

The Demand for Currency Substitution

John J. Seater

North Carolina State University

Abstract

A transactions model of the demand for multiple media of exchange is developed. Some results are expected, and others are both new and surprising. There are both extensive and intensive margins to currency substitution, and inflation may affect the two margins differently, leading to subtle incentives to adopt or abandon a substitute currency. Variables not previously considered in the literature affect currency substitution in complex and somewhat unexpected ways. In particular, the level of income and the composition of consumption expenditures are important, and they interact with the other variables in the model. Independent empirical work provides support for the theory.

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Correspondence: John J. Seater, Economics Department, North Carolina State University, Raleigh, NC 27612, john_seater@ncsu.edu

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I. INTRODUCTION

Currency substitution is the abandonment of one medium of exchange for another in the face of changing incentives. It is an important phenomenon in countries with high inflation rates, complicating forecasts of money demand and making monetary policy more difficult to conduct. Historically, the most important incentive for currency substitution has been change in the domestic inflation rate, though there have been episodes of currency substitution arising for other reasons. The basic intuition for the relation between inflation and currency substitution is simple enough. High domestic inflation makes holding the domestic money costly because the currency component (and perhaps others) pays a fixed nominal rate of interest of zero and so cannot offer compensation to offset the negative return caused by inflation. Domestic residents then may find it worthwhile to use currencies from countries with low inflation rates as substitutes for the domestic currency (Cagan, 1956; Barro, 1970).

Currency substitution is an aspect of money demand, with people choosing a combination of media of exchange to use in conducting market transactions. In particular, currency substitution is a type of demand for multiple media of exchange and therefore is amenable to analysis by the methods used in recent theoretical work on that subject. The present paper examines the demand for currency substitution within a money-demand framework that permits simultaneous holding of several media of exchange as well as a savings asset. The kinds of institutional factors discussed by Savastano (1996), such as availability of liquid assets, are naturally captured by different costs and rates of return attached to the alternative assets available for carrying out transactions. The analysis formalizes and supports Savastano's contention that financial institutional structure is important in determining a country's currency substitution behavior. The incentives to adopt or abandon a substitute currency are more subtle than the existing empirical literature assumes, involving calculations on both an extensive and an intensive margin. The theory also shows the importance of factors generally ignored by the currency substitution literature. In particular, the level of income per person and the composition of consumption expenditure for any given level of income affect the extent of currency substitution that a country experiences. The theory suggests that existing empirical investigations of currency substitution may have been misspecified by omitting relevant variables, and it offers guidelines on how to improve empirical specifications. It also gives guidance on what data is needed to carry out proper tests. Some aspects of the theory can be tested with household survey data, as discussed below. Indeed, in an independent study Stix (2007) applies the theory developed below to a small survey data set for Croatia, Slovenia, and Slovakia, and finds support for aspects of the theory. Testing the theory with market data would be more complicated than with household survey data because it would require further theoretical development first. The theory presented here is limited to the demand side. It is not even a partial equilibrium model, so one cannot test it with market-generated data, such as aggregate money holdings. Doing that would require coupling the theory with a complementary theory of the supply side of the currency substitution market. Developing such a theory and then deriving the market equilibrium is a worthwhile endeavor, but it is far beyond the scope of the present paper, which

should be regarded as a first step toward a complete theory of currency substitution.¹ Also, the present study examines currency substitution that results from domestic inflation. Currency substitution can occur for other reasons. It may arise as a response to a shortage of the domestic money (Colacelli and Blackburn, 2007) or as a means of establishing an optimal currency area (Alesina and Barro, 2001, 2002). Those motives for currency substitution are not addressed here.

II. MODEL FUNDAMENTALS

Most models of money demand based on tight micro foundations fall into two broad categories: search or matching models on the one hand and cash-in-advance models on the other. Members of the former class typically are used to study the origins of the medium of exchange (e.g., Jones, 1976; Niehans, 1978; Kiyotaki and Wright, 1989; Howitt and Clower, 2000), although some are used to study other questions (e.g., Cavalcanti and Wallace, 1999), including currency substitution (Craig and Waller, 2000). For our purposes, these models are not useful because either they are much too stylized to address the questions to be investigated here or they are analytically intractable and require numerical solution.² Instead, the analysis here is based on a type of cash-in-advance model, Santomero and Seater's (1996) model of the demand for smart cards and other new means of payment. Santomero and Seater extend the Baumol-Tobin transactions model of money demand to allow consumers to use several media of exchange simultaneously. Their approach to the demand for multiple media of exchange is much more general than their particular application to innovations in electronic means of payment and, with appropriate alteration, is well-suited to the study of currency substitution. The model simultaneously has the necessary structure and the analytical tractability to answer the questions of interest. Santomero and Seater consider the case of many media of exchange and many goods, but for our purposes, such generality merely adds tedious complications without providing any additional insights. We therefore will restrict attention to two media of exchange that can be used to buy either or both of two goods.

Before proceeding to the model and its solution, we should note that, although the Baumol-Tobin model is a type of cash-in-advance model, it differs substantially from the standard cash-in-

¹Jovanovic (1982), Romer (1987), and Leo (2006) present general equilibrium models of the transactions demand for money. Leo's approach is the simplest and seems the most likely to be amenable to generalization to multiple media of exchange and currency substitution.

²To use some of the works already cited as examples, Jones (1976) and Niehans (1978) examine existence of equilibria and the nature of the goods that emerges as the media of exchange. Their models are not suited to studying what characteristics determine which media a particular household will use. Kiyotaki and Wright (1989) and Craig and Waller (2000) restrict the analysis to households that demand only one good, making their models inappropriate for studying how the composition of consumption affects demands for multiple media of exchange. Cavalcanti and Wallace (1999) allows for two media but restricts a given household *a priori* to using one or the other of them. Howitt and Clower's (2000) model must be solved numerically.

advance model (Lucas and Stokey 1987). The Baumol-Tobin model has a much richer micro structure than the standard cash-in-advance model. The cost of that richness is that the Baumol-Tobin model is difficult to use in a general equilibrium framework, in contrast to the standard cash-in-advance model. At the end of the analysis we discuss some of the differences between the results obtained here and those emerging from standard cash-in-advance models.

A. Model Structure.

The model has the standard Baumol-Tobin structure except for the presence of more than one medium of exchange. The household receives a fixed real annual income Y^A each year, paid in J equal installments of amount $Y=Y^A/J$ at the beginning of J payments periods of equal length $1/J$. These J payments periods are the fundamental time periods of the analysis, and discussion will be couched in terms of them. Each payments period, the household exactly exhausts its income Y by buying fixed real amounts, X_g , of two different goods:

$$(1) \quad Y = X_1 + X_2$$

We distinguish between consumption and consumption expenditure. *Consumption* of goods occurs at a constant rate, whereas *consumption expenditures* (i.e., purchases of goods) occur at discrete sub-intervals of the payments period, chosen optimally by the household. Between such "shopping trips," the household holds inventories of the two goods, which it gradually consumes until exactly exhausting them at the moment it is time to make another shopping trip. A separate shopping trip is required for each type of good. Each type of commodity inventory pays a unique real rate of return, R_{X_g} . This rate may be implicit or explicit and may be positive or negative.

Purchases are subject to a cash-in-advance constraint: goods must be purchased with a medium of exchange that the household has on hand at the time of the purchase. There are two media of exchange, m_i , available to the household: m_1 is the nominal domestic currency, m_2 is a nominal foreign currency. The corresponding real quantities are denoted M_1 and M_2 . The household can use either or both types of money to buy each type of good. Denote the quantity of good g bought with money I by X_{gi} . The household may use medium M_1 on some shopping trips for good g and medium M_2 on others (although we will see momentarily that the household never chooses to do that, using only one medium to purchase a given good). Thus

$$(2) \quad X_g = X_{g1} + X_{g2}$$

There are Z_{gi} trips to purchase good g with money I . Each such trip has associated with it the real shopping cost B_{gi} , a lump-sum amount paid each trip but not depending on the amount spent. This cost may be explicit, such as a delivery charge or a check-cashing fee, or implicit, such as a time cost.

The household spends only a fraction of its income on any shopping trip. Unspent income is held in a single real savings asset, S , and in money balances. Savings earn the real rate of return

R_S , and the two kinds of money earn rates of return R_{M_i} . It is presumed that $R_S > R_{M_i} > R_{X_g}$.³ When the household exhausts its holdings of a medium of exchange, the cash-in-advance constraint forces it to replenish those holdings by converting some of its saving asset to the desired medium. There are T_i conversions to obtain m_i . Each conversion has associated with it the real conversion cost A_i , a lump-sum amount paid explicitly or implicitly each time a conversion is made that does not depend on the size of the conversion. As in the simple Baumol-Tobin model, optimal conversions are evenly spaced. Shopping trips occur between conversions and also are evenly spaced.⁴ There are N_{g_i} shopping trips to buy good g with money I per conversion of S into m_i . The total number of shopping trips, Z_{g_i} , to buy good g with money I is thus $T_i N_{g_i}$.

