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On the Explosive Nature of Hyper-Inflation Data

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Abstract:

Empirical analyses of Cagan's money demand schedule for hyper-inflation have largely ignored the explosive nature of hyper-inflationary data. It is argued that this contributes to an (i) inability to model the data to the end of the hyper-inflation, and to (ii) discrepancies between "estimated" and "actual" inflation tax. Using data from the extreme Yugoslavian hyper-inflation it is shown that a linear analysis of levels of prices and money fails in addressing these issues even when the explosiveness is taken into account. The explanation is that log real money has random walk behaviour while the growth of log prices is explosive. A simple solution to these issues is found by replacing the conventional measure of inflation by the cost of holding money.

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1 Introduction

The money demand equation for hyper-inflation of Cagan (1956) is a continuous time linear relationship between real money and the expected rate of change in prices. Cagan's own empirical work consists essentially of single equation regressions of log real money, $m_t - p_t$, regressed on the changes in log prices, $\Delta_1 p_t = p_t - p_{t-1}$, measured at a monthly frequency. If, as assumed in most of the literature, nominal money, m_t, and prices, p_t, were integrated of order two, $\mathsf{I}(2)$, the money demand relation could be found as a cointegrating relation. Here it is argued that in hyper-inflations nominal money and prices are typically not $\mathsf{I}(2)$, but explosive, as found by Juselius and Mladenović (2002). A different empirical analysis is called for. The problem arises since $\Delta_1 p_t$ as a measurement of the cost of holding money implicitly is motivated by a Taylor expansion of the logarithmic function, which has poor mathematical properties for large inflation rates. Using a different measure of the cost of holding money the difficulties can be overcome.

Most empirical studies have struggled with modelling hyper-inflationary episodes to the end. Cagan set the example of modelling for instance the German hyper-inflation until July of 1923 although the episode continued until november. Likewise, large discrepancies have been found between the "optimal" and the "actual" inflation tax, and, hence only little support for Cagan's theory for seigniorage. In the present analysis it is shown that the explosive behaviour of the data is the main source of the empirical problems.

The argument is based on an empirical analysis of the extreme Yugoslavian hyper-inflation of the early 1990s. This is one of the longest and most extreme episodes ever observed with monthly inflation rates above 50% for 24 months. Unlike the German government in 1920s the Yugoslavian government was unable to halt the inflation even temporarily in this period. These unfortunate features actually make it easier to analyse the Yugoslavian case than for instance the German case which was studied by Cagan. For a discussion of the resolution of the hyper-inflation puzzles it is therefore convenient to focus on the Yugoslavian case. With the analysis from this paper it should be possible to return to the more complicated German hyper-inflation in a later study. In the present analysis two econometrics models are used. The first model serves to show that a traditional linear econometric model linking the logarithm of real money, $m_t - p_t = \log(M_t/P_t)$, and inflation measured as the growth of log prices, $\Delta_1 p_t = p_t - p_{t-1}$, is indeed unbalanced. The second model shows that the puzzles are resolved by measuring inflation as the cost of holding money, $c_t = \Delta_1 P_t / P_t = 1 - \exp(-\Delta_1 p_t)$.

In the first model, the conventional variables, nominal money, m_t, nominal prices, p_t, and spot exchange rates, s_t, are analysed using a vector autoregression. Due to the accelerating nature of the data the vector autoregression is found to be explosive.

Using econometric methods developed in Nielsen (2005b) it is found that real money has random walk features while changes in log prices are explosive. This contrasts with the analyses of Sargent (1977) and Taylor (1991). A regression of real money on changes in log prices is therefore unbalanced which explains the puzzles.

In the second model real money is instead linked to the cost of holding money, c_t. A well-specified vector autoregressive model can now be made. The cointegration analysis leads to a linear relation between real money and the cost of holding money as expected from the Cagan model. This model does, however, fit throughout the full sample and the estimated "optimal" and "actual" inflation tax rates are now in line.

The outline of the paper is that §2 discusses Cagan's empirical puzzles, in the context of Cagan's own analysis and later empirical studies, as well as in the context of the Yugoslavian episode. The two econometric models are outlined in §3 and §5 with §4 describing the measure of cost of holding money. §6 concludes.

2 The Hyper-inflation Puzzles

A brief outline of the empirical literature on money demand in hyper-inflations is given. The theoretical and empirical work of Cagan (1956) is reviewed. The empirical puzzles identified by Cagan are then traced through the literature and are finally illustrated using data from the Yugoslavian hyper-inflation.

2.1 Cagan's Theory for Money Demand

Cagan's theory describes two aspects of hyper-inflations: the money demand schedule and the seigniorage. In his empirical work he noticed puzzles associated with both.

The money demand schedule is described in his equations 2 and 5. These are continuous time equations linking the log real cash balances with the expected rate of change in prices:

$$m_t - p_t = -\alpha E_t - \gamma, \tag{2.1}$$

$$\left(\frac{\partial E_t}{\partial t}\right)_t = \beta \left(C_t - E_t\right). \tag{2.2}$$

Here m_t and p_t represent the logarithm of money and prices, $C_t = \partial p_t / \partial t$ is the continuous rate of change in prices, while E_t represents an adaptive expectation of C_t. Other variables, like output, that are usually appearing in quantity theories for money are assumed to have a negligible influence. By solving equation (2.2) backwards from present time, t, to an initial value, $-T$, the expectations term E_t can be expressed as an exponentially weighted average of past values of C, that is

$$E_t = H \exp\left(-\beta t\right) + \beta \int_{-T}^{t} C_x \exp\left\{\beta\left(x - t\right)\right\} dx. \tag{2.3}$$

Inserting this in (2.1), Cagan could then estimate α and β from monthly data as follows. Letting $-T$ represent the beginning of the sample and assuming that prices had been almost constant before time $-T$, then H can be set to zero in (2.3). Cagan then made the crucial assumption that

$$C_t \text{ is constant within a month,} \tag{2.4}$$

in which case $C_t = \Delta_1 p_t = p_t - p_{t-1}$ and the latent expectations process E_t can be approximated by a sum. For a given value of β the parameter α can then be estimated from (2.1) by regression. By varying β a joint estimate for α, β can be found.

In the empirical analysis, Cagan considered data from seven hyper-inflations. The infamous German hyper-inflation from August 1922 to November 1923 was analysed using data until July 1923 only, due to difficulties in fitting the data from the last few months. This is puzzling in suggesting that the money demand schedule for hyper-inflations is not time invariant and may not even hold when the hyper-inflation is most extreme. In any case, he estimated the semi-elasticity α by $\widehat{\alpha} = 5.76$.

Cagan also analysed the seigniorage from printing money, arguing that the revenue from the inflation tax is the product of the rate of tax and the base

$$R = \left(\frac{dP}{dt}\frac{1}{P}\right)\frac{M}{P}, \tag{2.5}$$

where M and P are levels of money and prices, and the timing is left unspecified. He then made the counter factual assumption that the quantity of nominal money rises at a constant rate. This would eventually imply constancy of real money balances, which is contradicted by Cagan's own observation that real money balances tend to fall in hyper-inflation. It would also imply that E_t can be replaced by C_t in equation (2.1):

$$\frac{M}{P} = \exp(-\alpha C - \gamma) \tag{2.6}$$

Combining (2.5) and (2.6) gives a revenue of $R = C\exp(-\alpha C - \gamma)$, which has a unique maximum, with respect to C, when

$$C = \frac{1}{\alpha}.$$

The inverse of the semi-elasticity α is therefore interpreted as the rate of inflation that maximises the revenue from seigniorage under the above assumptions.

In the empirical analysis, Cagan's estimate for the German hyper-inflation is $\hat{\alpha}^{-1} = 0.183$. This is a continuously compounded rate corresponding to a monthly tax of $\exp(\widehat{\alpha}^{-1}) - 1 = 20\%$. He compared this with an average monthly rate of inflation of 322%, defining inflation as $\Delta_1 P_t / P_{t-1}$. Comparing the two shows a puzzling mismatch between an "optimal" tax rate and the "actual" inflation tax.

2.2 The $\mathsf{I}(2)$ Approach

While the time series methodology was in its infancy at the time of Cagan's study later work on hyper-inflation has been cast in an $\mathsf{I}(2)$-framework with nominal money, m_t, and prices, p_t, assumed $\mathsf{I}(2)$-series.

In this way Sargent and Wallace (1973) and Sargent (1977) revisited Cagan's analysis in part with a view towards explaining the discrepancy of the "optimal" and the "actual" inflation tax. The model of Sargent (1977) is a bivariate model for nominal money and prices involving a rational expectation, π_t, to future inflation, $\Delta_1 p_t$. Unlike Cagan's model it is discrete time model applied at a monthly frequency in the empirical work and therefore implicitely using the discretization assumption (2.4). Sargent further makes the assumptions:

$$m_t, p_t \sim \mathsf{I}(2), \qquad m_t - p_t, \Delta_1 m_t, \Delta_1 p_t \sim \mathsf{I}(1), \tag{2.7}$$

for the observables, whereas the rational expectations satisfy

$$\pi_t - \Delta_1 m_t, \pi_t - \Delta_1 p_t \sim \mathsf{I}(0).$$

Since Sargent's work predates the concept of co-integration this is not the focus of the work and $m_t - p_t$ and $\Delta_1 p_t$ are not cointegrating in his model. The causality structure in the model is that $\Delta_1(m_t - p_t)$ and hence $\Delta_1 m_t$ do not Granger-causes $\Delta_1^2 p_t$.

