Interactions in the New Keynesian DSGE Models: The Boltzmann–Gibbs Machine and Social Networks Approach

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Abstract   The Boltzmann–Gibbs distribution is currently widely used in economic modeling. One of the applications is integrated with the DSGE (Dynamic Stochastic General Equilibrium) model. However, a question that arises concerns whether the Boltzmann–Gibbs distribution can be directly applied, without considering the underlying social network structure more seriously, even though the social network structure is an important factor of social interaction. Therefore, this paper proposes two kinds of agent-based DSGE models. The first one belongs to mesoscopic modeling in formulating the social interaction with the Boltzmann–Gibbs machine, and the other one belongs to microscopic modeling in that it is augmented by the network-based ant machine. By comparing the population dynamics generated by those different agent-based DSGE models, we find that the Boltzmann–Gibbs machine offers a good approximation of herding behavior. However, it is difficult to envisage the population dynamics produced by the Boltzmann–Gibbs machine and by the network-based ant machine as having the same distribution, particularly in popular empirical network structures such as small world networks and scale-free networks. Thus, the social interaction behavior may not be replaced by the Boltzmann–Gibbs distribution.

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1   Introduction

While the Dynamic Stochastic General Equilibrium (DSGE) models have been widely used by central banks for policy analysis, the credibility of these models has apparently been challenged by the global financial crisis, and thus it may be risky for governments to use DSGE models as a tool in policy making. In fact, it is not easy to generate a crash or a bubble using a traditional DSGE model with its incredible assumptions such as those of a representative agent and rational expectations. To apply DSGE models to situations closer to real world situations, many researchers have *added heterogeneity, bounded rationality and adaptive learning mechanisms* to them, in the hope of calibrating the modified DSGE models to match the real world economy.[1] Besides the aforementioned modified models, Branch and McGough (2009) have also pointed out that further research should focus on the social interactions of learning behaviors.

Therefore, in order to describe how the social interactions affect the actions of agents, some economists have tried to introduce the statistical mechanics which have been developed by physicists into traditional economic models. The most popular form of statistical mechanics is the *Boltzmann–Gibbs distribution*. In the world of physics, the system is composed of many interacting particles and different statistical mechanics are developed to deal with the relationships between the macro and micro states. Thus, through the Boltzmann–Gibbs distribution, the proportion of different behavioral rules can evolve over time. For this reason, the Boltzmann–Gibbs distribution can be thought of as a tool for evolving the market-sentiment in the economic system. In general, the Boltzmann–Gibbs distribution is often used to deal with *expectation behavior*; it can, therefore, be applied to models incorporating expectations, such as the cobweb model, asset pricing model and positive versus negative feedback model, etc. Brock and Hommes (1997, 1998) can be regarded as the pioneers of this kind of research, also known as the *adaptive belief system model*. During the last decade, the Boltzmann–Gibbs distribution has been widely used for modeling financial markets especially in the study of financial markets' anomalies. A detailed survey of the use of the Boltzmann–Gibbs approach can be found in Chen et al. (2012).

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[1] See Bask (200, Chang et al. (2010), Wen (2010), Evans and Honkapohja (2001), Orphanides and Williams (2007a, b), Milani (2009), Branch and McGough (2009) and Chen and Kulthanavit (2010).

So far, the Boltzmann–Gibbs distribution has gradually been included in DSGE models (Bask, 2007; De Grauwe, 2010a, 2010b; Assenza et al., 2009; and Lengnick and Wohltmann, 2010). Bask (2007) combined a small open economic model with a Boltzmann–Gibbs distribution. He imposed technical and fundamental analyses as different rules in currency trade and found that chaotic dynamics and long swings may occur in the exchange rate. Assenza et al. (2009) combined human expectations in a standard DSGE model. They asked the subjects to provide two-period ahead forecasts of the inflation rate and the output gap for 50 periods. Thus, the realized inflation and the output gap could be determined by average individual expectations. In this experiment, subjects had only qualitative information about the macro economy, and did not know the underlying law of motion. They then separated the experimental data into four different forecasting rules: ADA (Adaptive Expectations), WTR (Weak Trend Followers), STR (Strong Trend Followers) and LAA (Learning Anchoring Adjustment). They found that the Boltzmann–Gibbs machine could successfully calibrate the macroeconomic variables dynamics generated by the human subjects experiment. Lengnick and Wohltmann (2010) combined the Boltzmann–Gibbs distribution and the DSGE macroeconomic model with the financial market. They found that stock market developments are more realistically described by the Boltzmann–Gibbs machine than rational DSGE models, and that the negative impact that the speculative behavior of financial market participants exerts on the macro economy can be reduced by the introduction of a transaction tax. In addition, a closed economic DSGE model is augmented with the Boltzmann–Gibbs distribution in De Grauwe (2010a, 2010b), who developed a stylized DSGE model in which agents use simple rules of heuristics to forecast the future inflation and output gap. The simulation results show that the dynamic behaviors of macroeconomic variables are more volatile in the Boltzmann–Gibbs machine than in stylized DSGE models, and endogenous economic cycles can be generated in the Boltzmann–Gibbs distribution.

The number of applications combining the Boltzmann–Gibbs distribution and the DSGE macroeconomic model has been increasing, but the question is whether the Boltzmann–Gibbs distribution can be directly applied, without considering the underlying social network structure more seriously, even though the social network structure is an important factor of social interaction. Methodologically, models connected with the Boltzmann–Gibbs distribution machine belong to the

mesoscopic genre, i.e., individual details are considered irrelevant. Of course, the social network structure is also not described in those models. However, the social interaction should generally be based on some kind of social network structure. In this case, we seem to know in-depth about the tool that we use. Thus, we need a deep fundamental insight into the economic system's dynamics and how it can be traced back to the structural properties of the underlying social interaction network.

In actual fact, the impact of social networks on economic behavior has become an important issue recently. In order to describe a specific network structure, a social network is broadly understood as a collection of nodes and links between nodes. The extant literature can be roughly classified into three kinds. The first kind treats networks as endogenously determined, and studies the process of formation of networks. In this regard, agents add or delete their links for maximizing utility (or profit) according to a network formation game. In this area, the social network can be applied to free trade networks, market sharing agreements, labor markets and the co-author model. A detailed survey can be found in Jackson (2005).

The second kind of literature regards networks as exogenous. In this case, network structures can be generated with different stochastic algorithms, such as random, scale-free or small world networks; these network structures have been applied to real social networks, i.e., collaborations (Vega-Redondo, 2007) and international trade and financial integration (Schiavo et al., 2010). According to the empirical results, economic networks may also reflect similar universality. Indeed, the connections of banks in an interbank network (Iori et al., 2008) show that the network structure of banks represents a scale-free system where only a few banks interact with many others. In this example, banks with similar investment behaviors cluster in the network. Similar regularities can be traced in many examples, including international trade networks and financial networks (Schiavo et al., 2010). In addition to the empirical approach, applying exogenous network structures to economic models and studying their economic implications is another direction of research. In the last few years, several macroeconomic models have combined heterogeneous expectations with social network structures for modifying the setting of interaction behaviors. For example, Westerhoff (2010) proposes a simple agent-based macroeconomic model with a scale-free and lattice network structure in which firms hold heterogeneous sales expectations. Thus,

each firm has fixed social relations with other firms, and they are either optimistic or pessimistic. The probability of a firm adopting an optimistic view increases not only during a boom, but also with the number of its optimistic neighbors. The change in firms' sentiment causing change in national income has been observed for both a square lattice network and a scale-free network. Alfarano and Milakovic (2009) and Alfarano et al. (2009) discuss how to overcome the N-dependence problem in agent-based financial models. By investigating a class of network structures in a generalized model that presumably reflect the institutional heterogeneity of financial markets, they show that these network structures in fact overcome the problem of N-dependence. However, at the same time they also increase system-wide volatility. Their results indicate that the network structure can be the source of volatility in addition to the behavioral heterogeneity of interacting agents.

