

## A Long-Run Structural Macroeconometric Model for Germany: An Empirical Note

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### **Abstract:**

To provide an modelling strategy with transparent and theoretically coherent foundation has been one of the targets of the paper by Garratt, Lee, Pesaran and Shin. They develop a core model for a small open economy based production technology, arbitrage conditions, flow identities and long-run solvency conditions. This leads to five long-run relations: the uncovered interest rate parity, the purchasing power parity, production function, trade balance and real money balance. Since the economic theory there is formulated generally but not restricted to the economy of UK for which their empirical model is implemented, we expect that this modelling strategy should be able to generate similar results for the data of other countries.

In this empirical note we apply the modelling strategy to German data to see in how far the economic theory formulated there can account for German data. We are able to identify five cointegration relations in a conditional vector error correction model and the overidentification restrictions of the five cointegration relations as, UIP, PPP, production function, trade balance and real money balance are not rejected by the data.

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## 1 The Model

The aim of the long run structural modelling strategy by Garratt et al. (2000), Garratt et al. (2003) and Garratt et al. (2004) is to develop a model with transparent and theoretically coherent foundation. The authors advanced the modelling framework of King R.G. and Watson (1991), Mellander et al. (1991) and Crowder et al. (1999), and developed a long-run framework suitable for modeling a small open macroeconomy. Their new strategy offers a practical approach to relationships suggested by economic theory in an otherwise unrestricted vector autoregressive (VAR) model. The structural model of UK in Garratt et al. (2003) includes theoretically founded long-run equations of the type exhibited by RBC models. The long-run relations are derived from production function, arbitrage conditions, solvency and portfolio balance conditions.

For the empirical analysis Garratt et al. (2003) used a log-linear approximation of the five long-run equilibrium relationships. In addition they introduced a “long-run forcing” variable as an exogenous variable: the oil prices. “Forcing” variable means that changes in oil prices have a direct influence on output, but they are not affected by the other variables in the model. This ends up with a conditional vector error correction model (VECMX) with nine variables and five structural cointegration relations, in which the five long-run relations correspond to the five cointegration relations and the oil prices as the conditioning variable.

The unconditional VAR model with the nine variables can be formulated as follows

$$\Delta \mathbf{z}_t = \mathbf{b} + \alpha \beta' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{u}_t \quad (1)$$

$$\mathbf{z}_t = (p_t^o, r_t, y_t, \Delta p_t, p_t - p_t^*, e_t, h_t - y_t, r_t^*, y_t^*)'$$

where  $p_t^o$  is log of the oil price;  $r_t = \log(1 + R_t)$  is the domestic continuously compounded interest rate;  $y_t$  is log of the domestic per capita output;  $\Delta p_t$  is the rate of inflation;  $p_t^*$  is the price of the world,  $p_t - p_t^*$  is the price gap between domestic and international prices;  $e_t$  is log of the exchange rate;  $h_t$  is log of the high power money.  $h_t - y_t$  is log of high power money per unit of output;  $r_t^* = \log(1 + R_t^*)$  is the world continuously compounded interest rate; and  $y_t^*$  is the world per capita output.

According to Garratt et al. (2003) the five log linearized long-run relations are<sup>1</sup>:

$$(p_t - p_t^*) - e_t = a_{10} + \xi_{1,t+1} \quad (2)$$

$$r_t - r_t^* = a_{20} + \xi_{2,t+1} \quad (3)$$

$$y_t - y_t^* = a_{30} + \xi_{3,t+1} \quad (4)$$

$$h_t - y_t = a_{40} + \beta_{42} r_t + \beta_{43} y_t + \xi_{4,t+1} \quad (5)$$

$$r_t - \Delta p_t = a_{50} + \xi_{5,t+1} \quad (6)$$

These structural long-run relations imply the following (over)-identification restrictions on the cointegration matrix  $\beta$  in the unconstrained VAR in equation (1).

