A Model of Eco-Efficiency and Recycling

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Abstract

This paper presents the model of an economy subject to the mass conservation principle. The economic system is related to the environment by a flow of virgin materials into the economy, and by the diffusion of waste into the environment. Eco-efficiency contributes to reducing material waste in all processes. Recycling can reduce the diffusion of waste by feeding it back into the economy. Human capital enhances productivity, eco-efficiency and the quality of all kinds of outputs. Recycling and human capital formation use productive factors and are rooted therefore, as all other activities, in the material basis of the economy. The paper studies an optimal material state of society.

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1   Introduction

The current anthropogenic pressure on the natural environment calls for effective control of the material dimensions of human activities in advanced industrial societies (van den Bergh 1996; Adriaanse et al. 1997; Fischer-Kowalski 1998; Fischer-Kowalski and Hüttler 1998; Ayres 1999b; Bouman et al. 2000). Different possible technological responses to the task of controlling anthropogenic material flows have been discussed in the literature as e.g.: dematerialisation, eco-efficiency, recycling, industrial ecology and industrial metabolism.

Although the analysis of physical constraints on economic activities (Ayres 1998, 1999a; Cleveland and Ruth 1997; Ruth 1993, 1999) is a much debated issue, the development of models combining physical insights with specific tools of economic analysis still remains a very fragmentary field of research, since existing economic-physical models make very different assumptions and focus on very different aspects of the complex interaction between economic and physical analysis.

Smith (1972) applies the law of mass conservation to recycling in a model without physical capital. He develops economic conditions for complete or zero recycling. Van den Bergh (1996) presents an experimental model, based on materials balances. Waste can be emitted, recycled or stored. Environmental quality depends on the stocks of renewables and on the stock of pollutants. Simulations are performed with exogenous scenarios. Di Vita (1997) studies the effects of recycling of imported raw materials on the balance of payments and on employment in an open economy. Huhtala (1999) investigates recycling as an alternative to resource extraction. Recycling reduces pollution, but labour is the only input and physical capital does not come into the picture. Nakamura (1999) applies an input-output approach to the study of waste recycling in a static setting without technical progress. Di Vita (2001) focuses on the effects of recycling on the rate of growth. His model includes technical progress, which is endogenously generated by research. Welfare is negatively affected by the dimension of the waste stock, but not by the scale of economic activities. Hosoda (2001) applies the corn-guano model of the post-Sraffian school (Bidard and Erreygers 2001) to an analysis of recycling. The residuals of a first process are used as inputs to a second process. In this way, waste is completely absorbed and unlimited growth is possible, even after exhaustion of landfills. Eichner and Pethig (2001) present a labour-only model based on mass conservation with recycling activities and waste treatment before disposal. They investigate waste markets and market failures in the waste sector. Highfill and Mc Asey (2001) study recycling as an alternative to landfilling in the framework of a growing economy. Economic growth is exogenous and no interrelationship between physical capital, recycling and technical progress is therefore addressed. Eichner (2005) applies a labour-only partial equilibrium model to the analysis of imperfectly competitive markets in the recycling sector. André and Cerdá (2006) analyse the intertemporal shift in the proportion between recyclable and non-recyclable production inputs in a dynamic economy, not constrained by the mass conservation principle and without capital accumulation and technical progress. An overview of applied materials flows models is given in Bouman et al. (2000).

The present paper attempts to study material flows in an economic model, constrained by the mass conservation principle, and in which technology is determined by the level of engineering knowledge, accumulated in the course of history. The flows

considered are the flows between the natural environment and the economy and the material flows within the economy. The stocks are: the stock of accumulated emissions and the stock of materials temporarily frozen within the economy in the shape of physical capital. Physical capital occupies natural spaces and determines the material scale of the economy. If a recycling sector exists, materials are re-directed from the flow of waste back into the economy as a second kind of material inputs, which is added to the flow of virgin materials, directly extracted from the environment.

There is some ambiguity in the literature on the nature of this second kind of material inputs (Converse 1997; Ayres 1999a). Can waste be recycled after diffusion into the environment, or is a previous sequestration of waste before diffusion necessary for recycling activities? If diffused waste is the source of recycling, the difference between virgin materials and recycled materials disappears, since both would have to be extracted from the environment. I shall adopt the view of a combined sequestration-recycling activity, because I assume that capturing waste materials before dissipation is economically more convenient than retrieving materials after dissipation. In this case, virgin materials and recycled materials are different inputs, since the first originate in nature, while the second originate from sequestration activities. This implies, that recycling is a two-stage process: sequestration comes first and recycling in the stricter sense follows, when materials are extracted, or 'mined' out of sequestered materials and transformed into inputs to final output.

The material structure of the economy is determined by knowledge and technology, and I shall consider three channels through which this may happen.

First of all, knowledge contributes to an increase in productivity. This aspect has been extensively investigated in economic theory (Lucas 1988; Romer 1989) and will also be considered here. Productivity gains imply however a rising throughput of materials, which has to be accounted for, both backwards, as an increase in material requirements, and also forwards, as a growing production of waste.

A second effect of knowledge is to contribute to a more efficient use of materials in the economic process, since technological improvements can reduce material losses in the process of transformation of material inputs into useful outputs (Reijnders 1998; Schmidt-Bleek 1993, 1997; Weizsäcker et al. 1997). In this way, eco-efficiency is enhanced and the environmental impacts of economic activities are reduced.

A third effect is on consumption. Consumption is not only a material, but also a cultural, aesthetic and social process and is therefore fundamentally influenced by the general level of accumulated social knowledge (Cogoy 1999).

The focus of this paper is on the optimal material state of the economy in a stationary equilibrium. In order to give the reader some insight into the potential dynamics of this model, I shall also introduce laws of motion of stocks in section 2 of this paper. The analysis in sections 3 to 5 is limited however to a stationary state, in which the time derivatives of stocks are set equal to zero. This implies that typical problems of dynamic analysis, as e.g. the description of a transitional path, technical change, the effects of a positive rate of discount and the stability properties of the stationary point remain outside the scope of the present paper. The laws of motion of stocks, introduced in section 2 are not very different from those adopted in other models of economic growth: capital can be accumulated by reducing consumption and technical progress can be achieved by allocating resources to the research sector. For this reason,

it is not to be expected, that the dynamics of this model would be much different from other familiar models of growth and technical change.

Sections 2 and 3 describe the model and derive first-order conditions for stationary optimality. Sections 4 and 5 investigate a simplified reference case and offers an analytical discussion of the solution. The final section concludes and formulates some warnings.

2 The Model

2.1 Material Stocks and Flows

I shall assume that human activities displace materials from the natural environment into two types of sinks: the physical capital stock[1] and accumulated emissions. At any point in time therefore:

$$S = K + D + V \tag{1}$$

S is the world's mass, considered to be constant in this paper. K is the physical capital stock. D are accumulated emissions. V is the world's "residual" which remains, after K and D have been subtracted from S. This "residual" is the source of future virgin material inputs to the economy. All quantities are measured in a mass-unit, e.g. tons. Physical capital is only considered as a container of materials in this section. Its role as a productive factor is discussed in section 2.3.

Since S is constant:

$$\dot{K} + \dot{D} + \dot{V} = 0 \tag{2}$$

(Points on variables denote time derivatives.)

Now consider flows. (Upper case notation denotes stocks, lower case denotes flows.) v is the flow of virgin materials currently extracted from the natural environment in order to be processed by the economic system, e is the flow of emissions from the economy, and a is the flow of materials reabsorbed by natural regeneration from the stock of discharged materials back into natural processes.

$$\dot{K} = v - e \tag{3}$$

$$\dot{D} = e - a \tag{4}$$

$$\dot{V} = a - v \tag{5}$$

I assume that absorption is a linear function of the stock of pollutants:

$$a = \tau D \qquad\qquad 0 < \tau < 1 \tag{6}$$

where τ is the rate of absorption.

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[1] I assume that physical capital is the only material stock within the economy. This implies, among other things, that consumer durables are not considered in this paper. All consumption goods are therefore transformed into waste during the same time period.

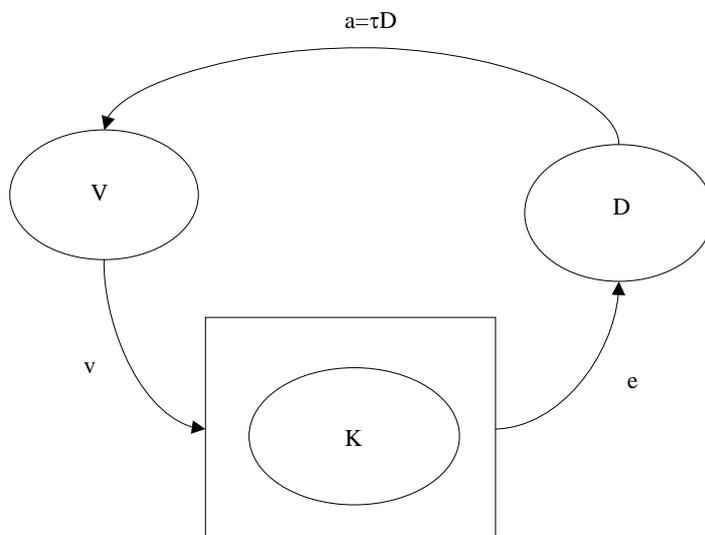The stock-flow relationships described above can be represented in Figure 1:

Figure 1: The Stock-Flow Model of the "World".