Sargent went on to fit the model to the data considered by Cagan. In the case of Germany, the estimate of α is virtually unchanged, $\widehat{\alpha} = 5.97$, but the uncertainty is judged differently with a standard error of 4.6 so the estimated confidence band for the "optimal" inflation tax covers nearly the whole positive real axis. Sargent's empirical analysis therefore lends support, albeit only weak support, to Cagan's model.

Around the same time Evans (1978) analysed the time series properties of m_t, p_t using Box-Jenkins analysis. That is an analysis based on inspection of the correlograms rather formal testing. This analysis lead Evans to conclude that for the German episode m_t and p_t are $\mathsf{I}(2)$ in line with Sargent. It should be noted that with the prevailing definition of correlograms explosive time series have an exponentially declining correlogram, see Nielsen (2006a). Christiano (1987) analysed variations of Sargents model a little further within an $\mathsf{I}(2)$ framework. This analysis found some, but not overwhelming, evidence against the Sargent and Wallace model. Here it should be noted that the reported mis-specification tests are based on the Box-Pierce statistic, which could suffer from the same problems as correlograms if there is explosive behaviour in the residuals.

A partial rational expectations formulation opens up for interesting interpretations. Then Cagan's equation is formulated in discrete time as

$$m_t - p_t = -\alpha \mathsf{E}_t (p_{t+1} - p_t) + \zeta_t. \tag{2.8}$$

Here E_t is the expectation conditional on $(m_s, p_s, \zeta_s)_{s\leq t}$ and thus a construct of a probability model rather than actually representing the expectations it is rational to form for the agents in the economy. The equation can be solved for p_t as a function of future values of m_t, see Turnovsky (2000, p. 86f & 91f) and Diba and Grossman (1988). To do this assume (i) that $\alpha > 0$ and $\alpha/(1+\alpha) < 1$ and (ii) that $|\{\alpha/(1+\alpha)\}^j \mathsf{E}_t(\tilde{m}_{t+j})|$ vanishes at a geometric rate. Then (2.8) is solved by

$$p_t = \frac{1}{1+\alpha}\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j \mathsf{E}_t\left(m_{t+j} - \zeta_{t+j}\right) + c\left(\frac{1+\alpha}{\alpha}\right)^t + \sum_{s=1}^{t}\left(\frac{1+\alpha}{\alpha}\right)^{t-s}\xi_s, \quad (2.9)$$

for any $c \in \mathbb{R}$ and random variables ξ_t satisfying $\mathsf{E}_t\xi_{t+j} = 0$. The first term inherits whatever stochastic properties m_t may have while the last two components are explosive and are referred to as an explosive bubble by Diba and Grossman (1988).

A difficulty with the solution (2.9) is that the model for m_t, p_t has to be formulated into the indefinite future and it has to capture the accelerating growth of the variables as well as the sudden halt in growth when the inflation stops. The literature has focused on somewhat less ambitious assumptions to m_t that gives cointegration properties. This is discussed in some detail in the Appendix. Here, three interesting examples are discussed. *First*, if m_t is a unit root process of order $\mathsf{I}(d)$ without bubble, so $c = \xi_t = 0$, then $p_t \sim \mathsf{I}(d)$, $m_t - p_t \sim \mathsf{I}\{\max(d-1, 0)\}$ and $m_t - p_t + \alpha\Delta_1 p_t \sim \mathsf{I}\{\max(d-2, 0)\}$. This idea has been pursued by Taylor (1991) as will be discussed below. *Secondly*, if m_t is explosive with a root ρ less than $1 + \alpha^{-1}$, without bubble, then p_t and $m_t - p_t$ are also explosive. *Thirdly*, due to the bubble component it is possible that p_t can be explosive without m_t being explosive. However, all three types of predictions will be contradicted by the empirical findings of §3. It is interesting to note though that explosive bubbles appear to be suited for an analysis stock prices and dividends as shown by Engsted (2006) using the co-explosive analysis that will discussed in §3.

Taylor (1991) looked at the possibilities for cointegration arising from the partial rational expectations model without bubble. He wrote the discrete time model as

$$m_t - p_t = -\alpha\Delta_1 p^e_{t+1} + \zeta_t, \quad (2.10)$$
$$\Delta_1 p^e_{t+1} = \Delta_1 p_{t+1} + \epsilon_{t+1}, \quad (2.11)$$

where the variable $\Delta_1 p^e_{t+1}$ measures the expected inflation in period $t+1$ and ζ_t, ϵ_{t+1} are stationary error terms. He showed that $\Delta_1 p^e_{t+1}$ can be interpreted as a rational expectation as above, or as an adaptive expectation or an extrapolative expectation, as long as equation (2.11) is satisfied. Inserting (2.11) into (2.10), adding $\alpha\Delta_1 p_t$ on both sides and then reorganising leads to

$$\Delta_1^2 p_{t+1} = -\alpha^{-1}\left(m_t - p_t + \alpha\Delta_1 p_t\right) - \left(\epsilon_{t+1} + \alpha^{-1}\zeta_t\right). \quad (2.12)$$

Assuming that m_t and p_t are both $\mathsf{I}(2)$ variables it can be tested whether real money $m_t - p_t$ is $\mathsf{I}(1)$ and in turn whether $m_t - p_t + \alpha\Delta_1 p_t$ cointegrates to $\mathsf{I}(0)$. In this cointegrated framework the coefficient to the expected inflation variable $\Delta_1 p^e_{t+1}$ therefore shows up as the coefficient to $\Delta_1 p_t$ in a cointegrating relation.

In the empirical work Taylor considered six of Cagan's episodes using 3 different data sources. As a justification for the $\mathsf{I}(2)$ framework, unit root tests were applied to levels, first, and second differences of $m_t - p_t$ and $\Delta_1 p_t$. For instance, for Germany it was concluded using three different data sources that $\Delta_1 p_t$ is $\mathsf{I}(1)$, possibly $\mathsf{I}(2)$. This was based on one-sided tests against the stationary alternative ignoring any structural breaks. Considering also the explosive alternative the test statistics of Taylor leads to the conclusion that $\Delta_1 p_t$ is explosive at least for two of the German data sets. Leaving that aside, Taylor (1991) found evidence for cointegration between $m_t - p_t$ and $\Delta_1 p_t$. For the German case Taylor estimated α by 5.31 in line with previous results.

Frenkel (1977) suggested linking real money balances with exchange rates and forward rates to overcome the problem of measuring expected inflation. The rationale is that agents hold real money in foreign currency and adjust holdings of real money to expected exchange rate depreciations. This idea was cast in Taylor's framework by Engsted (1996). Abel, Dornbusch, Huizinga and Marcus (1979) went one step further in suggesting that both inflation and depreciation in exchange rates may influence real money as in

$$m_t - p_t = -\alpha\Delta_1 p^e_{t+1} - \beta\Delta_1 s^e_{t+1} + \gamma + \epsilon_t.$$

Michael, Nobay and Peel (1994) addressed Cagan's two puzzles by adding real economy variables, notably real wages, to the money demand schedule, but found it necessary to separate periods of high inflation and periods of hyper-inflation. Their analysis of the German hyper-inflation was also done in an $\mathsf{I}(2)$ framework, justified with one-sided unit root tests against the stationary analysis. Once again, the unit root statistics of their Table 1 actually show that $\Delta_1 m_t$ and $\Delta_1 p_t$ are explosive, and that even for the high-inflation period prior to June 1923. It would be interesting to follow up the idea of including real economy variables, but for simplicity the presented analysis will ignore this aspect.

2.3 The Yugoslavian Hyper-inflation

Yugoslavia experienced two hyper-inflations in short time. The first had a long build-up during the 1980s and peaked in 1989 briefly reaching high, but not very extreme inflation. The second and very extreme hyper-inflation which is studied here developed from 1991 to January 1994. For the first Yugoslavian hyper-inflation, richer data are available such as wages. Juselius and Mladenović (2002) analysed this period seeking a link between wages and prices. They identified explosive behaviour in the data and set up an empirical model taking this into account. Since then econometric techniques have been developed for this situation, and these will be used in §3.

As an empirical example it is useful to look at the extreme Yugoslavian hyper-inflation of the 1990s. This is one of the longest and most extreme observed. Unlike the German episode the Yugoslavian government was unsuccessful in halting the inflation temporarily in the course of hyper-inflation. As a result the data appear smoother and are therefore more suited for addressing the puzzles and to show how they can be resolved. The data are taken from Petrović and Mladenović (2000) and are available from the *Journal of Money, Credit and Banking* online data archive. The data has previously been analysed by Petrović and Vujošević (1996), Petrović, Bogetić and Vujošević (1999) and Engsted (1998).