<sup>1</sup> See Garratt et al. (2003) for detailed derivation of the model.

$$\beta' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -\beta_{42} & \beta_{43} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7)$$

For oil prices to be the forcing variable, a necessary condition is that  $p_t^o$  is weakly exogenous for the parameter of the conditional process of VAR (1) given  $p_t^o$ . This implies that we have restrictions on the adjustment matrix  $\alpha$  :

$$\alpha = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix} \quad (8)$$

These restrictions make sure that the cointegration relations do not have influence on the “forcing” variable  $p_t^o$ .

The empirical implementation of the model includes then the following testing and estimating sequence: testing the order of the integration of the nine variables, modelling of a VAR with the nine variables, testing the rank of the cointegration of the nine variables, testing the weak exogeneity of  $p_t^o$  for the parameters of the conditional VAR process, testing the overidentification restrictions. We present the results in the next section.

## 2 Estimation and Testing of the Model

The variables for the model under consideration are  $y_t$ ,  $y_t^*$ ,  $r_t$ ,  $r_t^*$ ,  $e_t$ ,  $h_t - y_t$ ,  $p_t$ ,  $\tilde{p}_t$ ,  $p_t^*$  and  $p_t^o$ . A detailed description of these variables is given in Table 1. The data are taken from the OECD Statistical Compendium and the IMF Data Base. They are all quarterly and seasonally adjusted data. In the model we use data after logarithm transformation. They cover the period 1991Q1–2005Q4. Similar to the paper by Garratt et al. (2003), we use the producer price indices to construct deviations between the domestic and foreign price levels in the PPP relationship. Instead of the retail price index we use the consumer price index to measure domestic inflation in the Fisher interest rate parity relationship. Figure 1 shows the graphs of the data plots. The graphs suggest that these variables are I(1) variables, some time series show a linear trend, and some not.

Table 1: List of Variables and Their Descriptions in the Core Model

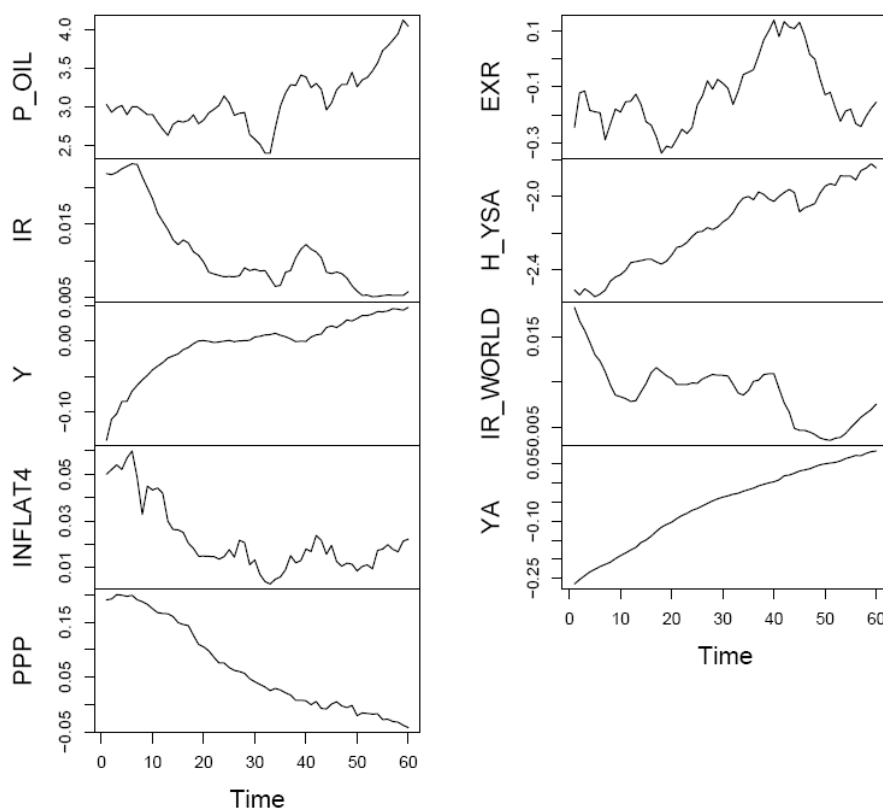
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$y_t$ :	natural logarithm of the German real per capita GDP (GDP deflator) (2000=100%).
$p_t$ :	natural logarithm of the German Producer Price Index (2000=100%).
$\tilde{p}_t$ :	natural logarithm of the German Consumer Price Index (2000=100%).
$r_t$ :	is computed as $r_t = 0.25\ln(1 + R_t/100)$ , where $R_t$ is the 90 day Interbank discount rate per annum.
$h_t$ :	natural logarithm of the German real per capita M1 money stock (2000=100%), Germany's share in M1 EMU (from 2003 without cash).
$e_t$ :	natural logarithm of the nominal DM/Euro exchange rate, monthly average (2000=100%).
$y_t^*$ :	natural logarithm of the foreign (OECD) real per capita GDP (GDP deflator) (2000=100%).
$p_t^*$ :	natural logarithm of the foreign (OECD) Producer Price Index (2000=100%).
$r_t^*$ :	is computed as $r_t^* = 0.25\ln(1 + R_t^a/100)$ , where $R_t^a$ is the weighted average 90 day interest rate per annum in the USA, UK, Japan and France.
$p_t^o$ :	natural logarithm of oil price, measured as the average price in US\$ per barrel oil.
$t$ :	time trend, taking the values 1,2,3,... in 1991Q1, ..., 2005Q4 respectively.