Physical capital has been included in a box in Figure 1, because *v* and *e* only represent flows to and from the borders of the economic system. Within the economy however, materials are processed and transformed, in order to achieve the goals of economic activities. I assume that economic activities transform materials into desired shapes and produce waste as an undesired by-product. Shaped materials are therefore only a fraction of the process inputs and the numerical value of this fraction measures the eco-efficiency of the material process. I shall denote eco-efficiency with the symbol η and assume that eco-efficiency is determined by the general level of knowledge and technology in society. If η is equal to one, the process is perfectly efficient and no materials are lost in transformation. If η is equal to zero, the process is perfectly inefficient, no output comes out of the process, and all inputs are transformed into waste.

η is a complex measure of eco-efficiency, since it evaluates more than one aspect of materials efficiency with one variable only. If fewer materials are lost in transformation, η will rise. But η will also rise, if process waste is directly channelled as input to other processes. In other words, η measures materials efficiency both at plant level and also at the level of interconnected plants (industrial ecology and industrial metabolism, cf. Ayres 1989; Ayres and Simonis 1994; Ayres and Ayres 1996; Erkman 1997). If materials are discarded from one production plant and directly channelled to another, they cannot be considered as waste in a strict sense. The definition of waste is confined in this paper to those materials which are either emitted into the environment, or sequestered for recycling. If industrial ecology were perfectly successful, process waste would be reduced to zero and η would be equal to one.

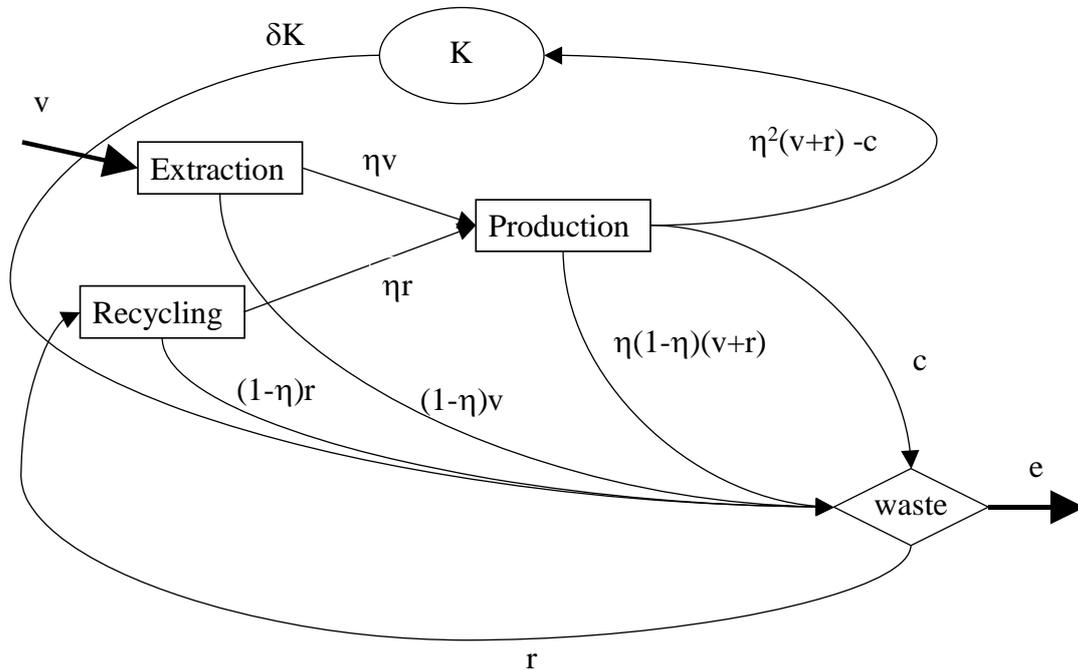Material flows within the economic box are represented in Figure 2:

Figure 2: The Material Structure of the Economy

I consider three materials processing sectors: a) extraction and refining of virgin materials, b) sequestration and recycling of waste and c) production of final output[2].

Virgin materials are extracted from the environment and processed up to the point that they may serve as inputs to the production of final output. From the materials balance point of view, the output of processed and refined virgin materials (ηv) is a fraction of the amount of materials extracted from the environment (v) and $(1-\eta)\,v$ is therefore waste from extraction.

If a recycling activity exists, a certain amount of materials (r) is "mined" from total waste and directed to the recycling sector, in a similar way, as virgin material inputs are extracted from the environment and submitted to the process of refining and transformation. ηr are recycled materials, which can be used as inputs to final output and $(1-\eta)r$ are materials redirected from the recycling sector to the flow of waste.

The same reasoning also applies to final output, where refined materials (virgin and recycled) are transformed to final goods. $\eta v + \eta r$ are material inputs to final output. Therefore, $\eta^2(v+r)$ is final output and $(1-\eta)\eta(v+r)$ is waste from the final output sector. The quadratic exponent of η for final output obviously follows from the assumption, that two stages are required: materials refinement (virgin and/or recycled) and final production. At each of these stages some materials are lost.

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[2] I neglect end-of-the-pipe waste treatment and storage, in order to reduce the complexity of the model. Waste treatment can improve environmental conditions however, both in the short and in the long run.

In this paper I focus on the overall effects of eco-efficiency on the economy and not on technological differences between sectors. I assume therefore that the eco-efficiency coefficient η is the same in all processes.

It can be seen from Figure 2 that there are in this model three sources of waste. A first source of waste is in the transformation process, where materials are moulded into the desired shape: if some materials are 'lost', while others are given a useful economic shape, these losses represent a source of waste. $v + r - \eta^2(v + r)$ is waste from transformation, since $v + r$ are materials (virgin and recycled) entering the transformation process, and $\eta^2(v + r)$ is the result of this process. The difference has been 'lost' in transformation. A second source of waste is consumption. Consumption is a materials processing sector of a particular kind: it transforms one fraction of final output into waste and yields welfare as its specific immaterial output. Therefore, eco-efficiency in consumption is zero by definition. A third source is from physical capital depreciation.

The materials balance equation for waste is therefore:

$$r + e = v + r - \eta^2(v + r) + c + \delta K \tag{7}$$

where c is aggregate consumption and δK is linear capital depreciation. Equation (7) implies, that the transformation of materials is completely accounted for: no piece of matter disappears or comes in unexplained at any point in the model.

It can be easily seen, that (7), together with (3) yields:

$$\dot{K} = \eta^2(v + r) - c - \delta K \tag{8}$$

This is the familiar equation, stating that net investment is equal to gross investment minus capital depreciation. It has to be interpreted here as a materials balance equation, since all quantities are expressed in mass units.

2.2   Stationary Equilibrium

In a stationary equilibrium: $\dot{K} = \dot{D} = \dot{V} = 0$ and $v = a = e$. Therefore:

$$v = \tau D \tag{9}$$

$$c = \eta^2(v + r) - \delta K \tag{10}$$

From the point of view of materials balances the role of capital on consumption is a negative one, since, for given eco-efficiency and given material inflows into the system, a larger capital stock will require a larger flow of materials replacing worn-out capital.

Equations (9) and (10) fully describe the materials balance conditions of the economic system in a stationary state.

2.3   The Economic Model

From the economic point of view, processes are described by production functions, which establish a relationship between material flows and productive factors.

Production functions are not materials balance equations, but rather flow-regulators: the larger the quantity of productive factors employed, the more materials will flow through the process, the proportion between useful output and waste being determined by eco-efficiency. I consider three factors: physical capital, measured in tons as in the preceding section, labour, measured in time-units, and accumulated engineering knowledge. Clearly, tons of physical capital make sense in materials balance equations, but they make less sense in economic equations, since a ton of capital mass may provide different productive services, depending on the 'shape' or 'form' given to capital-matter by historically accumulated engineering knowledge. For this reason, economic equations must contain some measure of the quality of this 'shape'. I shall measure the quality of capital mass by a productivity index π, depending on engineering knowledge. K_i is therefore mass capital in sector i, whereas πK_i is quality capital in sector i. π is assumed to be the same in all sectors.

From Figure 2 we know that refined virgin materials are equal to ηv. The production function for refined virgin materials can therefore be written as:

$$\eta v = \left(\pi K_V\right)^\lambda \left(\gamma\, l_V\right)^{1-\lambda} \qquad\qquad 0 < \lambda < 1 \qquad\qquad (11)$$

where K_V is physical capital (in tons), applied to the extraction and refinement of virgin materials, l_V is labour employed in the same sector, and γ is a productivity coefficient for labour, which is considered to be given and the same in all sectors. Assuming that π depends on engineering knowledge, whereas γ is constant, is tantamount to saying that technical progress is capital-mass-saving.

Recycled materials are equal to ηr. The economic equation for the recycling sector can be written therefore as:

$$\psi\eta\, r = \left(\pi K_R\right)^\lambda \left(\gamma\, l_R\right)^{1-\lambda} \qquad\qquad \psi > 1 \qquad\qquad (12)$$

K_R and l_R are capital and labour in the recycling sector and ψ measures the additional effort required for recycling as compared to virgin materials extraction. Recycling is assumed to be more costly than virgin materials extraction because it consists of two stages: sequestration and materials processing. Nevertheless, the additional costs of recycling may be worth incurring because of the positive impacts on the environment. If ψ were equal to one, recycling would require the same effort as the refinement of virgin materials and there would be no incentive to extract materials out of the natural environment.