According to the above, both the Boltzmann–Gibbs machine and the network approach have been important platforms for expressing social interaction behavior, although to this day it seems that few scholars have discussed the relationship between social networks and the Boltzmann–Gibbs distribution. In order to construct a social interactive DSGE model with a network structure, we have to choose a model which can be combined with different social network structures. The ant model of Kirman (1991, 1993), inspired by the ants' foraging behavior, is one of the choices. The ant model endogenously creates swings and herding behavior in aggregate expectations through interaction and has successfully replicated stylized facts of financial markets (Chen et al., 2012). Therefore, this paper proposes a network-based ant model and attempts to compare the population dynamics between the Boltzmann–Gibbs machine and network-based ant models that we apply to stylized New Keynesian DSGE models.

In order to focus on the population dynamics generated by the Boltzmann–Gibbs machine and network-based ant models, we follow De Grauwe (2010a, 2010b) and adopt the stylized New Keynesian DSGE model for simplicity. Nevertheless, our model leads to a number of interesting insights. We find that both the Boltzmann–Gibbs machine and network-based ant machine can generate herding behavior. However, it is rather difficult to envisage the population dynamics generated by the Boltzmann–Gibbs model and the network-based ant models with the same distribution, particularly in popular empirical network structures such as small world networks and scale-free networks. In addition, our

simulation results further suggest that the population dynamics of the Boltzmann–Gibbs model and the circle network ant model can be considered with the same distribution under specific parameters settings. This finding is consistent with the study of physics for which the Boltzmann–Gibbs distribution is based on the local interaction. Although the circle network structure is not the acknowledged social network structure, according to the relative entropy between the population dynamics of the Boltzmann–Gibbs machine and network-based ant machine, the Boltzmann–Gibbs model with intensity of choice equal to 10,000 is a good approximation of the herding behavior of our network-based ant model with any given network structure.

The remainder of this paper is organized as follows. In Section 2, we describe the stylized New Keynesian DSGE model. Next, we present a version of the agent-based DSGE model with the Boltzmann–Gibbs machine. In Section 4, we discuss the agent-based DSGE model with the network-based ant machine. Following that, we simulate different network structures and present the results. Section 6 concludes.

2　　The Stylized New Keynesian DSGE Model

This section describes the stylized New Keynesian DSGE model. New Keynesian DSGE models are widely used in macroeconomics because they are derived from individual optimization so that both parameters and shocks can be structural. The model consists of the following three equations:

$$y_t = a_1 E y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - E_t \pi_{t+1}) + \varepsilon_t \tag{1}$$

$$\pi_t = b_1 E_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t \tag{2}$$

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 y_t + c_3 r_{t-1} + u_t \tag{3}$$

Equation (1) is referred to as the standard aggregate demand that describes the demand side of the economy. It is derived from the Euler equation which is the result of the dynamic utility maximization of a representative household and market clearing in the goods market. The notation for aggregate demand is as follows: y_t denotes the output gap in period t, r_t is the nominal interest rate and

π_t is the rate of inflation. Here, we add a logged output gap in the aggregate demand equation to describe habit formation. E_t is the expectations operator, which we use to describe how people form their expectations. In the standard New Keynesian DSGE model, the representative agent always has rational expectations.

Equation (2) is a New Keynesian Phillips curve that represents the supply side in the economic system. Under the assumption of nominal price rigidity and monopolistic competition, the New Keynesian Phillips curve can be derived from the profit maximization of a representative final goods producer and the profit maximization of intermediate goods producers which are composed of a number of heterogeneous households. To reflect the price rigidity, the intermediate goods producers can adjust their prices through the Calvo pricing rule. By combining the first-order conditions of the final goods producer, the intermediate goods producer and the Calvo pricing rule, we can obtain the New Keynesian Phillips curve (Equation 2).

Equation (3) represents the Taylor rule commonly used to describe the behavior of the central bank in the standard New Keynesian DSGE model. The central bank reacts to deviations of inflation and output from targets. In Equation (3), π^* refers to the inflation target of the central bank. For convenience, π^* is set to be equal to 0. In addition, the lagged interest rate in Equation (3) represents the smoothing behavior.

Finally, as the DSGE model is the DGE (Dynamic General Equilibrium) model with stochastic terms, ε_t, η_t and u_t are all white noise disturbance terms.

According to the aforementioned equations, we can substitute Equation (3) into Equation (1) and rewrite the matrix notation. Thus, the reduced form can be written as:

$$
\begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & -a_2 c_2 \end{bmatrix} \times \begin{bmatrix} \pi_t \\ y_t \end{bmatrix}
$$

$$
= \begin{bmatrix} 1-b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \times \begin{bmatrix} E_t \pi_{t+1} \\ E_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 1-b_1 & 0 \\ 0 & 1-a_1 \end{bmatrix} \times \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix} \times r_{t-1} + \begin{bmatrix} \eta_t \\ a_2 u_t + \varepsilon_t \end{bmatrix} \tag{4}
$$

or

$$
AZ_t = BE_t Z_{t+1} + CZ_{t-1} + br_{t-1} + V_t \tag{5}
$$

According to the above, we can have the solution $\mathbf{Z_t}$ for the system.

$$\mathbf{Z_t} = \mathbf{A^{-1}}[\mathbf{BE_t Z_{t+1}} + \mathbf{CZ_{t-1}} + \mathbf{br_{t-1}} + \mathbf{V_t}] \tag{6}$$

We can derive the solution only if matrix $\mathbf{A}$ is non-singular. In other words, matrix $\mathbf{A}$ has to satisfy $(1 - a_2 c_2) \times a_2 b_2 c_1 \neq 0$. After obtaining the inflation rate (π_t) and output gap (y_t) through Equation (6), we have to substitute the solution for Equation (3) and to arrive at the interest rate (r_t).

Finally, we must emphasize that the difference between the stylized New Keynesian DSGE model and the agent-based DSGE models is the difference between the expectations of the output gap and inflation. Although agents also make forecasts of inflation, we simply assume that all agents perceive the central bank's announced inflation target π_t^* to be fully credible. In other words, we set $E_t \pi_{t+1} = \pi^* = 0$ in all simulation experiments, including the Boltzmann–Gibbs machine and the network-based ant model.

3   Agent-Based DSGE Model with Boltzmann–Gibbs Machine

To make the macroeconomic models more realistic, economists have started relaxing the standard New Keynesian DSGE model and built the agent-based version (De Grauwe, 2010a, 2010b). In actual fact, the Boltzmann-Gibbs distribution is developed by physicists. The beginning of the story is that some physicists found the collision of particles to be similar to the interaction of people. For example, Boltzmann (1872) showed that molecules are similar to many individuals. In addition, De Rosnay (1975) stated that: "In relation to society: we are the particles … our glance must be directed towards the systems which surround the particles in order to better understand their interactive and evolutionary dynamics.", and Ball (2004) also argued that to develop a physics of society one must use a model in which particles will become people to designate human particles in computer simulations. The logic that derives from the development of physics is the collision of constituent particles under specific structures that is analogous to the interaction of people under specific social networks. In addition, although each particle (agent) is affected by only a few closed particles (agents/friends), the aggregate outcome could result in a huge

change. This holds in both the world of particles and human society.[2] Since physicists have been dealing with the systems of many interacting particles for more than a century, they have developed many mature theories by using statistical mechanics such as the Boltzmann–Gibbs distribution that was developed to investigate the relationship between macroscopic and microscopic phenomena in the physical sciences. Thus, its focus has not been on the details of individual particles, but on the relationships and dynamics between particles. In terms of the methodology, the modeling concept is referred to as *mesoscopic*[3] which means that the individuals' details are considered to be irrelevant, i.e., interaction is what matters. Based on this, the setting of heterogeneity is relatively simple. Each cluster represents a behavioral rule, agents have the same behavior in the same cluster and the population dynamics can be evolved through the Boltzmann–Gibbs distribution over time. Therefore, the Boltzmann–Gibbs distribution can be thought of as a tool for evolving the micro structure of market participants. It can give the proportion of a particular rule of the system.