Finally, each of the assets, S and m_i , carries a real fixed cost F_i that must be paid if that asset is held at any time during the payments period. These fixed costs capture such things as monthly account fees. For currencies, the fixed costs may be zero; however, monies, even foreign ones, need not be currency but rather may be bank accounts which often do carry fixed fees. Consequently, fixed costs are carried through the analysis for all assets to maintain complete generality.

Figure 1, anticipating the solution to follow, shows the time paths of total wealth, the savings asset, two media of exchange, and two goods for the case of each good being purchased with a different medium of exchange. Total wealth W is the sum of the savings asset, the holdings of the media of exchange M_1 and M_2 , and the stocks of commodity inventories X_1 and X_2 . Each period's income Y has a value of 36, which the household divides between 6 units of the first good and 30 units of the second good. Wealth equals Y at the beginning of the payments period and falls at a constant rate (because of constant consumption), reaching zero exactly at the end of the period. The household puts some fraction Y (in this example, twenty-seven thirty-sixths of Y) into the saving asset S , which then falls in a stair-step pattern as it is converted into the two media of exchange, each step down corresponding to one conversion. The media of exchange also move in stair-step patterns, each step down corresponding to a shopping trip. In this example, the household initially puts three thirty-sixths of Y into M_1 and six thirty-sixths of Y into M_2 . It then immediately buys 1 unit of good 1 and 3 units of good 2. At time zero, then, the household has 2 units of M_1 , 3 units of M_2 , 1 unit of X_1 , and 3 units of X_2 . As time passes, the commodity inventories are exhausted, resulting in more shopping trips. Eventually, the household runs out of money and must convert some of the saving asset to the appropriate medium of exchange, leading to decreases in S . At the end of the payments period, total wealth is just exhausted, a new income installment is received, and the process repeats.⁵

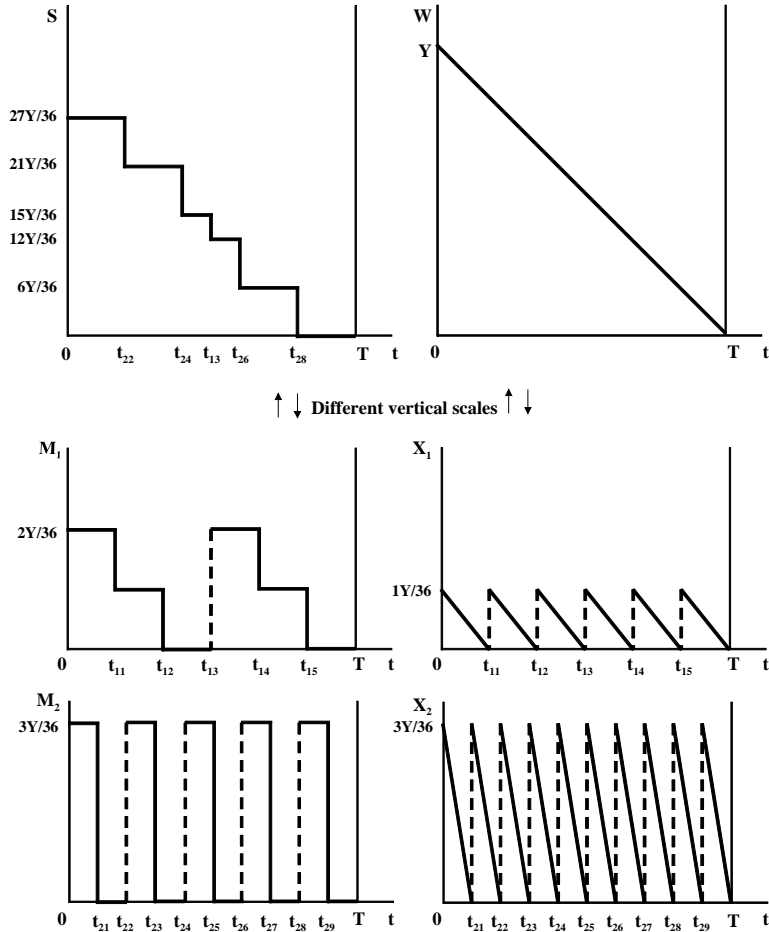
³It is not necessary that all money interest rates exceed all inventory rates of return, but imposing that requirement simplifies the discussion. It is trivial to show that in the general case, money i will not be used to purchase good g if the rate of return R_{X_g} on good g exceeds the rate of return R_{M_i} on money I .

⁴The proof that optimal asset conversions and shopping trips are evenly spaced is tedious and unimportant to the issues discussed, so it is omitted. See Tobin (1956) for details.

⁵The model assumes interest income is accumulated in an unmentioned asset and not spent during the period of analysis. That assumption greatly simplifies the analysis. Relaxing it has no effect on the qualitative conclusions, as shown by Jovanovic (1982) and Romer (1987).

Figure 1: Illustration of Asset Time Paths

Note that the vertical scales of the top two graphs differ from those of the bottom four graphs.



What remains to be determined is the initial division of income Y among the savings asset and the two types of currencies, how much of each type of currency to use in buying each type of good, how often to buy each good, and how often to replenish currency holdings by converting the savings asset to currency. Before we proceed to the solution, it is useful to note several assumptions that have been imposed on the problem.

First, income and the total quantities of goods bought are predetermined. A more complete model would allow the household to determine the values of X_1 and X_2 , and perhaps also Y through choice of labor supply, simultaneously with the quantities of m_1 and m_2 . Jovanovic (1982) and Romer (1987) provide such a model. As their analyses show, the qualitative conclusions concerning money demand are not changed by these generalizations, but the analysis is greatly complicated. We therefore have restricted attention to the simpler case where Y , X_1 , and X_2 are predetermined. Similarly, we do not consider the possibility that the types of goods produced are endogenous to the

problem. Second, there is no utility function in the model, so what distinguishes the two goods are the various costs and rates of return associated with them and the proportions of the household budget that they constitute.⁶ Those characteristics determine how much of each currency will be used to buy each good, as we will see momentarily. Third, all costs are assumed to be constant. In fact, it is possible that costs are related to the level of income or the volume of transactions (quantity of goods) bought. Mutual fund companies, for example, typically charge lower fees for large accounts, so that richer individuals or countries could face lower costs than their poorer counterparts. Fourth, there are no sunk costs in the model. Various studies (e.g., Guidotti & Rodriguez, 1992; Giacchino, 1996; Feige *et al.*, 2002, 2003; Havrylyshyn & Beddies, 2003) have found that currency substitution often persists after the inflation that induced it has subsided. Several contributions have shown that sunk costs, if they lead to a reduction in transactions costs, can result in such persistence (Ireland, 1995; Giacchino, 1996; Sturzenegger, 1997; Uribe, 1997). The mechanism is a straightforward application of the general theory of irreversible investment, and it seems to explain nothing beyond the persistence of currency substitution. Introducing sunk costs would complicate the analysis considerably without adding anything beyond persistence, so sunk costs have been omitted from the model for the sake of simplicity.

B. Model Solution.

The household seeks to maximize the profit from managing its assets:

$$(3) \quad V = R_S \bar{S} + \sum_{i=1}^2 R_{M_i} \bar{M}_i + \sum_{g=1}^2 R_{X_g} \bar{X}_g - \sum_{i=1}^2 T_i A_i \\ - \sum_{i=1}^L \sum_{g=1}^2 Z_{gi} B_{gi} - F_S I(S) - \sum_{i=1}^2 F_i I(M_i)$$

where $I(Q)$ is an indicator function that is 1 if average holdings of asset Q are positive and is 0 otherwise. The terms on the right side are, in order: (1) interest earnings on average holdings of the savings asset, (2) interest earnings on average holdings of the two media of exchange, (3) interest earnings on average inventories of goods, (4) total costs paid for converting between savings on the one hand and media of exchange on the other hand, (5) total shopping trip costs, (6) the fixed cost of holding savings assets, and (7) the fixed costs of holding the two media of exchange. To maximize this profit, the household chooses optimal values of average asset holdings, conversion and shopping trip frequencies, and the X_{gi} . This problem can be simplified in the usual way, by noting that the average asset values can be written in terms of the remaining variables (see the Appendix):

⁶Note that here both goods are assumed to have rates of return equal to zero for simplicity, but that need not be the case in general.

$$(4) \quad \bar{S} = \sum_g \frac{X_g}{2} - \sum_i \sum_g \frac{X_{gi}}{2T_i}$$

$$(5) \quad \bar{M}_i = \frac{\sum_g X_{gi}}{2T_i} - \frac{\sum_g X_{gi}}{2Z_{gi}}$$

$$(6) \quad \bar{X}_{gi} = \frac{X_{gi}}{2Z_{gi}}$$

Substituting these solutions in the profit expression gives

$$(7) \quad V = R_S \left[\sum_g \left(\frac{X_g}{2} - \sum_i \frac{X_{gi}}{2T_i} \right) \right] + \sum_i R_{M_i} \left[\sum_g \left(\frac{X_{gi}}{2T_i} - \frac{X_{gi}}{2Z_{gi}} \right) \right] \\ + \sum_g R_{X_g} \left(\sum_i \frac{X_{gi}}{2Z_{gi}} \right) - \sum_i T_i A_i - \sum_i \sum_g Z_{gi} B_{gi} \\ - F_S I(S) - \sum_i F_i I(m_i)$$