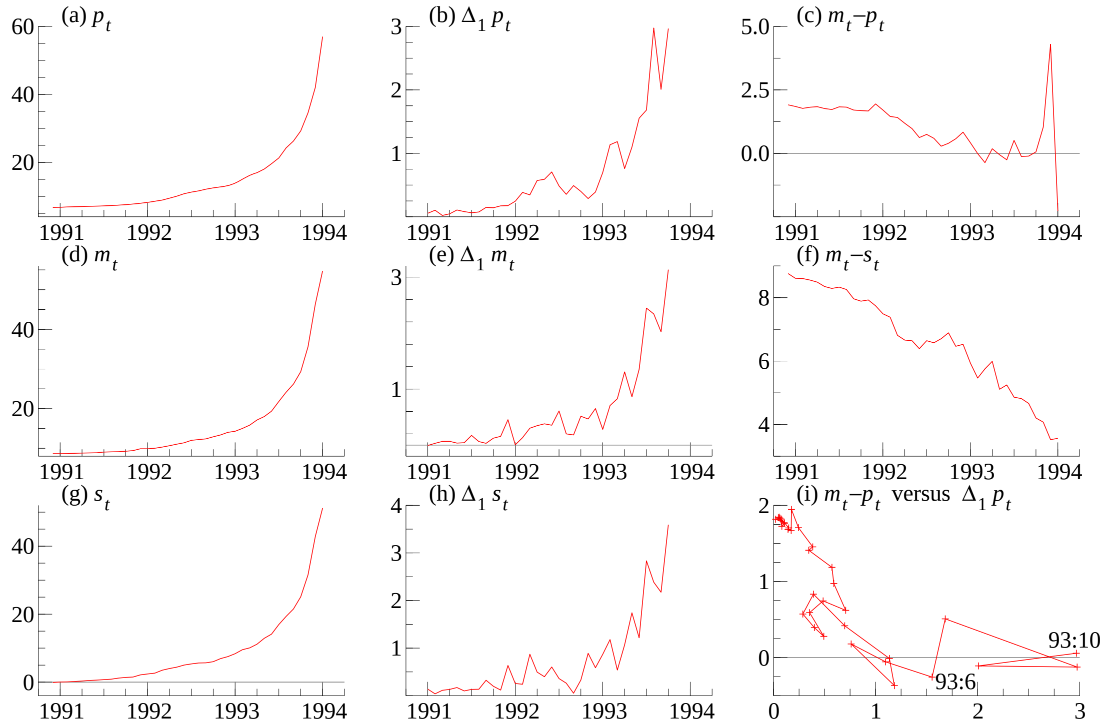
The institutional background for the extreme Yugoslavian hyper-inflation is described in Petrović and Vujošević (1996) and Petrović, Bogetić and Vujošević (1999). In short, the former federal republic of Yugoslavia was falling apart in 1991, the civil war started and United Nations embargo was introduced in May 1992. Output and fiscal revenue then decreased, while transfers to the Serbian population in Croatia and Bosnia-Herzegovina as well as military expenditure added to fiscal problems. The monthly inflation rose above 50% in February 1992 and accelerated further, a price freeze was attempted in August 1993 and the inflation finally ended on 24 January 1994 with a currency reform after prices had risen by a factor of 1.6×10^{21} over 24 months. This makes it the second longest recorded hyper-inflation and therefore, from an econometric perspective, the most promising in terms of sample length available.

Figure $1(a,d,g)$ shows three time series of monthly data relating to the period 1990:12 to 1994:1. The variables are the monthly retail price index, p_t, narrow money measured as M1, m_t, and a black market exchange rate for German mark, s_t, all reported on a logarithmic scale. The sources for the data are documented in Petrović and Mladenović (2000). They consider the prices for 1993:12 and 1994:1 to be unreliable and choose to end their analyses end at the latest 1993:11, sometimes even at 1993:6. This is in line with previous studies of hyper-inflation that mostly ignore the last few observations.

Figure $1(b,e,h)$ shows first differences of the series. Both in levels and in differences the series show an exponential growth over time and hence an accelerating inflation. Cross-plotting the variables against their lagged values would give approximately straight lines with slopes in the region 1.15-1.35, which would be another indication of explosive behaviour. This contradicts Cagan's assumption that nominal money rises at a constant rate.

Figure $1(c,d)$ shows real money series, $m_t - p_t$ and $m_t - s_t$, where money is discounted by the price level and the exchange rate, respectively. Both series are falling, matching the negative sign in equation (2.1). Since German prices only increase a few percent over the period the variable $p_t - s_t$ is essentially the real

Figure 1: The series p_t, m_t, s_t and linear transformations thereof



Note: the growth rates in panels (b, e, h) and the cross plot in (i) is only shown until 1993:10.

exchange rate, which is mostly falling; see Figure 2(a) below.

Figure 1(i) shows a cross-plot of real money versus price growth. This illustrates the puzzles Cagan was faced with in modelling the money demand schedule. There is a near linear relationship between the variables until 1993:6 but then a change in functional form. This is observed by Petrović and Mladenović (2000) who makes a linear analysis until this point and a non-linear analysis for the full sample. Michael, Nobey and Peel (1994) make a similar split the data for their analysis of the German hyper-inflation.

2.4 A Preliminary Analysis of the Yugoslavian Data

In the light of the structural model of Sargent (1977) it is interesting to construct simple descriptive time series models for the real money and inflation variables.

For real money discounted by the exchange rate, $m_t - s_t$, a very simple model fares well. The estimated model for the full sample is

$$\Delta_1 (m_t - s_t) = -\underset{(0.05)}{0.15} + 0.27\widehat{u}_t, \tag{2.13}$$

with standard error reported in parenthesis and $\widehat{u}_t$ denotes the standardised residuals. The residuals, $\widehat{u}_t$, pass mis-specification tests for normality, autocorrelation, and autoregressive conditional heteroskedasticity. This empirical model is consistent with the $\mathsf{I}(2)$ assumptions in (2.7). A similar result would be obtained for $m_t - p_t$ over the reduced period to 1993:10. The analysis presented in §5 will, however, reduce the residual standard error by a third by more careful modeling.

Turning to the log price growth $\Delta_1 p_t$ a second-order autoregression fares well for the sample until 1993:10,

$$\Delta_1^2 p_t = \underset{(0.09)}{0.15}\Delta_1 p_{t-1} - \underset{(0.19)}{0.67}\Delta_1^2 p_{t-1} + \underset{(0.08)}{0.04} + 0.32\widehat{u}_t. \tag{2.14}$$

Here, mis-specification tests for serial dependence pass, whereas normality cannot be accepted. The hypothesis of a unit root can be tested from the coefficient to $\Delta_1 p_{t-1}$. The t-statistic is the augmented Dickey-Fuller test statistic, taking a value of about 1.6, which is very large compared with the 95% quantile (against the explosive alternative) of -0.07. This suggests that $\Delta_1 p_t$ is an explosive process rather than a unit root process in contrasts to the $\mathsf{I}(2)$ assumptions in (2.7). This issue will be addressed more systematically through system analyses of the data.

3 A Linear Model for the Variables in Levels

In the following a linear vector autoregressive model is made for the levels of prices, p_t, money, m_t, and exchange rates, s_t. The focus of this model is to consider the standard $\mathsf{I}(2)$ assumptions within a multivariate model. Finding that these variables are actually explosive the analysis suggested by Nielsen (2005b) is needed. It can then be shown formally that m_t, p_t, s_t co-explode showing that the real variables like $m_t - p_t$ are $\mathsf{I}(1)$, but leaving the growth rate $\Delta_1 p_t$ explosive. The $\mathsf{I}(2)$ assumption is therefore found to be unhelpful when analysing hyper-inflations. Based on these findings an alternative way forward is found in §4 and §5. An simplified analysis along these lines was given for the bivariate system of m_t, p_t as an empirical illustration in Nielsen (2005b).

3.1 The Unrestricted Vector Autoregressive Model

A model with a constant, a linear trend and three lags is used for $X_t = (p_t, m_t, s_t)$:

$$X_t = \sum_{j=1}^{3} A_j X_{t-j} + \mu_c + \mu_l t + \varepsilon_t,$$

Table 1: Misspecification tests for the vector autoregressive model for p, m, s

Test	p	m	s	Test	(p, m, s)
$\chi^2_{normality}(2)$	1.3 [0.53]	6.0 [0.05]	4.5 [0.11]	$\chi^2_{normality}(6)$	3.1 [0.79]
$F_{AR(1)}(1, 20)$	1.8 [0.19]	1.0 [0.32]	0.1 [0.82]	$F_{AR(1)}(9, 39)$	1.5 [0.20]
$F_{AR(3)}(3, 18)$	0.6 [0.62]	0.8 [0.53]	0.3 [0.81]	$F_{AR(3)}(27, 29)$	1.1 [0.44]
$F_{ARCH(3)}(3, 15)$	0.1 [0.94]	0.2 [0.92]	0.1 [0.93]		

p-values are given in brackets

Table 2: Characteristic roots of unrestricted model

Re(z)	**1.21**	-0.42	-0.42	0.02	0.02	**0.75**	**0.75**	-0.31	0.09
Im(z)	**0**	0.84	-0.84	0.90	-0.90	**0.33**	**-0.33**	0	0
$\|z\|$	**1.21**	0.94	0.94	0.90	0.90	**0.81**	**0.81**	0.31	0.09

where the innovations ε_t are assumed independent normal $\mathsf{N}_3(0, \Omega)$ distributed. The lag length is chosen so as to ensure that the mis-specification tests pass. When it comes to the co-explosive analysis there is then one lag to each of the random walk component, the explosive component, and the stationary components. Adding the linear trend appears to help in capturing the variation in the data and matches Cagan's potentially counter factual assumption that M_t rises a constant rate. Due to the measurement problems of prices towards the end of the sample only the subsample 1990:12 to 1993:10 is analysed giving a sample size of $T = 35 - 3 = 32$. On the one hand, this gives a model that has admittedly few degrees of freedom in that each equation has 11 mean parameters. This issue is alleviated in the subsequent general-to-specific reduction. On the other hand, these explosively growing time series should be rather informative.

