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The macroeconomic consequences of artificial intelligence: a theoretical framework

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Abstract
The authors explore the impact of artificial intelligence on the economy by improving the neoclassical production function and the task-based model. Based on the capital accumulation of artificial intelligence and technological progress, they present a theoretical model that explores the effect of alternative and complementary artificial intelligence on wages, capital prices, labor share, capital share and economic growth. The model shows that artificial intelligence capital lowers the capital prices and increases wages. In addition, if artificial intelligence and labor force are complementary, artificial intelligence capital has a positive impact on labor share, but if artificial intelligence and labor force can substitute each other, labor share is negatively influenced by artificial intelligence capital. The authors extend the task-based model and find that technological progress increases both wages and labor share by generating new tasks. In the long run, without consideration of exogenous technology, as the artificial intelligence capital accumulates, per capita output, per capita traditional capital and per capita artificial intelligence capital grow at the same rate, and economic growth finally reaches steady state equilibrium. With exogenous technology considered, artificial intelligence technology improves, and sustained economic growth is achieved.

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1 Introduction

Artificial intelligence is manually designed system which possesses the ability of autonomous perception and decision making based on artificially designed algorithms and data. Artificial intelligence can help or even substitute humans to finish the work that required human intelligence previously. Automation is the process that machine or system achieve the desired goals set by humans through automatic detection, information processing, analysis and judgement, and control, with little human involvement. Therefore, automation is a kind of basic subjects, of which artificial intelligence is a branch. Indeed, few scholars distinguish between artificial intelligence and automation strictly, most studies regard them as substitutes for each other.

In terms of the relation with labor force, artificial intelligence can be classified into two categories: alternative and complementary. On the one hand, artificial intelligence has begun to replace many jobs and this trend is expected to continue (Frey and Osborne, 2017; Arntz et al., 2016; Acemoglu and Restrepo, 2017). For example, autonomous vehicles can speed transportation up, 3D printers can produce highly specialized products. On the other hand, artificial intelligence can assist humans in improving work efficiency. For example, Watson, a kind of AI product developed by IBM, incorporates more than 300 medical journals and various clinical guidelines, which can help doctors to make diagnoses and finish detailed reports.

There are two ways to study the theoretical effects of artificial intelligence on economic growth, and artificial intelligence is incorporated into the production function as a new factor in both ways. One is to improve the neoclassical production function (Hanson, 2001; Autor et al., 2013; Prettner 2017a, 2017b; Lankisch et al., 2019; Decanio, 2016), and the second is to extend the task-based model (Zeira, 1998; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018a-f; Acemoglu and Restrepo, 2019).

Improve the neoclassical production function

The most important feature of the neoclassical production function is the incorporation of the elasticity of substitution between labor and capital. Hanson (2001) improves the Cobb-Douglas production function and finds that computer capital and labor complement each other while machine intelligence has a displacement effect on labor, and the wide use of smart machines strongly stimulates economic growth. However, this model does not consider the creation of new jobs, therefore underestimate the impact of smart machines on the economy. Aghion et al. (2017) improves the CES production function and analyzes the changes in the share of automated goods by exploring the changes in the elasticity of substitution. The model explains the Baumol's Cost Disease, which indicates that when the productivity of automation sector improves, its share in GDP declines. It is shown that economic growth is constrained to its weakness rather than advantages.

Some studies combine the Cobb-Douglas function with the CES function to explain labor market polarization, wage inequality and unemployment (Autor et al., 2013; Lankisch et al., 2019). Some others adopt the two-level nested CES production function to discuss the effects of economic growth on wages and labor structure (Decaio, 2016), which no longer focuses on the substitution of artificial intelligence capital for a production factor only, but also pays attention to the elasticity of substitution between artificial intelligence capital and two production factors.

Recent literatures embed the Overlapping Generational Model (OLG) in neoclassical production functions, thus the studies on artificial intelligence move into the dynamic era (Benzell et al., 2015; Gasteiger and Prettner, 2017; Prettner and Strulik, 2017; Sachs et al., 2015). The most important feature of the OLG model is that it is iterative indefinitely, so it can be adopted to explore the impact of artificial intelligence on the economy in the long run. A common conclusion of the above literatures is that artificial intelligence slows down economic growth and reduces social welfare in the long run. For one reason, those models generally ignore the impact of newly created tasks on the economy, only consider the displacement effect of artificial intelligence for low-skilled labor and the complementary effect for high-skilled one. For another, the displacement effect of artificial intelligence for labor reduces wages and thus savings. Consequently, corporate investment which in fact comes from savings drops, and finally economic slowdown appears. In addition, artificial intelligence decreases both the total wages and the the share of labor, however, it increases both the return on capital and the capital share of income, and the inequality is exacerbated.