By solving for the optimal values of the T_i and Z_{gi} in terms of the X_{gi} (see the Appendix) and substituting in (3), we can write the profit expression as

$$(8) \quad V = R_S \frac{X_1 + X_2}{2} - [2A_1(R_S - R_{M1})(X_{11} + X_{21})]^{1/2} - [2A_2(R_S - R_{M2})\{(X_1 - X_{11}) + (X_2 - X_{21})\}]^{1/2} \\ - \{[2B_{11}(R_{M1} - R_{X1})X_{11}]^{1/2} + [2B_{21}(R_{M1} - R_{X2})X_{21}]^{1/2}\} \\ - \{[2B_{12}(R_{M2} - R_{X1})(X_1 - X_{11})]^{1/2} + [2B_{22}(R_{M2} - R_{X2})(X_2 - X_{21})]^{1/2}\} \\ - F_S I(S) - F_1 I(M_1) - F_2 I(M_2)$$

All that remains is to find the optimal values of X_{11} and X_{21} . However, the second-order conditions indicate that the profit expression is convex:

$$(9) \quad \frac{\partial^2 V}{\partial X_{ij} \partial X_{jl}} > 0 \quad \text{for } i, j = 1, 2 \\ \det H > 0$$

where H is the Hessian. Consequently, the interior extremum is a profit minimum, so the maximum occurs at a corner. This implies that the household always chooses to use only one medium of exchange to buy a given type of good.

The feature of the model that gives a corner solution is the requirement that each use of a currency to make a purchase requires paying the transactions cost B_{gj} . Using two types of money to buy a single good would require paying both transactions costs B_{g1} and B_{g2} instead of just one. There is no compensating benefit. If it is profit-maximizing to choose one type of money over the other to buy the first unit of the consumption good X_g , then it also is profit-maximizing to choose the same type of money for every other unit of X_g . This corner outcome is intuitively reasonable. It is hard to imagine that it would be worthwhile to pay for the groceries partly with cash and partly with a credit card. A payment pattern of that type would require that the net benefits of using a currency somehow decline in the amount of that currency being tendered. The mathematics of the problem at hand tell us that no such condition obtains here, so only the corner solutions emerge as profit-maximizing solutions.

A household that holds the savings asset S faces four possible choices:

$(S > 0, X_{11} = X_1, X_{21} = X_2)$	hold S , use M_1 to buy X_1 and X_2
$(S > 0, X_{11} = X_1, X_{21} = 0)$	hold S , use M_1 to buy X_1 and M_2 to buy X_2
$(S > 0, X_{11} = 0, X_{21} = X_2)$	hold S , use M_2 to buy X_1 and M_1 to buy X_2
$(S > 0, X_{11} = 0, X_{21} = 0)$	hold S , use M_2 to buy X_1 and X_2

The household, however, may choose not to hold the savings asset. It thus has four more possible choices:

$(S = 0, X_{11} = X_1, X_{21} = X_2)$	do not use S , use M_1 to buy X_1 and X_2
$(S = 0, X_{11} = X_1, X_{21} = 0)$	do not use S , use M_1 to buy X_1 and M_2 to buy X_2
$(S = 0, X_{11} = 0, X_{21} = X_2)$	do not use S , use M_2 to buy X_1 and M_1 to buy X_2
$(S = 0, X_{11} = 0, X_{21} = 0)$	do not use S , use M_2 to buy X_1 and X_2

The household therefore has eight possible usage patterns to consider. It chooses among them by comparing the profits associated with each of the eight possible patterns and picking the one with the highest profit. Stix (2007) provides evidence from household survey data that households actually do systematically use the domestic currency exclusively for some goods and a foreign currency exclusively for others, so there is good reason to explore the factors that determine how goods and currencies will be paired, that is, how the household chooses among the eight patterns just listed.

Before proceeding to a discussion of the household's choice and how that choice changes in response to various exogenous shocks, we modify the model slightly.

C. Convenience Yields.

To this point, the discussion has treated the two types of money in a completely general way. However, because the subject of this inquiry is currency substitution, henceforth we will use M_1 to denote the domestic money and M_2 to denote a foreign money that may circulate as an alternative medium of exchange. As long as some part of M_1 and M_2 consist of currency, the nominal interest rates r_1 and r_2 on those monies cannot fully adjust to inflation. It therefore simplifies matters to assume without loss of generality that no component of nominal money (cash, checking accounts, etc.) pays nominal interest.

A minor problem arises here. Actual episodes of substantial currency substitution are driven by high rates of inflation in M_1 -denominated prices, generally caused by printing excessive amounts of the nominal domestic money m_1 . If the only kind of return earned on a nominal asset is the explicit nominal interest rate, then currency earns no nominal interest rate and so earns the real rate of return $R_M = -\pi/(1+\pi)$, which has a lower bound of -1 (that is, negative 100 percent). The value of R_M can be made arbitrarily close to this lower bound, for inflation can be made arbitrarily high, and the yield spread $R_M - R_X$ can become negative. Once the spread becomes negative, people stop using M to buy X ; it is more profitable to hold inventories of X than inventories of M . If inflation rates in both M_1 -denominated prices and M_2 -denominated prices, denoted π_1 and π_2 , are sufficiently high, both types of money would be abandoned, and exchange would be conducted by barter. The problem with this outcome is that transactions models of the type used here are constructed under the implicit assumption that some form of money is used to buy goods; the models are incapable of handling barter.

There are two ways to address this difficulty. One approach is to build a model that accommodates barter explicitly. That is the type of model used to explore the origins of media of exchange. Although very interesting, as mentioned earlier such models either are so stylized that they cannot address the issues investigated here or are analytically intractable and can be solved only by numerical methods. The second possibility, adopted here, is to impose restrictions that prevent barter from arising. Two restrictions are needed.

First, we suppose that all types of financial assets pay an implicit real convenience yield R_C . This convenience yield captures the great savings in transactions costs achieved by using money instead of barter to buy goods. Behavior during hyperinflations suggests that R_C is a very large number. For example, in the hyperinflations that Cagan (1956) studied, *average* inflation rates often reached hundreds of percent per *month* (far more in the case of the second Hungarian hyperinflation), but people continued using the domestic medium of exchange even though foreign media began to circulate as alternatives.⁷ There clearly must be great value to using the domestic medium of exchange if people hold it in the face of such enormously negative real rates of return. We therefore assume here that R_C is sufficiently large that the real return to money

⁷In one month of the second Hungarian hyperinflation, the inflation rate was just over forty quadrillion (41.9×10^{15}) percent per month, yet people still did not abandon the domestic currency completely.

$$\begin{aligned}
 (10) \quad R_M &= R_C + \frac{r_M - \pi}{1 + \pi} \\
 &= R_C - \frac{\pi}{1 + \pi} \quad (\text{because } r_M \equiv 0)
 \end{aligned}$$

is positive for any rate of inflation that will be encountered. The savings asset S also pays R_C as part of its total return. Savings assets are held only if their rate of return exceeds that on money. In the absence of inflation, nominal interest rates on short-term assets equal real interest rates and generally are quite low (e.g., about 1 percent per year on one-year U.S. Treasury bills). For these assets to be held, it must be that they carry an implicit convenience yield that keeps their total return above that of money. We can suppose that S is denominated in terms of the domestic money, so the real rate of return can be written as

$$(11) \quad R_S = R_C + \frac{r_S - \pi_1}{1 + \pi_1}$$

In contrast to financial assets, physical inventories clearly do not carry a convenience yield. The convenience yield is precisely the extra value one gets by using money instead of goods to conduct transactions. The only rate of return on goods is the explicit real rate, which never is large in magnitude and usually is zero (and can be negative due to spoilage, theft, etc.). We therefore simplify the analysis by assuming that R_{X_g} is zero for all types of goods.⁸

The reader should bear in mind that the convenience yield R_C plays no role in the subsequent analysis and is merely a convenient way to avoid having to undertake an awkward analysis of barter. Its use here is similar in spirit to imposing Inada conditions in the Solow-Swan growth model to avoid uninteresting corner solutions.