Formal mis-specification tests are reported in Table 1. Interpreting these in the usual way indicates that the model is well specified. Graphical tests for mis-specification, which are not reported here, include Q-Q-plots for normality and are likewise supportive of the model. Note that the usual asymptotic theory is valid for general autoregressions with stationary, unit, as well as an explosive root. This has been proved for the test for autocorrelation in the residuals, see Nielsen (2006a,b), and for Q-Q plots for normality by Engler and Nielsen (2007). Some of the test statistics are reported in an F-form as advocated by Doornik and Hendry (2001) in an attempt to deal with finite sample issues for these tests even though it has not yet been argued whether this represents an improvement in the explosive case.

Table 2 reports the characteristic roots of the unrestricted vector regression. It appears as if there is one explosive root and two unit roots as marked with bold face. The explosive root of 1.21 is within the region of 1.15-1.35 discussed above. There

Table 3: Cointegration rank tests

Cointegration rank, r	0	1	2	3
Test	79.1 [0.00]	23.1 [0.11]	9.8 [0.14]	
Likelihood	15.30	43.27	49.94	54.84

p-values are given in brackets

Table 4: Characteristic roots of restricted model with rank one, $r = 1$

Re(z)	**1.19**	**1**	**1**	-0.37	-0.37	0.07	0.07	-0.54	0.07
Im(z)	**0**	**0**	**0**	0.88	-0.88	0.83	-0.83	0	0
$\|z\|$	**1.19**	**1**	**1**	0.95	0.95	0.83	0.83	0.54	0.07

is a further set of four complex roots near the unit circle. An interpretation of a seasonal pattern repeating itself every five months seems unlikely. In this analysis these four roots will be ignored, but it is a matter for further research to understand the nature of such roots.

3.2 Analysis of Cointegrating Properties

The next step of the analysis is a cointegration analysis using the approach suggested by Johansen (1996). For this purpose the model is re-parametrised as

$$\Delta_1 X_t = (\Pi, \Pi_l) X^*_{t-1} + \sum_{j=1}^{2} \Gamma_j \Delta_1 X_{t-j} + \mu_c + \varepsilon_t, \tag{3.1}$$

where $\Delta_1 X_t = X_t - X_{t-1}$ is the usual first difference and $X^*_{t-1} = (X'_{t-1}, t')'$. This likelihood can be maximised analytically under the reduced rank hypothesis

$$\text{rank}(\Pi, \Pi_l) \leq r \leq \dim X \qquad \text{so} \qquad (\Pi, \Pi_1) = \alpha\beta^{*\prime},$$

for matrices $\alpha \in \mathbf{R}^{p\times r}, \beta^* \in \mathbf{R}^{(p+1)\times r}$ with full column rank. Although the symbols α, β were used above to describe Cagan's model, they are used here in a different meaning to be consistent with Johansen's notation. The interpretation of the cointegrating vectors β is now that $\beta' X_t$ has no random walk component but it could have an explosive component. This statement will be made more precise in connection with the Granger-Johansen representation in (3.2) below. The usual asymptotic critical values are valid in the presence of explosive roots as argued by Nielsen (2001) for the univariate case and Nielsen (2005b) for the multivariate case.

The cointegration rank r is determined using the likelihood ratio tests reported in Table 3. It is relatively clear to conclude that $\hat{r} = 1$. The characteristic roots

Table 5: $\mathsf{I}(2)$ cointegration rank tests

	$I(2)$ roots			
$r = rank(\Pi, \Pi_l)$	3	2	1	0
0	140 [0.00]	108 [0.00]	89.7 [0.00]	79.1 [0.00]
1		63.2 [0.00]	35.8 [0.00]	23.1 [0.11]
2			25.5 [0.00]	9.8 [0.14]

p-values are given in brackets

Table 6: Estimated cointegrating vector

	p	m	s	t
H_1	1 (6.2)	−0.35 (−6.5)	−1 (−6.2)	0.065 (6.6)
H_1, H_ρ	1 (6.3)	−0.35 (−6.8)	−1 (−6.3)	−0.011 (−6.7)

Cointegrating vector, $\hat{\beta}^* = \hat{\beta}_1^*$, estimated under H_1 and under the joint hypothesis $\mathsf{H}_1, \mathsf{H}_\rho$. Signed likelihood ratio statistics, $\sqrt{LR}$, for insignificance in brackets

are only little changed by imposing this restriction as seen from comparing Table 4 with Table 2. The explosive root remains, so the cointegration rank test suggests that is not a realisation of a unit root with multiplicity one. There is the possibility that the explosive root could be a realisation of a unit root with multiplicity two. Thus, Table 5 applies the system $\mathsf{I}(2)$ analysis proposed by Rahbek, Kongsted and Jørgensen (1999) and shows that a non-explosive $\mathsf{I}(2)$ description of the data is firmly excluded. It should be noted though that the validity of the $\mathsf{I}(2)$ analysis has not been proved so far. If the $\mathsf{I}(2)$ restriction is successful in making the explosive root go away then the asymptotic theory of Rahbek, Kongsted and Jørgensen (1999) applies directly, but for the case the explosive root remains present a new asymptotic analysis is required following the steps of Nielsen (2005b, Theorem).

Once the rank is determined we can impose restrictions on the cointegrating vector β^*. A homogeneity restriction, H_1 say, between prices and exchange rates reduces the likelihood value slightly to 43.0 and such a restriction is therefore easily accepted when comparing the likelihood ratio statistics of 0.5 to a $\chi^2(1)$ distribution. The resulting cointegrating vector is reported in the first line of Table 6. As the cointegrating relation $\beta' X_t$ represents linear combinations that are explosively growing, but without a random walk component, it can be interpreted as the relation of nominal money, m_t, and real price, $p_t - s_t$, that generates the explosive trend.

3.3 Analysis of Co-explosive Properties

To investigate the influence of the explosive trend re-parametrise the model as

$$\Delta_1\Delta_\rho X_t = \alpha_1\beta_1^{*\prime}\Delta_\rho X_{t-1}^* + \alpha_\rho\beta_\rho'\Delta_1 X_{t-1} + \psi\Delta_1\Delta_\rho X_{t-1} + \mu_c + \varepsilon_t,$$

where $\beta_1^* = \beta_1$ is the cointegrating vector from before and $\Delta_\rho X_t = X_t - \rho X_{t-1}$ with ρ being an unknown scale parameter representing the explosive root. The matrix $\alpha_\rho\beta_\rho'$ has rank $\dim X - 1 = 2$ due to the single explosive root. Nielsen (2005b) shows that in this model the process X_t has Granger-Johansen representation

$$X_t \approx C_1\sum_{s=1}^{t}\varepsilon_s + C_\rho\sum_{s=1}^{t}\rho^{t-s}\varepsilon_s + y_t + \tau_c + \tau_l t + \tau_\rho\rho^t, \tag{3.2}$$

where y_t can be given a stationary initial distribution. The impact matrices C_1, C_ρ are functions of the parameters and satisfy $\beta_1'C_1 = 0$ and $\beta_\rho'C_\rho = 0$ whereas τ_l satisfies $\beta_1'\tau_l + \delta_1' = 0$ and the coefficients τ_c, τ_ρ are functions of parameters and initial values so $\beta_\rho'\tau_\rho = 0$. The explosive common trend $W_t = \sum_{s=1}^{t}\rho^{-s}\varepsilon_s$ converges almost surely to a random variable W as t increases according to the Marcinkiewicz-Zygmund result, see for instance Lai and Wei (1983).

Simple hypotheses on the co-explosive vectors β_ρ can be tested using χ^2-inference. The underlying asymptotic result, due to Lai and Wei (1985) and Nielsen (2005a) is that the stationary component, the random walk and the explosive trend are asymptotically uncorrelated. Nielsen (2005b) then uses this to show that simple hypotheses on the co-explosive vectors β_ρ can be tested using χ^2-inference under the normality assumption to the innovations, which was checked above.

The hypothesis, that β_ρ is known and given by

$$\mathsf{H}_\rho: \qquad \beta_\rho' = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix},$$

implies that each of $m_t - p_t$, $m_t - s_t$ and $s_t - p_t$ are co-explosive relations and are thus non-exploding random walks. Since β_ρ is completely specified, the model can be estimated by reduced rank regression for each value of ρ. This in turn results in a profile likelihood in ρ which can then be maximised by a grid search. This is done by constructing the variables $\Delta_\rho X_t$ and $\beta_\rho'\Delta_1 X_{t-1}$ for a given ρ. Standard software can then be used to perform a cointegration analysis on the variable $\Delta_\rho X_t$ with a linear trend restricted to the cointegrating space and a constant and $\beta_\rho'\Delta_1 X_{t-1}$ entered as unrestricted regressors. Searching in the region $\rho > 1$ there appears to be a unique maximum to the likelihood function of 41.3 with $\hat\rho = 1.174$ and a slightly changed cointegrating vector β_1 as given in Table 6. The test statistic for H_ρ against H_1 is

3.0 which is small compared to the $\chi^2(2)$ distribution. The sign to the linear trend component in β_1 appears to change but remains significantly different from zero.

