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Evaluating the New Keynesian Phillips Curve under VAR-based Learning

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Abstract

This paper proposes an econometric evaluation of the New Keynesian Phillips Curve (NKPC) in the euro area, under a particular specification of the adaptive learning hypothesis. The key assumption is that agents' perceived law of motion is a Vector Autoregressive (VAR) model, whose coefficients are updated by maximum likelihood estimation as the information set increases over time. Each time new data is available, likelihood ratio tests for the cross-equation restrictions that the NKPC imposes on the VAR coefficients are computed and compared with a proper set of critical values, which take the sequential nature of the test into account. The analysis focuses on the case in which the variables can be approximated as nonstationary cointegrated processes. Results on quarterly data relative to the period 1981−2006 show that: (i) the euro area inflation rate and the wage share are cointegrated, although their relationship does not appear stable during the eighties and first nineties; (ii) the cointegrated version of the 'hybrid' NKPC is sharply rejected under the rational expectations hypothesis; (iii) the NKPC is rejected also when the model is evaluated under a particular formulation of the adaptive learning hypothesis over the monitoring period 1986−2006.

JEL: C32, C52, D83, E10

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1 Introduction

There is growing awareness among applied researchers that the New Keynesian Phillips Curve (NKPC),[1] which plays a dominating role in the monetary policy literature, provides a poor explanation of inflation dynamics and persistence in developed countries, see Fuhrer and Moore (1995), Fuhrer (1997), Ruud and Whelan (2005*a*, 2005*b*, 2006), Boug *et al.* (2007) and Fanelli (2008), just to mention a few. Many explanations have been provided, including identification issues (Mavroeidis, 2005; Nason and Smith, 2008; Boug *et al.*, 2007), dynamic misspecification (Bårdsen *et al.* 2004; Dees *et al.*, 2008; Fanelli, 2008) and neglected nonstationarity (Juselius, 2006; Fanelli, 2008).

The NKPC is grounded on the rational expectations hypothesis (REH) and the econometric investigations generally maintain that such an hypothesis holds. Aside from the difficulties of disentangling forward- from backward-looking behaviour on empirical grounds (Hendry, 1988), many authors argue that in practice agents depart from the REH and display either imperfect knowledge (Goldberg and Frydman, 2007) or 'bounded' rationality, see Pesaran (1987), Sargent (1999) and Evans and Honkapohja (1999, 2001). For instance, focusing on exchange rate markets, Goldberg and Frydman (1996) argue that an alternative, more plausible assumption is that economic agents have only imperfect knowledge of the true relationship between exchange rates and fundamentals and appeal mainly to qualitative, rather than quantitative, knowledge about the economy. In the monetary policy framework, the idea that inflation expectations may not be rational and that deviations from the REH may represent a remarkable source of inflation persistence is well a recognized fact, see Roberts (1997) and Milani (2005). Sargent (1999), Orphanides and Williams (2005) and Primiceri (2006) provide examples in which rational policymakers learn about the behavior of the economy in real time and set stabilization policies conditional on their current beliefs.[2]

The traditional approach to modelling boundedly rational expectations assumes that agents behave as econometricians when making forecasts and use adaptive learning algorithms to update their beliefs (Evans and Honkapohja 1999, 2001). This means that they estimate and update the parameters of their forecasting model - the perceived law of motion - according to recursive rules. Replacing expectations in the forward-looking model, with the forecasts implied by the perceived law of motion, yields the so-called actual law of motion, which reads as the agents' data generating process. Under certain conditions, it has been found that expectations in these models can converge to a rational expectations equilibrium (or to a restricted perception equilibrium, Evans and Honkapohja, 2001), that means that in the limit the actual law of motion is indistinguishable from the model solution obtained under the REH.

[1] All acronyms used in the paper are reported in Table 1.

[2] Moreover, optimal policy rules designed under the REH may not perform satisfactorily if instead of having rational expectations private agents follow learning rules: Bullard and Mitra (2007) show that in this case the stability of the Taylor-type rules can not be taken for granted, see also Evans and McGough (2005). Evans and Honkapohja (2003*a*, 2003*b*) show how optimal monetary can be designed in these situations, provided that the mechanism generating private expectations is suitably incorporated into the model.

The notion of adaptive learning has been applied in the macroeconomic literature mainly as a selection criterion when the rational expectations model generates multiple solutions and in connection with the concept of 'learnability' and stability of rational expectations equilibria. On the econometric side, however, although the literature typically focuses on the problem of estimating the actual law of motion through recursive (possibly Bayesian) methods, little is known about inference in these models, including the issue of testing the model adequacy under the adaptive learning hypothesis (ALH).

The focus of this paper is not on the agents' estimation problem, but rather on the problem of testing the data adequacy of the NKPC under a simple formulation of the ALH.[3] If one finds that a forward-looking model is rejected by the data under the REH but is supported under the ALH, it can be reasonably argued that adaptive learning captures actual agents' behaviour and can potentially reconcile a class of forward-looking models with the data. Moreover, a test of the forward-looking model under the ALH, allows the researcher to assess whether the claimed process of convergence to a potential rational expectation equilibrium (or restricted perception equilibrium) is consistent with the data or not.

To our knowledge Fanelli and Palomba (2007) is the only contribution in which the econometric analysis of the NKPC has been addressed by using Vector Autoregressive (VAR) models and recursive likelihood-based methods, under a particular specification of the ALH.[4] In this paper we consider the case in which the inflation rate and/or its driving variable(s) can be approximated as nonstationary cointegrated processes; differently from Fanelli and Palomba (2007), we do not use simulation-based methods.

The investigation of the NKPC is carried out under the following assumptions: **(i)** agents use VAR models including inflation and its driving variables as their perceived law of motion; **(ii)** VAR coefficients are updated recursively through the reiterate application of maximum likelihood estimation.

We discuss in detail each of the two assumptions and related implications.

Assumption (i): the idea that agents use VARs to form their expectations is not new in the literature and is known as the 'VAR expectations' hypothesis (hereafter VEH), see Brayton *et al.* (1997), Johansen and Swensen (1999), Kozicki and Tinsley (1999), Branch (2004). Under the VEH, it is possible to derive a set of cross-equation restrictions between the VAR coefficients and the NKPC along the lines of Fanelli (2008). The VEH coincides with the REH when the (determinate) solution of the rational expectations model can be nested within the VAR for the data.[5]

Assumption (ii): we use a simple formulation of the ALH, based on the assumption that agents update coefficient estimates through Recursive Least Squares;

[3]See In-Koo and Kasa (2005) for an example in which the issue of learning and model validation is addressed.

[4]The literature provides several examples where dynamic stochastic general equilibrium models including the NKPC are investigated under the adaptive learning hypothesis, however, Milani (2005) is the only contribution where 'the econometric analysis of the NKPC under learning rules' is explicitly addressed. More on this in Section 4.