In summary, improving the neoclassical production function relies on the introduction of the elasticity of substitution to connect technology with employment. Employing the Cobb-Douglas production function or the CES production function to improve the neoclassical production function is simple and convenient, however, such approach is not general enough and ignores the traditional capital. By contrast, adopting the two-level nested CES production function is more general but more complicated. Furthermore, this approach can only perform static analysis, because there is often no dynamic equilibrium solution without the help of numerical simulation. The simultaneous adoption of both the Cobb-Douglas and the CES production function can take many determinants into consideration. It is also convenient to obtain the equilibrium solution and analyze long-term economic growth.

Task-based Model

Task-based model shows that automation takes place of existing tasks and creates new tasks, so that the change in job nature is reflected. Zeira (1998) is a pioneer in developing the task-based model. He divides work into tasks and employ intervals to denote tasks. Recent studies extend Zeira's model to explore the effect of artificial intelligence on economic growth (Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018a-f; Acemoglu and Restrepo, 2019). Although the models become more and more general, the main results stay unchanged. It is found that artificial intelligence has a displacement effect on labor, but it also brings four positive effects, which include productivity effect, capital accumulation, the deepening of automation deepening and the creation of new tasks. Among those four, the creation of new tasks has the most important effect, because humans have comparative advantages in performing new tasks. Automation reduces the labor share, but the new task increases it. Although the creation of new tasks can offset some negative effects of automation, the adjustment process in labor market is slow and painful. Because searching for new jobs takes time, and labor force need to develop new skills to adapt to new jobs. Moreover, the imperfection of labor market results in frictions and mismatches.

The theory developed by Acemoglu and Restrepo(2018a-f;2019) extends the research on economic growth theory and lays a foundation for later research on the impact of artificial intelligence on the economy. However, there is no strict definitions of "task" and no explanations on how new tasks are created. Additionally, the task-based model only considers the impact of automation capital on the economy, but ignores the traditional capital. Furthermore, the model focuses on the substitution between different tasks and simply assumes that capital substitutes labor perfectly. In fact, capital is not perfect substitute for labor in some given task, but if both the elasticity of substitution between different tasks and that between capital and labor are considered, the model will become too complicated. In order to overcome the above difficulties, some scholars introduce traditional capital into the task-based model, or combine the two types of models to study the impact of labor market polarization and taxing robots on the economy (Guerreiro et al., 2017).

By contrast, the neoclassical production function mainly focuses on the substitution between artificial intelligence capital and labor, while the task-based model emphasizes the substitution between tasks. The task-based model is more able to reflect the change in job nature, while the improved neoclassical production function can examine traditional capital and artificial intelligence capital simultaneously. The current trend is to combine these two types of models and develop new production function which is closer to reality.

It is worth noting that none of the above studies distinguishes between complementary artificial intelligence and alternative artificial intelligence, thus none discusses the different impacts on the economy. Based on the previous work (Acemoglu, 2018a-f, 2019; Aghion, 2017, Autor, 2013; Prettner, 2017), we for the first time introduce the elasticity of substitution, differentiate between complementary artificial intelligence and alternative artificial intelligence to study the impacts on the economy, and present a general framework in theory.

The results show that complementary artificial intelligence increases the the share of labor, while alternative artificial intelligence takes place of workforce and reduces the share of labor. In addition, ignoring exogenous technology, as artificial intelligence capital accumulates, per capita output, per capita traditional capital and per capita artificial intelligence capital all grow at the same rate, and economic growth finally reaches steady state equilibrium in the long run. With exogenous technology considered, artificial intelligence capital-augmenting technology improves, and sustained economic growth is achieved. Furthermore, we extend the task-based model and find that technological progress increases both wages and the share of labor by creating new tasks. It is possible to discuss other issue within this framework, such as the effect of artificial intelligence on unemployment, aging and the polarization of labor market.

2 Model

(1)We consider a setting with only one enterprise in a closed economy. All workforce is homogeneous and fully employed. There are two types of artificial intelligence. One is complementary to labor, and the other is a substitute for labor.

The production function of the enterprise is

(1) $$Y = K^{\alpha}(L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where K is traditional capital, L is labor, and P is artificial intelligence capital. σ is the elasticity of substitution between labor and artificial intelligence capital, and $\sigma \geq 0$. When $\sigma = 0$, $Y = K^{\alpha} \cdot \min\{L, P\}$, when $0 < \sigma < 1$, artificial intelligence capital is complementary to labor, when $\sigma = 1$, $Y = K^{\alpha} L^{\beta} P^{1-\alpha-\beta}$, when $\sigma > 1$, labor and artificial intelligence capital substitute each other, when $\sigma \to +\infty$, $Y = K^{\alpha}(L+P)^{1-\alpha}$. Define $Z = L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}$.