The second restriction we need to avoid barter is to assume that at least one of the profit expressions associated with the eight usage patterns is positive for some choice of holdings of M_1 , M_2 , and S. The profit expressions are given in Table 1.⁹ Given that we have included a convenience yield high enough to keep the R_{M_i} positive for any observed rate of inflation, this restriction is quite weak, only requiring that the rate of return on one of the monies be sufficiently high to make it profitable to make one conversion from money to goods during the payments period. In that case, the household always can choose to spend half its pay immediately to buy goods and hold the remaining half as money, run the inventories of goods down to zero, and then use the stock of money to buy goods again exactly half way through the payments period. As long as the interest earned

⁸The gain from this simplification is mathematical convenience. In some derivations below, we must manipulate expressions of the type $(R_{M_i} - R_{X_g})^x$, where x is an integer greater than one. The resulting expressions are cumbersome if R_{X_g} is non-zero because of the cross terms. Those terms are negligible in magnitude if R_{X_g} is small relative to R_{M_i} , so omitting R_{X_g} simplifies the calculations without altering the qualitative results.

⁹See Table A in the Appendix for the general form of the profit expressions when R_{X_g} is not constrained to zero.

TABLE 1
Profit Functions

$$\begin{aligned}
 V_{0,1,1} &= R_{M1} \frac{X_1+X_2}{2} - [2B_{11}R_{M1}X_1]^{1/2} - [2B_{21}R_{M1}X_2]^{1/2} - F_1 \\
 V_{0,1,2} &= R_{M1} \frac{X_1}{2} + R_{M2} \frac{X_2}{2} - [2B_{11}R_{M1}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_1 - F_2 \\
 V_{0,2,1} &= R_{M1} \frac{X_2}{2} + R_{M2} \frac{X_1}{2} - [2B_{21}R_{M1}X_2]^{1/2} - [2B_{12}R_{M2}X_1]^{1/2} - F_1 - F_2 \\
 V_{0,2,2} &= R_{M2} \frac{X_1+X_2}{2} - [2B_{12}R_{M2}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_2 \\
 V_{S,1,1} &= R_S \frac{X_1+X_2}{2} - [2A_1(R_S-R_{M1})(X_1+X_2)]^{1/2} \\
 &\quad - [2B_{11}R_{M1}X_1]^{1/2} - [2B_{21}R_{M1}X_2]^{1/2} - F_S - F_1 \\
 V_{S,1,2} &= R_S \frac{X_1+X_2}{2} - [2A_1(R_S-R_{M1})X_1]^{1/2} - [2A_2(R_S-R_{M2})X_2]^{1/2} \\
 &\quad - [2B_{11}R_{M1}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_S - F_1 - F_2 \\
 V_{S,2,1} &= R_S \frac{X_1+X_2}{2} - [2A_1(R_S-R_{M1})X_2]^{1/2} - [2A_2(R_S-R_{M2})X_1]^{1/2} \\
 &\quad - [2B_{21}R_{M1}X_2]^{1/2} - [2B_{12}R_{M2}X_1]^{1/2} - F_S - F_1 - F_2 \\
 V_{S,2,2} &= R_S \frac{X_1+X_2}{2} - [2A_2(R_S-R_{M2})(X_1+X_2)]^{1/2} \\
 &\quad - [2B_{12}R_{M2}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_S - F_2
 \end{aligned}$$

NOTE: The subscripts in the profit expression V_{ijk} have the following meanings:

I = S if the saving asset is used, 0 otherwise

j = 1 or 2 as M_1 or M_2 is used to buy good 1

k = 1 or 2 as M_1 or M_2 is used to buy good 2

on the money exceeds the cost of the single conversion that is made, the profit expression will be positive and positive money balances will be held. *A fortiori*, it also will be positive for the optimal choice of cash management, which must yield a profit at least as high as this minimal strategy. Money rather than barter then will be used to conduct all purchases.

III. DEMANDS FOR TRANSACTIONS ASSETS

We now examine the household's choice of medium of exchange and its dependence on interest rates, transactions costs, income, and expenditure patterns. Because the solution always is in a corner (i.e., where a given money is used either exclusively or not at all to purchase a given good),

we cannot use the standard first-order conditions that pertain to an interior solution. This fact complicates the analysis because, instead of just setting some first order conditions equal to zero and solving for the values of S , M_1 , and M_2 that simultaneously satisfy those conditions, we must carry out a rather tedious comparison of the eight possible solutions to see which one is best, something like what one does in moving from one node to another in seeking the solution to a linear programming problem. The method of analysis is to consider how a change in a variable of interest affects the difference between various pairs of profit expressions from Table 1. For example, suppose we wanted to study the effect of an increase in the real rate of return R_s on the choice of whether to use M_1 or M_2 to purchase good X_2 , given that the savings asset is held and that M_1 is used to purchase good X_1 . We would compute the profit difference $V_{S11}-V_{S12}$ and examine the sign of the partial derivative $\partial(V_{S11}-V_{S12})/\partial R_s$. If that sign were positive, then an increase in R_s raises the difference (i.e., makes it more positive), making it more likely that the household will choose to use M_1 to buy both goods.

A. Interest Rates and Currency Substitution.

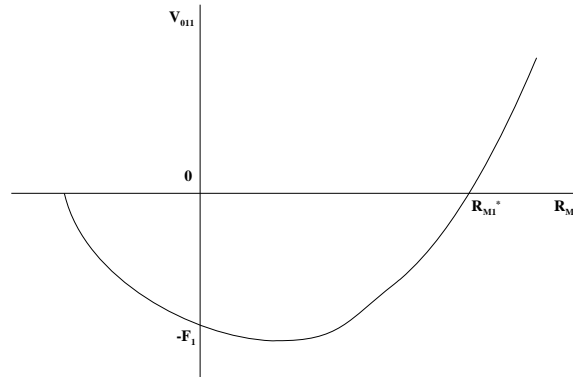
For currency substitution, the most important relation is between demands for domestic and foreign money on the one hand and domestic money's real rate of return on the other. That is because currency substitution arises almost exclusively in response to high domestic inflation rates, which appear in the model as reductions in the real rate of return R_{M1} on domestic money. The saving asset's nominal rate of return r_s presumably can adjust to changes in the inflation rate to satisfy the Fisher equation, leaving the real rate of return R_s unchanged. In contrast, the nominal rates of return on money r_{M1} and r_{M2} cannot make a full adjustment because part of each money stock is currency, which has a fixed nominal rate of zero. An increase in the domestic inflation rate π_1 therefore reduces R_{M1} , raises the yield spread R_s-R_{M1} , reduces the spreads $R_{M1}-R_{Xg} = R_{M1}$, and has no effect on R_{M2} or any of the spreads R_s-R_{M2} or $R_{M2}-R_{Xg} = R_{M2}$.

It may strike the reader as obvious that an increase in R_{M1} , holding constant everything else, would increase the demand for M_1 and reduce that for M_2 , thus reducing the degree of any currency substitution. An increase in R_{M1} would raise the demand for M_1 through the substitution effect, and, because total levels of consumption of X_1 and X_2 are given, there is no wealth effect to work in opposite direction. The net effect therefore seems straightforward. In fact, however, the relation between R_{M1} and the demand for M_1 is not at all straightforward, as we now see.

The issues are most easily explained by considering the simpler cases where S is not held. The same results apply in the cases where S is used. Begin with the profit expression for the case where M_1 is used to buy both goods:

$$(12) \quad V_{011} = R_{M1} \frac{X_1+X_2}{2} - [2B_{11}R_{M1}X_1]^{1/2} - [2B_{21}R_{M1}X_2]^{1/2} - F_1$$

The first derivative of this expression with respect to R_{M1} is

Figure 2: Shape of Profit Expression V_{011} 

$$(13) \quad \frac{\partial V_{011}}{\partial R_{MI}} = \frac{X_1 + X_2}{2} - \left[\left(\frac{B_{11}X_1}{2} \right)^{1/2} + \left(\frac{B_{21}X_2}{2} \right)^{1/2} \right] R_{MI}^{-1/2}$$

which can be positive or negative. The second term of this expression goes to zero as R_{MI} becomes large, so the derivative can be negative only for “small” values of R_{MI} . Indeed, the second derivative of V_{011} with respect to R_{MI} is everywhere positive:

$$(14) \quad \frac{\partial^2 V_{011}}{\partial R_{MI}^2} = \frac{1}{2} \left[\left(\frac{B_{11}X_1}{2} \right)^{1/2} + \left(\frac{B_{21}X_2}{2} \right)^{1/2} \right] R_{MI}^{-3/2} > 0$$

so that V_{011} has only one turning point. That turning point occurs at the profit minimum, which can be found easily by setting the first derivative to zero and solving for R_{MI} . Plugging the resulting value of R_{MI} into V_{011} shows that V_{011} is negative there. Also, V_{011} equals $-F_1$ if $R_{MI} = 0$. Taken together, these results indicate that the profit expression V_{011} has the form shown in Figure 2. All eight profit expressions have the same general shape, and there is a value R_{MI}^* for each profit expression, below which profit becomes negative. In the region where profit is positive (i.e., the region where it is relevant to the household's choice), the slope of profit with respect to R_{MI} is positive.