[5]We *do not assume* that the perceived law of motion *necessarily coincides* with the minimum state variable solution (McCallum, 1983, 2003) of the system.

this algorithm amounts to the re-iterated application of maximum likelihood with Gaussian disturbances. Given the recursive nature of the estimation problem, the cross-equation restrictions between the VAR coefficients and the NKPC have a sequential nature and can be recursively tested through likelihood-ratio tests obtained by estimating the VAR unrestrictedly and subject to the restrictions, over the monitoring period. From the inferential point of view, however, standard asymptotic theory can not be applied due to the law of iterated logarithms, hence the critical values must take the sequential nature of the test into account (Inoue and Rossi, 2005).

The proposed method, based on the assumptions above, is applied to investigate the NKPC in the euro area, using quarterly data for the period 1981-2006. We proxy firms real marginal costs by the wage share as in Galí *et al.* (2001) and include a short term nominal interest rate in the system, given the influence of interest rates on marginal costs through the so-called cost channel of monetary transmission (Chowdhury *et al.*, 2005). In previous research, Bårdsen *et al.* (2004), O'Reilly and Whelan (2005) and Fanelli (2008) have shown that the 'hybrid' formulation of the NKPC under the REH does not capture inflation dynamics successfully in the euro area. In this paper we find that the nonstationarity of variables is an issue and that the inflation rate and the wage share are cointegrated over the period 1981-2006, albeit the stability of their relationship is not fully supported by the data in the eighties and first nineties. Moreover, we find that the NKPC is sharply rejected under the VEH as well as under the chosen formulation of the ALH, over the period 1986-2006.

The rest of the paper is organized as follows. Section 2 introduces the NKPC and discusses two possible representations of the model when variables can be approximated as nonstationary processes. Section 3, which is divided into two subsections, proposes the econometric investigation of the cointegrated NKPC under the ALH. Section 4 applies the method to investigate the NKPC in the euro area data and discusses its relative implications. Section 5 contains some concluding remarks.

Table 1: Acronyms used in the paper.

Mnemonic	Definition
ALH	Adaptive Learning Hypothesis
AWM	Area Wide Model
BFGS	Broyden, Fletcher, Goldfarb, Shanno method, see Fletcher (1987)
MDS	Martingale Difference Sequence
NKPC	New Keynesian Phillips Curve
REH	Rational Expectations Hypothesis
VAR	Vector Autoregressive
VEH	VAR Expectations Hypothesis
VEqC	Vector Equilibrium Correction

2 Model and Cointegration Implications

A variety of pricing environments within the New Keynesian tradition give rise to the following 'hybrid' version of the NKPC

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda x_t + u_t \qquad (1)$$

where π_t is the inflation rate at time t, x_t is a scalar explanatory variable related to firms' real marginal costs and $E_t\pi_{t+1}$ indicates the expected value of π_{t+1} formed at time t on the basis of the available information summarized in the (non decreasing) sigma-field $\Omega_t \subseteq \Omega_{t+1}$, i.e. $E_t\pi_{t+1} = E(\pi_{t+1} \mid \Omega_t)$. γ_f, γ_b and λ are the structural parameters.

The properties of the process generating x_t in (1) are crucial for the derivation of the model solution and for the identification of the parameters; for ease of exposition, we leave first the process generating x_t unspecified; in the next section, the dynamics of x_t will follow a VAR model for π_t and x_t (and possibly other variables). The NKPC in (1) has been specified by including a disturbance term u_t. In the literature, it is a common practice to assume that the shock u_t obeys autoregressive dynamics (typically an AR(1) process) but we here interpret u_t as a term capturing unexplained (transitory) deviations from the theory (Kurmann, 2007) and assume that it follows a martingale difference sequence (MDS) with respect to Ω_t.

The structural parameters γ_f, γ_b and λ of model (1) can be generally expressed as nonlinear functions of 'primitive' parameters of the model, related to consumers and firms preferences. For instance, in the Calvo model of Galí and Gertler (1999) and Galí *et al.* (2001) one has

$$\gamma_f = \frac{\rho\theta}{\theta + \omega[1-\theta(1-\rho)]} \qquad (2)$$

$$\gamma_b = \frac{\varpi}{\theta + \varpi[1-\theta(1-\rho)]} \qquad (3)$$

$$\lambda = \frac{(1-\varpi)(1-\theta)(1-\theta\rho)}{\theta + \varpi[1-\theta(1-\rho)]} \qquad (4)$$

where ρ is firms' discount factor, $0 < \varpi < 1$ is the fraction of backward-looking firms in the economy that change their prices following rule-of-thumb behavior and $0 < \theta < 1$ is the probability that a firm will be unable to change its price in a given period, so that $(1-\theta)^{-1}$ is the average duration over which a price is fixed. By construction $\gamma_f + \gamma_b \leq 1$.

The empirical assessment of the NKPC (1) under the REH has attracted a great deal of research. The estimation through 'full-information' likelihood-based techniques requires the explicit specification of the process generating x_t and the derivation of the (possibly unique) reduced form solution associated with the model. The process generating x_t can be either a reduced form as in Pesaran (1987, Chap. 7) and Fuhrer and Moore (1995) and Fuhrer (1997), or a structural equation drawn from a 'small-scale' dynamic stochastic general equilibrium model of monetary policy as in Lindè (2005).

An alternative 'full-information' approach, based on the VEH, works under the (implicit) assumption that the data generating process belongs to the correctly specified VAR for $Z_t = (\pi_t \colon x_t)'$. The application of the method of undetermined coefficients allows to retrieve a set of cross-equation restrictions between the VAR coefficients and the NKPC parameters, which can be used to discuss the identifiability of the structural parameters and to estimate and test the model by maximum likelihood, see Kurmann (2007) and Fanelli (2008).

Aside from estimation methods, the empirical investigation of the NKPC (1) is usually carried out assuming that variables are generated by stationary processes, in line with the idea that the NKPC is derived from a dynamic stochastic general equilibrium model which is solved by log-linearizing around a steady state. We refer to Dees *et al.* (2008) for a comprehensive discussion of the perils of computing steady states through simple means or statistical procedures and for possible remedies. If stationarity is wrongly assumed, inference in the class of models of the form (1) may be misleading, see Johansen (2006) and Juselius and Franchi (2007).

Two formulations of the original NKPC model (1) are worth considering when variables are nonstationary and cointegrated. The former is based on the restriction $\gamma_f + \gamma_b = 1$ and is obtained by manipulating equation (1) in the form

$$\Delta\pi_t = \psi E_t \Delta\pi_{t+1} + \omega x_t + u_t^* \tag{5}$$

where

$$\psi = \frac{1-\gamma_b}{\gamma_b} \tag{6}$$

$$\omega = \frac{\lambda}{\gamma_b} \tag{7}$$

and $u_t^* = u_t/\gamma_b$. This representation emphasizes the role of the stationary variable x_t as driving force of the inflation acceleration rate, $\Delta\pi_t$. Equation (5) fits the data when π_t is integrated of order one (I(1)) and x_t is stationary (for example the output gap should be stationary by construction).

