac{\bar{X}_t}{W_t}\right)^{\frac{\lambda}{1-\lambda}}\left(1-\frac{a_{it}}{\theta f(K_t)}\right)\sum_{z=0}^{+\infty}\left(\frac{a_{it}^z \sigma^{z\left(\frac{\lambda}{1-\lambda}\right)}}{z!}\right)$$

where the summation on the right-hand side is the Taylor series expansion of $\exp\left(\sigma^{\frac{\lambda}{1-\lambda}} a_{it}\right)$ around $a_{it}=0$. Thus, we simply have

$$\mathbf{E}[\Pi_{it}] = \exp(-a_{it})(1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}}\left(\frac{\bar{X}_t}{W_t}\right)^{\frac{\lambda}{1-\lambda}}\left(1-\frac{a_{it}}{\theta f(K_t)}\right)\exp\left(\sigma^{\frac{\lambda}{1-\lambda}} a_{it}\right).$$

Rewriting this as in

$$\mathbf{E}[\Pi_{it}] = \exp\left[\left(\sigma^{\frac{\lambda}{1-\lambda}}-1\right)a_{it}\right](1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}}\left(\frac{\bar{X}_t}{W_t}\right)^{\frac{\lambda}{1-\lambda}}\left(1-\frac{a_{it}}{\theta f(K_t)}\right)$$

and defining $\Gamma \equiv \frac{\lambda}{1-\lambda}$, $\Lambda \equiv (1-\lambda)\lambda^{\frac{\lambda}{1-\lambda}}$, and $\Sigma \equiv \sigma^{\frac{\lambda}{1-\lambda}}-1$ complete the proof. *Q.E.D.*

Proof of Proposition 1:

The unique solutions of (17) and (25) satisfy

$$n_{wt} = n_{it} = n_t = \begin{cases} \frac{\phi}{P_t} & \text{if } \phi > P_t \\ 1 & \text{otherwise} \end{cases} \tag{A.1}$$

and

$$a_{it} = a_t = \begin{cases} 0 & \text{if } f(K_t) \leq \left[\theta\left(\sigma^{\frac{\lambda}{1-\lambda}}-1\right)\right]^{-1} \\ \theta f(K_t) - \left(\sigma^{\frac{\lambda}{1-\lambda}}-1\right)^{-1} & \text{otherwise} \end{cases} \tag{A.2}$$

where P_t depends on $(N_t, \bar{X}_t, X_{ft})$. At these solutions, (26) reduces into $W_t = \mathbf{E}[\Pi_{it}]$ to yield

$$W_t = \exp(\Sigma a_t)\Lambda\left(\frac{\bar{X}_t}{W_t}\right)^{\Gamma}\left(1-\frac{a_t}{\theta f(K_t)}\right).$$

This implies

$$W_t = \delta\left(a_t, K_t\right)\left(1-\lambda\right)^{1-\lambda}\lambda^{\lambda}\bar{X}_t^{\lambda} \tag{A.3}$$

where $\delta\left(a_t, K_t\right)$ is an auxiliary function defined as in

$$\delta\left(a_t, K_t\right) \equiv \exp\left[\left(1-\lambda\right)\left(\sigma^{\frac{\lambda}{1-\lambda}}-1\right)a_t\right]\left(1-\frac{a_t}{\theta f\left(K_t\right)}\right)^{1-\lambda}.$$

$a_t \geq 0$ is a function of K_t when it is strictly positive, and it does not depend on other endogenous state variables. Then, (A.2) and (A.3) solve W_t. a_t from (A.2) also solves $h_{rit} = h_{rt}$ via (10), and this solves $h_{mit} = h_{mt}$ via (20). Given $a_t \geq 0$, the realization of z_{it} for entrepreneur i follows from (9), and (8) solves X_{it} for entrepreneur i given z_{it} and $\bar{X}_t$. Given these solutions, the unique values of h_{wit}, y_{it}, and Π_{it} follow from (21), (6), and (22), respectively. Four equations that solve n_t, P_t, H_{ft}, and Y_{ft} given W_t and N_t are (4) with $L_f = 1$, (14), (27), and (A.1). Here, (27) reduces into $Y_{ft} = N_t n_t$ under $n_{it} = n_{wt} = n_t$. The solutions satisfy the following: For

$$\phi > P_t \Leftrightarrow X_{ft} > \frac{N_t}{\left(\frac{\phi}{W_t}\right)^{\frac{\eta}{1-\eta}}}, \tag{A.4}$$

we have

$$\begin{aligned} n_t &= \left(\frac{\phi}{W_t}\right)^{\eta}\left(\frac{X_{ft}}{N_t}\right)^{1-\eta},\ P_t = \frac{\phi}{\left(\frac{\phi}{W_t}\right)^{\eta}\left(\frac{X_{ft}}{N_t}\right)^{1-\eta}},\ H_{ft} = \left(\frac{\phi}{W_t}\right)N_t, \text{ and} \\ Y_{ft} &= X_{ft}^{1-\eta}\left(\frac{\phi}{W_t}\right)^{\eta}N_t^{\eta}. \end{aligned}$$