I assume that society chooses how to divide waste between dispersion and sequestration for recycling. Such a choice is a choice under technological constraint however, since waste materials can only be sequestered and recycled, if capital and labour are allocated to this purpose. Recycling substitutes virgin material inputs. At the same time the scale of the economy is affected, since recycling capital will be required. If society chooses to recycle, waste of different types will enter the sequestration process. It is certainly not meaningful to mix all sorts of waste and have a uniform mixture of materials, out of which recycled materials can be "mined". It will be probably more reasonable to have separate storage facilities for different types of materials (Craig 2001). Bent nails will have to be straightened, and not ground and mixed with sand before extracting iron out of the mixture. The recycling sector, as it is modelled in the present paper, uses capital and labour in order to reprocess waste of

different types, collected in different sequestration facilities. I assume that these operations deliver processed materials of the same quality as the output of the virgin materials sector, so that processed virgin materials and processed waste are perfect substitutes.

In a stationary state complete recycling implies: $v = a = e = 0$ and therefore $D = 0$. It will be shown in section 5, that this option is unlikely to be economically optimal. A consequence of this is that the issue of complete recycling (Bianciardi et al. 1993, 1996; Converse 1996, 1997; Washida 1998) turns out to be of little economic relevance. Even if complete recycling were technically possible, it can hardly be economically meaningful.

Since output is equal to $\eta^2(v + r)$ the production function for final output is:

$$\eta^2(v + r) = (\pi K_P)^\lambda (\gamma l_P)^{1-\lambda} \tag{13}$$

K_P and l_P are capital and labour in final output.

With (11) to (13) it is implicitly assumed, that the production function is the same in all sectors. The reason for this is, that the focus of the analysis is not on technological differences between sectors, but rather on the impacts of eco-efficiency and recycling on the overall material structure of the economy.

2.4   Consumption

Like other material flows, consumption is also measured in tons. In the same way as one physical unit of capital can be more or less productive, depending on how skilfully it is shaped, so can a physical unit of consumption deliver a higher contribution to welfare, if consumption goods are more conveniently 'shaped'. In this way, the benefits of physical consumption are enhanced by higher quality. Physical per capita consumption (b) is:

$$b = \frac{c}{N} \tag{14}$$

N is the number of identical individuals in a given population. Qualified per capita consumption (z) is:

$$z = \vartheta b \tag{15}$$

where ϑ is a quality index, depending on social knowledge, and measuring the 'shape' given to consumption mass. Using (10) qualified per-capita consumption can be also expressed as a function of the quality index ϑ, eco-efficiency, the flow of materials and the physical capital stock:

$$z = \frac{\vartheta}{N}\left[\eta^2(v + r) - \delta K\right] \tag{16}$$

Since both quality indexes for physical capital (π) and consumption goods (ϑ) depend on knowledge only, the quality of all goods does not depend on whether they are produced from virgin or recycled materials. Processed virgin materials and recycled materials can be indifferently used therefore as perfect substitutes in final output production.

The model discussed in this paper is the model of an affluent society. This means, that consumption is well above the level of survival and that its main role consists of enhancing life enjoyment. In this case, consumption mass can be substituted by quality, and z is a more adequate measure for the benefits of consumption than b. There is a lower bound to this kind of substitution however, since physical per capita consumption cannot decline below a minimum level of calories, necessary for survival.

2.5 The Research Sector

The role of knowledge is limited in this paper to its effects on productivity, eco-efficiency and the quality of consumption goods. Technological knowledge may be thought of as a collection of engineering information, developed by researchers in the course of time. I assume that this collection of information enhances productivity, eco-efficiency and consumption quality in a non-rival way. Research activities can add new items to the collection and are carried out in a special research sector, which uses capital and labour in a similar way, as other activities within the economy. I also assume that a given stock of engineering information requires an activation effort, since the collection of information must be maintained and kept operative, and its correct application must be supervised on the spot where it is utilized. In other words, not only the increase of knowledge, but also the unimpeded activation of a given information stock requires a continual effort in the research sector, and therefore the allocation of capital and labour to this purpose. I shall model knowledge accumulation together with maintenance, activation and supervision as:

$$\dot{H} + \varphi H = \left(\pi K_H\right)^\lambda \left(\gamma \, l_H\right)^{1-\lambda} \qquad\qquad \varphi > 0 \qquad\qquad (17)$$

where K_H and l_H are capital and labour requirements in the knowledge sector, $\dot{H}$ is knowledge accumulation and φ is a coefficient defining maintenance, activation and supervision costs. In stationary equilibrium:

$$\varphi H = \left(\pi K_H\right)^\lambda \left(\gamma \, l_H\right)^{1-\lambda} \qquad\qquad (18)$$

Equations (17) and (18) imply that the only material requirement in the research sector is physical capital, and ignore therefore other types of material inputs (e.g.: paper, fuel, etc.). The material body of knowledge (e.g.: paperboard, paper and ink for books, hard disks for electronic storage, etc.) is also neglected. From this it follows, that depreciating physical capital is the only kind of waste arising from the research sector. The reason for these simplifying assumptions is that the use of physical capital in the knowledge sector is sufficient to generate a material constraint on knowledge, and the introduction of other material constraints would only complicate, but not substantially modify the picture.

Knowledge begins to shape materials already at extraction and recycling level, when materials to be extracted and recycled are selected and shaped in such a way, as they can best serve their purpose further down in the production process. The same is true of eco-efficiency: already at the stage of extraction, materials can be selected in such a way, that losses at later stages of the process can be minimized. Knowledge thus

permeates all stages of the economic process, although its results are only measurable when output reaches its final destination as capital or consumption good.

The productivity, eco-efficiency, and consumption quality coefficients π, η and ϑ are determined by the level of accumulated social knowledge (H):

$$\pi = \pi(H) \qquad\qquad \pi' > 0 \qquad\qquad \pi'' < 0 \qquad\qquad (19)$$

$$\eta = \eta(H) \qquad\qquad \eta' > 0 \qquad\qquad \eta'' < 0 \qquad\qquad (20)$$

$$\vartheta = \vartheta(H) \qquad\qquad \vartheta' > 0 \qquad\qquad \vartheta'' < 0 \qquad\qquad (21)$$

η has a logical upper bound of one, if perfect eco-efficiency is considered to be possible. Otherwise, the upper bound of eco-efficiency will be lower than one. π and ϑ may, or may not, have empirical upper bounds. Equations (19) to (21) imply that knowledge is non rival in this paper: the **same** stock of knowledge increases productivity in all capital using sectors; at the same time it raises eco-efficiency and consumption quality.

Assuming that eco-efficiency and commodity quality are jointly enhanced by an increase in social knowledge is obviously a strong assumption, since negative cross effects may occur in the real world. A shaped piece of marble, a statue for example, is more likely to produce aesthetic pleasure on the beholder than an unshaped piece of the same matter. Sculptured marble will "loose" however more matter in transformation than unsculptured one, and eco-efficiency will be lower as a consequence of an improved aesthetic quality of marble, so that eco-efficiency is negatively affected by quality. In a baroque Pietà, for example, quality is high, and eco-efficiency is likely to be rather low. Assuming a joint progress of eco-efficiency and quality means that, in the aggregate, as product quality improves, also the proportion of materials finding any kind of useful employment (either as statue or as marble dust) will increase. The assumption of joint progress at aggregate level is therefore less unrealistic than it may seem for individual processes.

2.6   Model Reduction

There are in the above described model four types of physical capital and labour in extraction, recycling, final output and maintenance of accumulated knowledge. Total capital is therefore equal to:

$$K = K_V + K_R + K_P + K_H \qquad\qquad (22)$$

One unit of labour is inelastically supplied by each of the identical individuals in the population, so that total labour supply is equal to N. Therefore:

$$N = l_V + l_R + l_P + l_H \qquad\qquad (23)$$

Equations (11) to (13) and (18) can be reduced to a single equation, by noticing that with the same technologies everywhere, efficient capital and labour allocation requires:

$$\frac{K_V}{l_V} = \frac{K_R}{l_R} = \frac{K_P}{l_P} = \frac{K_H}{l_H} \tag{24}$$

Using (24) one obtains for the stationary state:

$$(\pi K)^\lambda (\gamma N)^{1-\lambda} = \eta(v + \psi r) + \eta^2(v + r) + \varphi H \tag{25}$$

Equation (25) is the aggregate production function of the economy in the stationary state. It states that aggregate factors $(\pi K)^\lambda (\gamma N)^{1-\lambda}$ can be applied to three types of productive effort. $\eta(v + \psi r)$ represents extraction and refinement of materials (virgin and recycled), suitable to enter the final output process. $\eta^2(v + r)$ represents final output. φH are maintenance and activation services to the knowledge sector. The assumption that production functions are the same in all sectors implies that one ton of refined virgin materials requires the same effort as one ton of final output and as $\dfrac{1}{\psi}$ tons of recycled materials. The factor allocation to final output production, relative to materials processing (virgin and recycled), i.e. the expression: $\dfrac{\eta(v + r)}{v + \psi r}$ increases with η. This means, that with rising eco-efficiency factors are shifted from materials processing to final output. Low eco-efficiency implies heavy losses of materials during processing. As a consequence, a greater portion of aggregate factors will have to be invested in the materials processing sectors, and less will be left for final output. For this reason, eco-efficiency plays an important role at aggregate level, since, given aggregate capital and labour, the system will be more productive if a greater portion of aggregate factors is employed in final output production.