For describing the different behavioral rules of the expected output gap, we assume that the agents do not fully understand how the output gap is determined, and so the agents use simple rules, say, the optimistic rule and the pessimistic rule, to forecast the future output gap. Therefore, in our Boltzmann–Gibbs machine DSGE model, forecasts of optimistic agents systematically bias the output upwards and forecasts of pessimistic agents systematically bias the output downwards. Specifically, the optimists' rule is defined by $E_{o,t} y_{t+1} = g$ and the pessimists' rule is defined by $E_{p,t} y_{t+1} = -g$, where $g > 0$ denotes the degree of bias in the estimation of the output gap.

Furthermore, the population dynamics is not static. It evolves over time in most cases. According to the Boltzmann–Gibbs machine, the population dynamics

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[2] We, of course, are aware that whether or not human agents can be regarded as atoms can be an issue that is much more subtle and controversial then the statement presented above. See Galam (2012) for a thorough discussion.

[3] In this type of study, how individual agents decide what to do may not matter very much. What happens as a result of their actions may depend much more on the interaction structure through which they act—who interacts with whom, and according to what rules. Therefore, they ignore the decision details of human beings and only assume that agents follow some simple rules and care about how individual forecasting rules interact at the micro level and which aggregate outcome they co-create at the macro level.

and the fractions of optimists ($\alpha_{o,t}$) and pessimists ($\alpha_{p,t}$) can be derived from the following equations:

$$\text{prob}(x(t) = \text{o}) = \alpha_{o,t} = \frac{\exp(\lambda V_{o,t})}{\exp(\lambda V_{o,t}) + \exp(\lambda V_{p,t})} \tag{7}$$

$$\text{prob}(x(t) = \text{p}) = \alpha_{p,t} = 1 - \alpha_{o,t} = \frac{\exp(\lambda V_{p,t})}{\exp(\lambda V_{o,t}) + \exp(\lambda V_{p,t})} \tag{8}$$

There are two alternatives o (optimist) and p (pessimist) in the two-type agent-based DSGE model. Each will produce some gains to the agent. In this formulation, the agent's current choice is mainly determined by the utilities which he experienced when choosing different alternatives. In this model, these experienced utilities have been constantly updated with time t, and *Vo,t* and *Vp,t* represent the experienced utility of being an optimist and pessimist, respectively, updated at time t. Equations (9) and (10) show how the agents compute the utility, *Vo,t* and *Vp,t,* for the optimists' and pessimists' rules. The parameters ρ_k govern the geometrically declining weights.

$$V_{o,t} = -\sum_{k=1}^{\infty} \rho_k \left(y_{t-k} - \text{E}_{o,t-k-1} y_{t-k} \right)^2 \tag{9}$$

$$V_{p,t} = -\sum_{k=1}^{\infty} \rho_k \left(y_{t-k} - \text{E}_{p,t-k-1} y_{t-k} \right)^2 \tag{10}$$

Parameter λ is carried over from the assumed random component. In addition, there is a new interpretation for parameter λ, namely, the *intensity of choice*, because it basically measures the extent to which agents are sensitive to additional profits gained from choosing optimism instead of pessimism. According to the above, we can obtain the aggregate expected output gap of period t+1 through Equation (11).

$$\text{E}_t y_{t+1} = \alpha_{o,t} \text{E}_{o,t} y_{t+1} + \alpha_{p,t} \text{E}_{p,t} y_{t+1} \tag{11}$$

In sum, the agent-based DSGE model with the Boltzmann–Gibbs distribution machine can be regarded as the first step in preparing the standard DSGE model based on its agent-based variants.

4   Agent-Based DSGE Model with Network-Based Ant Machine

According to the above, the structure of networks is hidden in our economic lives and a vast amount of research has been carried out during the last few decades. For example, network analysis is not only applied to examine the transmission of information regarding job opportunities, trade relationships, how diseases spread, how people vote and which languages they speak, but is also used in empirical works, such as the World Trade Web, the Internet, ecological networks and co-authorship networks. There is no doubt that a network structure is quite important for social interaction. Thus, we would like to introduce a network-based ant model for the New Keynesian DSGE framework[4]. Kirman characterized the switching potential of each individual by two parameters, namely, a probability of self-conversion and a probability of imitation. The self-conversion probability represents the probability that the agent changes the rule for personal reasons, whereas the probability of imitation refers to the agent changing the rule because of the influence of friends. Thus, the probability of agent i switching from the pessimistic rule to the optimistic rule could be represented by Equation (12):

$$\text{prob}(\text{p} \rightarrow \text{o}) = s_i + m_i \omega_{ij} \sum_{j \neq i} D_o\,(i,j) \tag{12}$$

where s_i denotes the self-conversion (due to idiosyncratic factors) rate, and m_i refers to the imitation rate. To simplify our model, we let both the self-conversion rate and imitation rate be constant. In other words, $s_i = s_j$ and $m_i = m_j$ for each $i \neq j$, and ω_{ij} denotes the interaction strength between i and friend j. Equation (13), $D_o(i,j)$, is an indicator function that counts the number of i's friends who are optimists.

$$D_o(i,j) = \begin{cases} 1, & \textit{if } j \textit{ is an optimistic neighbor of } i \\ 0, & \textit{otherwise} \end{cases} \tag{13}$$

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[4] There are many agent-based models that can be embodied with network structures. However, one of the purposes in introducing the Boltzmann-Gibbs distribution to DSGE models is to calibrate the crash and the bubble. In the research on agent-based modeling, both the Boltzmann-Gibbs machine and Kirman's ant model can easily generate herding behavior (Chen et al. 2012). For this reason, we build up the network-based ant model.

Symmetrically, if the agent uses the optimistic rule in period *t*, the probability of agent *i* converting to a pessimist person could be represented by Equation (14):

$$\text{prob}(o \rightarrow p) = s_i + m_i \omega_{ij} \sum_{j \neq i} D_p(i,j) \tag{14}$$

$$D_p(i,j) = \begin{cases} 1, & if\ j\ is\ a\ pessimistic\ neighbor\ of\ i \\ 0, & otherwise \end{cases} \tag{15}$$

Finally, variable $\omega_{i,j}$ is used to describe the interaction strength between *i* and *j*. In this paper, we assume that the interaction scheme should follow some specific network structures and the details of social network topologies will be described in the appendices. To consider the utility of different rules for each agent, we connect the interaction strength between *i* and friend *j*, ω_{ij}, and the performances of different rules for each agent. Therefore, according to Equations (9) and (10), we can assign different scores for each rule and then have the score matrix **S**, with dimensions N×N. In this case, if the agent is an optimist, it gets the score for the optimistic rule, and vice versa. By using the score matrix and the specific social network structure recorded by **N**, we can have ω_{ij} through Equation (16).

$$\omega_{ij} = \frac{N.\times S}{\sum_{i=1}^{N}(N.\times S)} \tag{16}$$

N. ×**S** means that the element of S is multiplied by the corresponding element of N and, therefore, we can have a new matrix which contains only friends' scores. Then, each agent assigns a weight to all its friends. Thus, the agent has to sum up the scores of all friends, i.e., we have to compute $\sum_{i=1}^{N}(N.\times S)$ for each row. Finally, the friends' score matrix should be divided by $\sum_{i=1}^{N}(N.\times S)$, and after that, ω_{ij} can be generated.