For

$$\phi \leq P_t \Leftrightarrow X_{ft} \leq \frac{N_t}{\left(\frac{\phi}{W_t}\right)^{\frac{\eta}{1-\eta}}}, \tag{A.5}$$

on the other hand, we have

$$n_t = 1,\ P_t = W_t\left(\frac{X_{ft}}{N_t}\right)^{-\frac{1-\eta}{\eta}},\ H_{ft} = \left(\frac{X_{ft}}{N_t}\right)^{-\frac{1-\eta}{\eta}}N_t, \text{ and } Y_{ft} = N_t.$$

Now, given n_t, P_t, W_t, and Π_{it}, (16) and (18) solve C_{wt} and C_{it}, respectively. Thus, only E_t remains to be solved.

What solves E_t is basically (29). To see this, recall that the arrival rate a_t is common across entrepreneurs. Since invention events are independent across entrepreneurs, the symmetry with respect to a_t implies—via (Borel's version of) the law of large numbers—that the ex post fraction of entrepreneurs with $z \geq 0$ inventions for any given a_t is equal to the ex ante Poisson probability $[a_t^z \exp(-a_t)]/z!$ of achieving $z \geq 0$ inventions. This property allows us to write

$$\int_0^{E_t} h_{wit} \mathrm{d}i = E_t \sum_{z=0}^{+\infty} \left[\frac{a_t^z \exp(-a_t)}{z!}\right] h_{wt}(z) \tag{A.6}$$

where $h_{wt}(z)$ reads

$$h_{wt}(z) \equiv \left(\frac{\lambda}{1-\lambda}\right) \exp\left[-\left(\sigma^{\frac{\lambda}{1-\lambda}} - 1\right) a_t\right] \sigma^{\left(\frac{\lambda}{1-\lambda}\right)z}$$

as implied by (21) and the solution of W_t. Applying, now, the reasoning of the proof of Lemma 1 to the right-hand side of (A.6) yields

$$\int_0^{E_t} h_{wit} \mathrm{d}i = E_t \left(\frac{\lambda}{1-\lambda}\right).$$

This last equation and (29) then solve E_t as in (34). *Q.E.D.*

Proof of Proposition 2:

The existence and uniqueness of period-t SGE from Proposition 1 and that the laws of motion for endogenous state variables, i.e., (1), (5), (13), and (37), are all real-valued functions imply the existence and uniqueness of the DGE for the entire history from $t = 0$ to $t \to +\infty$. *Q.E.D.*

Proof of Lemma 2:

This proof has two parts: First, it shows why G_{Wt} is increasing in t for $a_t > 0$. Second, it derives the unique limit $G_W^\star$.

Equation (32) implies

$$G_{Wt} = \left[\frac{\delta(a_{t+1}, K_{t+1})}{\delta(a_t, K_t)}\right] \left(\frac{\bar{X}_{t+1}}{\bar{X}_t}\right)^{\lambda}.$$

We also have $\bar{X}_{t+1}/\bar{X}_t > 1$ from (37) with $a_t > 0$. Then, it is sufficient to show that $\delta(a_t, K_t)$ is increasing in t. Substituting $a_t > 0$ from (30) in (33) implies

$$\delta_t \equiv \delta(a_t, K_t) = \left[\frac{\exp\left[\left(\sigma^{\frac{\lambda}{1-\lambda}} - 1 \right) \theta f(K_t) - 1 \right]}{\left(\sigma^{\frac{\lambda}{1-\lambda}} - 1 \right) \theta f(K_t)} \right]^{1-\lambda}, \tag{A.7}$$

and it follows, given $\Sigma = \sigma^{\frac{\lambda}{1-\lambda}} - 1$, that

$$\frac{\partial \delta_t}{\partial f(K_t)} = (1-\lambda)\delta_t^{-\frac{\lambda}{1-\lambda}} \left[\frac{\Sigma\theta \exp\left[\Sigma\theta f(K_t) - 1\right] (\Sigma\theta f(K_t) - 1)}{(\Sigma\theta f(K_t))^2} \right] > 0$$

since $a_t > 0$ requires $\Sigma\theta f(K_t) - 1 > 0$. As $f(\bullet)$ is a strictly increasing function, showing that K_t is also increasing in t completes this part of the proof. To see this, rewrite (13) with (34) to get

$$K_{t+1} = K_t + \omega E_t = K_t + \omega(1-\lambda)\left(1 - \frac{H_{ft}}{N_t}\right) N_t$$

where $H_{ft}/N_t < 1$ since both sectors operate in equilibrium. Thus, $K_{t+1} > K_t$ for $a_t > 0$.