The latter formulation of the NKPC is based on the assumption that π_t and x_t are both I(1) and share a common stochastic trend.[6] Provided that $\gamma_f + \gamma_b < 1$ and assuming that π_t and x_t are cointegrated with coefficients $(1, -\phi)$, the model is obtained by manipulating equation (1) in the form

$$\Delta\pi_t = \psi E_t \Delta\pi_{t+1} + \omega(\pi_t - \phi x_t) + u_t^* \tag{8}$$

[6] In Section 4 we find that the euro area inflation rate and the wage share (log of real unit labour costs) are cointegrated over the period 1981-2006. We do not attach any particular interpretation to this finding, see Fanelli (2008) for details.

where

$$\psi = \frac{\gamma_f}{\gamma_b}, \tag{9}$$

$$\omega = -\frac{[1-(\gamma_f+\gamma_b)]}{\gamma_b}, \tag{10}$$

$$\phi = \frac{\lambda}{[1-(\gamma_f+\gamma_b)]}. \tag{11}$$

The interesting feature of the cointegrated NKPC (8) is that equation (11) establishes a link between the slope parameter λ and the cointegration parameter ϕ: in general, for given ϕ, γ_f and γ_b, $\lambda = \phi[1-(\gamma_f+\gamma_b)]$ is automatically determined, see Fanelli (2008).

Disentangling between the formulation (5)-(7) and the formulation (8)-(11) of the cointegrated NKPC is an empirical question, that can be addressed by investigating the cointegration properties of the system $Z_t = (\pi_t \colon x_t : a_t')'$, where a_t is a $q_a \times 1$ vector of additional variables which are deemed to be relevant for the analysis. If, for instance, it is found that Z_t embodies a single cointegrating relation,[7] one can test whether $\beta' Z_t$ is consistent with the structure $\beta' = (0, 1, 0')$ (model (5)), or with the structure $\beta' = (1, -\phi, 0')$ (model (8)).

The two representations (8) and (5) can be nested in the more general specification

$$\Delta\pi_t^d = \psi\ E_t \Delta\pi_{t+1}^d + \omega(\beta' Z_t)^d + u_t^* \tag{12}$$

where u_t^* is a MDS and ψ and ω are determined according to (6)-(7) or (9)-(10), depending on the structure of β. In (12) we have used the superscript 'd' to remark that the variables entering the model represent deviations from constant (deterministic steady state) levels.

To investigate the NKPC (12) we derive the restrictions that this model imposes on the VAR for Z_t, where the latter is assumed to capture the statistical properties of the observed time series. Provided that the VAR is identifiable under the restrictions, one can compute likelihood-ratio tests as in Fanelli and Palomba (2007) and Fanelli (2008). In the next section we briefly review the method and extend it to a recursive framework to account for the econometric implications of the ALH.

3 The Cointegrated NKPC Under the Adaptive Learning Hypothesis

For ease of exposition we divide this section into two parts. In Subsection 3.1 we review the analysis of the NKPC under the VEH and in Subsection 3.2 we derive and discuss the implications that arise when the ALH is assumed.

[7] Of course, when the cointegration rank is greater than one, the researcher faces the problem of identifying the additional long run relationships.

3.1 Cross-Equation Restrictions Under the VEH

Assume that the law of motion for the $p \times 1$ vector of "raw" observable variables, $Z_t = (\pi_t : x_t : a_t')'$, is given by

$$Z_t = \sum_{i=1}^{k} A_i Z_{t-i} + \Theta D_t + \varepsilon_t \tag{13}$$

where k is the lag length, Z_0, $Z_{-1}, ..., Z_{1-k}$ are fixed, A_i, $i = 1, 2, ..., k$ are $p \times p$ matrices of parameters, D_t is an $d_0 \times 1$ vector of deterministic terms (constant, linear trend, deterministic dummies, etc.) with associated $p \times d_0$ matrix of parameters, Θ and ε_t is a MDS with respect to the sigma-field $\mathcal{H}_t = \sigma(Z_t, Z_{t-1}, ..., Z_1) \subseteq \Omega_t$ and has Gaussian distribution and covariance matrix Σ_ε.

Given the VAR characteristic polynomial $A(L) = I_p - A_1 L - \cdots - A_k L^k$, where L is the lag operator, it is assumed that the roots, s, of $\det[A(s)] = 0$ are $\mid s \mid \geq 1$, hence explosive roots are ruled out. Moreover, we maintain that when there are (unit) roots at $s = 1$, their exact number is $p - r$, where r, $0 < r < p$, is the cointegration rank of the system; this means that the focus is on I(1) cointegrated processes, see Johansen (1996).

The VAR (13) is here treated as the agents' forecast model (perceived law of motion) and it is assumed that the VAR lag length, k, fulfils the restriction $pk \geq 3 + (p - r)$. As it will be clear below, this restriction is necessary to guarantee the local identifiability of the constrained VAR.

Given the cointegrated VAR (13), we consider the corresponding Vector Equilibrium Correction (VEqC) counterpart

$$\Delta Z_t = \alpha\beta' Z_{t-1} + \sum_{i=1}^{k-1} \Phi_i \Delta Z_{t-i+1} + \mu + \varepsilon_t \tag{14}$$

where $\alpha\beta' = -(I_p - \sum_{i=1}^{k} A_i)$ is the long run impact matrix with α and β two $p \times r$ full rank matrices and $\Phi_j = -\sum_{i=j+1}^{k} A_i$, $j = 1, ..., k-1$; for ease of exposition and in line with the results of Section 4, we consider the case where $D_t = 1$ and $\Theta = \mu$, admitting the presence of deterministic linear trends in the system. Under a suitable set of identifying restrictions, the elements in the $r \times 1$ vector $\beta' Z_t$ define the cointegration relations of the system (Johansen, 1996; Juselius, 2006).

Finally, to derive the cross-equation restrictions between the VEqC and the representation (12) of the NKPC, it is convenient to re-arrange the variables in 'triangular' form, introducing the vector

$$W_t = \begin{pmatrix} \beta' Z_t \\ v' \Delta Z_t \end{pmatrix} \equiv \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} \quad \begin{matrix} r \times 1 \\ (p-r) \times 1 \end{matrix} \tag{15}$$

where v is a $p \times (p - r)$ matrix such that $\det(v'\beta_\perp) \neq 0$ and $\beta_\perp$ is the orthogonal complement of β (Johansen, 1996). It can be proved (Mellader *et al.*, 1992; Paruolo, 2003, Theorem 2) that W_t admits the VAR representation

$$W_t = \sum_{i=1}^{k} B_i W_{t-i} + \mu^0 + \varepsilon_t^0 \tag{16}$$

where $\varepsilon_t^0 = (\beta,v)'\varepsilon_t$ is a MDS with covariance matrix Σ_{ε^0}, B_i, $i = 1, ..., k$ are $p \times p$ matrices and μ^0 is a $p \times 1$ vector; the elements in B_i and μ^0 depend on the elements in α, Φ_is and μ. Furthermore, it can be proved that B_k is restricted as

$$B_k = B_k^* \equiv \left[\underset{p\times r}{B_{w1,k}} \vdots \underset{p\times(p-r)}{O} \right] \tag{17}$$

and that the VAR (16)-(17) is (asymptotically) stable, i.e. the roots of $\det(I_p - \sum_{i=1}^{k} B_i s^i) = 0$ are $\mid s \mid > 1$.