In summary, this analysis indicates, that m_t, p_t, s_t have common unit root components and explosive components

$$m_t, p_t, s_t \sim \mathsf{I}(1, x).$$

whereas the possibility that the variables are $\mathsf{I}(2)$ was rejected. Thus, differencing removes the unit root component giving pure explosive variables

$$\Delta_1 m_t, \Delta_1 p_t, \Delta_1 s_t \sim \mathsf{I}(x),$$

while the real variables are co-explosive

$$m_t - p_t, m_t - s_t, s_t - p_t \sim \mathsf{I}(1).$$

Since $\mathsf{I}(2)$-ness is rejected the Yugoslavian episode gives evidence against the $\mathsf{I}(2)$ assumptions of Sargent (2.7) as well as the rational expectations solution (2.9) without bubble component, so $c = \xi_s = 0$, and where m_t is assumed to be $\mathsf{I}(2)$ as proposed by Taylor (1991). Since m_t and p_t co-explode the rational expectations solution (2.9) without bubble component and where m_t is assumed explosive is also ruled out.

The bubble solution (2.9) in which $c \neq 0$ and $\xi_s \neq 0$ was discussed by Diba and Grossman (1988) and successfully implemented in a co-explosive analysis of stock prices and dividends by Engsted (2006). In the context of hyperinflation the bubble solution predicts that p_t is explosive, whereas m_t is not. The fact that m_t, p_t, s_t have an explosive common trend and their differences co-explode is evidence against this hypothesis. The hypothesis could also be formulated directly as the co-exploive relation $\beta_\rho = (0, 1)$ in an analysis of the bivariate system of $X_t^{(2)} = (m_t, p_t)'$. It turns out to be awkward to estimate the model under that hypothesis, essentially because the hypothesis does not fit with the data. Since the restricted model cannot be maximised analytically and the hypothesis does not appear to be valid the estimated restricted model is somewhat complicated. Mimicking the analysis above for the two variables $X_t^{(2)}$ gives more or less the same results: cointegration rank of one, rejection of $\mathsf{I}(2)$-ness, and $m_t - p_t$ is a co-exploding relation. In particular, imposing a cointegration rank of one gives a likelihood of 10.8 and an explosive root of 1.205. The model satisfying $\beta_\rho = (0, 1)$ can be estimated by a reduced rank regression of $\Delta_1\Delta_\rho X_t^{(2)}$ on $\Delta_\rho X_t^{(2)}$ and a time trend correcting for intercept, $\Delta_1\Delta_\rho X_{t-1}^{(2)}$ and $\beta_\rho' \Delta_1 X_{t-1}^{(2)} = \Delta_1 m_{t-1}$. For $\rho = 1.205$ the likelihood is 1.3 so twice the likelihood distance is 19.0. While this is not the likelihood ratio statistic the statistic would quite possibly be χ_1^2 if the hypothesis were valid, giving another indication that the hypothesis is not valid. Searching over ρ the maximum is found at $\rho = 1$ with likelihood 3.7. A detailed analysis of the estimated model shows that it actually has an I(2) root, corresponding to $\rho = 1$, as well as an explosive root in the short term dynamics to pick up the explosiveness of m_t. The likelihood ratio statistic of 14.1 should probably be judged against an I(2)-distribution, which is not tabulated in the literature, but then the I(2) was already rejected.

In summary, this empirical analysis follows in its principles of the I(2) analyses in the literature in that the variables are analysed in levels with a view towards establishing cointegration and if possible (polynomial) cointegration between real money, $m_t - p_t$, and $\Delta_1 p_t$. As in previous studies the hyper-inflation episode is not modelled to the end due to difficulties in capturing the properties of the data. It is found that the three variables p_t, m_t, s_t have a common explosive trend and two common random walk trends. The series co-explode so $m_t - p_t$, $m_t - s_t$ and $p_t - s_t$ are all non-exploding random walks. Thereby the rational expectations solution (2.9) is contradicted. The conclusion that $m_t - p_t$, $m_t - s_t$ and $p_t - s_t$ are non-exploding random walks is, however, in line with the assumptions of Sargent (1977) and Taylor (1991). The differenced series $\Delta_1 p_t, \Delta_1 m_t, \Delta_1 s_t$ are, however, explosive with no random walk component. This indicates that linking for instance $m_t - p_t$ with $\Delta_1 p_t$ will not give a balanced regression in this situation and explains why linear modelling of the variables in levels is not giving an adequate empirical model. In the following a solution is found by abandoning the discretization assumption (2.4).

4 Measuring Inflation

The assumption (2.4) of piece wise constant rate of change in prices, C_t, appears more and more unrealistic as the inflation progresses. This is apparent from Figure 1(b) where the line pieces connecting the points of the time series become steeper and steeper. By discretization of the continuous rate of change in a different way this problem can be overcome and the puzzles resolved.

As an alternative measure of the cost of holding money consider

$$c_t = \frac{\Delta_1 P_t}{P_t} = 1 - \frac{P_{t-1}}{P_t} = 1 - \exp\left(-\Delta_1 p_t\right),$$

showing the relative loss in purchasing power over one period and the relative gain if c_t is negative. This measure can be motivated by an argument inspired by Hendry and von Ungern-Sternberg (1981). The nominal money stock grows according to

$$M_t = M_{t-1} + \delta_t,$$

where δ_t represents net money issues. Dividing through by P_t gives

$$\frac{M_t}{P_t} = \frac{M_{t-1}}{P_{t-1}}\left(\frac{P_{t-1}}{P_t}\right) + \frac{\delta_t}{P_t},$$

where the coefficient $c_t = 1 - P_{t-1}/P_t$ is the proportion of the real money stock that is lost from period to period.

The variable c_t is bounded by 1 indicating that in each period one can at most loose all money. This fits nicely with interpreting inflation as seigniorage, giving a maximal tax rate of 100%. When the quantity $\Delta_1 p_t = p_t - p_{t-1} = \log(P_t/P_{t-1})$ is small, a Taylor expansion shows $c_t \approx \Delta_1 p_t$. Once the inflation rise above about 20% per period there will be a substantial difference between c_t and $\Delta_1 p_t$. Note, that $c_t = \Delta_1 P_t/P_t$ is different from the percentage change $\Delta_1 P_t/P_{t-1}$. The measure c_t is closely related to the inflation measure $\Delta_1 p_t/(1 + \Delta_1 p_t)$, which, however, has an asymptote for $\Delta_1 p_t = -1$. Such a fall was for instance experienced in the dollars/German mark exchange rate in the second quarter of 1920.

It seems more conceivable that $c_t^e - c_t$ is stationary than $\Delta_1^e p_t - \Delta_1 p_t$ is stationary. Likewise, agents in the economy can handle and perhaps even forecast a variable like c_t rather than $\Delta_1 p_t$. This is illustrated by a numerical example in which prices could go up 10- or 20-fold. This translates into a c_t of 0.9 or 0.95 and a $\Delta_1 p_t$ of $\log 10 = 2.3$ or $\log 20 = 3.0$. In the latter case the uncertainty is exploding with $\Delta_1 p_t$ whereas the bounded nature of c_t ensures that the increasing uncertainty about the economy has a bounded impact.

The proposal is therefore to use c_t as a discrete time proxy for the continuous time cost of holding money, C_t, appearing in Cagan's model. Inspired by the setup of Taylor, see §2.2, the testable assumptions are that $m_t - p_t$ and c_t are $\mathsf{I}(1)$. The usual rational expectations machinary does no longer apply since the difference variable $\Delta_1 p_t$ is now replaced by c_t which is not linear in p_t and p_{t-1}. It is, however, possible that the agent's of the economy may form somewhat accurate forecasts, c_{t+1}^e, of the cost of holding money that cointegrate with the actual cost, c_{t+1}, as long as the inflation runs. This leads to the following discrete time version of Cagan's model