Equations (9), (16) and (25), together with technology functions (19) to (21) describe a feasible stock-flow equilibrium in a stationary state, constrained by the mass conservation principle.

2.7 Preferences

I assume that welfare is determined for each individual by the stream of qualified per-capita consumption z, and by the state of the environment. The state of the environment is a public good and affects all of the identical N individuals in the same way.

Physical capital affects welfare in two ways: a direct, and an indirect one. The indirect effect is positive and has been extensively analysed in economic theory: capital enhances labour productivity and contributes in this way to output and consumption. On the other hand however, physical capital encroaches upon natural spaces, spoils landscapes, destroys biotopes, and in this way negatively affects plant and animal life. In a finite world no anthropogenic physical stock can grow without limits, since natural space is limited and an unbounded expansion of industrial settlements negatively affects natural life. This is true for physical capital K and not necessarily for quality capital πK. For this reason, an increase in π is not assumed to have any direct negative impact on welfare. I assume therefore $U_K < 0$, and also $U_{KK} < 0$. This means, that the

marginal impact of physical capital is low at low levels of capital accumulation in society when natural spaces are abundant. At high levels of capital accumulation it is reasonable to assume significant negative impacts of further capital accumulation, since plant and animal life is endangered, if a significant portion of natural space is already occupied by industrial settlements. This justifies the assumption of low negative marginal impacts at low levels of physical capital accumulation, and of rising negative marginal impacts as the quantity of capital increases. Physical capital K is, in other words, a necessity, not a pleasure[3]. This suggests the idea of an optimal level of physical capital, which will have to be determined by a compromise between its positive contribution to production and its negative effects on the natural environment.

The stock of accumulated emissions D is another argument of the welfare function, since pollution negatively affects welfare.

With these premises in mind, preferences can be modelled as:

$$U = U(z, K, D)$$
$$U_z > 0, \quad U_{zz} < 0, \quad U_K < 0, \quad U_{KK} < 0, \quad U_D < 0, \quad U_{DD} < 0 \qquad (26)$$

At this point, the difference between physical capital and knowledge may be summarized as follows. Both enhance productivity in all sectors, in which they are employed. Physical capital directly encroaches upon the environment by occupying natural spaces, whereas the research system only affects the environment, insofar as physical capital is needed to support its operations.

3   The Social Planner's Problem

3.1   The Problem

In order to determine the optimal stationary material state of society, the social planner maximizes (26), subject to (9), (16), (25) and technology functions (19) to (21). The solution to this problem determines, together with the other variables, the socially desirable levels of the physical capital stock and of accumulated knowledge. The knowledge sector requires physical capital to support its operations, and is therefore constrained by environmental considerations, as are all activities requiring physical stocks. For this reason, the optimal level of knowledge is endogenous in this paper.

In a decentralized market economy the social optimum may, or may not, be attained, depending on the structure of markets and on market imperfections. The goal of this paper is only to characterize the social optimum and an analysis of market operations is outside the scope of this paper[4].

In order to give an intuitive interpretation of the first-order conditions, I shall use a more general form for (16) and (25):

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3 I am only concerned with productive capital in this paper. Matters are obviously different for architectural capital, works of art, architectured landscapes, gardens, etc. as expressions of aesthetic and cultural values.

4 For an analysis of markets in the recycling sector cf. Eichner/Pethig, 2001 and Eichner, 2005.

$$z = z(\vartheta, \eta, v, r, K) \tag{16'}$$

$$F(\pi, \eta, v, r, K, H) = 0 \tag{25'}$$

3.2 First Order Conditions

First-order conditions for this problem can be derived by using Lagrange's method:

$$U_z z_v = \frac{F_v}{F_K}(U_K + U_z z_K) - \frac{U_D}{\tau} \tag{27}$$

$$U_z z_r = \frac{F_r}{F_K}(U_K + U_z z_K) \tag{28}$$

$$U_z(z_\eta \eta' + z_\vartheta \vartheta') = \left(\frac{F_\pi \pi' + F_\eta \eta' + F_H}{F_K}\right)(U_K + U_z z_K) \tag{29}$$

First-order condition (27) equates marginal benefits to consumption from an increase in virgin materials inflows (v) to marginal environmental degradation deriving from induced increases in stocks K and D. $U_z z_K$ is the direct negative marginal effect on consumption, deriving from the burden of capital depreciation. Similarly, (28) compares benefits and costs of a marginal increase in recycling. The lack of a second member on the right hand side is due to the fact that recycling does not increase the pollution stock. Equation (29) compares the consumptive benefits deriving from a marginal increase in social knowledge to the negative effects of an induced marginal increase in physical capital. Of course, this negative effect is mitigated by a raise in productivity π.

First order conditions, together with the problem's constraints, determine the optimal values for variables: K, D, H, v, r, z, π, η and ϑ.

4 Reference Case

Given the number of variables and the non-linearity of the system, finding a solution may become quite an intricate business. In the next section I shall present a graphic discussion of a simplified reference case, which allows to study the basic structure of the model.

For the simplified reference case I shall make following assumptions:

a) *Non-Depreciating Physical Capital*

Non-depreciating physical capital implies:

$$\delta = 0 \tag{30}$$

b) Technology Functions

I shall assume that the upper bound of η is 1, and model eco-efficiency as:

$$\eta = \frac{H}{\alpha + H} \qquad\qquad \eta' = \frac{\alpha}{(\alpha + H)^2} \tag{31}$$

From (31):

$$H = \alpha \frac{\eta}{1 - \eta} \tag{32}$$

Productivity and consumption quality are assumed to be bounded and fixed multiples of eco-efficiency:

$$\pi = \bar{\pi} \frac{H}{\alpha + H} \qquad\qquad \pi' = \bar{\pi} \frac{\alpha}{(\alpha + H)^2} \tag{33}$$

$$\vartheta = \bar{\vartheta} \frac{H}{\alpha + H} \qquad\qquad \vartheta' = \bar{\vartheta} \frac{\alpha}{(\alpha + H)^2} \tag{34}$$

$\bar{\pi}$ and $\bar{\vartheta}$ are upper bounds of productivity and consumption quality.

c) Preferences

I shall model utility as a logarithmic function of qualified consumption, net of quadratic environmental damage from stocks:

$$U = q_z \log z - \frac{1}{2}\left(q_K K^2 + q_D D^2\right) \tag{26'}$$

where q_i are weights of the arguments in the utility function.

With these assumptions, and using (9), (16) and (32) to (34) to eliminate ϑ, π, z, D and H, equations (25) and (27) to (29) become:

$$\left(\bar{\pi}\eta K\right)^\lambda \left(\gamma N\right)^{1-\lambda} = \eta(v + \psi r) + \eta^2(v + r) + \alpha\varphi \frac{\eta}{1 - \eta} \tag{35}$$

$$\lambda\left(\bar{\pi}\eta K\right)^\lambda \left(\gamma N\right)^{1-\lambda}\left[q_z \tau^2 - q_D v(v + r)\right] = q_K \tau^2 \eta(1 + \eta)(v + r)K^2 \tag{36}$$

$$\lambda q_z\left(\bar{\pi}\eta K\right)^\lambda \left(\gamma N\right)^{1-\lambda} = q_K \eta(\psi + \eta)(v + r)K^2 \tag{37}$$

$$3\lambda q_z\left(\bar{\pi}\eta K\right)^\lambda \left(\gamma N\right)^{1-\lambda} =$$
$$= q_K\left[\alpha\varphi \frac{\eta}{(1 - \eta)^2} + \eta(1 + 2\eta)v + \eta(\psi + 2\eta)r - \lambda\left(\bar{\pi}\eta K\right)^\lambda \left(\gamma N\right)^{1-\lambda}\right]K^2 \tag{38}$$

For a given size of the population, (35) to (38) yield optimal values for v, r, η and K.

For a graphic analysis of the solution, (35) to (38) can be more conveniently manipulated into:

$$q_D(\psi + \eta)v^2 + q_D(\psi + \eta)rv - q_z\tau^2(\psi - 1) = 0 \tag{39}$$

$$v = \frac{\alpha\varphi(1 - \lambda + \lambda\eta) - (1 - \eta)^2[(2 + \lambda)\psi + (1 + \lambda)\eta]r}{(1 - \eta)^2[3\psi - (1 - \lambda) + (1 + \lambda)\eta]} \tag{40}$$

$$\lambda^\lambda q_D{}^2 \bar{\pi}^{2\lambda}(\gamma N)^{2(1-\lambda)}(1 - \eta)^{2-\lambda} v^2 =$$
$$= [q_K\tau^2(\psi - 1)]^\lambda \eta^{2(1-\lambda)}[q_z\tau^2(\psi - 1)(1 - \eta) + q_D\alpha\varphi v - q_D(\psi - 1)(1 - \eta)v^2]^{2-\lambda} \tag{41}$$

$$\lambda q_D(\psi - 1)(1 - \eta)v^2 - \lambda q_D\alpha\varphi v + \tau^2(\psi - 1)(1 - \eta)(q_K K^2 - \lambda q_z) = 0 \tag{42}$$

Physical per-capita consumption can be derived from (10), (14) and (39):

$$b = \frac{\eta^2 q_z\tau^2(\psi - 1)}{Nq_D(\psi + \eta)v} \tag{43}$$

Equation (43) defines a carrying-capacity limit for the economy, since physical per-capita consumption cannot be lower than a minimum calories level, necessary for survival.