As the VAR (16)-(17) is stationary by construction, we define the demeaned process $W_t^d = W_t - E(W_t)$, where $E(W_t) = B(1)^{-1}\mu^0$ and $B(L)W_t^d = \varepsilon_t^0$. Hereafter the analysis will be developed with respect to the W_t^d process, in line with the formulation (12) of the NKPC, which is also expressed in demeaned form. In this setup, ℓ-step ahead forecasts of W_t^d can be computed as

$$\widehat{E}_{t-h} W_{t+\ell}^d = E(W_{t+\ell}^d | \mathcal{H}_{t-h}) = g_w' \ B^{\ell+h} \ \widetilde{W}_{t-h} \tag{18}$$

where h is an integer, $\widetilde{W}_t = (W_t^{d\prime} \vdots W_{t-1}^{d\prime} \vdots ... \vdots W_{t-k+1}^{d\prime})'$ is the $pk \times 1$ state vector associated with the VAR (16),

$$B = \begin{bmatrix} B_1 & B_2 & \cdots & B_{k-1} & B_k^* \\ I_p & O & \cdots & O & O \\ \vdots & \ddots & & \vdots & \vdots \\ O & \cdots & & I_p & O \end{bmatrix} \tag{19}$$

is the $pk \times pk$ associated companion matrix and g_w is a $pk \times p$ selection matrix such that $g_w'\widetilde{W}_t = W_t^d$. The symbol '$\widehat{\ }$' above expectations in (18) is used to recall that in the present context the agents' forecasts do not necessarily coincide with the expectations taken under the REH.

From (18) it turns out that for $h = 1$ and $\ell = 1$ and $\ell = 0$, the forecasts of the variables in W_t^d are given by the expressions

$$\widehat{E}_{t-1}\Delta\pi_{t+1}^d = g_\pi' \widehat{E}_{t-1} W_{t+1}^d = g_\pi' B^2 \ \widetilde{W}_{t-1} \tag{20}$$

$$\widehat{E}_{t-1}\Delta\pi_t^d = g_\pi' \widehat{E}_{t-1} W_t^d = g_\pi' B \ \widetilde{W}_{t-1} \tag{21}$$

$$\widehat{E}_{t-1}(\beta' Z_t)^d = g_{w_1}' \widehat{E}_{t-1} W_t^d = g_{w_1}' B \ \widetilde{W}_{t-1} \tag{22}$$

where g_π is a selection vector such that $g_\pi'\widetilde{W}_t = \Delta\pi_t^d$ and g_{w1} is a selection matrix such that $g_{w1}'\widetilde{W}_t = W_{1t}^d = (\beta' Z_t)^d$. Condition both sides of the NKPC (12) upon information $\mathcal{H}_{t-1}$ and using the MDS property of u_t^*, yields the relation

$$g_\pi' \widehat{E}_{t-1} W_t^d = \psi \ g_\pi' \widehat{E}_{t-1} W_{t+1}^d + \omega \ g_{w_1}' \widehat{E}_{t-1} W_t^d \tag{23}$$

which using (20)-(22) and observing that $\widetilde{W}_t \neq 0$ almost surely for each t, implies the following set of cross-equation restrictions

$$g_\pi' B(I_{pk} - \psi B) - \omega g_{w_1}' B = 0_{1\times pk}. \tag{24}$$

The restrictions in (24) generalize the approach originally proposed by Sargent (1979) and Campbell and Shiller (1987) for 'exact' rational expectations models (i.e. not including unobservable disturbances). As stated in Assumption (i), these restrictions coincide with the restrictions implied by REH only when the rational expectations solution of the NKPC can be nested within the VAR in (13) (for a given specification of x_t).[8]

Fanelli and Palomba (2007) discuss in detail the nature of nonlinear restrictions of the form (24) and show the conditions under which the VAR in (16) is locally identifiable under these constraints; Fanelli (2008) focuses on the restrictions (24) and show how these can be solved. It turns out that the cross-equation restrictions (24) can be written in explicit form so that the VAR coefficients of one of the equations of the system (16) depend nonlinearly on ψ and ω and on the remaining VAR coefficients. The number of cross-equation restrictions is $n = [p(pk-p+r)] - [(p-1)(pk-p+r)+2] = pk-p+r-2$, where the first term in square brackets is the number of free parameters of the unrestricted VAR (16)-(17) and the second term in square brackets is the number of free parameters (including ψ and ω) of the VAR (16)-(17) under the restrictions. The condition $pk \geq 3 + (p-r)$ guarantees that $n \geq 1$, i.e. that the cross-equation restrictions are testable. A likelihood-ratio test can be computed by estimating the system (16)-(17) without (additional) restrictions and subject to the cross-equation restrictions, using $\chi^2_{n,1-\eta}$, where $\chi^2_{n,1-\eta}$ is the $1-\eta$ quantile of the χ^2 distribution with n degree of freedom and η is the level of the test.

3.2 Cross-Equation Restrictions Under the ALH

When it is assumed that agents behave as econometricians and form their beliefs following adaptive learning rules, the analysis can be opportunely adapted. Since at time $t-1$ the VAR coefficients must be estimated from $\mathcal{H}_{t-1}$, in practice the agents' expectations are formed according to

$$\widehat{E}_{t-1}\pi^d_{t+1} = g_\pi'(B_{t-1})^2\,\widetilde{W}_{t-1} \tag{25}$$

$$\widehat{E}_{t-1}\pi^d_t = g_\pi'(B_{t-1})\,\widetilde{W}_{t-1} \tag{26}$$

$$\widehat{E}_{t-1}(\beta_t' Z_t)^d = g_{w_1}'(B_{t-1})\,\widetilde{W}_{t-1} \tag{27}$$

where now the elements in $\widetilde{W}_t$ are defined as $W_t^0 = (Z_t'\beta_t\ ,\ \Delta Z_t' v)'$, and with the notation β_t and B_{t-1} we conventionally denote the counterparts of the cointegration matrix and the companion matrix B in (19) when the coefficient estimates are recursively updated with the increase of the information set. Obviously, $\beta_t' Z_t = \beta' Z_t$ and $W_t^0 = W_t$ if the estimator of the cointegration parameters is not recursively

[8]Thus it would be in principle correct to disentangle between the REH and the VEH, although in the current literature the two notions are treated interchangeably.

updated.

Given initial coefficient estimates of the VAR (16)-(17) based on the sample from $t = 1$ to T_0 ($\mathcal{H}_{T_0}$), imagine a situation in which at time $t = T_0 + 1$, $T_0 + 2$, ..., agents observe new data and update the estimates of VAR coefficients following their learning algorithm. For instance, the recursive least squares algorithm amounts to the re-iterated application of Gaussian maximum likelihood estimation.[9] In this setup, also the cross-equation restrictions between the NKPC and the VAR coefficients must be updated and evaluated in correspondence of $t = T_0 + 1$, $T_0 + 2$, ... From (25)-(27), the recursive counterpart of the restrictions in (24) take the form

$$g'_{\pi}B_{t-1}(I_{pk} - \psi B_{t-1}) - \omega g'_{w_1}B_{t-1} = 0_{1\times pk} \quad , \quad t = T_0 + 1, T_0 + 2, ... \tag{28}$$

where $T_0 + 1$ can be regarded as the first monitoring time, meaning that the model will be recursively estimated and tested from $T_0 + 1$ onwards. Clearly, also the restrictions in (28) admit, for each t, a unique explicit form representation.

