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The delimitation of Giffenity for the Wold-Juréen (1953) utility function using relative prices: a note

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Abstract

In the study of Giffen behavior or "Giffenity", there remains a paradox. On one hand, the Wold-Juréen (*Demand analysis: A study in Econometrics*, 1953) utility function has been touted as the progenitor of a multi-decade search for those two-good, particular utility functions, which exhibit Giffenity. On the other hand, there is no evidence that the Wold-Juréen (1953) utility function has ever been fully evaluated for Giffenity, with perhaps one minor exception, Weber (*The case of a Giffen good: Comment*, 1997). But there, Weber (1997) showed that the Giffenity of Good 1 depends upon the relative magnitude of income vis-à-vis the price of Good 2. Weber's precondition is so vague that it lacks broad appeal. This paper offers a new and a clear cut precondition for Giffen behavior under the Wold-Juréen (1953) utility function. That is, the author shows that if the price of Good 1 is greater than or equal to the price of Good 2, then Good 1 is a Giffen good.

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1. Introduction

Within the domain of consumer theory, there has been a multi-decade search for two-good, particular utility functions which exhibit Giffen's paradox or "Giffenity" (to use modern-day parlance). This exploration began with Wold and Juréen (1953), and it has gone on to include such papers as Vandermeulen (1972), Spiegel (1994), Weber (1997), Nachbar (1998), Moffatt (2002), Sørensen (2007), Doi et al. (2009), Heijman and van Mouche (2011a), Moffatt (2011), Haagsma (2012), Biederman (2015), and Landi (2015).

This multi-decade endeavor offers a paradox. On one hand, the Wold-Juréen (1953) utility function has been touted as the progenitor [viz., Moffatt (2011, page 127) stated that: "(e)ver since Wold and Juréen's attempt to illustrate the Giffen paradox by specifying a particular direct utility function, there has been a stream of contributions from authors pursing similar objectives"]. On the other hand, the research literature provides no evidence that the Wold-Juréen (1953) utility function has ever been fully evaluated for Giffenity, except for Weber (1997). Weber showed that the Giffenity of Good 1 (the inferior good) is dependent on the relative magnitudes of the decision maker's (DM) income and the price of Good 2. He wrote: "Giffen behavior is *more likely* for higher … incomes" and that the Giffenity of Good 1 is *more likely* at lower values of the price for Good 2 [Weber (1997, page 40)]. We hold that Weber's precondition is so vague that it lacks broad appeal.

The present note breaks new ground by presenting a new precondition for Giffenity when the utility function is the Wold-Juréen (1953) utility function. First, we define a new property of the Wold-Juréen (1953) utility function. Second, we then exploit this new property to sign the total effect of a a change in the price of Good 1 on the demand for Good 1. We are able to show that if the price of Good 1 is greater than or equal to the price of Good 2, then Good 1 is a Giffen good. We maintain that our precondition is more appealing than Weber's in that ours accords with a core tenet of microeconomics, viz., that economic decision-making is predicated on (changes in) relative prices.

This note is organized as follows. In Section 2, we will lay out the context for the present discussion of the Slutsky decomposition, including our detailed review of all relevant prior research. This context for the present discussion will span two cases: (a) the case of an arbitrary utility function, and (b) the case of the Wold-Juréen (1953) utility function. When we consider the case of the Wold-Juréen (1953) utility function in Section 2, we shall review the findings of Weber (1997). In Section 3, we shall begin defining a new property of the Wold-Juréen (1953) utility function, and then (using it) we shall show that if the price of Good 1 is greater than or equal to the price of Good 2, then Good 1 is a Giffen good. Final comments are offered in Section 4.

2. Previous Research

In this section, we shall offer an overview to the previous research on the Slutsky decomposition. This will serve as the backdrop for the development of our contribution reported in Section 3 below.

The present overview is comprised of two parts. The first offers a review of the literature on the Slutsky decomposition for an arbitrary utility function, while the second offers a review of the literature on the Slutsky decomposition for the Wold-Juréen (1953) utility function.

2.1. The Slutsky Decomposition for an Arbitrary Utility Function: Let $U = U(x_1, x_2)$ denote an arbitrary, well-behaved utility function, where x_1 and x_2 denote the amounts of Good 1 and Good 2. By "well-behaved", we mean a utility function, which has positive marginal utilities and diminishing marginal utilities, and which is quasi-concave.