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Figure 1: Data Plots

## German Data



### Investigation of the Non-Stationarity of the Data

The Augmented Dickey-Fuller (ADF) test statistics for the levels and first differences of the core variables are reported in Table 2.

The results of the tests suggest that it is reasonable to treat  $y_t$ ,  $y_t^*$ ,  $r_t$ ,  $r_t^*$ ,  $e_t$ ,  $h_t - y_t$ ,  $p_t$ ,  $p_t^*$  and  $p_t^o$  as I(1) variables. For these variables the unit root hypothesis is rejected when applied to their first differences for all variables at the significance level of 1% excepting  $y_t^*$  and  $r_t^*$ . For these two variables the unit root hypothesis is rejected at the significance level of 5%. When the tests are applied to levels of the variables unit root hypothesis cannot be rejected. There is, however, an exception regarding the order of integration of the price variable  $p_t$ . The application of the ADF test to  $\Delta p_t$  does not reject the unit root hypothesis, but  $\Delta \Delta p_t$  is identified as a stationary variable. This corresponds to the fact, that  $p_t$  is an I(2) variable. However, this variable appears in the model only in differences in the Fisher equation.

Table 2: Augmented Dickey-Fuller Unit Root Test Applied to Variables in the Core Model; 1991Q1–2005Q4

Variable	For the levels		For the first differences	
	t-statistic	Lags	t-statistic	Lags
$y_t$	-2.43	4	-4.03	3
$y_t^*$	-1.07	1	-2.97	2
$r_t$	-1.90	2	-3.93	1
$r_t^*$	-3.03	2	-3.14	1
$e_t$	-2.45	4	-6.96	4
$h_t - y_t$	-2.67	1	-6.89	1
$p_t$	-2.08	2	-4.39	1
$\tilde{p}_t$	-3.18	5	-1.77	7
$p_t^*$	-2.15	2	-4.83	1
$p_t^o$	-1.63	6	-4.35	5
$p_t - p_t^*$	-0.80	1	-7.57	1
$\Delta \tilde{p}_t$	-2.57	4	-11.46	3

Notes: The t-statistics are computed using ADF regressions with an intercept, a linear time trend and s lagged depended variables, when applied to the levels; and with an intercept and s lagged first differences of dependent variable, when applied to the first difference. The order of augmentation in the Dickey-Fuller regressions is chosen using the Akaike Information Criterion, with a maximum lag order of ten. The critical values for the t-Test: -4.12 (level of significance 1%) and -3.49 (level of significance 5%) for the levels. -3.55 (level of significance 1%) and -2.91 (level of significance 5%) for the differences. The critical values are from MacKinnon (1996).