We are not really interested in how the individual profits V_{ijk} respond to a change in R_{MI} but rather how the profit differences respond. It is the changes in those differences that tell us whether the household switches from the domestic money to the foreign money or vice versa. For example, consider the difference

$$(15) \quad V_{011} - V_{012} = R_{MI} \frac{X_1 + X_2}{2} - (2B_{21}R_{MI}X_2)^{1/2} \\ - R_{MI} \frac{X_1}{2} - R_{M2} \frac{X_2}{2} + (2B_{22}R_{M2}X_2)^{1/2} + F_2$$

which determines whether the domestic money will be used to buy good X_2 given that it is used to buy good X_1 .¹⁰ The derivative of this expression with respect to R_{M1} is

$$(16) \quad \frac{\partial(V_{0,1,1} - V_{0,1,2})}{\partial R_{M1}} = \frac{X_2}{2} - \left(\frac{B_{21}X_2}{2} \right)^{1/2} R_{M1}^{-1/2}$$

which can be positive or negative but which is increasing in R_{M1} and positive for sufficiently large R_{M1} . The second derivative of the profit difference with respect to R_{M1} is everywhere positive, so there is only one turning point in the profit difference. That point occurs at the value of R_{M1} where the first derivative is zero, which is

$$(17) \quad R_{M1} = \frac{2B_{21}}{X_2}$$

At this value of R_{M1} , the profit difference $V_{011} - V_{012}$ is

$$(18) \quad (V_{011} - V_{012}) \Big|_{R_{M1} = \frac{2B_{21}}{X_2}} = -R_{M2} \frac{X_2}{2} + (2B_{22}R_{M2}X_2)^{1/2} + F_2$$

which is not necessarily negative. So, although the profit difference has the same general shape as V_{011} alone, it can be positive at its minimum (and therefore at all values of R_{M1}). Also, both the individual profit expressions V_{011} and V_{012} can be positive at the value of R_{M1} at which $V_{011} - V_{012}$ reaches its minimum.

The upshot is that it is possible for the profit difference to fall with an increase in R_{M1} even if the profit difference and the individual profit expressions are positive. That means that an increase in R_{M1} could lead to a *reduction* in the use of M_1 (the domestic money), even though there are no wealth effects at work. This seemingly perverse behavior arises from the interplay of the two profit expressions being compared. Each function depends on R_{M1} in a nonlinear way, positively through the direct effect of interest earnings on money balances and negatively through the indirect effect of R_{M1} on the optimal number of conversions between money and goods (and therefore on the amount of transactions costs paid). The two profits depend on R_{M1} in similar but not identical ways, so that the difference between them shows complicated non-linear behavior. The seeming perversity is the result. This perversity is present for sufficiently low values of R_{M1} and disappears for R_{M1} sufficiently large because (16) is increasing in R_{M1} . Consequently, the response of demand for M_1 to a change in R_{M1} (and therefore to domestic inflation π_1) may be different for high initial rates of inflation (and correspondingly low values of R_{M1}) than for low initial rates.

It is important to distinguish between two different effects at work here on the demand for domestic money. The effect we have just been discussing concerns whether or not M_1 is used to buy either or both of the consumption goods. We can think of this as an extensive margin. There also is an intensive margin. Given that M_1 is being used to buy at least one good, the amount that is held

¹⁰The same qualitative results arise from other such profit differences, so we do not consider them explicitly in this study.

is given by (see the Appendix)¹¹

$$(19) \quad \bar{M}_i = \left[\frac{A_i}{2(R_S - R_{M_i})} \sum_g X_{gi} \right]^{1/2} - \sum_g \left[\frac{B_{gi} X_{gi}}{2R_{M_i}} \right]^{1/2}$$

which is unambiguously increasing in R_{M_i} . A change in R_{M_i} can move in opposing directions along the intensive and extensive margins. An increase, for example, could reduce the number of goods purchased with M_i but at the same time raise the amount that is held for the purchase of those goods still bought with M_i . The net change in the total demand for M_i in this case would be ambiguous. If movement along the two margins is in complementary directions, of course, there is no ambiguity in what happens to the demand for M_i .

Note also that the precise behavior of the profit differences, their derivatives, and the quantities of each type of money held all depend on the values of the conversion costs and fixed costs. These costs capture the financial institutional structure that Savastano (1996) discusses. Savastano argues from the empirical evidence that the extent of currency substitution depends on how well developed a country's financial sector is. Countries with well-developed financial institutions can offer a wide range of transactions assets that can adjust to inflation and protect their owners from inflation, whereas countries that are financially "repressed" offer a much more restricted set of transactions assets with much less inflation protection. In terms of our model, financially developed countries will have many types of domestic money (such as demand deposits and money market mutual funds) that offer nominal interest rates, thus allowing them to adjust to inflation at least in part and help insulate the real rate of return from inflation. The fixed costs of holding those types of money will be low, as will the conversion costs associated with using them. The result will be much less incentive to substitute foreign media of exchange for domestic money in the presence of high inflation than in a financially repressed country.¹²

These conclusions provide the fundamental results for currency substitution. What we have just seen is that there are two dimensions to currency substitution - an extensive margin and an intensive one. Past discussions seem not to have made this distinction, measuring currency substitution solely by something like the (change in the) ratio of aggregate holdings of foreign money to holdings of domestic money holdings or the (change in the) ratio of foreign holdings to total holdings.¹³ Such a measure will be totally satisfactory only if the economy is moving along the intensive and extensive margins in a complementary way, something it need not necessarily do. Suppose, for example, that the economy is in the region where an increase in R_{M_i} reduces the

¹¹The expression obtained in the Appendix is more general than the following because here we have imposed the restriction that $R_{X_g} = 0$.

¹²However, there is at least one type of financial development that can increase the demand for currency substitution, namely, permission for citizens to hold and use foreign currency (Bahmani-Oskooee and Domac 2003).

¹³See Feige *et al.*, (2002, 2003) for a careful discussion of alternative methods of measuring currency substitution.

number of goods bought with M_1 but raises the amount of M_1 held for those goods still bought with it. We then have something similar to a “mixed market” on a stock exchange, where the number of losers, say, is larger than the number of gainers but the market as a whole gains. In the currency substitution analog, fewer goods are bought with M_1 but more M_1 is held. Should we say that currency substitution has increased or decreased? This distinction could be a useful one to explore for episodes of very high inflation, when the economy is most likely to be in the region of “perverse” behavior.

Having established the possibility of “mixed market” behavior, we now will simplify the subsequent discussion by assuming henceforth that R_{M1} initially is large enough (i.e., inflation is initially low enough) that an increase in its value makes use of M_1 more attractive. Then a reduction in R_{M1} caused by an increase in domestic inflation unambiguously will lead to a substitution of M_2 for M_1 . Nonetheless, even though we now have an unambiguous qualitative response, there still is the important issue of the quantitative response, which depends on other variables we have ignored so far.

B. Conversion Costs and Fixed Costs.

The effects of the conversion costs A_j and B_{ij} and the fixed costs F_j are straightforward and do not require detailed discussion. Increases in any of these costs reduces the value of using the associated asset S , M_1 , or M_2 ; all profit differences $V_{ijk}-V_{ij'k}$ change in the direction that indicates reduced value to using the asset whose associated cost has risen. Similarly, an increase in A_1 , B_{11} , and B_{12} makes the derivatives of all profit differences with respect to R_{M1} move in a direction that indicates reduced likelihood to use M_1 after an increase in π_1 occurs, so that currency substitution is more likely the more costly the domestic money already is to use. These results further support Savastano's contentions regarding the importance of financial institutional structure to the extent of currency substitution experienced by a country.

We now turn to two variables that affect currency substitution usually ignored in the existing literature.

C. Level of Income.

The level of income affects currency substitution in subtle and complex ways. Income equals purchases of goods, X_1+X_2 , so we can study the effects of income on currency substitution by examining the dependence of the various profit differences $V_{ijk}-V_{ij'k}$ on X_1+X_2 . It is easiest to start with the simpler case where households choose not to hold the savings asset S . We will examine the case where S is positive shortly.

Consider the choice between using domestic money M_1 exclusively or using M_1 to buy X_1 and M_2 to buy X_2 . The choice depends on the sign of the profit difference $V_{011}-V_{012}$, given in (15). We can rearrange the terms in that expression slightly to give the following more convenient form:

$$(20) \quad V_{011} - V_{012} = (R_{M1} - R_{M2}) \frac{X_2}{2} - [2B_{21} R_{M1} X_2]^{1/2} + [2B_{22} R_{M2} X_2]^{1/2} + F_2$$

The first three terms all depend positively on X_2 , but the first term increases linearly in X_2 whereas the other two terms increase in the square root of X_2 . Clearly, as X_2 rises, the first term dominates, and the whole expression becomes unambiguously positive. If X_2 is a normal good, then it increases when income increases, and we can conclude that a sufficiently high income will guarantee that the first term in (20) dominates everything else. The sign of that term depends directly on the sign of $R_{M1} - R_{M2}$. If $R_{M1} - R_{M2}$ is positive, then a high-income household will use only the domestic money M_1 ; if it is negative, the household will use M_1 to buy X_1 and M_2 to buy X_2 . In the latter case, we would observe currency substitution. The intuition is straightforward. The first term in (20) captures the interest income gained by using M_1 instead of M_2 (this “gain” will be negative if $R_{M1} < R_{M2}$), the second and third terms reflect the importance of conversion costs, and the fourth term captures the extra fixed cost associated with using M_2 along with M_1 .¹⁴ As in all transactions models of this type, the number of conversions increases in the square root of expenditure, whereas interest income increases linearly in income. Consequently, as income rises, the interest income becomes dominant, so that high income households base their money usage decision almost exclusively on the interest income term and tend to use only the money that pays higher interest. In particular, if their government is inflating the currency, then $R_{M1} - R_{M2}$ can become negative, and higher income households should start abandoning M_1 for M_2 . The same results hold for the profit differences corresponding to other usage choices: $V_{011} - V_{021}$, $V_{012} - V_{022}$, and so forth.