The logic of the solution is as follows. Equations (39) and (40) yield an efficient locus of points in v/η space. Each point on the locus is associated with a different level of recycling. Equation (41) identifies the optimal point on the efficient locus (and therefore optimal recycling) for an exogenously given size of the population. Equation (42) can then be solved for optimal physical capital. Equations (43) determines the value of physical per-capita consumption.

5 A Study of Equations (39) to (43)

5.1 Efficient Locus of Non-Negative Recycling in v/η Space

Consider first (39) and (40). Setting $r = 0$ yields:

$$v = \tau\sqrt{\frac{q_z(\psi - 1)}{q_D(\psi + \eta)}} \tag{44}$$

$$v = \frac{\alpha\varphi(1 - \lambda + \lambda\eta)}{(1 - \eta)^2[3\psi - (1 - \lambda) + (1 + \lambda)\eta]} \tag{45}$$

Since the graph of (44) is monotonically declining, whereas the graph of (45) is monotonically rising, the two equations will only have a $v > 0$, $\eta > 0$ solution, if

$$\tau\sqrt{\frac{q_z}{q_D}\frac{\psi - 1}{\psi}} > \frac{\alpha\varphi(1 - \lambda)}{3\psi - (1 - \lambda)} \tag{46}$$

It can be also shown, that if (46) is **not** satisfied, then a solution $\eta = 0$, $v > 0$, $r \geq 0$ exists. Setting $\eta = 0$ into (39) and (40) yields:

$$v = -\frac{r}{2} + \sqrt{\left(\frac{r}{2}\right)^2 + \tau^2 \frac{q_z(\psi - 1)}{q_D \psi}} \tag{47}$$

$$v = \frac{\alpha\varphi(1 - \lambda) - (2 + \lambda)\psi r}{3\psi - (1 - \lambda)} \tag{48}$$

The value of v in (47) tends to zero, as r tends to infinity, whereas in (48) v declines to zero for a finite value of r. Therefore (48) cuts (47) from above if the inequality in (46) is reversed. From this I conclude that (39) and (40) have **either** a $r = 0$, $v > 0$, $\eta > 0$ **or** a $\eta = 0$, $v > 0$, $r \geq 0$ solution, depending on the direction of the inequality in (46). I shall call this point the ***origin*** of the solution locus.

For increasing values of r the crossing points of (39) and (40) generate an efficient locus in v / η space (Figure 3).
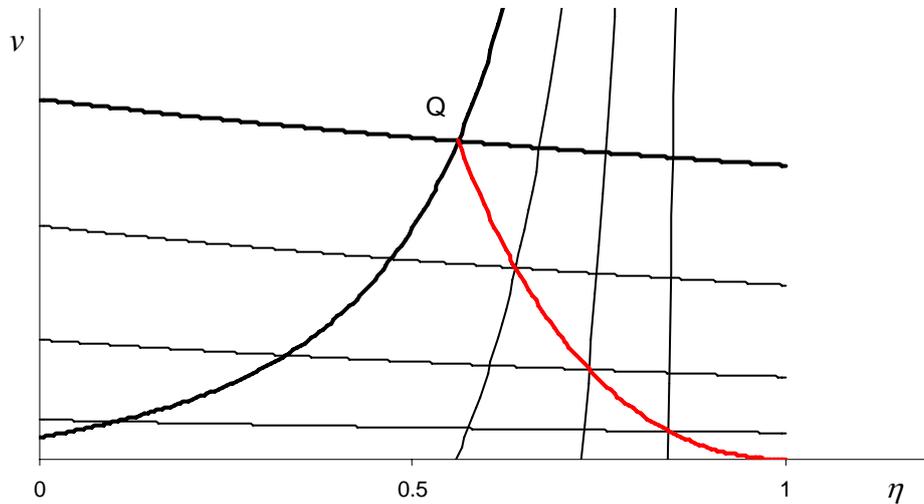

Figure 3: The Locus of Efficient Points (red) if Inequality (46) Holds

Point Q represents the origin of the locus in the case where (46) holds. If (46) does **not** hold, point Q is on the vertical axis. Pairs of v and η on this locus satisfy:

$$v \leq \frac{\alpha\varphi(1 - \lambda + \lambda\eta)}{(1 - \eta)^2 [3\psi - (1 - \lambda) + (1 + \lambda)\eta]} \tag{49}$$

and also:

$$v \leq \tau \sqrt{\frac{q_z(\psi - 1)}{q_D(\psi + \eta)}} \tag{50}$$

Inequalities (49) and (50) define an area of non-negative recycling in v / η space.
Inserting r from (39) into (40) yields the equation of this locus:

$$q_D(1-\lambda)(\psi-1)(\psi+\eta)(1-\eta)^2 v^2 - q_D\alpha\varphi(\psi+\eta)(1-\lambda+\lambda\eta)v +$$
$$+ q_z\tau^2(\psi-1)(1-\eta)^2[(2+\lambda)\psi+(1+\lambda)\eta] = 0 \tag{51}$$

Derivation of the implicit function (51) yields:

$$\frac{dv}{d\eta} = \frac{g_1(v,\eta)}{g_2(v,\eta)} \tag{52}$$

where:

$$g_1(v,\eta) = q_D(1-\lambda)(\psi-1)(1-\eta)(2\psi-1+3\eta)v^2 + q_D\alpha\varphi[1+(\psi-1)\lambda+2\lambda\eta]v +$$
$$+ q_z\tau^2(\psi-1)(1-\eta)[2(2+\lambda)\psi-(1+\lambda)+(3-\lambda)\eta] \tag{53}$$

$$g_2(v,\eta) = q_D(\psi+\eta)[2(1-\lambda)(\psi-1)(1-\eta)^2 v - \alpha\varphi(1-\lambda+\lambda\eta)] \tag{54}$$

It can be seen, that: $g_1(v,\eta) > 0$, and that: $g_2(v,\eta) < 0$ if :

$$v < \frac{\alpha\varphi(1-\lambda+\lambda\eta)}{2(1-\lambda)(\psi-1)(1-\eta)^2} \tag{55}$$

A quick check shows, that if (49) is satisfied, (55) is satisfied as well. This implies, that the locus is monotonically falling: at higher levels of knowledge and technology, virgin material inputs are substituted by recycled materials.

It can be seen that complete recycling ($D = v = 0$) can only be efficient at $\eta = 1$. The reason for this is simple: at $D = 0$ marginal damage from pollution is zero and there is no incentive therefore to prevent some materials to diffuse into the environment.

5.2 The Solution Point as a Function of the Size of the Population

An easy graphic interpretation of (41) can be given after manipulating it into:

$$\lambda^\lambda q_D^2 \bar{\pi}^{2\lambda}(\gamma N)^{2(1-\lambda)} =$$
$$= [q_K\tau^2(\psi-1)]^\lambda \frac{\eta^{2(1-\lambda)}}{v^\lambda}\left[\frac{q_z\tau^2(\psi-1)}{v} + q_D\frac{\alpha\varphi}{1-\eta} - q_D(\psi-1)v\right]^{2-\lambda} \tag{56}$$

For any value of η such that $0 < \eta < 1$, a unique value of v exists, setting the right hand side of the equation equal to the left hand side, since the right hand side is monotonically declining from infinity to zero as v increases from zero to the positive value reducing the expression in square brackets to zero. Without the necessity of calculating derivatives, one can also see, that $\frac{dv}{d\eta} > 0$, and that the graph shifts downwards for rising values of N.

For the case where (46) holds, the graph of (56) passing through the origin of the locus of efficient points is represented in Figure 4:
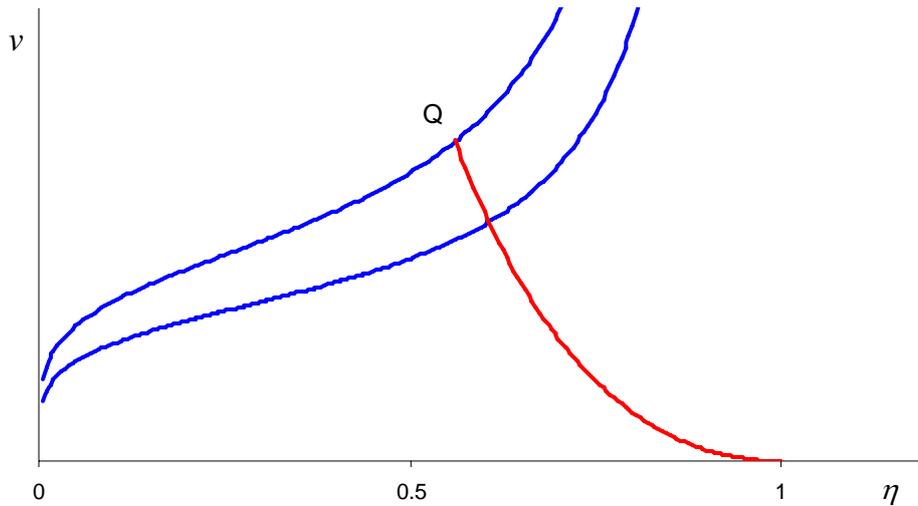
Figure 4: Equation (56) (blue) with Increasing *N*

If the population increases, the graph shifts downwards. Eco-efficiency increases and virgin material inputs into the economic system are substituted by recycled materials. It must be noted, that the size of the population in the origin of the efficient locus is strictly positive, and recycling is zero. Therefore, if (46) holds, there is for non-negative recycling a lower limit to the size of the population. In the case where (46) does **not** hold, the origin of the efficient locus is on the vertical axis and the size of the population in the origin is zero.