Next let $x_i^M = x_i^M(p_1, p_2, m)$ denote the DM's Marshallian demand function for the ith good (where i = 1,2), let $x_i^H = x_i^H(p_1, p_2, U)$ denote the DM's Hicksian demand function for the ith good, let p_1 and p_2 denote the prices of Good 1 and Good 2, and let m denote the DM's income. After Cook (1972), the Slutsky decomposition states,

$$\frac{\partial x_1^M(p_1, p_2, m)}{\partial p_1} = \frac{\partial x_1^H(p_1, p_2, U)}{\partial p_1} - x_1^M \frac{\partial x_1^M(p_1, p_2, m)}{\partial m}$$

where $\frac{\partial x_1^M(p_1, p_2, m)}{\partial p_1}$ denotes the total effect (hereafter TE) of the change in p_1 on the demand for Good 1, where $\frac{\partial x_1^H(p_1, p_2, U)}{\partial p_1}$ denotes the Hicksian substitution effect (hereafter SE) of the change in p_1 on the demand for Good 1, and $- x_1^M \frac{\partial x_1^M(p_1, p_2, m)}{\partial m}$ denotes the income effect (hereafter IE) of the change in p_1 on the demand for Good 1.

2.2. The Slutsky Decomposition for The Wold-Juréen (1953) Utility Function: The Wold-Juréen (1953) utility function is defined as $U(x_1, x_2) = (x_1 - 1)(x_2 - 2)^{-2}$ where, by assumption, $x_1 > 1$ and $0 < x_2 < 2$ [Wold and Juréen (1953, page 102), Vives (1987, page 99), Weber (1997, pages 39-40), and Chipman and Lenfant (2002, page 579, footnote 47)]. Like the arbitrary utility function, the Wold-Juréen (1953) utility function is quasi-concave. But unlike the arbitrary utility function, the Wold-Juréen (1953) utility function does not exhibit diminishing marginal utility in both goods. Thus, Weber (1997, page 39) stated that $\frac{\partial^2 U(x_1, x_2)}{\partial x_1^2} = 0$ and $\frac{\partial^2 U(x_1, x_2)}{\partial x_2^2} > 0$, where $U(x_1, x_2) = (x_1 - 1)(x_2 - 2)^{-2}$ and where $x_1 > 1$ and $0 < x_2 < 2$.

The Marshallian demand functions associated with the Wold-Juréen (1953) utility function are: [2]

$$x_1^M = x_1^M(p_1, p_2, m) = 2 + \frac{2p_2 - m}{p_1}. \tag{1}$$

$$x_2^M = x_2^M(p_1, p_2, m) = 2\left(\frac{m - p_1}{p_2} - 1\right) \tag{2}$$

Likewise, it can be shown that the Hicksian demand functions associated with the Wold-Juréen (1953) utility function are:[3]

$$x_1^H = x_1^H(p_1, p_2, U) = 1 + \frac{(p_2)^2}{4U}(p_1)^{-2} \tag{3}$$

$$x_2^H = x_2^H(p_1, p_2, U) = 2\left(1 - \frac{p_2}{4p_1U}\right) \tag{4}$$

Given Equations (1) and (3), we can state the components of the Slutsky decomposition for the Wold-Juréen (1953) utility function. In particular, it follows from Equation (1) that:

$$\frac{\partial x_1^M}{\partial p_1} = \frac{m - 2p_2}{(p_1)^2} = TE \tag{5}$$

[see Weber (1997, page 40, Equation (15))]. Likewise, it follows from Equation (3) that:

$$\frac{\partial x_1^H}{\partial p_1} = -\frac{(p_2)^2}{2U(p_1)^3} = SE < 0 \tag{6}$$

Finally, it follows from Equation (1) that:

$$-x_1^M \frac{\partial x_1^M}{\partial m} = -x_1^M\left(-\frac{1}{p_1}\right) = IE > 0 \tag{7}$$

Question: What then is the present state of the literature on the Slutsky decomposition for the Wold-Juréen (1953) utility function? Answer: This literature offers just two findings. One, the sign of the TE for Good 1 is ambiguous since the SE and the IE have opposite signs [see Equations (6) and (7)]. Two, in view of Equation (5), it is clear that the sign of the TE is ambiguous. This is echoed by Weber (1997), viz.,

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[2] Three Notes: (a) Recall that the Marshallian demand functions originate from the DM's decision to maximize utility subject to a budget constraint. (b) Equations (1) and (2) above appear in Vives (1987, page 99), Weber (1997, pages 39-40), and Chipman and Lenfant (2002, page 579, footnote 47). (c) Finally, Weber (1997, page 39) has shown that (in the case of the Wold-Juréen (1953) utility function) the second-order condition for this constrained-maximization problem holds.

[3] Three Notes: (a) Recall that the Hicksian demand functions originate from the DM's decision to minimize expenditure subject to a utility constraint. (b) It can be shown that (in the case of the Wold-Juréen (1953) utility function) the second-order condition for this constrained-minimization problem holds.