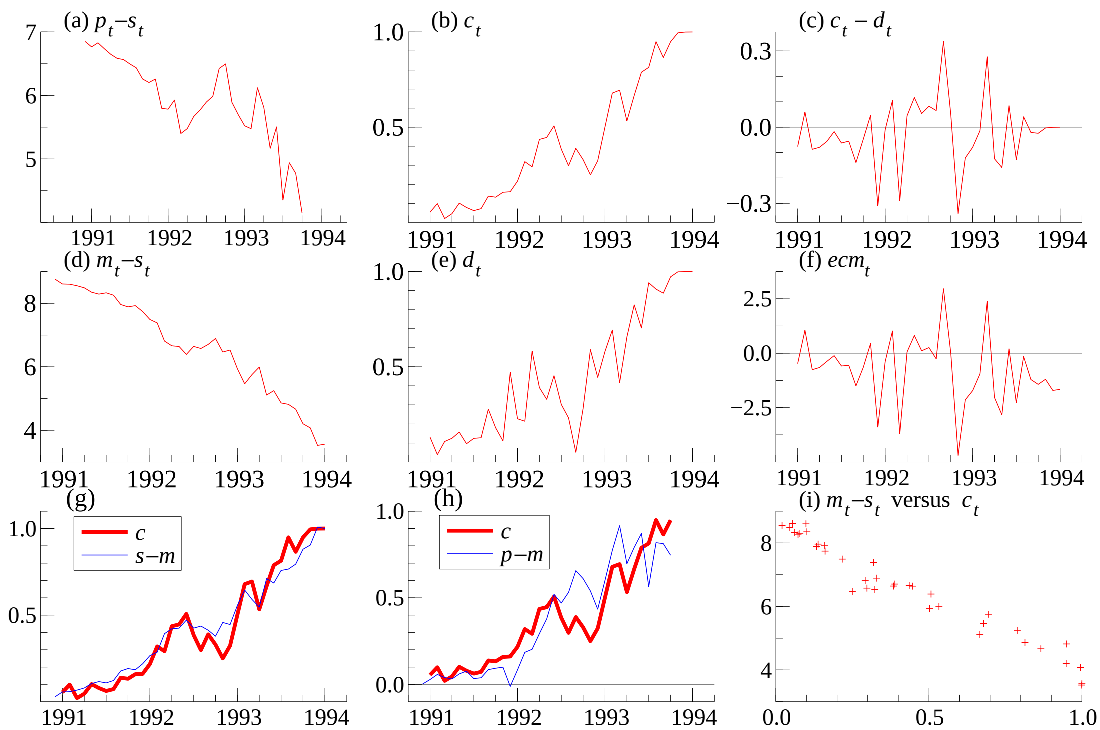
$$\begin{aligned} m_t - p_t &= -\alpha c_{t+1}^e + \zeta_t, \\ c_{t+1}^e &= c_{t+1} + \epsilon_{t+1}, \end{aligned}$$

where α corresponds to Cagan's continuous time semi-elasticity. Following the manipulations of Taylor this implies the equilibrium correction model

$$\Delta_1 c_{t+1} = -\alpha^{-1}(m_t - p_t + \alpha c_t) + \left(\epsilon_{t+1} + \alpha^{-1}\zeta_t\right).$$

While $\Delta_1 p_t$ is the standard inflation measure when analysing economies without severe inflation the choice of measure becomes increasingly important as the inflation accelerates. As the price series p_t accelerates, c_t approaches 1 indicating a nearly complete loss in value of money. This type of transformation is related to the non-linear models suggested by Frenkel (1977) linking real money, $m_t - p_t$, with either $\log(\Delta_1 p_t)$ or $(\Delta_1 p_t)^\gamma$. These measures do, however, not approximate $\Delta_1 p_t$ even for small values of inflation, so they do not fit easily with the Cagan setup. A measure like c_t appears to give a more direct measure of the cost of holding money and can more easily be used in a linear model. Finally, the cost of holding money has the added benefit of reducing the impact of measurement error as prices accelerate. In the Yugoslavian case the measurement issues for the last few observations of p_t can therefore be ignored when using c_t rather than $\Delta_1 p_t$ as inflation measure.

Figure 2: The series $m_t - p_t$, $m_t - s_t$, c_t, d_t and linear transformations thereof



Note: the series in (a, h) are shown only until 93:10.

The transformed variable c_t as well as a depreciation rate $d_t = 1 - \exp(-\Delta_1 s_t)$ are plotted in Figure 2(b, e). Real money will be measured as $m_t - s_t$ rather than $m_t - p_t$. This is partly due to measurement problems in prices as shown in Figure 1(c), and partly due to a considerable currency substitution. Moreover, the exchange rate is in effect a price index for a single 'good', whereas the price index p_t is an average over goods which will have very different inflation rates if there are price controls on some of the goods. The cross-plot in Figure 2(i) shows a near linear relationship between $m_t - s_t$ and c_t in contrast to Figure 1(i). Concentrating on the variables $m_t - s_t, c_t, d_t$ at first it is possible to set up a model for the entire period up to 1994:1. This will be done in the following.

5 A Linear Model for Transformed Variables

A vector autoregressive model is set up for the transformed variables $m_t - s_t$, c_t, d_t. This model can be analysed using standard $\mathsf{I}(1)$ cointegration techniques. Here money is chosen to be deflated by the exchange rate instead of the price index, partly because the exchange rate measures the price of just one 'good', namely German mark, and partly to avoid measurement problems for p_t in the end of the sample.

5.1 Model and Rank Determination

A third order vector autoregression with a restricted constant is fitted to the data 1991:1 to 1994:1 giving a sample size of $T = 37 - 3 = 34$. The lag length is chosen so as to ensure that the mis-specification tests pass. As the explosive component will now be eliminated two lags would perhaps have been preferable on grounds of parsimony. Modelling the data right until the end of the hyper-inflation in 1994:1 resolves the first puzzle set out in §2.1. While this only represents a modest gain in degrees of freedom, the importance lies in the ability to analyse the hyper-inflation to the end. This is where Cagan's theory is meant to work best. Mis-specification tests supporting the model are reported Table 7. Graphical tests, not reported here, include recursive tests and they are likewise supportive of the model. This shows that a well-specified joint model with time-invariant parameters can be established

Table 8 shows tests for stationarity of individual variables. No variable can be considered stationary on its own, not even the linear combination $c_t - d_t$.

Table 7: Misspecification tests for model for transformed data

Test	$m_t - s_t$	c_t	d_t	Test	$(m_t - s_t, c_t, d_t)$
$\chi^2_{normality}(2)$	0.1 [0.95]	1.2 [0.54]	1.9 [0.38]	$\chi^2_{normality}(6)$	2.8 [0.83]
$F_{AR(1)}(1, 23)$	0.1 [0.71]	0.1 [0.70]	1.4 [0.25]	$F_{AR(1)}(9, 46)$	0.5 [0.87]
$F_{AR(3)}(3, 21)$	0.8 [0.49]	1.3 [0.31]	2.1 [0.13]	$F_{AR(3)}(27, 38)$	0.9 [0.59]
$F_{ARCH(3)}(3, 18)$	1.4 [0.28]	0.2 [0.91]	0.2 [0.88]		

p-values are given in brackets

Table 8: Test for stationarity of individual variables

$m_t - s_t$	c_t	d_t	$c_t - d_t$
32.4	32.4	30.6	21.9

The stationarity tests are fore the restriction that $\beta = (e_3, 1)'$ where e_3 is a unit vector of dimension three. All tests are asymptotically χ^2_2, and therefore strongly rejected

There is now one characteristic root at 1.035 while the remaining roots are well inside the unit circle, see Table 9. The cointegration rank tests reported in Table 10 point to a rank of 1. Under that hypothesis the slightly explosive root is restricted to 1 and all characteristic roots, but two unit roots, are well inside the unit circle, see Table 11. In other words the apparent explosive root in the unrestricted model is not significantly different from one. The issue of explosiveness then disappears and the standard cointegration analysis of Johansen (1996) is applicable with the conventional interpretation.

Table 9: Characteristic roots of unrestricted model for transformed data

Re(z)	1.035	0.61	0.61	-0.01	-0.01	-0.04	-0.33	-0.33	-0.44
Im(z)	0	0.21	-0.21	0.57	-0.57	0	0.64	-0.64	0
$\|z\|$	1.035	0.65	0.65	0.57	0.57	-0.04	0.72	0.72	0.44

Table 10: Cointegration rank tests for transformed data

Hypothesis	$H(0)$	$H(1)$	$H(2)$	$H(3)$
Test	60.1 [0.00]	15.5 [0.20]	4.2 [0.40]	
Likelihood	80.03	102.31	107.97	110.06

p-values are given in brackets

Table 11: Characteristic roots when the rank is restricted to one

Re(z)	1	1	0.81	0.09	-0.14	-0.14	-0.17	-0.40	-0.40
Im(z)	0	0	0	0	0.68	-0.68	0	0.54	-0.54
$\|z\|$	1	1	0.81	0.09	0.70	0.70	0.17	0.67	0.67

5.2 The Cointegrating Vector

The cointegrating relation estimated from the Johansen approach is given by

$$\underset{(\sqrt{LR})}{ecm_t} = \underset{(2.8)}{1}(m_t - s_t) + \underset{(5.1)}{13.5}c_t - \underset{(5.7)}{10.3}d_t - \underset{(-2.7)}{8.48} \quad (5.1)$$

$$= \underset{(2.8)}{1}(m_t - s_t) + \underset{(2.0)}{3.26}c_t - \underset{(5.7)}{10.3}(d_t - c_t) - \underset{(-2.7)}{8.48} \quad (5.2)$$

$$= \underset{(2.8)}{1}(m_t - s_t) + \underset{(2.0)}{3.26}d_t - \underset{(5.1)}{13.5}(d_t - c_t) - \underset{(-2.7)}{8.48} \quad (5.3)$$

The signed log-likelihood ratio test statistics for individual exclusion restrictions are reported in brackets and are asymptotically standard normal distributed, so one-sided tests 5% level tests would have a critical value of about plus or minus 1.65.

This cointegrating vector shows that real money, deflated by exchange rates, moves both with c_t and d_t. It is formulated in three equivalent ways involving two of the three variables c_t, d_t and $d_t - c_t$. By construction the coefficients to c_t and d_t in (5.2) and (5.3) are identical and are interpreted as the semi-elasticity for the expected future cost of holding money as discussed in §4. Exclusion of the differential $d_t - c_t$ is strongly rejected whereas the decision to keep c_t or d_t is marginal. In order not to distort the subsequent analysis by making a marginal decision no restrictions are made on the cointegrating relation.