Consider now the objective of constructing a test for the null hypothesis that the sequence of cross-equation restrictions in (28) holds. For each given $t = T_0 + 1$, $T_0 + 2$,..., let

$$\log \widehat{L}_t^{\max} = -\frac{t}{2}\log(\det(\widehat{\Sigma}_{\varepsilon^0,t})) \tag{29}$$

be the log-likelihood (a part from a constant) of the VAR (16)-(17) evaluated at the maximum, where $\widehat{\Sigma}_{\varepsilon^0,t}$ is the estimated covariance matrix based on $\mathcal{H}_t$; let

$$\log \widetilde{L}_t^{\max} = -\frac{t}{2}\log(\det(\widetilde{\Sigma}_{\varepsilon^0,t})) \tag{30}$$

be the log-likelihood (a part from a constant) of the VAR (16)-(17) subject to the cross-equation restrictions, where $\widetilde{\Sigma}_{\varepsilon^0,t}$ is the estimated covariance matrix of the restricted system, based on $\mathcal{H}_t$. Given (29) and (30), likelihood-ratio statistics for the null hypothesis can be obtained through the sequence

$$LR_t = -2(\log \widetilde{L}_t^{\max} - \log \widehat{L}_t^{\max}) \quad , \quad t = T_0 + 1, T_0 + 2, ... \tag{31}$$

Although the ALH hinges on a perpetual learning mechanism whose ultimate effect can be evaluated only in the limit, in practice the sequence (31) will be calculated over a finite number of periods, i.e. from $t = T_0 + 1$ until $t = T^{\max}$, where $T^{\max}$ is the length of the available sample. In order to control the size of the tests over the entire sequence $T_0 + 1$- $T^{\max}$, while preserving power against backward-looking (unrestricted) alternatives, a procedure which works successfully can be obtained by comparing each LR_t in (31) with the asymptotic critical values $IR_t^{n,\eta} = (c_{n,\eta})^2 + n\log(t/T_0)$ provided by Inoue and Rossi (2005) for sequences of recursive tests for predictive ability; for given n and η, the quantity $c_{n,\eta}$ can be taken

[9] A widely used recursive updating algorithm is the 'constant gain' version of recursive least squares (Sargent, 1999). This estimator discounts past observations at a geometric rate and is consequently more robust to structural changes (Branch and Evans, 2006).

from their Table 1.[10]

4 Results

We consider quarterly data relative to the euro area, using the last release of the Area-wide Model (AWM) data set, see Fagan *et al.* (2001). The variables used in the analysis cover the period 1980:4-2006:4. To measure inflation we use the GDP deflator, i.e. $\pi_t = 400 \times (p_t - p_{t-1})$, where p_t is the log of the GDP deflator. As in Gali *et al.* (2001), firms' average marginal costs are proxied by the wage share (log of real unit labour costs), $x_t = ws_t$. A short term interest rate, $a_t = i_t$ $(q_a = 1)$, has been included in the system, as interest rates can influence the marginal costs through the cost channel, see Chowdhury *et al.* (2005). In this context, a VAR for $Z_t = (\pi_t : ws_t : i_t)'$ can be regarded as the unrestricted reduced form of a typical small-scale (including three equations) dynamic stochastic general equilibrium model of monetary policy.

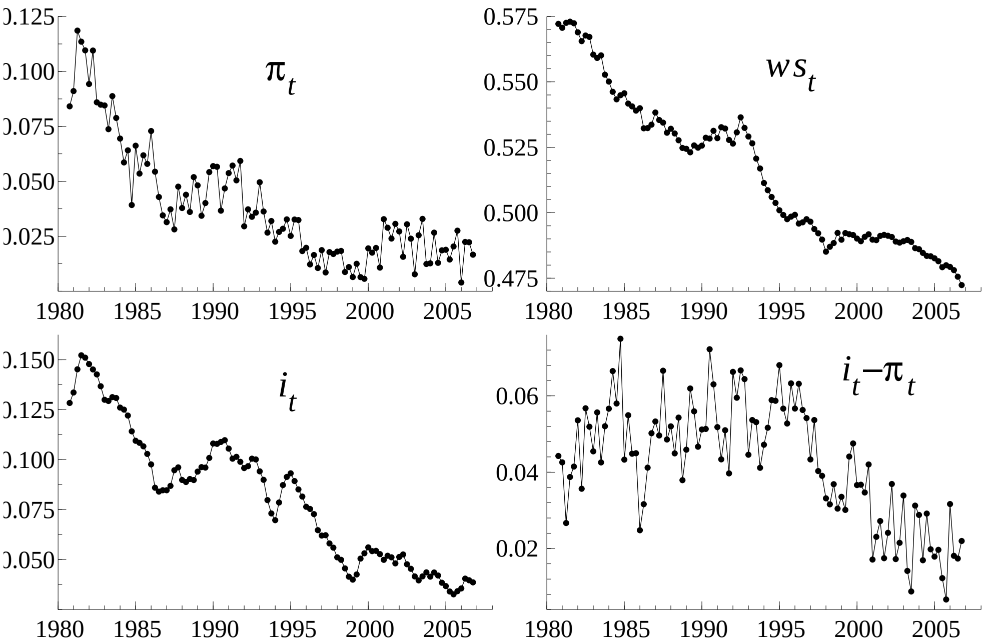
The plots of π_t and ws_t are reported in the upper panel of Figure 1, whereas in the lower panel we have plotted the short term nominal interest rate, i_t, and the real (ex-post) interest rate, $i_t - \pi_t$, respectively.[11] The graphical inspection seems to question the stationarity of the ex-post real interest rate in the euro area.

The analysis is based on the VAR $Z_t = (\pi_t : ws_t : i_t)'$ with two lags ($k = 2$), including an unrestricted constant. Conditional on initial values, the model is estimated over the period 1980:4-2006:4. Table 2 investigates the cointegration properties of the system. The likelihood ratio trace test (Johansen, 1996) suggests the presence of $p - r = 2$ common stochastic trends in system, though this evidence is not clear-cut. The determination of the cointegration rank is a difficult choice in finite samples and can be supported by other information in the model (Juselius K., 2006). In the middle panel of Table 2 we have reported the inverse roots of the characteristic equation of the VAR, obtained in correspondence of the three possible values of r. It turns out that with $r = 0$, one should switch to a model in first differences to eliminate the unit roots in the system, complicating the analysis of the NKPC. On the other hand, with $r = 2$ (implying a single common stochastic trend), one root remains close to one, suggesting that imposing two cointegrating relations and identifying one of the cointegration vectors as a relation involving i_t and π_t, would leave serious doubts about the stationarity of the identified relations. On the other hand, the option $r = 1$ suggested by the trace test, seems to remove all roots close to one from the system and is consistent with a scenario where i_t and π_t do not cointegrate at all, see Figure 1.