5   Collaborations and Simulation Results

5.1   Parameters Setting

In simulations, we follow the parameters setting of De Grauwe (2010a) for the stylized New Keynesian DSGE model. Details of parameters in the stylized New Keynesian DSGE model, Boltzmann–Gibbs machine, network-based ant model and parameters values of different network structures can be found in Table 1. In

Table 1: Parameters Setting of the Calibrated Models

Parameters setting of the stylized New Keynesian DSGE model		
π^*	0	the central bank's inflation target
a_1	0.5	coefficient of expected output in output equation
a_2	-0.2	the interest elasticity of output demand
b_1	0.5	coefficient of expected inflation in inflation equation
b_2	0.05	coefficient of output in inflation equation
c_1	1.5	coefficient of inflation in Taylor equation
c_2	0.5	coefficient of output in Taylor equation
c_3	0.5	interest smoothing parameter in Taylor equation
g	0.01	output forecasts of optimists
ρ_k	0.5	the speed of declining weights omega in mean squared errors
$\varepsilon_t, \eta_t, u_t$	0.005	standard deviation shocks of output gap, inflation and Taylors' rule
Parameters setting of the Boltzmann–Gibbs machine		
λ	100 500 1,000	intensity of choice
	5,000 10,000 50,000	
Parameters setting of the network-based ant machine		
s	0.15	self-conversion rate
m	0.7	imitation rate

Table 1 continued

Parameters setting of the network-based ant machine		
k	1	number of neighbors from the left (right) in circle network structure
k	2	number of neighbors from the left (right) in regular network structure
m_0	20	initial nodes of scale free network structure
p	0.1 0.3 0.5 0.7 0.9 1	cutting (rewiring) probability of small world network structure
Others		
N	100 /1000	number of agents
T	100	number of simulation periods for each experiment of calibrations
R	100	Number of experiments for each calibration

order to find out the distribution of the population dynamics, we run 100 experiments for a given collaboration. For each experiment of a specific collaboration, we set the number of agents equal to 100 (1,000[5]) and run 300 periods. In addition, one of the purposes in combining the Boltzmann–Gibbs machine and the stylized New Keynesian DSGE model is to generate booms and busts. For this reason, the focus is on self-conversion and imitation rates which can produce the herding behavior in the network-based ant model. In this case, the self-conversion rate equals 0.15 and the imitation rate equals 0.7, which meet the requirements. The details can be found in Table 1.

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[5] To make the model easier to operate with large number of runs, a size of 1,000 agents seems to be a practical choice. In fact, in the current state of agent-based computational economics, this size seems to be above the average. It is true that some agent-based models do encounter the scaling-up problem (see Lux and Schornstein, 2005), and whether the property which we derive from this economy can be carried over to a larger economy is an issue for a further study.

5.2   Simulation Results

For the Boltzmann–Gibbs machine design, we try different values of intensity of choice. In this case, if we increase the intensity of choice (λ), then the strength of social interaction is increased. According to Figure 1, we can observe that if λ is low enough, say, λ = 100, the fraction of optimists is very close to 0.5. As λ gets larger, the states of the probability density function of the optimistic ratio in the Boltzmann–Gibbs machine become divergent. Therefore, we can obtain bell-shaped probability density functions if λ is between 500 and 1,000. In such cases, herding behavior (animal spirits) cannot be generated. However, if the value of λ is larger than 5,000, the probability density functions of the optimistic ratio are U-shaped. In other words, to generate the herding behavior (or animal spirits),[6] the value of λ has to be set above 5,000. Then, we can have a boom or bust situation easily. The similarity of the two population dynamics generated by the Boltzmann–Gibbs machine and the network-based ant model can be explained in three different ways. Firstly, the probability density function of the optimistic ratio for different models is sketched in order to observe the distribution types. Secondly, the Kolmogorov–Smirnov test is applied for all models. Finally, the relative entropy is introduced to measure the similarity between the two population dynamics distributions.

5.2.1   Qualitative Analysis: Distribution Types

Figure 1 presents the probability density function of the optimistic ratio for the Boltzmann–Gibbs machine. The first row refers to the probability density function of the optimistic ratio in the 100th period, the second row represents the probability density function of the optimistic ratio in the 200th period, and the third row denotes the probability density function of the optimistic ratio in the 300th period. Next, Figures 2 and 3 depict the probability density function of the optimistic ratio for the network-based ant model with 100 agents and 1,000 agents, respectively.

A comparison of Figures 1 and 2 shows the difference between the probability density functions of optimistic ratios for the Boltzmann–Gibbs machine and the

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[6] It means that all agents adopt the same behavior, and the phenomenon is referred to as 'animal spirits' in De Grauwe (2010a, 2010b).

Figure 1. Probability Density Function of the Optimistic Ratio
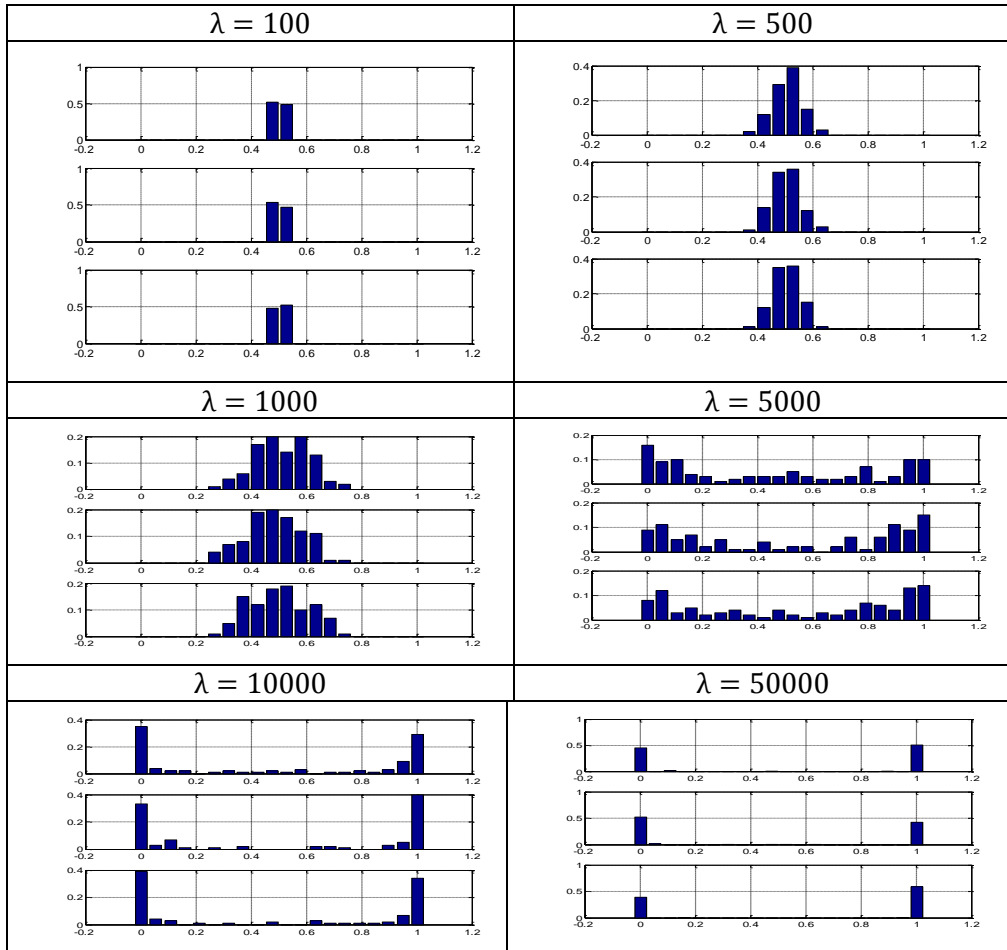of the Boltzmann–Gibbs Machine.