Proposition 1 [Weber (1997)]: If the DM's utility function is the Wold-Juréen (1953) utility function, then $sign\left(\frac{\partial x_1^M}{\partial p_1}\right) = sign(m - 2p_2) = sign(TE)$.

Proof: Equation (5). ●

Thus, when commenting on Equation (5) or on Proposition 1, Weber (1997, page 40) wrote: "Giffen behavior is *more likely* for higher … incomes" and that the Giffenity of Good 1 is *more likely* at lower values of the price for Good 2.

In the next section, we will offer an improvement over Weber's (1997) Proposition 1. That is, we will show that Good 1 is a Giffen good, if the price of Good 1 is greater than or equal to the price of Good 2.

3. Using Relative Prices to Delimit Giffenity for The Wold-Juréen (1953) Utility Function

In this section, we define a new property of the Wold-Juréen (1953) utility function, and then exploit this property to sign the TE. That new property is defined by the last of the following three lemmas.

Lemma 1: If the DM's utility function is the Wold-Juréen (1953) utility function, then by definition $m < p_1 + 2p_2$.

Proof: If the DM's utility function is the Wold-Juréen (1953) utility function, then by definition $x_1^M(p_1, p_2, m) > 1$. This implies that $2 + \frac{2p_2 - m}{p_1} > 1$ [Equation (1)], that $\frac{2p_2 - m}{p_1} > -1$, and that $m < p_1 + 2p_2$. ●

Lemma 2: If the DM's utility function is the Wold-Juréen (1953) utility function, then by definition $p_1 + p_2 < m < p_1 + 2p_2$.

Proof: If the DM's utility function is the Wold-Juréen (1953) utility function, then by definition $0 < x_2^M(p_1, p_2, m) < 2$. This implies that $0 < 2\left(\frac{m - p_1}{p_2} - 1\right) < 2$ [Equation (2)], that $0 < \frac{m - p_1}{p_2} - 1 < 1$, that $1 < \frac{m - p_1}{p_2} < 2$, that $p_2 < m - p_1 < 2p_2$, and that $p_1 + p_2 < m < p_1 + 2p_2$. ●

Lemma 3: If the DM's utility function is the Wold-Juréen (1953) utility function, then by definition $p_1 - p_2 < m - 2p_2 < p_1$.

Proof: By Lemmas 1 and 2, $p_1 + p_2 < m < p_1 + 2p_2$, which in turn implies $p_1 - p_2 < m - 2p_2 < p_1$. ●

With Lemma 3 in place, we turn next to the task of signing the TE [see Propositions 2 and 3 below]. There we show that the relative magnitudes of prices, p_1 and p_2, can be used to sign the TE positive or to delimit Giffen behavior. In particular:

Proposition 2: If the DM's utility function is the Wold-Juréen (1953) utility function, and if $p_1 \geq p_2$, then $TE = \frac{\partial x_1^M}{\partial p_1} > 0$.

Proof: If the DM's utility function is the Wold-Juréen (1953) utility function, then $p_1 - p_2 < m - 2p_2 < p_1$ [Lemma 3]. If $p_1 \geq p_2$, then $p_1 - p_2 \geq 0$, $0 < m - 2p_2$, and $TE = \frac{\partial x_1^M}{\partial p_1} = \frac{m - 2p_2}{(p_1)^2} > 0$. ●

Proposition 3: If the DM's utility function is the Wold-Juréen (1953) utility function, and if $p_1 < p_2$, then the sign of the TE is ambiguous.

4. Conclusion

As we noted at the outset, there is a paradox in the literature on Giffenity. On one hand, the Wold-Juréen (1953) utility function has been touted as the progenitor of a multi-decade search for those two-good, particular utility functions, which exhibit Giffenity. On the other hand, there is no evidence that the Wold-Juréen (1953) utility function has been fully evaluated for Giffenity, except for Weber (1997). But the problem with Weber's paper is that it does not provide a clear cut precondition for Giffenity.

This note has broken new ground in the study of the properties of the Wold-Juréen (1953) utility function by presenting such a precondition. In particular, this note has shown that if the price of Good 1 is greater than or equal to the price of Good 2, then Good 1 is a Giffen good.

In Section 2 of this note, we reviewed the present-day discussion of the Slutsky decomposition for two cases: (a) the case based on an arbitrary utility function, and (b) the case based on the Wold-Juréen (1953) utility function. In Section 3, we were able to define a new property of the Wold-Juréen (1953) utility function, and we were able to show that this property offers our clear cut precondition for Giffen behavior. That is, we show that if the price of Good 1 is greater than or equal to the price of Good 2, then Good 1 is a Giffen good.

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