[10] An alternative route for recursively testing the NKPC under the ALH is proposed in Fanelli and Palomba (2007) who use simulation-based methods. Using the local Monte Carlo technique ('parametric bootstrap') formalized in Dufour and Jouini (2006), they compute the simulated p-value associated with each LR_t in (31). As shown by those authors, this testing method has the advantage of working successfully in finite samples but is computationally intensive.

[11] In the graph the inflation rate has been reported in the form $(p_t - p_{t-1})$, and the nominal interest rate has not been divided by 100.

Figure 1: Euro area data. Inflation rate (π_t), wage share (ws_t), short-term nominal interest rate (i_t) and short-term real (ex-post) interest rate ($i_t - \pi_t$).



Thus, the choice $r = 1$ seems the best compromise between data properties and *a priori* expectations. We have tested the two formulations (5) and (8) of the NKPC in the lower panel of Table 2; the likelihood ratio tests for over-identifying restrictions on β seem to favour form (8), rejecting the hypothesis of stationary wage share. This results is in line with the findings in Fanelli (2008) obtained on a previous release of the AWM data set, on a different span of data. The estimated cointegration coefficient between π_t and ws_t obtained over the entire sample period is equal to $\widehat{\phi} = 0.81$.

The graph in Figure 2 recursively tests whether once fixed $r = 1$, the constancy of the (unrestricted) estimated cointegration matrix can be accepted. The test is based on a sequence of LR tests that recursively control whether the estimated $\widetilde{\beta} = \widehat{\beta}_{T^{\max}} = (1, -\widehat{\beta}_{1,T^{\max}}, -\widehat{\beta}_{2,T^{\max}})'$ based on the entire sample is in the space spanned by $\widehat{\beta}_t = (1, -\widehat{\beta}_{1,t}, -\widehat{\beta}_{2,t})'$, for $t = T_0 + 1, ..., T^{\max}$, see e.g. Juselius K., (2006), Chap. 9. Figure 2 suggests that using the full sample as the reference period and $T_0 + 1$ =1986:1 as first monitoring time (see below), the stability of the cointegration relationship can not be taken for granted during the eighties and first nineties.

Considering the VEqC representation of $Z_t = (\pi_t : ws_t : i_t)'$ and fixing β at the super-consistent estimate of Table 2, such that $\widehat{\beta}' Z_t = \pi_t - \widehat{\phi} ws_t = \pi_t - 0.81 ws_t$, we

Table 2: Likelihood ratio trace test for cointegration rank, roots of the system for different values of cointegration rank and estimated cointegrating relation for r=1.

VAR for $Z_t = (\pi_t, ws_t, i_t)'$, $k = 2$, 1981:2–2006:4

Cointegration rank test

$H_0 : r \leq j$	Trace	p-val
j=0	30.39	0.04
j=1	11.51	0.18
j=2	1.99	0.16

Roots of the system for $r = 0$: 0.986, 0.813, 0.667±0.241i, -0.391, 0.111
Roots of the system for $r = 1$: 0.623±0.105i, -0.396, 0.106
Roots of the system for $r = 2$: 0.815, 0.666±0.242i, -0.393, 0.118

Estimated cointegrating relation, $r = 1$

$$\widehat{\beta}' Z_t = W_{1t} = \pi_t - \underset{(0.092)}{0.81}\, ws_t \quad , \text{LR: } \chi^2(1) = \underset{[0.07]}{3.29}$$

$$\widehat{\beta}' Z_t = W_{1t} = ws_t \quad , \quad \text{LR: } \chi^2(2) = \underset{[0.000]}{16.68}$$

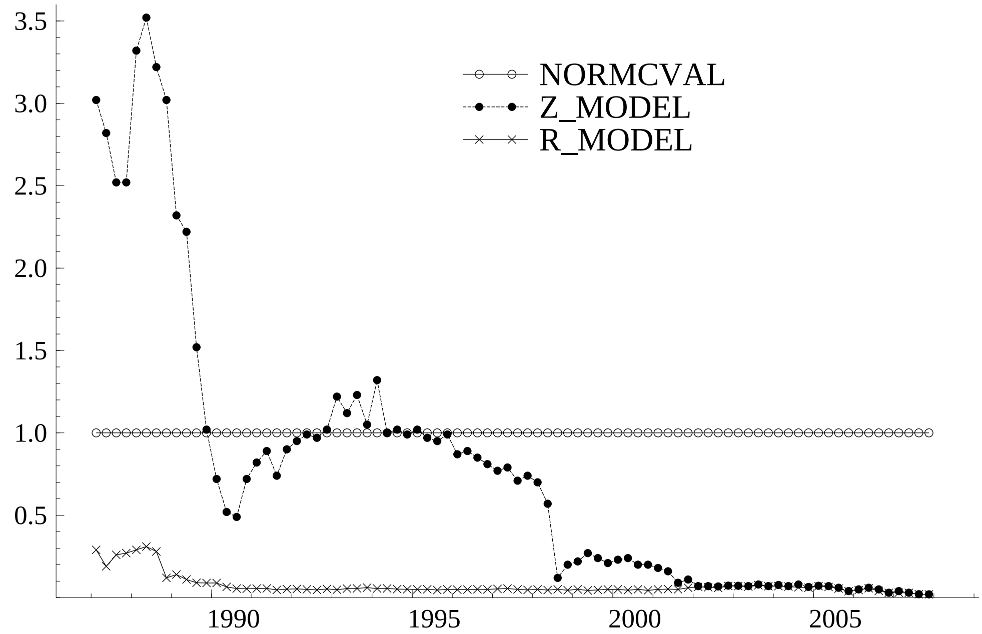
Standard errors in parentheses, p-values in square brackets.

define the vector

$$W_t = \begin{pmatrix} \widehat{\beta}' Z_t \\ v' \Delta Z_t \end{pmatrix} = \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} = \begin{pmatrix} \pi_t - 0.81 ws_t \\ \Delta \pi_t \\ \Delta i_t \end{pmatrix} \tag{32}$$

where it can be recognized that for each t, $\det(v' \widehat{\beta}_\perp) \neq 0$, where $v' = (e_1' : e_3')$ and e_i is a $p \times 1$ vector of zeros except in the i-th entry, which contains one. As shown in Section 3, W_t in (32) admits a VAR representation of the form (16)-(17) with $k = 2$ lags and a constant. This VAR will be the statistical model upon which the NKPC (8) will be tested under the ALH. Table 3 reports some vector residual diagnostic tests relative to the VAR for W_t over the period 1981:2-2006:4 and remarks that the hypothesis of Gaussian uncorrelated disturbances can be taken as a reasonable approximation for this model.