### Estimation and Testing of the Long-Run Relations

The first stage of the modelling sequence is to select the order of the underlying VAR in these variables. Here we find that a VAR of order two appears to be appropriate when using the AIC and SIC as the model selection criteria .

The next step is to test the cointegration rank. As some time series show linear trend in the data, we specified the VECM with an unconstrained constant term. The maximal eigenvalue statistic of the Johansen test shows that there are 5 cointegration relations among the 9 variables at 5% significance level. The corresponding Johansen test result is given in Table 3.

Table 3: Johansen Test of Cointegration Rank

	Max eigen statistic	Critical value
$r \leq 0$	81.653500	58.51
$r \leq 1$	65.240583	52.41
$r \leq 2$	47.079322	46.31
$r \leq 3$	40.893145	40.19
$r \leq 4$	35.835442	34.03
$r \leq 5$	27.096278	27.80
$r \leq 6$	18.244107	21.49
$r \leq 7$	14.042794	15.02
$r \leq 8$	2.797498	8.19
	Trace statistic	Critical value
$r \leq 0$	332.882669	198.72
$r \leq 1$	251.229169	160.87
$r \leq 2$	185.988586	127.05
$r \leq 3$	138.909264	97.26
$r \leq 4$	98.016119	71.44
$r \leq 5$	62.180677	49.64
$r \leq 6$	35.084399	31.88
$r \leq 7$	16.840292	18.11
$r \leq 8$	2.797498	8.19

However, the trace test identifies seven cointegration relations at 5% level. One grate obstacle in our empirical analysis is that the length of the time series used is quit short. 200 observations would be a more preferable situation to conduct the cointegration analysis in a system with nine variables. To assess the influence of the numbers of observations on the analysis we have conducted simulation studies. The required numbers of observations depend, on the scale of the system, the number of independent stochastic trends and the magnitude of the stationary roots<sup>2</sup>.

Since five cointegration relations are in line with our a priori expectation based on the long-run theory, we proceed our further analysis under the assumption that there are five independent cointegrating vectors.

The next step is to test the weak exogeneity of  $p_i^o$ . The likelihood ration test statistic of the five restrictions in equation (8) is 9.630373. The corresponding p-value of  $\chi^2(5)$  distribution is 0.086. This means we cannot reject the null of weak exogeneity of  $p_i^o$ . To take the DGP of this particular data set into account we conduct also a non-parametric bootstrap test of the weak exogeneity. This is done in the following way. We estimate the VECM with 5 cointegration relations under the restrictions of (8) and

<sup>2</sup> See Chen et al. (2008) for more details.

obtain the estimated residuals. Then we reconstruct the bootstrapped series using the estimated parameter of the VECM and the random draws from the estimated residual vectors. The bootstrap test is carried out with 1000 replications. The quantiles of bootstrapped test statistic under the null of weak exogeneity is: 15.20 at 90% level, 18.67 at 95% level and 24.98 at 99% level. Also the bootstrap test cannot reject the null of weak exogeneity of  $p_t^o$ . Under the valid assumption of the weak exogeneity of  $p_t^o$  the unconditional VECM (1) can be decomposed as a conditional VECM (9) given  $p_t^o$ , and the marginal process of  $\Delta p_t^o$  and the inference in the conditional VECM is efficient<sup>3</sup>.