Lower income households can behave quite differently. For them, the second and third terms of (20) become important because the transactions costs and fixed costs are relatively more important than interest earnings when the volume of savings on which interest is earned is small. Low income households may continue to use M_1 exclusively even if R_{M1} falls below R_{M2} if B_{22} is large relative to B_{21} or if F_2 is high. We can conclude, then, that two households facing the same interest rates, conversion costs, and fixed costs but with different income levels may make opposite choices on which type of money to use in buying a particular good.

Despite the foregoing qualifications, we do seem to have the result that currency substitution is more likely among higher-income households than lower-income households. This prediction is consistent with Dotsey's (1988) evidence that higher income reduces the fraction of household expenditures paid with currency. Dotsey's data pertain only to domestic money and explore the split between currency on the one hand and all other forms of domestic money (e.g., demand deposits) on the other. He also has only imperfect measures of transactions costs (he uses the wage rate as a proxy for the value of time) and no measure at all of any fixed costs. His findings therefore are not a direct test of the relations predicted by the theory developed herein, but they are suggestive. Further evidence of the income effect on currency substitution comes from a study by Stix (2007)

¹⁴Presumably this fixed cost would be zero if M_2 consisted only of foreign currency and did not include any bank accounts, such as M_2 -denominated demand deposits, which might carry monthly fees or other fixed costs.

of survey data on households in Croatia, Slovenia, and Slovakia. Stix finds that households with higher education (which seems to be positively correlated with income) are more likely to use foreign currencies in making domestic purchases. Stix does not include measures of either transactions costs or fixed costs, so Stix's findings, like Dotsey's, must be treated as only suggestive. However, it is important to recognize the positive aspect of Dotsey's and Stix's findings. They are consistent with the theory presented herein and strongly suggest that an effort to measure and collect data on transactions costs and fixed costs would be worthwhile.

Presumably, our theoretical results carry over to entire countries. Citizens of a rich country have higher incomes on average than citizens of a poor country, so the representative agent for a rich country should behave more like a rich household than does the representative agent for a poor country. We thus are left with something of a mixed result for the relation between income and currency substitution. On the one hand, if R_{M1} exceeds R_{M2} , there should be less use of M_2 in rich countries than in poor countries. On the other hand, if R_{M1} is less than R_{M2} , the opposite is true.

So far, we have assumed that X_2 is a normal good. If X_2 is inferior, then its use falls with an increase in income, reversing all the conclusions concerning comparisons of rich and poor countries. Although possibly important for analyzing individual household behavior, this complication seems unlikely to be relevant for analysis of aggregate data. It seems almost certain that broad categories of goods, such as food or clothing, are normal even though individual items within those categories are inferior. We thus ignore inferior goods hereafter.

The foregoing results were derived for the case where the savings asset S is not used. When S is used, the relation between income and currency substitution is even more complex. Consider the profit difference analogous to (20) when S is held:

$$(21) \quad \begin{aligned} V_{S11} - V_{S12} = & -[2A_1(R_S - R_{MI})(X_1 + X_2)]^{1/2} - [2B_{21}R_{MI}X_2]^{1/2} \\ & + [2A_1(R_S - R_{MI})X_1]^{1/2} + [2A_2(R_S - R_{M2})X_2]^{1/2} \\ & + [2B_{22}R_{M2}X_2]^{1/2} + F_2 \end{aligned}$$

This expression is more complicated than (20) because of the presence of R_S and A_1 , but the really important difference is that now all income-related terms - X_1+X_2 , X_1 , and X_2 - enter as square roots, so that no one of them obviously becomes dominant as income rises. We therefore cannot make easy comparisons between rich and poor countries (or households). Taking the derivative of (21) with respect to X_1+X_2 yields

$$(22) \quad \begin{aligned} \frac{\partial V_{S11} - V_{S12}}{\partial (X_1 + X_2)} = & - \left[\frac{A_1(R_S - R_{MI})}{2(X_1 + X_2)} \right]^{1/2} + \left[\frac{A_1(R_S - R_{MI})}{2X_1} \right]^{1/2} \left(\frac{X_1}{X_1 + X_2} \right) \left(\left[\frac{dX_1}{X_1} \right] \left[\frac{d(X_1 + X_2)}{(X_1 + X_2)} \right]^{-1} \right) \\ & + \left\{ \left[\frac{A_2(R_S - R_{MI})}{2X_2} \right]^{1/2} + \left[\frac{B_{22}R_{M2}}{2X_2} \right]^{1/2} - \left[\frac{B_{21}R_{MI}}{2X_2} \right]^{1/2} \right\} \left(\frac{X_2}{X_1 + X_2} \right) \left(\left[\frac{dX_2}{X_2} \right] \left[\frac{d(X_1 + X_2)}{(X_1 + X_2)} \right]^{-1} \right) \end{aligned}$$

$$= - \left[\frac{A_1(R_S - R_{MI})}{2(X_1 + X_2)} \right]^{1/2} + \left[\frac{A_1(R_S - R_{MI})}{2X_1} \right]^{1/2} \sigma_1 \varepsilon_1$$

$$+ \left\{ \left[\frac{A_2(R_S - R_{MI})}{2X_2} \right]^{1/2} + \left[\frac{B_{22}R_{MI}}{2X_2} \right]^{1/2} - \left[\frac{B_{21}R_{MI}}{2X_2} \right]^{1/2} \right\} \sigma_2 \varepsilon_2$$

where σ_i and ε_i are the expenditure share and income elasticity of good I. This expression can be positive or negative, depending in part on the relative values of expenditure shares and income elasticities. In general, we can draw no conclusion about the sign of this expression, except that it may be different for different countries facing the same conversion costs, interest rates, and even levels of income because expenditure shares and income elasticities may well differ across countries.

There is still one more complication with the relation between income and currency substitution. Both (20) and (21) assume the use or non-use of S is given exogenously. In fact, however, the decision to use S is endogenous and depends on the level of income. Consider, for example, the profit difference $V_{S11} - V_{011}$:

$$(23) \quad V_{S11} - V_{011} = (R_s - R_{MI}) \left(\frac{X_1 + X_2}{2} \right) - [2A_1(R_S - R_{MI})(X_1 + X_2)]^{1/2} - F_S$$

This equation has much the same character as (20) in terms of how income enters. The first term increases linearly in income, whereas the second increases in the square root of income. Consequently, for sufficiently large income, the first term dominates and the whole expression is positive because $R_s > R_{MI}$. The intuition is the same as before; for high income, the extra interest earned dominates the additional conversion costs and fixed costs from using S. We thus should expect to see more people in high income countries using S than in low income countries. The implication for currency substitution is that it is not just the value of the objective function (that is, the profit difference) that varies with income but the form of that function itself. Rich countries are likely to be using profit differences like (21) to decide which money to use, whereas poor countries are more likely to use profit differences like (20). The two types of profit differences have very different sensitivities to changes in the domestic inflation rate (which cause R_{MI} to change in the opposite direction). We can see the differences clearly by examining the derivatives of the two profit differences with respect to R_{MI} . The derivative of $V_{011} - V_{012}$ with respect to R_{MI} is given by (16), repeated here for reference:

$$(16) \quad \frac{\partial(V_{011} - V_{012})}{\partial R_{MI}} = \frac{X_2}{2} - \left(\frac{B_{21}X_2}{2} \right)^{1/2} R_{MI}^{-1/2}$$

whereas the same derivative for (21) is

$$(24) \quad \frac{\partial(V_{S11} - V_{S12})}{\partial R_{MI}} = \left[\frac{A_1(X_1 + X_2)}{2(R_S - R_{MI})} \right]^{1/2} - \left[\frac{B_{21}X_2}{2R_{MI}} \right]^{1/2} - \left[\frac{A_1X_1}{2(R_S - R_{MI})} \right]^{1/2}$$

The latter depends on everything in the former and also on X_1 , A_1 , and R_S , and the signs of the two expressions can be different for several reasons.