Suppose the enterprise makes profits π, the worker's wage is ω, the rent of traditional capital is R_K, and the rent of artificial intelligence capital is R_P, thus

$$\pi = Y - \omega L - R_K K - R_P P .$$

The enterprise maximizes profits, so the worker's wage is

$$\omega = \frac{\partial Y}{\partial L} = (1-\alpha)K^{\alpha} Z^{\frac{1-\sigma\alpha}{\sigma-1}} L^{-\frac{1}{\sigma}}, \tag{2}$$

the rent of traditional capital is

$$R_K = \frac{\partial Y}{\partial K} = \alpha K^{\alpha-1} Z^{\frac{\sigma(1-\alpha)}{\sigma-1}}, \tag{3}$$

the rent of artificial intelligence capital is

$$R_P = \frac{\partial Y}{\partial P} = (1-\alpha)K^{\alpha} Z^{\frac{1-\sigma\alpha}{\sigma-1}} P^{-\frac{1}{\sigma}} . \tag{4}$$

The no arbitrage condition is $R_K = R_P$, hence

$$K = \frac{\alpha}{1-\alpha} Z P^{\frac{1}{\sigma}} . \tag{5}$$

Substitute the Eq. (5) into the Eq. (2), (3) and (4), we have

$$\omega = aL^{-\frac{1}{\sigma}} Z^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}}, \tag{6}$$

where $a = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Thus,

$$R = R_K = R_P = aZ^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}} . \tag{7}$$

Proposition 1 (i) The accumulation of artificial intelligence capital increases wages and lowers capital rent.

Proposition 1 (ii) The share of traditional capital stays unchanged at α. When $0<\sigma<1$, artificial intelligence capital is complementary to labor, it increases the share of labor but reduces the share of artificial intelligence capital. When $\sigma>1$, labor and artificial intelligence capital substitute each other, artificial intelligence capital lowers the the share of labor but promotes the share of artificial intelligence capital. When $\sigma=1$, both the share of labor and the share of artificial intelligence capital remain fixed.

All proofs are contained in the appendix.

The economic intuition of Proposition 1 is that if unemployment is ignored, artificial intelligence capital lowers the price of capital, so workers are relatively scarcer and wages are raised. When artificial intelligence capital is complementary to labor, automation assists in human working and improves productivity. Therefore, artificial intelligence capital is beneficial to labor by improving the the share of labor. When artificial intelligence capital and labor substitute each other, competition is formed. Artificial intelligence capital reduces the demand for labor, and lowers the the share of labor, thus is to the disadvantages of labor. In fact, technological progress does not always bring benefits. Take the first industrial revolution as an example, the British worker suffered and even started to smash machines during the Luddite movement. This indicated that in order to eliminate the negative effect of technological advancement, countries can choose to support the develop of complementary artificial intelligence, providing protection for labor. It is worth noting that the above conclusions are obtained without considering unemployment. If some industries have large demand for labor, especially low-skilled labor, it is advantageous to develop alternative artificial intelligence, which can make up the shortfall in labor supply.

Extension 1: When capital is used to improve the level of artificial intelligence capital-augmenting technology

Suppose that capital is invested to improve the level of artificial intelligence and the production function is

(8) $$Y = K^{\alpha}[L^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where A_P is artificial intelligence capital-augmenting technology. Define $Z_P = L^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}$.

Similarly, apply the no arbitrage condition, we have

(9) $$K = \frac{\alpha}{1-\alpha} A_P^{\frac{1-\sigma}{\sigma}} Z_P P^{\frac{1}{\sigma}},$$

(10) $$\omega = a A_P^{\frac{(1-\sigma)\alpha}{\sigma}} L^{-\frac{1}{\sigma}} Z_P^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}},$$

(11) $$R = R_K = R_P = a A_P^{\frac{(1-\sigma)(\alpha-1)}{\sigma}} Z_P^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}}.$$

Proposition 2 (i) When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, artificial intelligence capital-augmenting technology increases wages. When $\sigma > \frac{1}{\alpha} > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology lowers wages.

Proposition 2 (ii) When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology raises the price of capital. When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, the relation between artificial intelligence capital-augmenting technology and the price of capital is uncertain.