$$\Delta \mathbf{y}_t = \mathbf{a}_y + \alpha_y \beta' \mathbf{z}_{t-1} + \Gamma_0 \Delta p_t^o + \sum_{i=1}^{p-1} \Gamma_{yi} \Delta \mathbf{z}_{t-i} + \mathbf{u}_{yt} \quad (9)$$

$$\mathbf{y}_t = (r_t, y_t, \Delta p_t, p_t - p_t^*, e_t, h_t - y_t, r_t^*, y_t^*)'$$

The unconstrained cointegration matrix  $\beta$  in the VECMX (9) is the same as that in VECM (1). After normalizing the cointegration vectors we have 20 free parameters for the unconstrained cointegration relations as shown in (10).

$$\beta' = \begin{pmatrix} \beta_{11} & 0 & 0 & \beta_{14} & 1 & \beta_{16} & 0 & \beta_{18} & 0 \\ \beta_{21} & 1 & 0 & \beta_{24} & 0 & 0 & 0 & \beta_{28} & \beta_{29} \\ \beta_{31} & 0 & 1 & 0 & \beta_{35} & 0 & \beta_{37} & 0 & \beta_{39} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} & 0 & 0 & 1 & 0 & 0 \\ \beta_{51} & \beta_{52} & 0 & -1 & 0 & 0 & \beta_{57} & 0 & \beta_{59} \end{pmatrix} \quad (10)$$

The five long-run relations derived from the economic theory contains after normalizing the cointegration vectors only two free parameters as shown in equation (7). This implies that the economic theory of the five long-run relations put altogether 18 restrictions on the  $\beta$  matrix. The likelihood ratio test statistic for these 18 restrictions is 87.5831. The corresponding p-value based on  $\chi^2(18)$  distribution is 0. This would reject the null imposed by the economic theory. However, as argued by Garratt et al. (2003), the bootstrap test is a more relevant test in this case. Following this argument we conduct also a non-parametric bootstrap test for the 18 restrictions on  $\beta$  matrix. This is done by estimating a VECMX under the assumption that the theoretical restrictions hold. Then based on the estimated VECMX and the random draws from the estimated residual vectors we reconstruct the bootstrap series. Using the bootstrapped series we calculated the likelihood ratio test statistic. The following quantiles are calculated using 1000 bootstrap replications: 87.80 at 90%, 93.43146 at 95% and 104.39559 at 99%. The bootstrap test cannot reject the null of weak exogeneity of  $p_t^o$  at 10% significance level.

We proceed further with the estimation of the VECMX under the overidentification restrictions. The estimated VECMX model under the restrictions on  $\beta$  gives the following estimated long-run relations.

<sup>3</sup> For detailed information of the notion of exogeneity see Engle et al. (1983).

$$(p_t - p_t^*) - e_t = -0.08 + \hat{\xi}_{1,t+1}$$

$$r_t - r_t^* = -0.0019 + \hat{\xi}_{2,t+1}$$

$$y_t - y_t^* = -0.147 + \hat{\xi}_{3,t+1}$$

$$h_t - y_t = -5.513 - 126.36r_t + 42.07y_t + \hat{\xi}_{4,t+1}$$

$$r_t - \Delta p_t = -0.009 + \hat{\xi}_{5,t+1}$$

The estimated parameters are reasonable, and they have the similar magnitudes as the parameters in the model of Garratt et al. (2003). It is to note that our model differs slightly from that of Garratt (2003) in that we have unconstrained intercept in the VECMX, while they have an additional constrained trend in the cointegration space to capture the effect in the real balance equation.

### Impulse Response Analysis

To conduct an impulse response analysis we use the same identification scheme as in Garratt et al. (2003), namely we choose the Cholesky decomposition in the order of  $(p_t^o, e_t, r_t^*, r_t, \Delta p_t, p_t - p_t^*, y_t, h_t - y_t, y_t^*)$ . These variables correspond to the variables P\_OIL,EXR, IR\_WORLD, IR, INFLAT4, PPP, Y, H\_YSA, YA in the impulse response function graph respectively.