5.3   Physical Capital

Solving equation (42) for *v* yields:

$$v = \frac{\lambda q_D \alpha \varphi \pm \sqrt{(\lambda q_D \alpha \varphi)^2 - 4 \lambda q_D \tau^2 (\psi - 1)^2 (1 - \eta)^2 (q_K K^2 - \lambda q_z)}}{2 \lambda q_D (\psi - 1)(1 - \eta)} \qquad (57)$$

For values of $K \geq \sqrt{\dfrac{\lambda q_z}{q_K}}$, (57) yields two branches, one ascending, the other descending. The locus of bifurcation points is given by:

$$v = \frac{\alpha \varphi}{2(\psi - 1)(1 - \eta)} \qquad (58)$$

Figure 5 represents the graph of (57) together with the locus of bifurcation points (58) and the efficient locus (51).
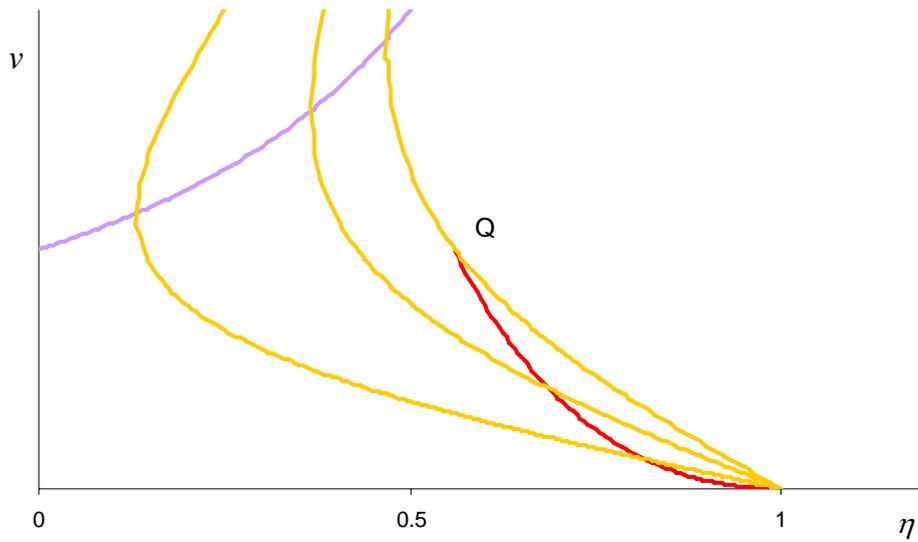
Figure 5: Bifurcation Locus (violet) and Different Levels of Physical Capital (gold)

The graph shows, that physical capital declines as the growth of the population shifts the solution point downwards along the efficient locus. This is because physical capital is substituted by labour and knowledge as the population increases.

If physical capital and virgin material inputs both decline, this means, that the state of the environment improves. It may seem strange that in the stationary state the quality of the environment is better with a larger size of the population. This result follows however from the assumption of an inelastic labour supply. With an increasing population, the supply of labour increases in all sectors, research included. Labour
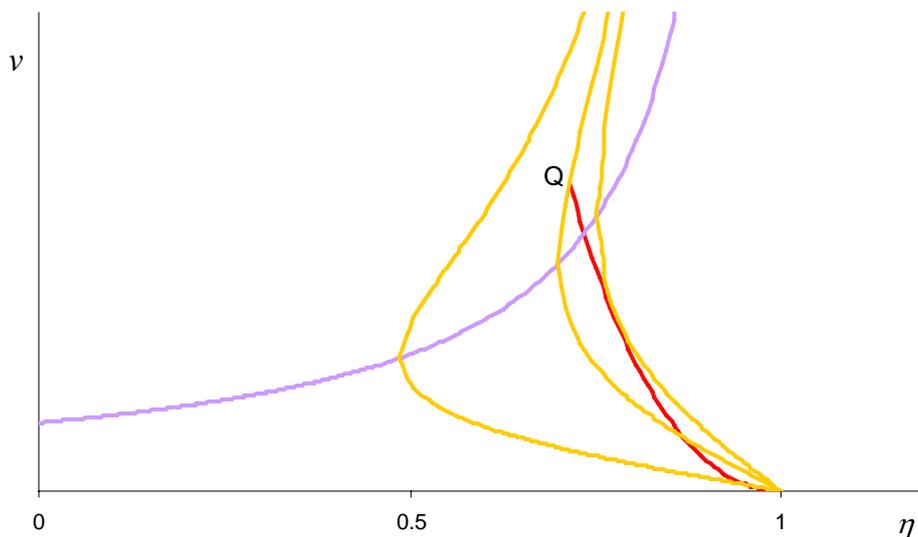

Figure 6: Origin of the Efficient Locus Q above the Bifurcation Locus

substitutes capital and recycling substitutes virgin materials extraction. Since good substitutability between consumption and environmental quality has been assumed in the social welfare function, an improvement in environmental conditions helps reduce the loss in welfare when individual consumption declines.

There is an exception however to the decline of physical capital with an increase of the population. If the origin of the efficient locus is above the locus of bifurcation points, physical capital rises first and later declines as the population increases, as is shown in Figure 6.

5.4   Physical Per Capita Consumption

Substituting N from (43) into (41) yields:

$$
\left(\lambda q_D{}^2 \overline{\pi}{}^2\right)^{\lambda} \left(\gamma q_z\right)^{2(1-\lambda)} \left[\tau^2 \left(\psi - 1\right)\right]^{2-3\lambda} \left(\frac{\eta}{\psi + \eta}\right)^{2(1-\lambda)} \left(1 - \eta\right)^{2-\lambda} v^{2\lambda} =
$$
$$
= q_K{}^{\lambda} b^{2(1-\lambda)} \left[q_z \tau^2 \left(\psi - 1\right)\left(1 - \eta\right) + q_D \alpha\varphi v - q_D \left(\psi - 1\right)\left(1 - \eta\right)v^2\right]^{2-\lambda}
$$

(59)

This equation identifies points in v/η space for given values of physical per capita consumption. Any point on the efficient locus is associated with one level of per capita consumption. The graphs of (51) and (59) must therefore cross. How this looks like, depends on the value of the Cobb-Douglas parameter λ.

The first derivative of (59) is:

$$
\frac{dv}{d\eta} = \frac{v}{\eta\left(1 - \eta\right)\left(\psi + \eta\right)} \ \frac{g_3\left(v,\eta\right)}{g_4\left(v,\eta\right)}
$$

(60)

where:

$$
g_3\left(v,\eta\right) = 2q_D\left(1 - \lambda\right)\psi\left(\psi - 1\right)\left(1 - \eta\right)^2 v^2 +
$$
$$
+ q_D\alpha\varphi\left[\left(4 - 3\lambda\right)\psi\eta + \left(2 - \lambda\right)\eta^2 - 2\left(1 - \lambda\right)\psi\right]v - 2q_z\tau^2\left(1 - \lambda\right)\psi\left(\psi - 1\right)\left(1 - \eta\right)^2
$$

(61)

$$
g_4\left(v,\eta\right) = 4q_D\left(1 - \lambda\right)\left(\psi - 1\right)\left(1 - \eta\right)v^2 + q_D\alpha\varphi\left(3\lambda - 2\right)v + 2\lambda q_z\tau^2\left(\psi - 1\right)\left(1 - \eta\right)
$$

(62)

v/η space can be divided in sections, depending on the signs of $g_3\left(v,\eta\right)$ and $g_4\left(v,\eta\right)$.