Figure 2. Probability Density Function of the Optimistic Ratio
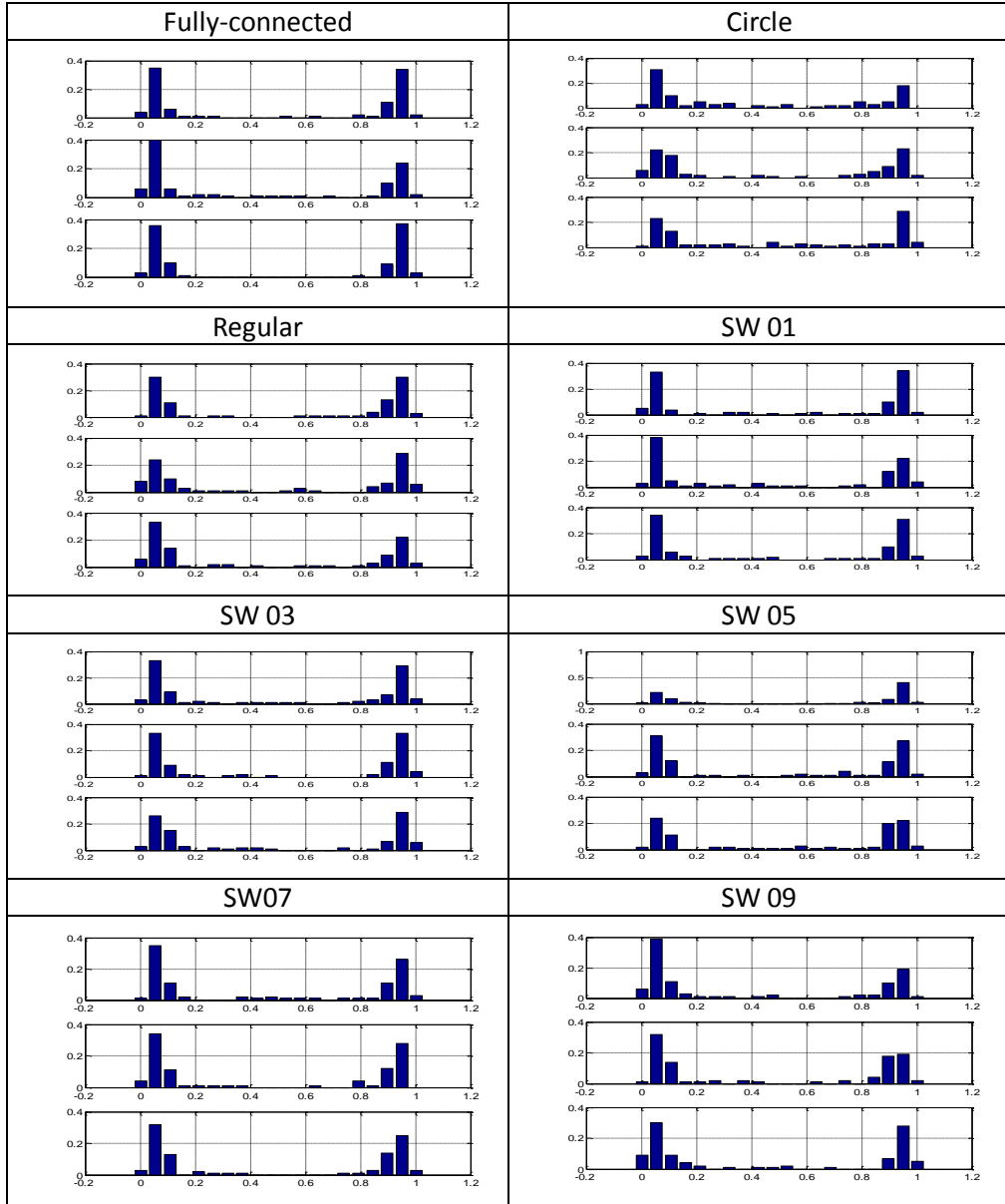of the Network-Based Ant Machine (N=100).
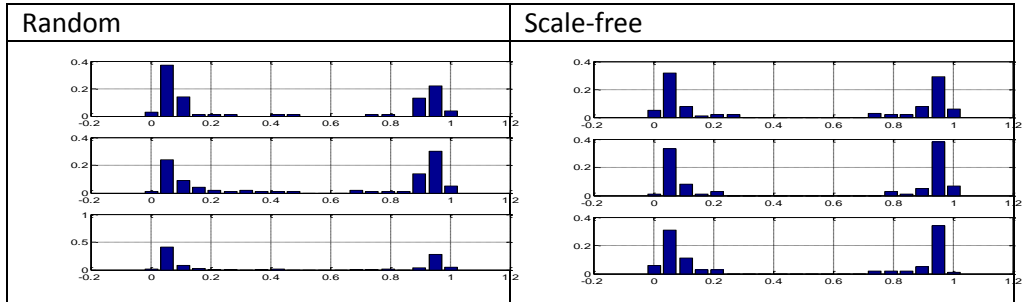
Figure 2 continued

Figure 3. Probability Density Function of the Optimistic Ratio
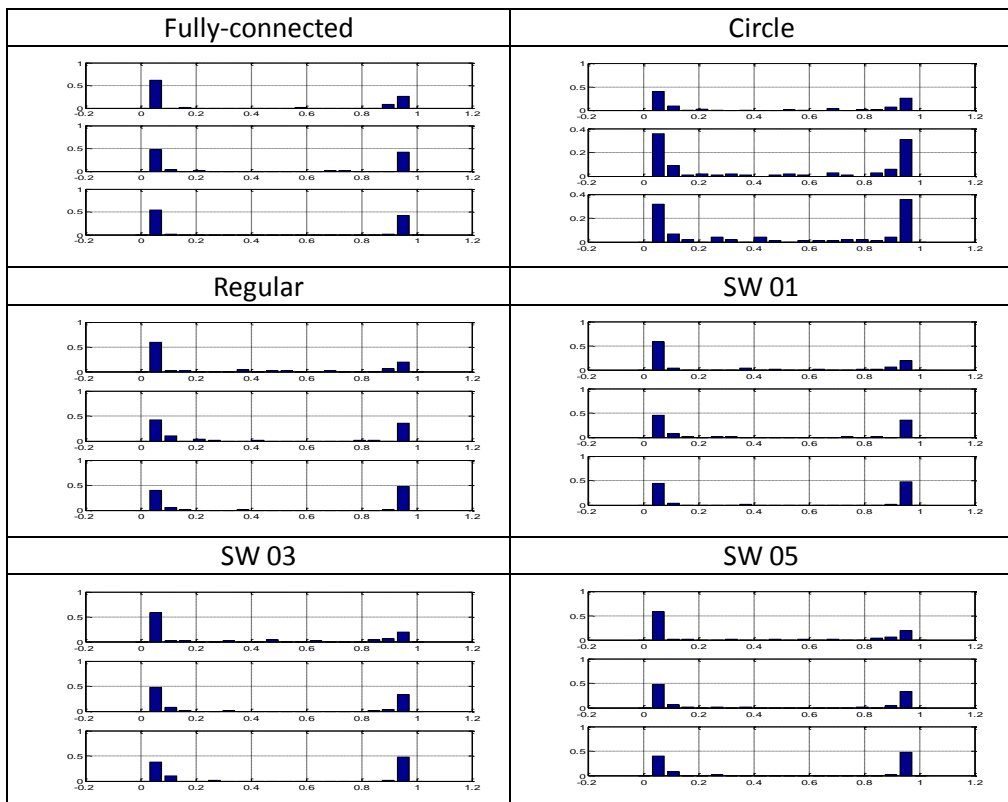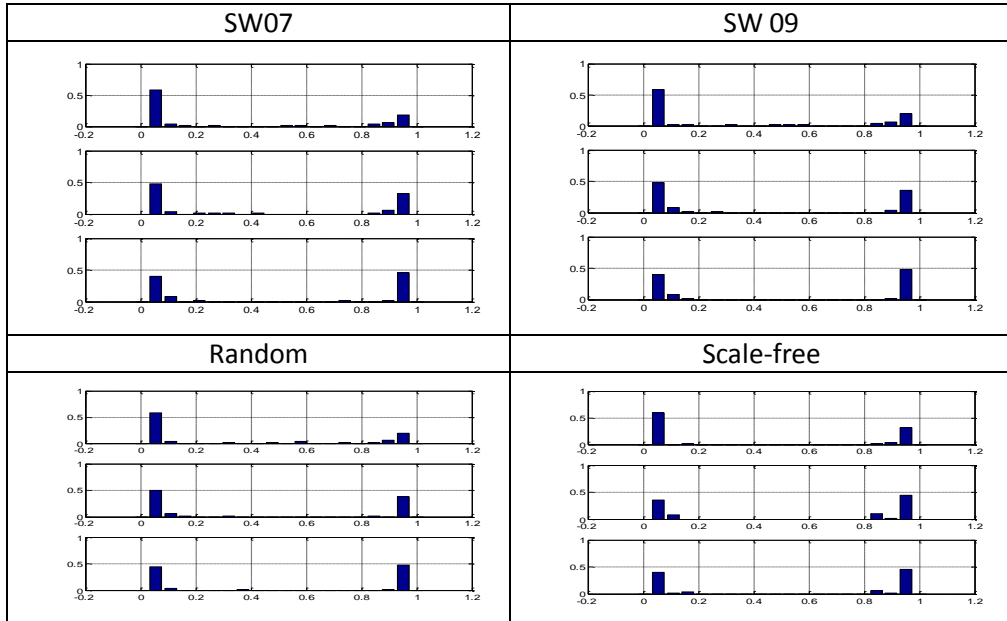of the Network-Based Ant Model (N=1000).