The main implication is that there may not be a simple relation between currency substitution and inflation rates in a given data set. Our theory is not nihilistic, however. It does not say there is no relation between currency substitution and inflation or even that there is a relation that is vague or indefinite. There is a very specific relation dictated by the theory. It just is not simple.

D. Relative Expenditures.

An interesting implication of the theory presented here is that the degree of currency substitution depends on not just the level of income but also on the composition of expenditure. Two countries with the same income X_1+X_2 may choose to adopt different mixes of monies for a given inflation rate and definitely will have different sensitivities to changes in the inflation rate. These implications are easily seen from (20). Holding total income X_1+X_2 fixed, the first term on the right side of (20) becomes dominant as X_2 grows relative to X_1 , as explained earlier. The sign of (20) then depends on the sign of $R_{M1}-R_{M2}$. Consequently, as X_2 becomes a large part of total expenditure, the country is increasingly likely to use the higher-interest money to buy it; for large enough X_2 , that outcome is guaranteed. For example, if $R_{M1}-R_{M2}$ is positive, a country with a large value of X_2 is guaranteed to have a positive value of (20) and thus to use M_1 to buy X_2 , whereas a country with a lower value of X_2 (but the same value of X_1+X_2) may have a negative value of (20) and so may make the opposite choice.¹⁵ This example is enough to illustrate the point that the choice of which monies to use depends on expenditure's composition as well as its level.¹⁶

It should come as no surprise by now that the amount of currency substitution a country experiences in response to a change in its domestic inflation rate also depends on its composition of expenditure. Two countries with the same income level X_1+X_2 and that had chosen the same money usage pattern (such as, for example, holding no S and using M_1 to finance all purchases) could have different marginal responses to a change in R_{M1} . For example, the right side of equation (16) is increasing in X_2 ; it is guaranteed to be positive for X_2 sufficiently large but can be negative for smaller values of X_2 . Consequently, two countries that had equal incomes, that had chosen to hold no S , and that had chosen to use their domestic money M_1 to buy both goods (that is, they have the usage pattern $[0,1,1]$) could respond in opposite ways to an increase in domestic inflation

¹⁵Furthermore, because there are only two goods in this model, an increase in X_2 with X_1+X_2 held fixed necessarily entails a matching decrease in X_1 . As X_1 decreases, the logic we have just used to discuss the choice of which money to use in buying X_2 works in reverse for the choice of which money to use in buying X_1 . The country (or household) with a low value of X_1 is more likely to buy it with M_2 than M_1 , compared to a country with high X_1 . The result is a further difference from a country that has high X_1 and low X_2 .

¹⁶Other interesting implications of expenditure composition for usage patterns are discussed by Seater (2002).

(equivalently, a decrease in R_{M1}) if they have different relative values of X_2 that cause (16) to have opposite signs for the two countries.¹⁷

We thus see that the relation between currency substitution, inflation, and income is even more complex than it appeared in the previous section.¹⁸

E. Comparison with Standard Cash-in-Advance Models

Comparing the results obtained here with those obtained from standard cash-in-advance models is useful to clarify the differences between the two approaches and the results obtained from them.

The standard cash-in-advance (hereafter, CIA) model requires agents to accumulate cash before they can make purchases. There is no discussion of specific transactions costs in converting one type of asset into another or in making purchases. Higher income means higher purchases, which in turn means higher accumulations of cash to make the purchases. In the Baumol-Tobin (hereafter, BT) class of models, such as ours, agents hold more than one kind of transactions asset - cash and also an interest-earning savings asset. The amounts of each type of asset to hold and the timing for liquidating them depends on the various transactions costs that are absent from CIA models. The presence of an interest-earning savings asset breaks the one-to-one link between income and cash holdings. Higher income does imply higher purchases, but the higher purchases do not require higher money holdings. A household can accumulate more of the savings asset and then make many conversions between the savings asset and cash as it carries out its purchases. Keeping cash constant as income grows turns out to be sub-optimal, but that is an outcome of the household's optimization rather than of a rigid financing constraint. The savings asset also makes the incentive to engage in currency substitution weaker in BT models than in CIA models. In CIA models, the incentive is straightforward: cash accumulated ahead of purchases loses value in the presence of inflation and so is costly relative to a foreign currency that is not experiencing inflation. It is not surprising that there then is a relation between income and currency substitution. In BT

¹⁷It is easy to see that such a situation is possible, even if there are no fixed costs. For any given level of income, we can rule out the use of S by making A_1 and A_2 very large. We also can rule out the use of M_2 to buy X_1 by making B_{12} very large. That leaves only the usage patterns (0,1,1) and (0,1,2) to consider. We have not yet specified the values of R_{M1} , R_{M2} , B_{11} , or B_{21} , or X_2 (we are not free to choose X_1 because it is determined by choice of X_2 given that we already have fixed X_1+X_2), which gives us five degrees of freedom to make (15) barely positive and to make (16) either positive or negative as we choose. We thus can create a country that initially chooses the usage pattern (0,1,1) and that either retains or alters that pattern in response to a reduction in R_{M1} .

¹⁸It seems possible that some of the variables affecting currency substitution also would be relevant to the decision to join a currency union. The value of a currency union arises from a reduction in transactions costs compared to maintaining an independent currency. Our theory suggests that value may depend on variables, such as the composition of expenditures, not previously considered in the currency union literature (e.g., Alesina and Barro, 2001, 2002).

models, the interest rate on the savings asset adjusts to compensate for inflation, which partly insulates the households from the effects of inflation and so reduces the incentive to engage in currency substitution. The household can reduce the effects of inflation further by making more conversions from the savings asset to cash, thus reducing average cash holdings and the costs imposed by inflation.

Sturzenegger (1997) constructs an extended CIA model that allows agents to have different levels of income and buy a variety of goods. Each good is indivisible, only one unit of each good can be bought in any period, and higher quality goods (in terms of marginal utility of one unit) are more expensive to buy. Agents may use the domestic currency to buy any good at no cost. They also may use a foreign currency to buy goods. However, use of the foreign currency is costless only for goods that previously have been bought with it. Adding a good to the set bought with the foreign currency requires payment of a sunk cost. With this structure, higher-income agents buy a greater variety of goods, with the additional goods being those that are more costly. Cash holdings are positively related to income because expenditures rise with income and cash must be accumulated to make all purchases. Consequently, the inflation tax increases with income. The sunk cost and the positive relation between income and the benefits of using the foreign currency lead to the result that currency substitution will be more prevalent among high income households and that those households will use foreign currency to buy the most expensive goods.¹⁹ In our model, the relation among income, the consumption bundle, and currency substitution is completely different. Quantities purchased are determined outside the model, so no distinction need be made between expensive and inexpensive goods, and indeed none is made. There simply are two goods bought in some predetermined quantities. There is no unit restriction on the quantities purchased, the goods are divisible, and there is no restriction on the relative quantities of the goods. The only question is which currency to use to buy each good. That choice depends on the vector of transactions costs, absent from standard CIA models in general and Sturzenegger's model in particular. In our model currency substitution is not necessarily positively related to income. The relation depends on the opportunity costs of holding cash, the transactions costs associated with converting the savings asset to cash and making purchases, and whether some of the goods bought are normal or inferior. Also, in our model two countries (or households) with equal incomes but different consumption bundles (presumably because of different utility functions) can respond in opposite ways to an increase in domestic inflation, something that Sturzenegger's framework does not permit by construction.

Finally, as already emphasized earlier, the results obtained here pertain only to the demand for currency substitution. They are not general or even partial equilibrium results, in contrast to the general equilibrium results obtained from CIA models.

¹⁹The model also generates persistence of currency substitution. Once the household has paid the sunk cost to use foreign currency to buy a good, it has no incentive to revert to use of the domestic currency because the transactions costs for continuing to use the foreign currency is the same as for using domestic currency, namely, zero.

IV. CONCLUSION

We have used an extended transactions model of the demand for money to provide a theory of the demand for currency substitution. Our theory predicts that factors generally ignored by the currency substitution literature are important in determining the extent of currency substitution. Besides the usual only interest rates and transactions costs, fixed costs, the level of income, and the composition of the consumption bundle affect the demand for currency substitution. Even some of the results pertaining to traditionally included variables, such as interest rates, are surprising. Specifically, we derived the following results:

- (A) For example, the qualitative (not just quantitative) response of demand for currency substitution to an increase in inflation can be different for low inflation rates than high rates. At high rates, demand always increases with inflation, but at low rates, demand for currency substitution may fall with a small increase in inflation.
- (B) Demand for substitution takes place on both extensive and intensive margins - how many goods bought with each currency and how much money is held to buy goods for which that money is used. The demand on the two margins does not necessarily respond to an increase in inflation in the same direction.
- (C) Financially developed countries offer more ways to insulate from inflation, so there would be less currency substitution for a given level of inflation.
- (D) Currency substitution generally should be more prevalent among high income than low income households or countries.
- (E) At the household level, the fraction of the consumption bundle that comprises inferior goods affects the decision to substitute.
- (F) Both qualitative and quantitative predictions concerning the household's decision to engage in currency substitution depend on whether or not the household uses the savings asset.
- (G) The composition of consumption bundle matters, apart from normality or inferiority of goods.