5.4.1   The $g_3\left(v,\eta\right) = 0$ Locus

The first derivative of $g_3\left(v,\eta\right) = 0$ is:

$$
\frac{dv}{d\eta}\bigg|_{g_3\left(v,\eta\right)=0} = -\frac{q_D\alpha\varphi v^2 \left[2\left(2 - \lambda\right)\eta + \left(4 - 3\lambda\right)\psi\eta + \lambda\psi\right]}{2\left(1 - \lambda\right)\psi\left(\psi - 1\right)\left(1 - \eta\right)^3 \left(q_D v^2 + q_z\tau^2\right)}
$$

(63)

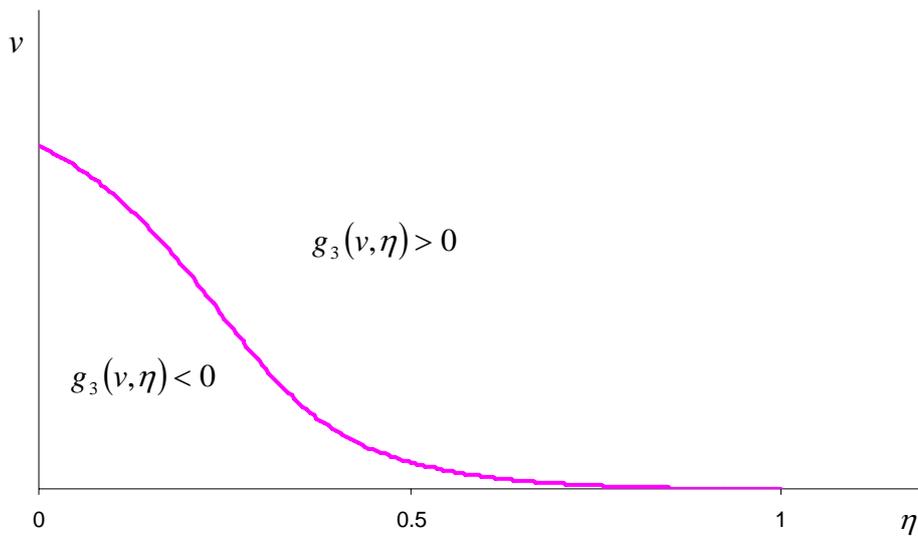The graph of $g_3\left(v,\eta\right) = 0$ is downwards sloping, as is shown in Figure 7.

Figure 7: The $g_3(v,\eta)=0$ Locus (pink)

5.4.2 The $g_4(v,\eta)=0$ Locus

If $\lambda \geq \dfrac{2}{3}$, $g_4(v,\eta)$ is always positive. If $\lambda < \dfrac{2}{3}$ the locus: $g_4(v,\eta)=0$ is given by:

$$v = \frac{q_D\alpha\varphi(2-3\lambda)\pm\sqrt{[q_D\alpha\varphi(2-3\lambda)]^2-32\lambda(1-\lambda)q_z q_D\tau^2(\psi-1)^2(1-\eta)^2}}{8q_D(1-\lambda)(\psi-1)(1-\eta)} \qquad (64)$$

Equation (64) consists of two branches. The bifurcation is located at the value of η which sets the expression under the square root equal to zero, i.e.:

$$\eta = 1 - \frac{\alpha\varphi}{4\tau(\psi-1)}\sqrt{\frac{q_D(2-3\lambda)^2}{2q_z\lambda(1-\lambda)}} \qquad (65)$$

The expression under the square root rises monotonically from zero to infinity as λ declines from $\lambda = \dfrac{2}{3}$ to zero. The bifurcation point shifts therefore from $\eta = 1$ for $\lambda = \dfrac{2}{3}$ to $\eta = -\infty$ for $\lambda = 0$.

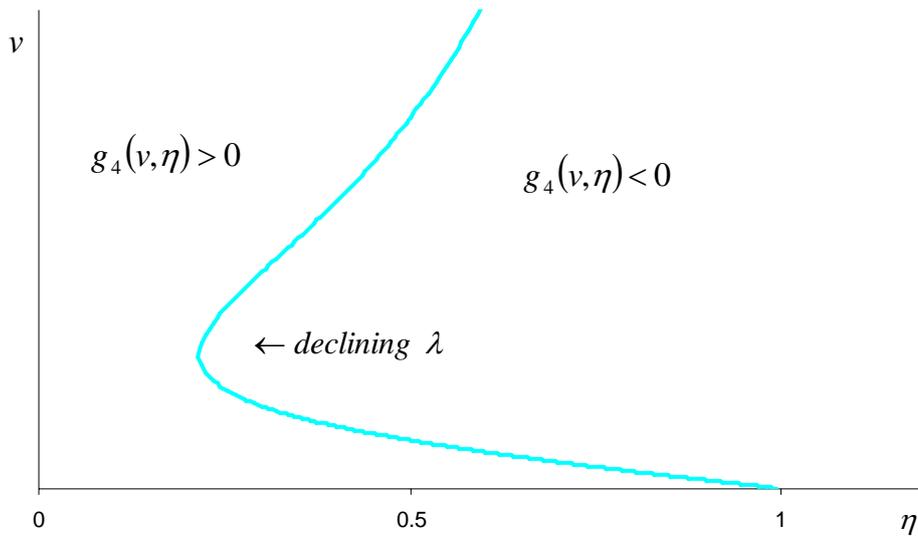Figure 8 represents the graph of the $g_4(v,\eta)=0$ locus with $\lambda < \dfrac{2}{3}$.

Figure 8: The $g_4(v,\eta) = 0$ Locus (cyan)

5.4.3   The Division of v/η Space

The graphs of $g_3(v,\eta) = 0$ and of $g_4(v,\eta) = 0$ can be combined and generate different patterns, depending on the value of λ. Three possible patterns can be distinguished. a) If $\lambda \geq \dfrac{2}{3}$, the graph of (59) only depends on the $g_3(v,\eta) = 0$ locus. I shall call this the case of a "large" value of λ. b) If λ is small enough for the two graphs to interfere, the pattern will be called that of a "small" value of λ. c) In the "intermediate" case, λ is smaller than $\dfrac{2}{3}$, but the two graphs do not interfere.

5.4.4   A "Large" Value of λ

The graph of $g_3(v,\eta) = 0$ (pink curve), and the graph of (59) (green curves for two different values of physical per capita consumption) are represented in Figure 9.

   The direction of a shift as a consequence of a change of b can be determined by calculating $\dfrac{dv}{db}$ from (59) for a given value of eco-efficiency:

$$\frac{dv}{db} = \frac{2(1-\lambda)v}{b} \quad \frac{q_z\tau^2(\psi-1)(1-\eta) + q_D\alpha\varphi v - q_D(\psi-1)(1-\eta)v^2}{g_4(v,\eta)} \tag{66}$$

   With $g_4(v,\eta) > 0$, this expression is positive. For this reason, the graph shifts downwards as per capita consumption declines, and this means that with increasing population physical per capita consumption declines.
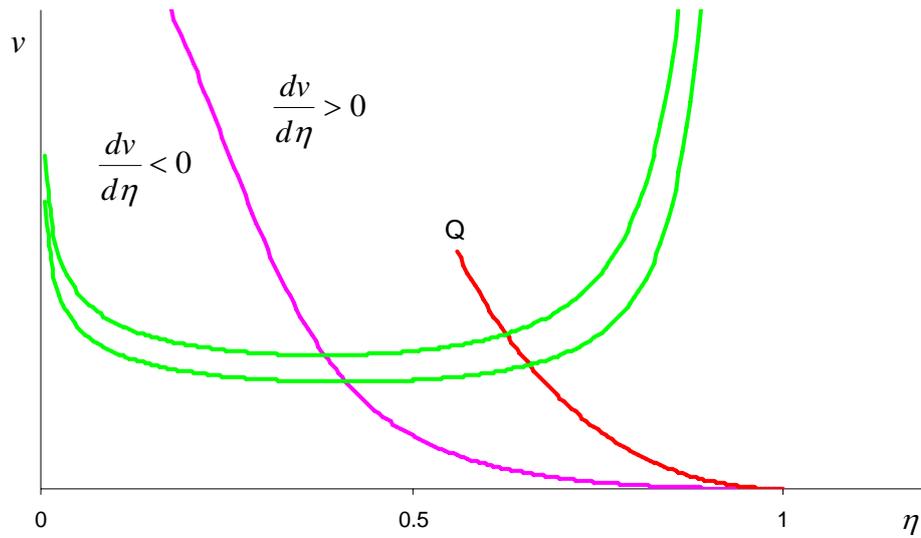
Figure 9: Physical Per Capita Consumption with a "Large" λ

5.4.5 A "Small" Value of λ

The division of v/η space with respect to the derivatives and the graph of (59) for three different levels of physical per capita consumption are shown for a "small" value of λ in Figure 10.
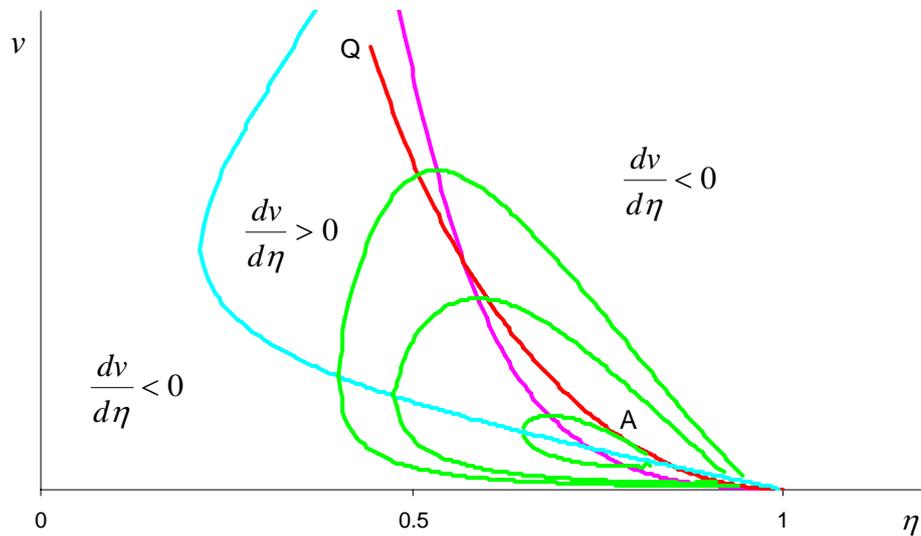

Figure 10: Physical Per Capita Consumption with a "Small" λ

The graph of (59) is a loop, and the relevant feature is that an increase in b will not shift the graph upwards, as in the "large" λ case, but rather contract the loop. This is because the upper part of the loop is in the $g_4(v,\eta)<0$ section, whereas the lower part is in the $g_4(v,\eta)>0$ section of v/η space. According to (66) therefore, the upper part shifts downwards, and the lower part shifts upwards (and the loop therefore contracts)

as *b* increases. This means, that physical per capita consumption increases with increasing population up to the tangency point (point A) between the locus of efficient points and the loop representing (59). For increases in the size of the population beyond the tangency point A, obviously physical per capita consumption declines again. As long as per capita consumption and the population both grow, aggregate consumption increases. This can happen without disrupting the environment, because capital is substituted by labour and recycling is increased.