Figure 3 continued

network-based ant model. According to Figure 1, the herding behavior can be observed when the value of the intensity of choice is large enough, say, larger than 5,000. However, Figure 2 shows that if the values of the self-conversion rate and imitation rate are, say, 0.15 and 0.7, respectively, it is not difficult to produce herding behavior (animal spirits) in the network-based ant model. Figure 3 (which results from a large sample) shows the same property as Figure 2. In other words, the proposed network-based ant model can generate U-shaped probability density functions of the optimistic ratio with any given network structure.

5.2.2   Quantitative Analysis:The Kolmogorov–Smirnov Statistic

However, we wonder which network structure can generate population dynamics closest to the Boltzmann–Gibbs machine. Then, we can reconsider whether the Boltzmann–Gibbs machine is a reliable tool for describing social interaction. In order to answer this question, we compare asymptotic distribution of the optimists' ratios for the Boltzmann–Gibbs machine with that for the network-based ant

model, by conducting the Kolmogorov–Smirnov test. The statistical Kolmogorov–Smirnov test can be used to compare distributions of the values in the two data vectors **x1** and **x2**.

Here, **x1** could be regarded as the 100 optimist ratios of the 300th period in the Boltzmann–Gibbs machine, and **x2** is based on the network-based ant model. According to the definition of the Kolmogorov–Smirnov test, the null hypothesis is that **x1** and **x2** are from the same distribution. The alternative hypothesis is that they are from different distributions. Therefore, if the p-value of the Kolmogorov–Smirnov test is larger than 0.05, the hypothesis of **x1** and **x2** coming from the same distribution cannot be rejected. The results of the Kolmogorov–Smirnov test are presented in Tables 2 and 3. The simulation results show that the circle network can produce population dynamics most similar to the Kolmogorov–Smirnov test. This finding is consistent with the study of physics for which the Boltzmann–Gibbs distribution is based on the local interaction. However, it is difficult to treat the population dynamics generated by the Boltzmann–Gibbs machine and network-based ant machine as being from the same distribution, particularly in the popular empirical network structures such as the small world network and scale-free network. Furthermore, we have to mention that if we decrease the number of bins to 2, the results indicate that the p-values of most cases are larger than 0.05. Otherwise, if we increase the number of bins to 100, none of the cases can pass the null hypothesis. Since the results of the Kolmogorov–Smirnov test will be significantly affected by the number of bins, we further employ a similarity measure, relative entropy, to check whether the Boltzmann–Gibbs machine is a good approximation for the herding behavior for any given network structure.

Table 2: Kolmogorov–Smirnov Test Results (N=100)

	Fully	Circle	Regular	SW01	SW03
λ=100	3.70E-12	2.95E-11	4.41E-15	3.70E-12	1.06E-11
λ=500	3.70E-12	8.08E-11	1.43E-14	1.06E-11	1.06E-11
λ=1000	2.95E-11	3.70E-09	1.40E-13	2.95E-11	2.17E-10
λ=5000	0.013112	0.193042	0.000322	0.008216	0.008216
λ=10000	2.75E-07	2.21E-08	1.33E-06	6.12E-07	5.22E-08
λ=50000	4.52E-14	4.41E-15	4.52E-14	4.52E-14	4.52E-14
	SW05	SW07	SW09	Random	Scale-free
λ=100	1.40E-13	3.70E-12	4.41E-15	3.96E-16	1.40E-13
λ=500	4.26E-13	3.70E-12	4.41E-15	3.96E-16	1.40E-13
λ=1000	2.95E-11	1.06E-11	4.52E-14	3.96E-16	1.40E-13
λ=5000	0.193042	0.003031	0.001029	0.000174	0.001785
λ=10000	2.85E-06	5.22E-08	1.21E-07	2.45E-05	5.96E-06
λ=50000	1.40E-13	1.34E-15	1.43E-14	1.40E-13	3.96E-16

Table 3: Kolmogorov–Smirnov test results (N=1,000)

	Fully	Circle	Regular	SW01	SW03
λ=100	1.43E-14	3.70E-12	1.06E-11	1.06E-11	1.06E-11
λ=500	1.43E-14	2.95E-11	1.06E-11	1.06E-11	1.06E-11
λ=1000	1.43E-14	1.47E-09	1.06E-11	1.06E-11	1.06E-11
λ=5000	4.81E-05	0.099376	0.008216	0.005043	0.008216
λ=10000	9.12E-09	2.21E-08	9.12E-09	9.12E-09	9.12E-09
λ=50000	3.96E-16	3.96E-16	3.96E-16	3.96E-16	3.96E-16
	SW05	SW07	SW09	Random	Scale-free
λ=100	1.06E-11	1.06E-11	1.06E-11	1.06E-11	1.40E-13
λ=500	1.06E-11	1.06E-11	1.06E-11	1.06E-11	1.40E-13
λ=1000	1.06E-11	1.06E-11	1.06E-11	1.06E-11	1.40E-13
λ=5000	0.008216	0.003031	0.001785	0.005043	0.008216
λ=10000	9.12E-09	9.12E-09	9.12E-09	9.12E-09	2.21E-08
λ=50000	3.96E-16	3.96E-16	3.96E-16	3.96E-16	1.15E-16