The cointegrating equation is approximately of the same form as Cagan's with real money stock measured in foreign currency falling with depreciation rate d_t. Indeed, ignoring the significant component $d_t - c_t$ in (5.3) and replacing d_t by $\Delta_1 s_t$ the relation (5.3) appears close to the relation $m_t - s_t = -3.4\Delta_1 s_t + 8.4$ found by Petrović and Mladenović (2000, Table 2) in an analysis until 1993:6. A similar analysis using d_t gives $m_t - s_t = -6.0\Delta_1 s_t + 8.9$ with the difference stemming from the discrepancy between d_t and $\Delta_1 s_t$ culminating at $\Delta_1 s_{93:6} = 1.22$ and $d_{93:6} = 0.70$ in this sub-sample.

The term $d_t - c_t$ can be interpreted as the real appreciation rate of the German mark. It enters positively so that if the German mark appreciates faster than prices rise goods become relative cheaper and the real money circulation rises. This is a variation of the combination of transactions and portfolio demand discussed by Ando and Shell (1975), Goldfeld and Sichel (1990), Baba, Hendry, and Starr (1992). Comparing the Figures 2(c, d) shows how the sign of $c_t - d_t$ varies over time so $m_t - s_t$ tends to increase when $c_t - d_t$ is negative. The cointegrating relation itself, normalised on real money is plotted in Figure 2(f) .

5.3 The Inflation Tax

Ignoring the differential of the cost of holding money and the depreciation, Cagan's semi-elasticity α can be estimated by $\widehat{\alpha} = 3.26$. This value is in line with both Cagan's and Sargent's estimates for the German hyper-inflation. According to Cagan the maximal revenue from seigniorage, assuming money rises at a constant rate, is then estimated by $\exp(\widehat{\alpha}^{-1}) - 1 = 36\%$. It seems natural to compare this with the average cost of holding money for a month, $c_t = \Delta_1 P_t / P_t$, rather than the average of inflation measure through $\Delta_1 P_t / P_{t-1}$ since the former is precisely a measure for how much value is lost over a month. For the full sample this average is 42.6%. The likelihood ratio test statistic for the hypothesis that the coefficient to c_t is $\{\log(1 + 0.426)\}^{-1}$ is 0.43 [$\mathsf{p} = 0.51$]. Likewise the average of d_t is 44.9%. The test statistic for the coefficient to d_t being $\{\log(1 + 0.449)\}^{-1}$ is 0.58 [$\mathsf{p} = 0.45$]. While the assumption underlying Cagan's theory of money rising at a constant rate is violated and the idea of taking average over time of a trending variable is somewhat contrived, the predictions of his theory are not rejected this way.

Table 12: The adjustment vector $\hat{\alpha}$ for the transformed model

$m_t - s_t$	c_t	d_t	$c_t - d_t$
0.31	−0.092	−0.065	−0.028
(4.6)	(−5.3)	(−1.7)	(−0.7)

Signed likelihood ratio statistic, $\sqrt{LR}$, for insignificance is given in brackets

5.4 Weak Exogeneity Properties

Having the cointegrating relation in place, the short term dynamics of the system can be analysed in order to understand how the variables adjust. The notion of weak exogeneity introduced by Engle, Hendry and Richard (1983) is helpful and can be implemented in the cointegration analysis by restricting the adjustment vector α, see Johansen (1996, §8). After exploration of weak exogeneity properties the approach of Hendry (1995, §16.8) is followed in obtaining parsimonious vector autoregressions by simultaneous equation methods using the estimated cointegrating relation as regressor. This will go a step towards uncovering the causality structure.

An advantage of Johansen's method for cointegration analysis is its invariance to linear transformations of the variables, hence it is equivalent to consider the variable vectors $(m_t - s_t, c_t, d_t)$ and $(m_t - s_t, c_t, c_t - d_t)$. Table 12 reports the four different adjustment coefficients related to this model. While it is rejected that real money, $m_t - s_t$, or the cost of holding money, c_t, could be weakly exogeneous, there is a marginal indication that the depreciation rate, d_t, could be weakly exogenous, and stronger evidence that the real depreciation rate, $c_t - d_t$ could be weakly exogenous. In the following weak exogeneity is imposed for $c_t - d_t$. This has the interpretation that the fluctuations in the foreign exchange rate, $c_t - d_t$, are exogenous to the demand for money.

In the sub-sequent analysis weak exogeneity of $c_t - d_t$ is imposed in the context of a model for $(m_t - s_t, d_t, c_t - d_t)$. In this way the endogenous variables $m_t - s_t$ and d_t are balanced in that d_t is the cost, in terms of the depreciation rate, of holding money deflated by the exchange rate. When weak exogeneity is imposed the cointegrating vector (5.3) changes slightly to

$$ecm_t^d = m_t - s_t + 3.22 d_t - 13.5\,(d_t - c_t) - 8.50.$$

Including this as a regressor the conditional system can be reduced to

$$\begin{aligned}\Delta_1 d_t &= -\underset{(0.009)}{0.087} ecm_{t-1} + \underset{(0.04)}{0.12}\Delta_1(m-s)_{t-1} + \underset{(0.03)}{0.17}\Delta_1(m-s)_{t-2} \\ &\quad -\underset{(0.05)}{0.89}\Delta_1^2(c-d)_t - \underset{(0.03)}{0.27}\Delta_1^2(c-d)_{t-1} + 0.044\hat{\varepsilon}_t, \qquad (5.4)\\ \Delta_1(m-s)_t &= +\underset{(0.04)}{0.29} ecm_{t-1} - \underset{(0.16)}{0.64}\Delta_1(m-s)_{t-1} + \underset{(0.22)}{0.74}\Delta_1(c-d)_t \\ &\quad -\underset{(0.28)}{1.22}\Delta_1(c-d)_{t-1} - \underset{(0.21)}{0.90}\Delta_1(c-d)_{t-2} + 0.177\hat{\varepsilon}_t, \qquad (5.5)\end{aligned}$$

where the over-all likelihood ratio test statistic is 6.8 [$\mathsf{p} = 0.34$] compared to a $\chi^2(6)$-distribution. The marginal model for $(c_t - d_t)$ can likewise be reduced to

$$\Delta_1(c-d)_t = -\underset{(0.14)}{0.47}\Delta_1(c-d)_{t-1} - \underset{(0.14)}{0.42}\Delta_1(c-d)_{t-2} - \underset{(0.08)}{0.25}\Delta_1(m-s)_{t-1} + 0.139\hat{\varepsilon}_t, \qquad (5.6)$$

where the likelihood ratio for the reduction is 1.3 [$\mathsf{p} = 0.74$] compared to a $\chi^2(3)$-distribution. The weak exogeneity of $c_t - d_t$ fits with the combined transactions and portfolio demand interpretation of the cointegrating vector discussed in §5.2 with the real depreciation rate $d_t - c_t$ being a driving force for inflation.

The empirical model indicates that the (weakly) endogenous variables, real money and the cost of holding money, are determined simultaneously. This suggests a more complicated relationship than in single cause models like Sargent's model where inflation causes money and models where money causes inflation. Moreover, the equation for the exogenous variable $c_t - d_t$ shows an ongoing feedback from the changes in real money into the foreign exchange market, which is not unreasonable. The residual standard error in the equation for $m_t - s_t$ is 0.18 compared to 0.27 in the simple time series model in §2.4 that form the basis for Sargent's model. It is interesting to note that due to the new measures c_t and d_t of the cost of holding money and the depreciation rate the emphasis in this model is on real money, whereas in Sargent's model the role of nominal and real money is more interchangeable.

A similar analysis could also be carried out with $m_t - p_t$ instead of $m_t - s_t$ as measure for real money, were it not for the measurement errors of p_t in the end of the sample and an attempted prize freeze in July 1990. Even when taking these issues into account the cointegration analysis is less clear. This point can be illustrated graphically. In Figure 2(g, h), the negative of the the real money variables, $s_t - m_t$ and $p_t - m_t$, respectively, are plotted with c_t with ranges and means adjusted to the latter. It is clear that $s_t - m_t$ follows c_t nicely with discrepancies matched by $d_t - c_t$ of Figure 2(c) as in the analysis above while $p_t - m_t$ does not track c_t well. Further research would be needed to see whether this is a feature particular to the Yugoslavian case, or whether the relative ease of measuring exchange rates rather than prices makes $m_t - s_t$ a better measure for real money in hyper-inflations. The issue at hand could be that $m_t - s_t$ involves the price of a single "good", whereas $m_t - p_t$ involves the price index, which is constructed by averaging over goods which can have very different inflation rates in a hyper-inflation.

6 Discussion

The discretization assumption (2.4) and the use of $\Delta_1 p_t$ as the cost of holding money have been identified as the main sources for the puzzles in the empirical analysis of hyper-inflations. Since $m_t - p_t$ has random walk-like behaviour while $\Delta_1 p_t$ has explosive behaviour regressions of $m_t - p_t$ on $\Delta_1 p_t$ will be unbalanced. The proposed solution is straightforward in replacing $\Delta_1 p_t$ by the cost of holding money, c_t. This variable has desirable statistical properties in that it is bounded and it has random walk-like behaviour. Its interpretation is simple and similar to that of the rate of change in prices appearing in Cagan's continuous time model.