Figure 2: Recursively calculated test of $\widetilde{\beta} \subseteq sp(\beta_t)$, $t = 1986 : 1, ..., 2006 : 4$, where $\widetilde{\beta} = \widehat{\beta}_{T^{\max}}$ is estimated on the full sample period 1981:2-2006:4 (Sp(b) stands for Sp(β)). With the 'Z-model' the test is based on the recursive estimation of both long run and short run parameters, whereas with the 'R-model' the short run parameters are concentrated from the log-likelihood and the test is based on the recursive estimation of α and β.



In light of Figure 2, we evaluate the NKPC considering also a VAR for $W_t^0 = (Z_t'\widehat{\beta}_t\ ,\ \Delta Z_t'v)'$, where $\widehat{\beta}_t = (1,\ -\widehat{\phi}_t, 0)'$ is recursively estimated over the monitoring period.

In order to compute likelihood-ratio tests for the cross-equation restrictions implied by the NKPC under the ALH, the VAR for W_t (W_t^0) must be estimated recursively both unrestrictedly and subject to the restrictions, for $t = T_0 + 1$, $T_0 + 2$,... We have chosen the sample 1986:1-2006:4 as the monitoring period; in terms of the notation of Section 3, $T_0 + 1$ =1986:1 and $T^{\max}$ =2006:4. The choice of the sample 1986:1-2006:4 is motivated by the observation that after 1984/1985, the nature of the European Exchange Rate Mechanism characterizing the majority of European countries changed from a 'soft' to a 'hard' exchange rate parity arrangement (Batini, 2006); moreover, from 1986 onwards almost all European central banks adopted a more aggressive approach toward fighting inflation, with the objective of converging towards the European Monetary Union. It can be argued thereby that the econometric evaluation of the NKPC over the period 1986:1-2006:4 involves a relatively 'homogeneous' monetary policy regime. Accordingly, the VAR for W_t (as well as the

Table 3: Diagnostic tests on the vector of residuals of the VAR in (32) and estimated roots.

	VAR for $W_t = (W_{1t}, \Delta\pi_t, \Delta i_t)'$, $k = 2$, 1981:2-2006:4
Autocor.	F(45,241)=1.39 [0.06]
Normality	$\chi^2(6)$=0.60 [0.36]
Roots	$0.623 \pm 0.105i$, -0.396, 0.106

Autocor= LM vector test for residual autocorrelations up to 5 lags; Normality = LM vector test for residual normality; p-values in square brackets; Roots = estimated inverse roots of the characteristic equation.

VAR for W_t^0) has first been estimated on the period 1981:1-1985:4 to form initial coefficients beliefs, and then has been estimated recursively over the period 1986:1-2006:4 to evaluate the NKPC, as detailed in Subsection 3.2.[12] While β is fixed at the full sample estimates and only the estimates of the short run coefficients are updated recursively in the VAR for W_t, both short run and long run parameters are updated recursively over the monitoring period in the VAR for W_t^0.

The maximization of the Gaussian likelihood of the (partially) unconstrained VAR is standard for $t = T_0 + 1, ..., T^{\max}$: since the VAR coefficients are subject to the zero constraints in (17), the maximum likelihood estimation of the model amounts to a feasible version of generalized least squares. The maximization of the Gaussian likelihood of the constrained VAR has been performed by combining the BFGS method (Fletcher, 1987) with a grid search for the structural parameters ψ and ω, for $t = T_0 + 1, ..., T^{\max}$.[13] In practice, we selected a grid of points for γ_f and γ_b (recall that $\lambda = \widehat{\phi}(1 - (\gamma_f + \gamma_b)$ because of (11)), imposing the constraint $\gamma_f + \gamma_b < 1$ and then used the mapping (9)-(10) to recover the corresponding values of ψ and ω; the parameter γ_f has been chosen in the range 0.6-0.98 and the parameter γ_b in the range 0.0-0.30, using common incremental value 0.02.[14] The number of cross-equation restrictions is $n = pk - p + r - 2 = 2$.

Figure 3, upper panel, plots the sequence of likelihood ratio statistics computed over the monitoring period using the VAR for W_t, along with the 5% ($\eta = 0.05$) critical values $\chi^2_{n,0.95}$ and $IR_t^{n,\eta}$, where $IR_t^{n,\eta}$ is taken from Table 1 in Inoue and Rossi (2005). Figure 3, lower panel, reports the recursive maximum likelihood estimates of the structural parameters γ_f, γ_b and $\lambda = \widehat{\phi}(1 - (\gamma_f + \gamma_b)$, obtained from the estimation of the constrained VAR.

[12] Carceles-Poveda and Giannitsarou (2007) show that the initialization issue may be relevant in finite samples; our testing results, however, are robust to different choices of $T_0 + 1$.

[13] Although the VAR includes a constant, the cross-equation restrictions have been derived considering only the coefficients in the matrices B_i, $i = 1, .., k$, see Subsection 3.1.

[14] In preliminary analysis, we used smaller values for γ_f and larger values for γ_b in the grid, without changes in the results.

Figure 3 shows that if one considers a 'one-shot' test of the NKPC, namely compares the value of the likelihood-ratio statistic obtained at the date 2006:4 (using all available information from $t = 1$ until $t = T^{\max}$) with $\chi^2(2)$, the model is sharply rejected. This results is consistent with Bårdsen *et al.* (2004), O'Reilly and Whelan (2005) and Fanelli (2008), who provide some direct and indirect evidence against the NKPC. Nevertheless, taking into account the sequential nature of the recursive test and comparing each likelihood-ratio statistics in the sequence with the critical values $IR_t^{n,\eta}$, it seems that the model is not rejected. As concerns the structural parameters reported in the lower panel of Figure 3, the magnitude of the estimated, γ_f, dominates the magnitude of the backward-looking parameter, γ_b, over the entire monitoring period; on the other hand, in this setup the slope parameter λ depends on γ_f and γ_b and the estimated cointegration parameter $\widehat{\phi}$, as implied by the relation (11). Referring to the 'primitive' parameters of the Calvo model and using the mapping (2)-(4), the values of γ_f, γ_b and λ reported in Figure 3 imply that the discount factor, ρ, fluctuates in the range 0.76-0.79, that the average duration over which prices are kept fixed, $(1-\theta)^{-1}$, fluctuates in the range 3.3-3.5 (quarters) and that the fraction of backward-looking firms that change their prices following rule-of-thumb behavior, ϖ, fluctuates in the range 0.014-0.25, over the monitoring period. The large magnitude of γ_f can be explained by observing that the agents' perceived law of motion, which enters the NKPC (12) through the term $E_t\Delta\pi_{t+1}^d$, already incorporates lagged values of the variables. This kind of evidence is a typical by-product of the adaptive learning algorithms.[15]

When we consider the formulation $W_t^0 = (Z_t'\widehat{\beta}_t \, , \, \Delta Z_t'v)'$ of the VAR, however, the results change sharply. In this model the coefficient characterizing the cointegrating relation between π_t and ws_t is not fixed at the full sample estimate but is recursively updated. Figure 4 shows that in this case the NKPC is rejected.