5.2.3 Quantitative Analysis: Relative Entropy

According to the distribution type analysis, it seems that the Boltzmann–Gibbs machine is a robust approximation of herding behavior in a network-based ant model. However, none of the population dynamics produced by the network-based ant model with different network structures could pass the Kolmogorov–Smirnov test, besides the circle network structure. For this reason, we introduce the relative entropy. Before we refer to the relative entropy for measuring the similarity between two population dynamics distributions, we have to introduce the concept of Shannon entropy (Shannon, 1948), used to describe the uncertainty in the information theory represented by Equation (17).

$$H(p_1, p_2, \cdots, p_n) = -\sum_{i=1}^{n} p_i \log_2 p_i \qquad (17)$$

where $H(p_1, p_2, \cdots, p_n)$ is a continuous function, and p_i is the frequency (probability) of state i. If $p_1 = p_2 = \cdots = p_n = \frac{1}{n}$, we obtain the maximum H. It means the highest uncertainty exists in the system. However, if p_i=1 and $p_{j \neq i}$=0, H will equal zero, and in this case, state i always occurs and the degree of uncertainty in the system is 0. In our population dynamics case, we group the optimistic ratio into 10 groups and calculate the frequency for each group. The 1st group represents an optimistic ratio larger than 0 and less than 0.1, the 2nd group includes an optimistic ratio between 0.1 and 0.2,…, and so on. Therefore, we can obtain the Shannon entropy of our model through Equation (17), where n=10.

Based on the definition of Shannon entropy, Kullback and Leibler (1951) proposed relative entropy, which is also known as cross entropy or Kullback–Leibler divergence. Relative entropy is a measure of similarity, where it is assumed that the baseline distribution is **G** and the alternative distribution is S. However, it is not entirely clear if S is a good approximation of the distribution of **G**. Thus, the relative entropy can be used to measure the similarity between two population dynamics distributions. The more dissimilar **G** and **S** are, the larger the relative entropy is.

Therefore, if we have two density vectors $\mathbf{G} = (g_1, g_2, g_3, \cdots, g_n)$ (G is the frequency of the optimistic ratio derived by 100 experiments with the Boltzmann–Gibbs machine) and $\mathbf{S} = (s_1, s_2, s_3, \cdots, s_n)$ (S is the frequency of the optimistic ratio derived by the network-based ant model of a given social network structure), the definition of relative entropy is given as Equation (18). In this case, H(G|S)

will always be larger than or equal to zero; if **G** and **S** are identical, *H(G/S)* equals zero.

$$H(G|S) = \sum_{i=1}^{n} g_i \log_2\left(\frac{g_i}{s_i}\right) = \sum_{i=1}^{n} g_i \log_2 g_i - \sum_{i=1}^{n} g_i \log_2 s_i \qquad (18)$$

where $g_i \geq 0, s_i \geq 0$ and $\sum_{i=1}^{n} g_i = \sum_{i=1}^{n} s_i = 1$

However, relative entropy is asymmetric. In other words, *H(G|S)≠H(S|G)*. This is why it is the Kullback–Leibler divergence rather than the Kullback–Leibler distance.

Table 4 shows the results of the relative entropy procedure. The absolute values of relative entropy are all less than 0.5 for all different social network structures if the intensity of choice equals 10,000. Therefore, the result indicates that the Boltzmann–Gibbs machine (with an intensity of choice equal to 10,000) offers a good approximation of the herding behavior of our network-based ant model with any given network structure.

Table 4: Relative Entropy

Intensity of choice	λ=100	λ=500	λ=1000	λ=5000	λ=10000	λ=50000
Fully	0.810	0.597	-0.267	-1.160	-0.318	0.501
Circle	1.448	1.235	0.372	-0.522	0.320	1.139
Regular	1.066	0.852	-0.011	-0.905	-0.062	0.757
SW01	0.977	0.763	-0.100	-0.993	-0.151	0.668
SW03	1.000	0.787	-0.076	-0.970	-0.128	0.692
SW05	1.082	0.869	0.006	-0.888	-0.046	0.773
SW07	1.088	0.875	0.012	-0.882	-0.040	0.780
SW09	0.974	0.761	-0.102	-0.996	-0.154	0.666
SW10	0.985	0.772	-0.091	-0.985	-0.143	0.677
Scale-free	0.797	0.583	-0.280	-1.174	-0.331	0.488

6 Conclusion

This paper compares the population dynamics between the Boltzmann–Gibbs machine and network-based ant model under a stylized New Keynesian DSGE framework. We find that both the Boltzmann–Gibbs model and network-based ant model can generate herding behavior. However, as stated earlier, it is hard to envisage population dynamics generated by the Boltzmann–Gibbs model and the network-based ant model being from the same distribution, particularly in the popular empirical network structures such as a small world network and scale-free network. In addition, our simulation results further suggest that the population dynamics of the Boltzmann–Gibbs model and circle network ant model can be considered to be from the same distribution under specific parameter settings. The finding is consistent with the study of physics for which the Boltzmann–Gibbs distribution is based on the local interaction. Although the circle network is not the acknowledged social network structure, according to the relative entropy between the population dynamics of the Boltzmann–Gibbs machine and the network-based ant model, the Boltzmann–Gibbs machine with an intensity of choice equal to 10,000 is a good approximation of the herding behavior of our network-based ant model with any given network structure. In addition to the population dynamics, there are some other questions regarding the use of the Boltzmann–Gibbs machine to describe social interaction in the stylized New Keynesian DSGE model. For example, the frequency of herding behavior in financial markets and macroeconomic systems may be different. The change of opinion could occur very rapidly in financial markets but could be slower in the macroeconomic system. In this case, we have to consider whether an intensity of choice equal to 10,000 produces a change of opinion that is too heavy. Thus, we may have to further confirm whether the Boltzmann–Gibbs machine is a suitable tool for calibrating social interaction under a stylized New Keynesian DSGE framework.

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Appendices

In order to depict the social network's formation and its structure, we apply the concept of graph theory. Thus, a network **G** (**V, E**) is defined by a set of agents N and a set of links E. More specifically, **V** = {1,...,n} denotes all agents connected in some network relationship, and the number n refers to the size of the network. E denotes which pairs of agents are linked to each other so that $\mathbf{E} = \{b_{ij} : i, j \in \mathbf{V}\}$ encodes the relationship between any two agents in the network. Customarily, we use $b_{ij} = 1$ to indicate that there exists an edge (connection, relation) between i and j; otherwise it is zero. For this reason, we can use an N×N matrix to describe the network structure. However, we set $b_{ij} = b_{ji}$, which is known as a non-directed network in our model. Therefore, we can have a symmetric network matrix and the network formation algorithm for each specific social network structure as follows. The graph of these different network structures can be found in Figure A.1.

(1) Fully-connected network structure

The fully-connected network has the feature that agents are completely connected with each other. In other words, each agent has (n-1) links.

(2) Circle and regular network structures

In a regular network structure, all agents are connected to their respective k-nearest neighbors and k is a constant number. Thus, each agent connects with k neighbors on both the left and the right. The simplest case, k=1, would be a circle network structure. In our model, the regular network structure refers to k=2, i.e., each agent makes friends with the 2-nearest neighbors from the left and the 2-nearest neighbors from the right.

(3) Small world and random network structures

Watts and Strogatz (1998) first proposed a model of small-world networks. They started with random and regular graphs, and looked at two properties of these graphs, namely, clustering and path length. Clustering is a measurement of the set of friends who all know each other. Thus they developed a clustering coefficient which provides the number of pairs of two nodes that are connected to the same node, and are also connected to each other. Path length is used to measure the average distance between two nodes, which corresponds to the degrees of separation in a social network. Their initial results showed that regular graphs have high clustering and high path lengths; random graphs of the same size tend to have low clustering and low path lengths. However, neither of these was considered to be a good model of social networks which seem to combine high clustering with short path lengths. Therefore, Watts and Strogatz tried to create a network generating algorithm to establish a network which has the same property as a social network in the real world. First, they started with a regular graph with n nodes and k neighbors. Then, each agent had a rewiring probability, p, to cut off the link with each neighbor and build up a new link with one of the strangers. The probability, *p*, controls how random the graph is. With *p*=0, the network structure is regular; with *p*=1, it is random. In our simulations, we consider the regular network structure and set the rewiring rate, *p*, equal to 0.1, 0.3, 0.5, 0.7, 0.9 and 1 to generate different random network structures.