Many or even all these predictions are testable, given appropriate data. Several of the important variables are observable, at least in principle, so the theory suggests both new tests and new data needed to undertake those tests.

Countries typically will have different levels of income, different distributions of income, different aggregate consumption bundles, and of course different transactions and fixed costs. We

should expect to see different amounts of currency substitution across countries even in the face of similar inflation rates. The theory thus suggests possible explanations for the quite different histories of currency substitution that Feige (2003) and Feige *et al.* (2002, 2003) document for Latin America and the countries of the former Soviet Union. The relation between income and currency substitution is complex and cannot be summarized as a simple relation such as “high income increases (or decreases) the extent of currency substitution.” By providing an explicit model for the demand for multiple media of exchange, the theory offers guidance in how to construct empirical specifications. The dependence of currency substitution on the composition of expenditures is perhaps the most surprising result derived. Again, the relation is complex, but the explicit solutions provided by the theory give guidance on how to handle that relation in empirical work. The data required to test the predictions of the theory are not readily available, but they are not impossible to assemble. The best tests probably would be based on household data, similar in nature to the data that Colacelli and Blackburn (2007) collected for their study of secondary currency use in Argentina. Stix (2007) using a limited survey of households in Croatia, Slovenia, and Slovakia, finds support for one of the new predictions of the theory, that currency substitution is positively related to household income. Developing more complete household survey data sets would be a valuable extension that would allow further tests.

Finally, our theory offers a rigorous foundation for Savastano's (1996) suggestion that the extent of currency substitution in a country depends on the financial institutional framework in that country. Savastano argued that currency substitution would be less extensive in countries with well-developed financial systems that offer media of exchange that can compensate users for inflation. Our framework captures that element of institutional framework in the real rates of return, transactions costs, and fixed costs it attaches to alternative monies. Our results support Savastano's contention; financial structure does matter in determining the extent of currency substitution.

APPENDIX

TABLE A
General Form of Profit Functions: R_{Xg} Not Constrained to Zero

$$V_{0,1,1} = R_{M1} \frac{X_1 + X_2}{2} - [2B_{11}(R_{M1} - R_{X1})X_1]^{1/2} - [2B_{21}(R_{M1} - R_{X2})X_2]^{1/2} - F_1$$

$$V_{0,1,2} = R_{M1} \frac{X_1}{2} + R_{M2} \frac{X_2}{2} - [2B_{11}(R_{M1} - R_{X1})X_1]^{1/2} - [2B_{22}(R_{M2} - R_{X2})X_2]^{1/2} - F_1 - F_2$$

$$V_{0,2,1} = R_{M1} \frac{X_2}{2} + R_{M2} \frac{X_1}{2} - [2B_{21}(R_{M1} - R_{X2})X_2]^{1/2} - [2B_{12}(R_{M2} - R_{X1})X_1]^{1/2} - F_1 - F_2$$

$$V_{0,2,2} = R_{M2} \frac{X_1 + X_2}{2} - [2B_{12}(R_{M2} - R_{X1})X_1]^{1/2} - [2B_{22}(R_{M2} - R_{X2})X_2]^{1/2} - F_2$$

$$V_{S,1,1} = R_S \frac{X_1 + X_2}{2} - [2A_1(R_S - R_{M1})(X_1 + X_2)]^{1/2} \\ - [2B_{11}(R_{M1} - R_{X1})X_1]^{1/2} - [2B_{21}(R_{M1} - R_{X2})X_2]^{1/2} - F_S - F_1$$

$$V_{S,1,2} = R_S \frac{X_1 + X_2}{2} - [2A_1(R_S - R_{M1})X_1]^{1/2} - [2A_2(R_S - R_{M2})X_2]^{1/2} \\ - [2B_{11}(R_{M1} - R_{X1})X_1]^{1/2} - [2B_{22}(R_{M2} - R_{X2})X_2]^{1/2} - F_S - F_1 - F_2$$

$$V_{S,2,1} = R_S \frac{X_1 + X_2}{2} - [2A_1(R_S - R_{M1})X_2]^{1/2} - [2A_2(R_S - R_{M2})X_1]^{1/2} \\ - [2B_{21}(R_{M1} - R_{X2})X_2]^{1/2} - [2B_{12}(R_{M2} - R_{X1})X_1]^{1/2} - F_S - F_1 - F_2$$

$$V_{S,2,2} = R_S \frac{X_1 + X_2}{2} - [2A_2(R_S - R_{M2})(X_1 + X_2)]^{1/2} \\ - [2B_{12}(R_{M2} - R_{X1})X_1]^{1/2} - [2B_{22}(R_{M2} - R_{X2})X_2]^{1/2} - F_S - F_2$$

NOTE: The subscripts in the profit expression V_{ijk} have the following meanings:

I = S if the saving asset is used, 0 otherwise

j = 1 or 2 as M_1 or M_2 is used to buy good 1

k = 1 or 2 as M_1 or M_2 is used to buy good 2

1. Average assets. Average total real assets are

$$\bar{W} = \bar{S} + \sum_{i=1}^L \bar{M}_i + \sum_{g=1}^G \bar{X}_g = \bar{S} + \sum_{i=1}^L \left(\bar{M}_i + \sum_{g=1}^G \bar{X}_{gi} \right)$$

We also have, because asset conversions and shopping trips are evenly spaced and the rate of consumption is constant,

$$\bar{W} = \sum_g \frac{X_g}{2}$$

$$\bar{M}_i + \sum_g \bar{X}_{gi} = \sum_g \frac{X_{gi}}{2T_i}$$

$$\bar{X}_{gi} = \frac{X_{gi}}{2Z_{gi}}$$

We then have

$$\bar{M}_i = \left(\bar{M}_i + \sum_g \bar{X}_{gi} \right) - \sum_g \bar{X}_{gi} = \frac{\sum_g X_{gi}}{2T_i} - \frac{\sum_g X_{gi}}{2Z_{gi}}$$

$$\bar{S} = \bar{W} - \sum_i \left(\bar{M}_i + \sum_g \bar{X}_{gi} \right) = \sum_g \frac{X_g}{2} - \sum_i \sum_g \frac{X_{gi}}{2T_i}$$

2. Optimal values of T_i and Z_{gi} . Substituting the foregoing expressions in the profit expression gives

$$\begin{aligned} V = & R_S \left[\sum_g \left(\frac{X_g}{2} - \sum_i \frac{X_{gi}}{2T_i} \right) \right] + \sum_i R_{M_i} \left[\sum_g \left(\frac{X_{gi}}{2T_i} - \frac{X_{gi}}{2Z_{gi}} \right) \right] + \sum_g R_{X_g} \left(\sum_i \frac{X_{gi}}{2Z_{gi}} \right) \\ & - \sum_i T_i A_i - \sum_i \sum_g Z_{gi} B_{gi} - F_S I(S) - \sum_i F_i I(M_i) \end{aligned}$$

The first-order conditions for T_i and Z_{gi} are

$$\frac{\partial V}{\partial T_i} = 0$$

$$\frac{\partial V}{\partial Z_{gi}} = 0$$

which give the solutions

$$T_i = \left[(R_S - R_{Mi}) \sum_g \frac{X_{gi}}{2A_i} \right]^{1/2}$$

$$Z_{gi} = \left[(R_{Mi} - R_{Xg}) \frac{X_{gi}}{2B_{gi}} \right]^{1/2}$$

If $X_{gi}=0$ for all g , money I is not used and drops out of the maximization problem. In that case, of course, both T_i and Z_{gi} automatically are zero.

These first-order conditions assume that T_i and Z_{gi} can be treated as continuous variables, whereas in fact they must be integers. Barro (1976) shows that the correct values are found by computing the continuous solutions as above and then choosing the integers on either side of those solutions that maximize profit. He also shows that the qualitative results are the same for the continuous approximation as for the exact solution, so it is standard procedure to simplify the analysis by discussing only the continuous solution, as done in the analysis here.

3. Average assets again. Substituting these last expressions into those above for average assets gives

$$\bar{X}_{gi} = \left[\frac{X_{gi} B_{gi}}{2(R_{Mi} - R_{Xg})} \right]^{1/2} = \left[\frac{X_{gi} B_{gi}}{2R_{Mi}} \right]^{1/2}$$

$$\bar{M}_i = \left[\frac{A_i}{2(R_S - R_{Mi})} \sum_g X_{gi} \right]^{1/2} - \sum_g \left[\frac{B_{gi} X_{gi}}{2(R_{Mi} - R_{Xg})} \right]^{1/2}$$

$$\bar{S} = \sum_g \frac{X_g}{2} - \sum_i \left[\frac{A_i}{2(R_S - R_{Mi})} \sum_g X_{gi} \right]^{1/2}$$

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