The impulse response function is calculated through re-transforming the conditional VECM to the unconditional VECM through adding a model for the forcing variable  $p_t^o$ . The impulse response functions with respect to an oil price shock are reported in Figure 2 and Figure 3. They show that the oil price shock has permanent effects on the level of some series. This corresponds to the I(1) property of the model.

The one standard error of oil price shock corresponds to an increase of oil price by 12%. The oil price shock has at the beginning a negative effect on the domestic output, reducing the output up to 0.5% per year. After three and half years the economy will recover from the oil price shock, and return to the previous level of output. The foreign output raises by approximately 0.6% per year. However all these responses are statistically insignificant. The effect of the oil price shock on the domestic rate of inflation is permanent. After two year of up and down adjustment the domestic rate of inflation will increase by 0.8%. and eventually it will stabilize at 0.4% per year permanently. Due to the permanent higher domestic rate of inflation, the oil price shock generates a permanent depreciation of the nominal exchange rate at the level of 8%. Further, the oil price shock is accompanied by an increases in domestic interest rate and a decrease in foreign interest rate, though all these responses are also statistically insignificant.



Figure 2: Impulse Response Function

## VECM Orthogonal Impulse Responses

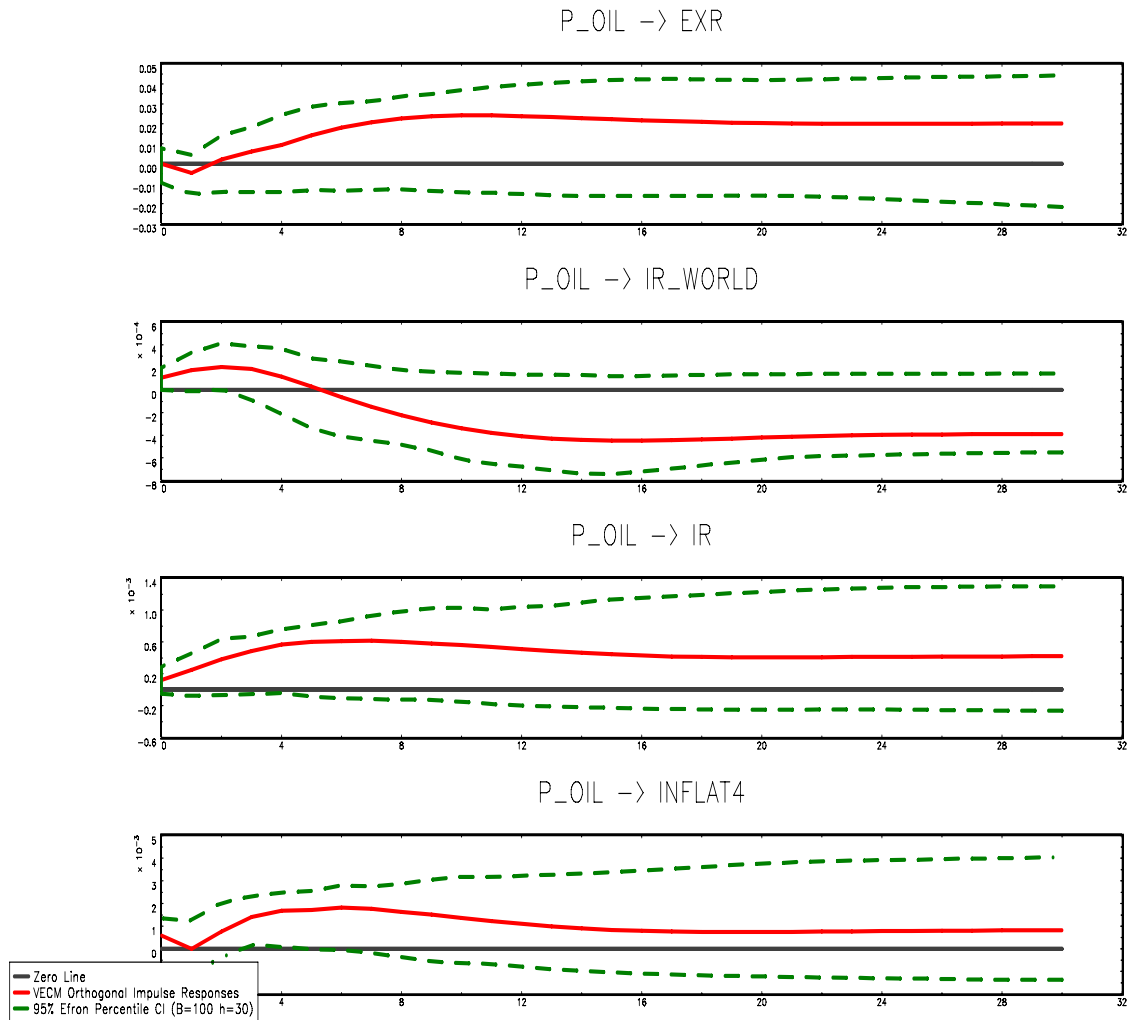
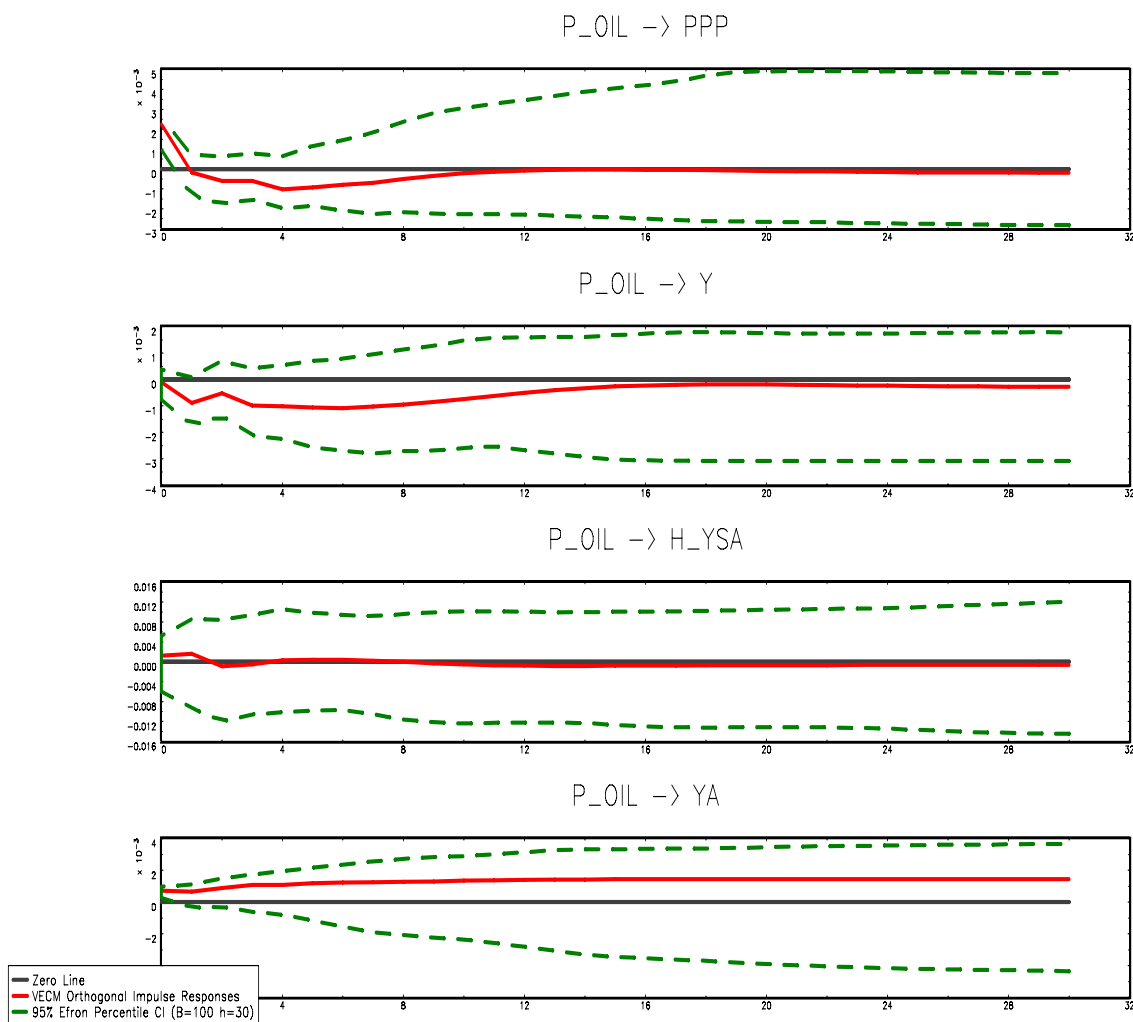