Regarding the limit $G_W^\star$, notice from (A.7) that δ_t converges to a positive constant since we have $K_{t+1} > K_t$ for all t and $f(K_t) \to 1$ from (12). This implies $G_{Wt} \to \lim_{t\to+\infty}(\bar{X}_{t+1}/\bar{X}_t)^\lambda$, and (37) leads to

$$G_W^\star = \lim_{t\to+\infty} \left(\frac{\bar{X}_{t+1}}{\bar{X}_t} \right)^\lambda = \lim_{t\to+\infty} \{\exp[(\sigma-1)a_t]\}^\lambda = \{\exp[(\sigma-1)a^\star]\}^\lambda$$

with $a^\star \equiv \lim_{t\to+\infty} a_t = \theta - (\sigma^{\frac{\lambda}{1-\lambda}} - 1)^{-1}$ given (30) and (12). *Q.E.D.*

Proof of Lemma 3:

Equation (41) simply follows from $v_t = (\phi/W_t)^{\frac{\eta}{1-\eta}}$. For x_{ft+1}/x_{ft}, we have

$$\frac{x_{ft+1}}{x_{ft}} = \frac{X_{ft+1}/N_{t+1}}{X_{ft}/N_t} = \frac{X_{ft+1}/X_{ft}}{N_{t+1}/N_t} = \frac{X_{ft+1}/X_{ft}}{n_t}$$

by definition. For $n_t > 1$, (4), (5), (35), and (36) imply

$$\frac{X_{ft+1}}{X_{ft}} = 1 + \frac{\psi X_{ft}^{1-\eta} H_{ft}^\eta}{X_{ft}} = 1 + \psi \left(\frac{\phi}{W_t} \right)^\eta x_{ft}^{1-\eta} \left(\frac{1}{x_{ft}} \right) = 1 + \frac{\psi n_t}{x_{ft}},$$

which yields the desired result given (38). For $n_t = 1$, the result follows from $Y_{ft} = N_t$. *Q.E.D.*

Proof of Proposition 3:

Since v^q in (43) uniquely exists given any $\bar{X}_0$, the only task is to show that the equation within the arg solve term of (44) is solved for a unique and strictly positive x given v^q. Rewrite this equation as in

$$x = (v^q)^{-(1-\eta)}x^{\eta} + \psi. \tag{A.8}$$

Since the right-hand side of (A.8) is strictly increasing, strictly concave, and equal to ψ for $x = 0$, a unique $x > 0$ solves the equation. *Q.E.D.*

Proof of Proposition 4:

Since we have $K_{t+1} > K_t$ for all t from the proof of Lemma 2, the growth of K_t throughout Regimes 1 and 2 eventually implies $K_t > \hat{K}$ at some $\hat{t}$. *Q.E.D.*

Proof of Proposition 5:

The task is to show that, if the economy starts its evolution at $t = 0$ in Regimes 1, 2, or 3, it eventually enters Regime 4, and that, if the economy is in Regime 4, it stays in Regime 4 to converge to the unique asymptotic equilibrium.

Starting in Regime 1 (with $n_0 = 1$ and $a_0 = 0$), $v_t = v^q = v_0$ is constant, and x_{ft} and K_t are growing. There are, then, three possibilities for a regime change at a future period: If the growth rate of x_{ft} is sufficiently larger than the growth rate of K_t, the economy enters Regime 2 with $x_{ft} > 1/v^q$ and $K_t \leq \hat{K}$. Conversely, if the growth rate of x_{ft} is not sufficiently larger than the growth rate of K_t, the economy enters Regime 4 with $K_t > \hat{K}$ and $x_{ft} \leq 1/v^q$. Finally, if x_{ft} and K_t grow in such a way that the conditions $K_t > \hat{K}$ and $x_{ft} > 1/v^q$ are satisfied for the same $t > 0$, then the economy enters Regime 3.

Next, starting in Regime 2 (with $n_0 > 1$ and $a_0 = 0$), $v_t = v^q = v_0$ is constant, K_t is growing, and x_{ft} is either increasing or decreasing toward its quasi-steady-state value of $x_f^q > 0$ depending on x_{f0} as the point (x_{f0}, v_0) may reside either below or above the $x_f x_f$ locus. The economy then only transits to Regime 3 when K_t is large enough to imply $K_t > \hat{K}$ since we have $n_t > 1$ for all t in Regime 2.