The result of a growing physical per capita consumption with an increasing population seems to be rather counter-intuitive, and requires therefore some comment. If λ has a "low" value, this means that $1-\lambda$, that is the share of labour in the Cobb-Douglas production function, is "high". This is true in all sectors where labour is employed, therefore also in research. With an increase in the labour supply, the positive contribution of eco-efficiency can temporarily offset the decline in marginal returns to labour. Finally however, declining marginal returns and a decline in per capita consumption must prevail.

This allows to characterize the logic structure of the present model. For a **given** level of the population the model is bounded by the environment, since physical capital cannot grow beyond a certain level without causing environmental disruption. With an increase in the labour force, there is no environmental disruption, since labour is considered in this model to be an environmentally "clean" factor. The growth of the population is bounded however by declining marginal returns to labour, and therefore by declining per capita consumption.

5.4.6   The "Intermediate" Case

In the intermediate case $\lambda < \dfrac{2}{3}$, but $g_3(v,\eta)=0$ and $g_4(v,\eta)=0$ do not cross. The graph is represented in Figure 11.
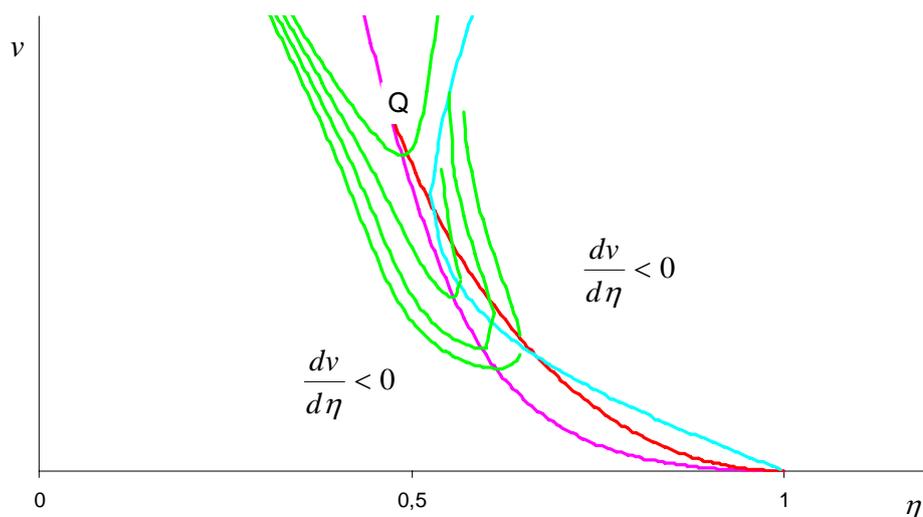

Figure 11: Physical per capita consumption with an "intermediate" value of λ

Although the graph is somewhat different, the result is similar to the "large" λ case. On the efficient locus $g_4(v, \eta)$ is positive, and therefore b declines as the population increases.

5.5   Carrying Capacity

The present model is built on the assumption that consumption quantity can be substituted by quality. There are obvious lower limits to such a kind of substitution, since, once consumption per head is reduced to a minimum caloric level, a further substitution of quantity with quality is no longer possible. For this reason, there is an upper bound to the size of the population, given by **carrying capacity,** and this means, that there is a point on the efficient locus, below which a growth of the population is no longer sustainable.

6   Conclusions and Some Warnings

The model discussed in the previous sections investigates a material stock-flow equilibrium, supporting a sustainable economy. The constraints imposed by the environment upon the accumulation of physical capital and upon the growth of the research sector are described in this paper by two basic assumptions. The first is that the mere existence of physical capital negatively affects the environment. Thus, physical capital directly enters the social welfare function with a negative first derivative. Increasing marginal environmental damage prevents physical capital from growing without bounds and requires that human activities should not encroach upon natural capital beyond a reasonable level (Ekins 2003; Ekins et al. 2003). Clearly, the quality of physical capital is enhanced by knowledge, and in this way the benefits of capital accumulation can be expanded beyond its physical limits. The second basic assumption is that the aggregate production function is subject to declining marginal returns to knowledge, whereas maintenance and activation costs in the research sector linearly rise. For this reason, the stock of engineering knowledge cannot grow without limits, and an optimal level of this stock has to be endogenously determined.

Enhancing eco-efficiency is an important social response to the growing encroachment of the economy upon the environment, but high levels of eco-efficiency require a well developed system of knowledge and technology, and knowledge and technology cannot be acquired at zero environmental costs. Productive factors are necessary in order to support the research sector, and the natural environment cannot absorb unlimited quantities of physical capital. On the other hand, also an increase of the factor labour, which has been optimistically assumed to be an environmentally "clean" factor in this paper, must finally result in declining per capita consumption and in an overshooting of the carrying capacity potential of the natural environment.

Recycling reduces flows of waste materials to the environmental sink, but achieves this by expanding other types of stocks, such as sequestration and recycling capital. For this reason, although recycling can be useful to a certain extent, it cannot fully overcome the basic environmental limitation of the economic system.

The model results, which most diverge from standard intuition may be listed as follows:

- Aggregate physical capital may decline as the population increases;
- physical per capita consumption may increase with an increase of the population, if the Cobb-Douglas exponent is biased towards labour;
- the state of the environment may improve for a larger size of the population, if consumption and environmental quality are good substitutes in the social welfare function;
- recycling may be useful, but complete recycling is unlikely to be a meaningful option.

A stationary state, in order to be sustainable, must also satisfy some basic ecological conditions that have remained outside the scope of the present paper. A fundamental sustainability condition requires that the chemical elements entering the system must exit the system in the same proportions; otherwise the economic system would work as a filter, increasing over time the concentration of some chemical elements at the expense of others. For this reason, a stationary state, defined at aggregate level, will only be sustainable if dispersed waste is qualitatively recombined by natural processes in such a way as to preserve a balance between nature and the economy for a long time period. If this is not the case, an economic stationary state may turn out to be environmentally disruptive in the long run, and therefore ecologically unsustainable. An important condition also concerns the ecological quality of the economy, and in particular of resources and energy. It is not unrealistic to assume abundant resources in an aggregate model, although individual resources may face exhaustion in a shorter time period. Obviously, a source of energy is necessary in order to set flows in motion. If some of the materials here described are fossil fuels, they may provide the necessary source of energy. Alternatively, some of the capital goods of this paper may be thought of as consisting of appliances able to capture solar energy. A stationary state based on fossil fuels is however completely different from a stationary state based on solar energy. In a fossil fuel system carbon is extracted from below the earth crust and is emitted into the atmosphere. In the terminology of this paper eco-efficiency is low in such a system, since it is based on carbon dispersion. A fossil system will be capable of producing a stationary state of some duration, if an adequate technology for carbon recovery from the atmosphere becomes available (Holloway 2001). Low eco-efficiency and storage of waste is therefore a characteristic feature of such a system. In a solar economy waste production is low, since it is basically reduced to the wear and tear of solar capital stock. At high levels of eco-efficiency the energy system in this paper may be best thought of therefore as being based on solar energy.

A sustainable equilibrium of human activities with the environment may have to be rearranged as time goes by for at least two reasons. The first reason is, as was argued by one of the anonymous reviewers of this paper, that nature evolves in historical time and therefore the interaction of the economy with the biosphere may need adaptation in the course of time. These types of adaptation will have to take place at longer time intervals, since the pace of natural evolution is slow, compared to the rate of change in human societies. There is an additional reason however for the necessity of rearranging the economy-biosphere interaction in shorter time intervals, and that is, that our idea of sustainability is always conditional upon a given state of eco-systems knowledge. The

experience of the past has shown, that threats to sustainability have remained covered for long periods of time and have only been uncovered by eco-systems research at a point in time, when considerable damages to the ecosystem had already been produced. For this reason, society may have to rearrange the interaction between the economy and the environment in shorter time intervals, due to improvements in eco-systems knowledge.

For these reasons, we should be very cautious when we talk about sustainability and we should always be aware that all of our statements on this matter are conditional upon the present state of natural evolution and of eco-systems knowledge. The sensitivity of society to environmental warnings and the flexibility to adapt to new environmental insights remains therefore the basic social resource, needed to keep anthropic activities in balance with the natural environment.

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