Figure 3: Impulse Response Function

VECM Orthogonal Impulse Responses



### 3 Conclusions

In this paper we apply the methodology developed by Garratt et al. (2003) to German data. As we expected the estimated VECMX shows a similar model structure as the UK model by Garratt et al. (2003). This provides an evidence that the economic theory applied in Garratt et al. (2003) can be generally applied to other economy.

The modelling process starts with a presentation of a set of long-run relations between the macroeconomic variables. These long-run relations are based on production, arbitrage, solvency and portfolio balance conditions, together with stock-flow and accounting identities. Further, these long-run relationships are embedded in an unrestricted VECM model with nine variables, augmented appropriately by intercept. The VECM model is estimated over the period 1991Q1–2005Q4, subject to the theoretical restrictions on the long-run coefficients.

An important component of this modelling approach is the possibility for testing formally the validity of restrictions suggested by economic theory in the context of a macroeconomic model. The underlying economic theory provides five long-run relations or equilibrium conditions among the nine variables of the macro-model. The statistical tests provide little evidence to reject this view. Under the assumption that there are five long-run relationships we carry out further, the likelihood ratio tests of over-identifying restrictions suggested by economic theory, so that we conclude that the estimated model is both theory and data consistent.

Due to the German reunification in 1990, the available official data for unified German economy started from 1991. Since we have only 60 observations to estimate and test the model, the result obtained in our exercise should be handled critically. Because of the length of the observations the model sensitivity becomes a important issue in this empirical implementation. In order to assess the sensitivity of the model we have conducted the same analysis with 50 observations. This leads to the same model structure: we have 5 cointegration relations, the  $p_i^?$  is weakly exogenous and the over-identification restrictions cannot be rejected by the bootstrap test (see Appendix). However, if the length of the observations and henceforth the reliability of the results is an issue, the result of the analysis cannot be strengthened by an analysis with an even shorter data set.

As further research project we are considering to expand the time span of the analysis by using VECM with structural breaks<sup>4</sup>.

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<sup>4</sup> Thank one of the referees who has suggested this possibility to do the analysis.

## Appendix

The Johansen test of cointegration rank with a subset of data (1991:Q1–2003:Q2)

	Test statistic	Critical value
r = 0	79.281227	58.51
r = 1	56.855807	52.41
r = 2	48.705150	46.31
r = 3	40.729891	40.19
r = 4	34.992156	34.03
r = 5	25.399038	27.80
r = 6	16.737414	21.49
r = 7	8.125556	15.02
r = 8	4.902986	8.19

Test of weak exogeneity of the oil price

LR test statistic = 8.893361 p-value = 0.1133940

Bootstrap quantiles of the test of overidentification restrictions

LR test statistic = 109.8766

	90%	95%	99%
	89.11723	97.66611	118.22057

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