[15]These findings can be related to other studies based on adaptive learning. Focusing on the U.S. economy, Milani (2005) considers a specification of the NKPC of the form (1) and a univariate autoregressive forecast model which is estimated recursively by using a 'constant gain' version of recursive least squares (see footnote 9). He finds that when such a learning mechanism replaces the REH, structural sources of inflation persistence such as indexation are no longer essential to fit the data. Learning is thus interpreted as the major source of persistence in inflation. Differently from Milani (2005), our testing approach shows that adaptive learning behaviour does not suffice alone to explain the inertia in the data captured by the lagged inflation term in (1).

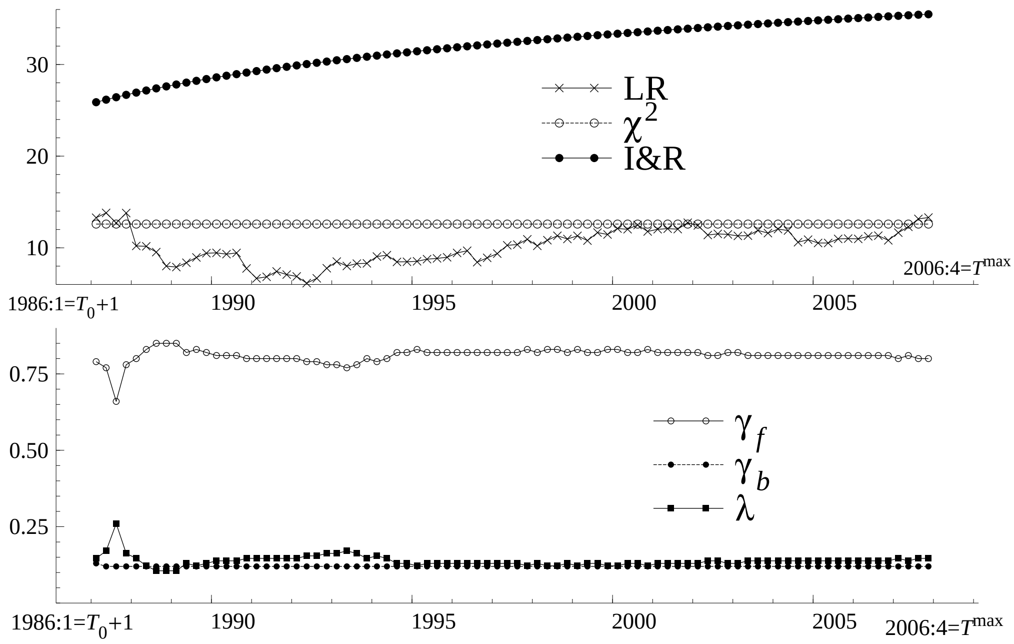


Figure 3: Upper panel: Sequence of recursively computed likelihood ratio (LR) statistics for the cross-equation restrictions implied by the cointegrated NKPC under the ALH, over the monitoring period 1986:1-2006:4, with corresponding 5% critical values (Section 3.2). Lower panel: recursively estimated structural parameters of the NKPC obtained through the grid-search. Results are here obtained through a VAR for the vector W_t defined in (32), based on two lags ($k = 2$). I&R is the critical value taken from Inoue and Rossi (2005), see Section 3.2.

Overall, the evidence in favour of the NKPC under the chosen formulation of the ALH is not clear-cut. Indeed, the restrictions implied by the model are not rejected only when the cointegration coefficient linking the inflation rate and the wage share is fixed to the full sample estimates, but such an assumption is not entirely supported by the data.

5 Concluding Remarks

In this paper we have provided a method for evaluating the data adequacy of the NKPC under a particular formulation of the ALH. The idea is that agents use VAR models as their perceived law of motion and update coefficients recursively through maximum likelihood estimation (recursive least squares) as new data enter the information set. The ALH gives rise to a sequence of cross-equation restrictions between the VAR coefficients and the NKPC, rather than a single set of constraints. The cross-equation restrictions can be recursively tested by computing sequences of likelihood-ratio statistics over the monitoring period, but standard (time invariant) critical values can not be applied.

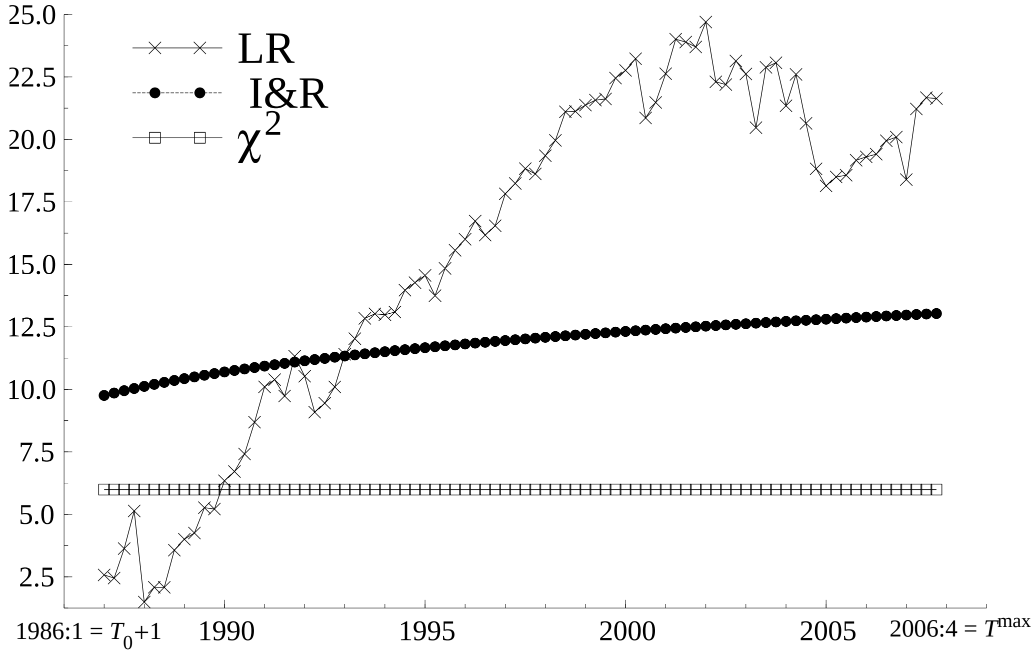


Figure 4: Sequence of recursively computed likelihood ratio (LR) statistics for the cross-equation restrictions implied by the cointegrated NKPC under the ALH, over the monitoring period 1986:1-2006:4, with corresponding 5% critical values (Section 3.2). Results are here obtained through a VAR for the vector W_t^0, based on two lags ($k = 2$). I&R is the critical value taken from Inoue and Rossi (2005), see Section 3.2.

The empirical analysis based on quarterly data for the euro area has shown that the inflation rate and the wage share can be approximated as nonstationary cointegrated processes over the period 1981-2006, although their relationship does not appear stable during the eighties and first nineties. The NKPC is sharply rejected with the cointegrated VAR under the REH. Assuming that the agents deviate from the REH and learn gradually the parameters of the model over the period 1986-2006, does not improve convincingly the evidence in support of the model.

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