The rational expectations model of the kind discussed by Taylor (1991) and analysed in some detail by Engsted (1993) does not appear to match these data. That model essentially expresses p_t as a function of m_t, so that the properties of p_t are derived from whatever properties m_t is thought to have. The appendix analyses four different assumptions for m_t with and without bubbles, and none of these examples appear to be supported by the data. Thus, the Yugoslavian hyper-inflation does not appear to give evidence in favour of rational expectations model.

With the new inflation measure various lines of future research are opened up. First, a comparative analysis of hyper-inflation episodes in different countries using the cost of holding money as inflation measure can provide new insights, notably for the classic episodes studied by Cagan. Secondly, Cagan's assertion that variables like productivity and wages are irrelevant in hyper-inflation can be reviewed as done in the work by Michael, Nobay and Peel (1994) and Juselius and Mladenović (2002). Thirdly, on the structural side it would be interesting to reconsider the structural models in literature, recognising that it does not appear valid to exploit that the cost of holding money is linear in p_t and p_{t-1}.

Appendix: Explosive Bubbles

The rational expectations model (2.8), that is

$$m_t - p_t = -\alpha \mathsf{E}_t (p_{t+1} - p_t) + \zeta_t, \tag{A.1}$$

allows expressing prices, p_t in terms of expectation to future money stocks, $\tilde{m}_t = m_t - \zeta_t$. In the following it is discussed how properties assumed for $\tilde{m}_t$ transmits into properties p_t.

The solution to (A.1) was given in (2.9) as

$$p_t = \frac{1}{1+\alpha}\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j \mathsf{E}_t\left(\tilde{m}_{t+j}\right) + c\left(\frac{1+\alpha}{\alpha}\right)^t + \sum_{s=1}^{t}\left(\frac{1+\alpha}{\alpha}\right)^{t-s}\xi_s. \qquad \text{(A.2)}$$

This is seen to be a solution by deriving $\mathsf{E}_t p_{t+1}$ from (A.2) and inserting in (A.1), see also Turnovsky (2000, p. 86f & 91f) and Diba and Grossmann (1988). The solution is well-defined when $\alpha > 0$ and $|\{\alpha/(1+\alpha)\}^j \mathsf{E}_t(\tilde{m}_{t+j})|$ vanishes at a geometric rate.

It is convenient to derive equations for $p_t - \tilde{m}_t$ and $\tilde{m}_t - p_t + \alpha\Delta_1 p_{t+1}$. First, subtracting $\tilde{m}_t$ on both sides of (2.9) and using that $\sum_{j=0}^{\infty}\{\alpha/(1+\alpha)\}^j = 1+\alpha$ it follows that

$$p_t - \tilde{m}_t = \frac{1}{1+\alpha}\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j e_{t,j} + c\left(\frac{1+\alpha}{\alpha}\right)^t + \sum_{s=1}^{t}\left(\frac{1+\alpha}{\alpha}\right)^{t-s}\xi_s, \qquad \text{(A.3)}$$

where $e_{t,j} = \mathsf{E}_t\left(\tilde{m}_{t+j} - \tilde{m}_t\right)$, whereas differencing gives

$$\alpha\Delta_1 p_{t+1} = \sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^{j+1} f_{t,j} + c\left(\frac{1+\alpha}{\alpha}\right)^t + \sum_{s=1}^{t}\left(\frac{1+\alpha}{\alpha}\right)^{t-s}\xi_s + \alpha\xi_{t+1}. \qquad \text{(A.4)}$$

where $f_{t,j} = \mathsf{E}_{t+1}\left(\tilde{m}_{t+1+j}\right) - \mathsf{E}_t\left(\tilde{m}_{t+j}\right)$. This implies the observable version of (A.1) is

$$\tilde{m}_t - p_t + \alpha\Delta_1 p_{t+1} = \frac{1}{1+\alpha}\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j \left(\alpha f_{t,j} - e_{t,j}\right) + \alpha\xi_{t+1}. \qquad \text{(A.5)}$$

To describe the properties of p_t some assumptions must be made to $\tilde{m}_t, c,$ and ξ_s. It is interesting to consider a few examples.

Example 1. An $\mathsf{I}(1)$ model without bubble. Assume that $\Delta_1\tilde{m}_t = \eta_t$ with $\mathsf{E}_t\eta_{t+j} = 0$, and that $c = \xi_s = 0$. The equation for $\tilde{m}_t$ implies

$$\tilde{m}_{t+j} = \sum_{u=1}^{j}\eta_{t+u} + \tilde{m}_t,$$

so $\mathsf{E}_t\left(\tilde{m}_{t+j}\right) = \tilde{m}_t$ implying $e_{t,j} = 0$ and $f_{t,j} = \Delta_1\tilde{m}_{t+1}$. Then (A.2) shows

$$p_t = \frac{1}{1+\alpha}\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j \tilde{m}_t = \tilde{m}_t$$

since $\sum_{j=0}^{\infty}\{\alpha/(1+\alpha)\}^j = 1+\alpha$ so $p_t \sim \mathsf{I}(1)$. In the same way (A.3) and (A.5) show that $\tilde{m}_t - p_t, \tilde{m}_t - p_t + \alpha\Delta_1 p_{t+1} \sim \mathsf{I}(0)$

Example 2. An $\mathsf{I}(2)$ model without bubble as in model (2.12), originally suggested by Taylor (1991). Assume, for simplicity that $\Delta_1^2 \tilde{m}_t = \eta_t$ with $\mathsf{E}_t \eta_{t+j} = 0$, and that $c = 0$ and $\xi_s = 0$. The equation for $\tilde{m}_t$ implies

$$\tilde{m}_{t+j} = \sum_{v=1}^{j} \sum_{u=1}^{v} \eta_{t+u} + \tilde{m}_t + j \Delta_1 \tilde{m}_t,$$

so $\mathsf{E}_t(\tilde{m}_{t+j}) = \tilde{m}_t + j\Delta_1 \tilde{m}_t$ implying $e_{t,j} = j\Delta_1 \tilde{m}_t$ and $f_{t,j} = \Delta_1 \tilde{m}_{t+1} + j\Delta_1^2 \tilde{m}_{t+1}$. Then (A.2) shows that $p_t \sim \mathsf{I}(2)$, (A.3) shows that $\tilde{m}_t - p_t \sim \mathsf{I}(1)$ so $\Delta_1 \tilde{m}_t - \Delta_1 p_t \sim \mathsf{I}(0)$, whereas (A.5) shows $\tilde{m}_t - p_t + \alpha \Delta_1 p_{t+1} \sim \mathsf{I}(0)$ since $\sum_{j=0}^{\infty} \{\alpha/(1+\alpha)\}^j (\alpha - j) = 0$. Combining these results shows that also $\tilde{m}_t - p_t + \alpha \Delta_1 m_{t+1} \sim \mathsf{I}(0)$ as pointed out by Engsted (1993).

Example 3. An $\mathsf{I}(1)$ model with bubble. Assume that $\Delta_1 \tilde{m}_t = \eta_t$ with $\mathsf{E}_t \eta_{t+j} = 0$, and that $c \neq 0$ and $\xi_s \neq 0$. Then p_t and $\tilde{m}_t - p_t$ and $\tilde{m}_t - p_t + \alpha \Delta_1 p_{t+1}$ have the same number of unit roots as in Example 1 and p_t and $\tilde{m}_t - p_t$ and $\tilde{m}_t - p_t + \alpha \Delta_1 p_t$ also have explosive components, whereas $\tilde{m}_t - p_t + \alpha \Delta_1 p_{t+1}$ has no explosive component, see also Engsted (1993, p. 354).

Example 4. An $\mathsf{I}(1)$ model with explosive roots, that is $\mathsf{I}(1, x)$ say, but no bubble. Assume $\Delta_1 \Delta_\rho \tilde{m}_t = \eta_t$ where $1 < \rho < (1+\alpha)/\alpha$ with $\mathsf{E}_t \eta_{t+j} = 0$, and that $c = \xi_s = 0$. Then

$$\tilde{m}_{t+j} = \frac{1}{1-\rho} \sum_{u=1}^{j} \eta_{t+u} + \frac{1}{\rho - 1} \rho^j \sum_{u=1}^{j} \rho^{-u} \eta_{t+u} + \tilde{m}_t + \Delta_1 \tilde{m}_t \rho \frac{1-\rho^j}{1-\rho},$$

so $(1-\rho)\mathsf{E}_t(\tilde{m}_{t+j}) = \Delta_\rho \tilde{m}_t - \rho^{j+1} \Delta_1 \tilde{m}_t$ implying $(1-\rho) e_{t,j} = \Delta_1 \tilde{m}_t \rho (1 - \rho^j)$ and $(1-\rho) f_{t,j} = (1-\rho^{j+1}) \Delta_1 \Delta_\rho \tilde{m}_{t+1} + (1-\rho)\rho^{j+1} \Delta_1 \tilde{m}_t$. Then $p_t \sim \mathsf{I}(1, x)$, $\tilde{m}_t - p_t \sim \mathsf{I}(x)$, and $\tilde{m}_t - p_t + \alpha \Delta_1 p_{t+1} \sim \mathsf{I}(x)$ since $\sum_{j=0}^{\infty} \{\alpha/(1+\alpha)\}^j \{\alpha(1-\rho)\rho^{j+1} - \rho(1-\rho^j)\} = (1+\alpha)(1-\rho) \neq 0$. So the variables have the same number of unit roots as in Example 1 combined with an explosive component.

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