Starting in Regime 3 (with $n_0 > 1$ and $a_0 > 0$), the economy does not transit to Regime 1 or Regime 2 because the growth of K_t implies $a_t > 0$. This leaves only two possibilities, transiting to Regime 4 or staying in Regime 3. The claim is that the economy enters Regime 4 at some finite t. Since we have $x_{ft}v_t > 1$ in Regime 3 and $x_{ft}v_t \leq 1$ in Regime 4, the transition requires $x_{ft}v_t$ to decrease and to intersect with the $N^{\star}$ locus of the bottom panel of Figure 2. Denoting by $G_{x_f v,t}$ the gross growth rate of $x_{ft}v_t$, the transition may occur in two ways:

- The state of the economy in Regime 3, i.e., (x_{ft}, v_t), may reside above the nn_t locus. This leads to

$$v_t > \left(\frac{x_{ft}^{\eta}}{G_{Wt}^{\frac{\eta}{1-\eta}} x_{ft} - \psi} \right)^{\frac{1}{1-\eta}} \Leftrightarrow G_{Wt}^{-\frac{\eta}{1-\eta}} \left(\frac{1}{x_{ft}^{1-\eta} v_t^{1-\eta}} + \frac{\psi}{x_{ft}} \right) = G_{x_f v,t} < 1.$$

  $x_{ft} v_t$ then decreases toward unity to lead (x_{ft}, v_t) to intersect with the $N^{\star}$ locus, and the economy enters Regime 4.

- (x_{ft}, v_t) in Regime 3 may reside below the nn_t locus to imply $G_{x_f v,t} > 1$. In this case, the state of the economy does not (directly) move toward the $N^{\star}$ locus because $x_{ft} v_t$ is growing. But since x_{ft} is increasing and v_t is decreasing, the state of the economy moves toward the nn_t locus while the nn_t locus itself is gradually shifting toward the origin given G_{Wt} is increasing in t (see Lemma 2). Basically because the nn_t locus and the $N^{\star}$ locus intersect for a unique and a strictly positive x_{ft}, denoted by $\tilde{x}_{ft}$, the state of the economy satisfies $G_{x_f v,t} < 1$ after some finite t. Then, $x_{ft} v_t$ decreases toward unity as in the previous case, and the economy enters Regime 4.

Finally, starting in Regime 4 (with $n_0 = 1$ and $a_0 > 0$), the economy does not transit to Regime 1 or Regime 2 because the growth of K_t, again, implies $a_t > 0$. There exist two scenarios through which the economy ends up staying in Regime 4 (with $x_{ft} v_t \leq 1$) to converge to the asymptotic equilibrium:

- First, if we have $x_{ft} > \tilde{x}_{ft}$, (42) implies that $G_{x_f v,t} = G_{Wt}^{-\frac{\eta}{1-\eta}} \left[1 + (\psi / x_{ft})\right] < 1$. The state of the economy thus resides below the $N^{\star}$ locus. That is, the economy stays in Regime 4 for $t \to +\infty$.

- If, however, we have $x_{ft} \leq \tilde{x}_{ft}$, (42) now leads to $G_{x_f v,t} \geq 1$ and the economy might enter Regime 3 at some finite t if it does not stay in Regime 4. But if the economy enters Regime 3, it eventually enters Regime 4 as discussed above.

Consequently, the economy always ends up in Regime 4 for large t—as it transits only to Regime 3 while in Regime 2 and only to Regime 4 while in Regime 3 (see Figure A.1). In Regime 4, v_t keeps decreasing toward zero, and x_{ft} keeps increasing toward $+\infty$. The economy thus converges to its asymptotic equilibrium with $a_t \to a^{\star} > 0$, $G_{Wt} \to G_W^{\star}$, $n_t = n^{\star} = 1$, $N_t = N^{\star} > 0$, and $H_{ft}/N_t \to 0$. *Q.E.D.*

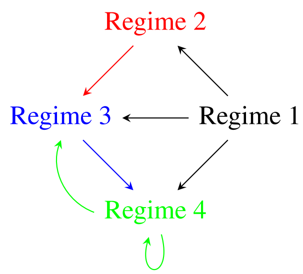

Figure A.1: Regime Transitions

Proof of Proposition 6:

Starting in Regime 1, the economy does not enter Regime 3 or Regime 4 since $\hat{t}$ is assumed to be sufficiently large; see the proof of Proposition 5. There exists, then, $\check{t} > 0$ at which the economy enters Regime 2. Then, the economy enters only Regime 3 at some $\hat{t} > \check{t}$, and it transits from Regime 3 to Regime 4 at some $t^{\star} > \hat{t}$; see, again, the proof of Proposition 5. *Q.E.D.*

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