(4) Scale-free network structure

A scale free network is a network with the power law property. Thus, the number of links originating from a given node denotes a power law distribution represented by $p(k) = k^{-\gamma}$ where k denotes the number of links. The idea of a scale-free network comes from observations of many social contexts, e.g., the

citation network among scientific papers (Redner, 1998), the World Wide Web and the Internet (see, e.g., Albert et al., 1999; Faloutsos et al., 1999), telephone call and e-mail graphs (Aiello et al., 2002; Ebel et al., 2002), or the network of human sexual contacts (Liljeros et al., 2001). All of them show that only a few agents have many friends; most agents in the network have only a few friends. The most popular method to construct a scale-free network is the preferential attachment of Barabási and Albert (1999), which starts with m_0 agents and then progressively adds one new agent, i, to an existing network and builds links to existing agents with preferential attachment, according to Equation (19). That describes the rich getting richer; the probability of linking to a given agent is proportional to the number of existing links that a node has.

$$\text{prob(linking to agent i)} = \frac{k_i}{\sum_{j}^{N-1} k_j} \tag{19}$$

Furthermore, we have to mention that if we decrease the number of bin to 2 then the result indicates that the p-values of most cases are less than 0.05. Otherwise, if we increase the number of bin to 100 then the result shows that none of the cases can pass the null hypothesis. Since the results of Kolmogorov–Smirnov test will be significant affected by the number of bins, we, therefore, further employ a similarity measure and relative entropy to check whether the Boltzmann–Gibbs machine is a good approximation for the herding behavior for any given network structure.
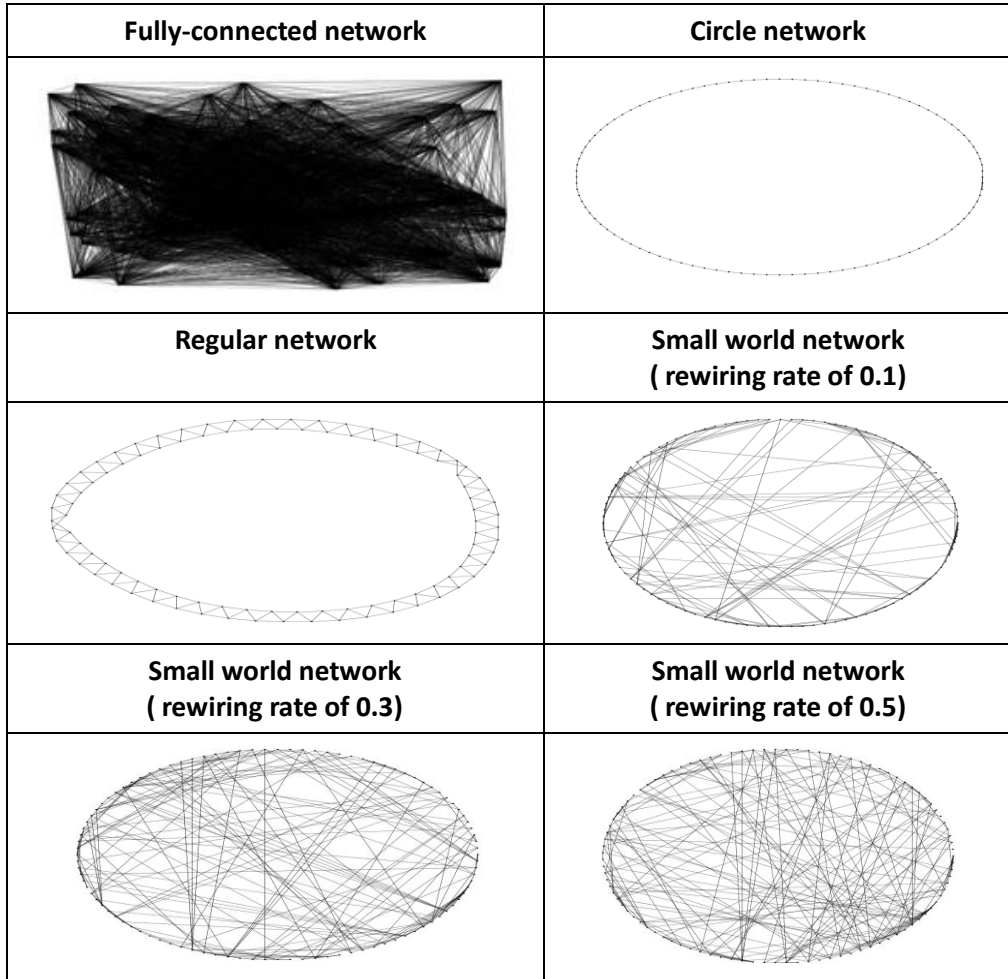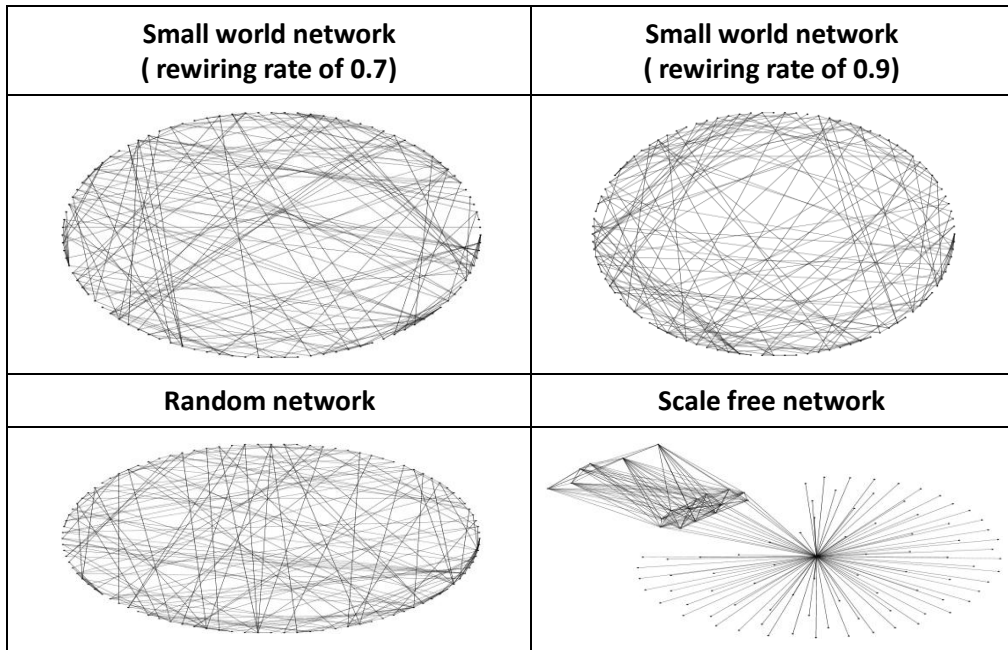
Figure A.1 Social Network Structures

Fully-connected network	Circle network
Regular network	**Small world network (rewiring rate of 0.1)**
Small world network (rewiring rate of 0.3)	**Small world network (rewiring rate of 0.5)**

Figure A.1 continued

Small world network (rewiring rate of 0.7)	Small world network (rewiring rate of 0.9)
Random network	Scale free network

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