Proposition 2 (iii) When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, artificial intelligence capital-augmenting technology improves the labor share but reduces the share of artificial intelligence capital, and the share of traditional capital remains unchanged.

When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology reduces the labor share but improves the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma = 1$, both the labor share and the share of artificial intelligence capital are fixed.

All proofs are contained in the appendix.

The economic intuition of Proposition 2 is that while ignoring unemployment and assuming full employment of labor, the impacts of artificial intelligence capital-augmenting technology on wages and labor share depend on whether it is complementary or alternative. If artificial intelligence capital is complementary to labor, it is good for wages and labor share. If artificial intelligence capital and labor substitute each other, artificial intelligence capital-augmenting technology worsens the competitive situation of labor, as more workers are replaced, and both wages and labor share drop.

Facing the challenge of machines, it is more favorable for human to choose collaboration than competition, because artificial intelligence has extremely high productivity. Therefore, making artificial intelligence helpers for humans is a good idea. Humans may engage in the jobs which need creativity, decision-making and abstract thinking, leaving the dull and repetitive work to machines. Humans get rid of boring work and focus on what they are good at. Meanwhile, artificial intelligence improves productivity largely and promotes economic growth. In this way, labor force spends less time on work, enjoys more leisure but earns higher wages, thus the social welfare is improved.

Extension 2: When capital is used to improve labor-augmenting technology

When capital is invested to improve labor-augmenting technology, the production function is

(12) $$Y = K^{\alpha}[(A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where A_L is the labor-augmenting technology. Define $Z_L = (A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}$.

Similarly, apply the no arbitrage condition,

(13) $$K = \frac{\alpha}{1-\alpha} Z_L P^{\frac{1}{\sigma}}$$

(14) $$\omega = a A_L^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} Z^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}}$$

(15) $$R = R_K = R_P = a Z_L^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}}$$

Proposition 3 (i) Labor-augmenting technology raises the price of capital. When $0 < \sigma < \alpha$, that is, artificial intelligence capital is complementary to labor, labor-augmenting technology lowers wages. When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, labor-augmenting technology increases wages.

Proposition 3 (ii) When $0 < \sigma < 1$, that is, artificial intelligence capital is complementary to labor, labor-augmenting technology lowers labor share but increases the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma > 1$, that is, artificial intelligence capital and labor substitute each other, labor-augmenting technology raises labor share but reduces the share of artificial intelligence capital, and the share of traditional capital remains unchanged. When $\sigma = 1$, both labor share and the share of artificial intelligence capital are fixed.

All proofs are contained in the appendix.

Proposition 3 has essentially the same logic as Proposition 2. When artificial intelligence capital is complementary to labor, the increase in labor-augmenting technology indeed helps humans to complete more tasks in less time, which saves labor and raises wages. Conversely, if labor-augmenting technology improves, humans are actually doing more for machines. Therefore, machines rather than humans are benefited, both wages and the share of labor. This is called Baumol's Cost Disease, which means that the share of one sector declines when its productivity improves.

When artificial intelligence capital and labor substitute each other, there exists competition between them. Higher productivity helps labor to win competitive advantages and earn higher wages. If artificial intelligence takes the place of a large quantity of jobs, labor will be required to improve productivity.

Combining Proposition 1 with Proposition 3, when artificial intelligence capital is complementary to labor, higher artificial intelligence capital leads to higher labor share, but higher labor-augmenting technology results in lower labor share. When artificial intelligence capital and labor substitute each other, labor-augmenting technology has positive effect on labor share while the effect of artificial intelligence capital is negative. Given that the productivity of artificial intelligence is much higher than humans, it is not wise for humans to compete with artificial intelligence by improving productivity. In sum, assuming no new jobs are created, it is optimal to develop complementary artificial intelligence.

The following conclusions can be obtained from Propositions 1, 2, and 3, as shown in Table 1, Table 2, and Table 3.

Table 1: Impact of artificial intelligence on workers' wages

	P	A_L	A_P
Complementary artificial intelligence	+	-	+
Alternative artificial intelligence	+	+	-

Note: "+" means that when the factors increase, the wages of workers will increase, "-" means that when the factors increase, the wages of workers will decrease, same as below.

Table 2: The impact of artificial intelligence on labor share

	P	A_L	A_P
Complementary artificial intelligence	+	-	+
Alternative artificial intelligence	-	+	-

Table 3: Impact of artificial intelligence on capital share

	P	A_L	A_P
Complementary artificial intelligence	-	+	-
Alternative artificial intelligence	+	-	+

（4） The develop of artificial intelligence and the creation of new tasks

One of the shortcomings of the neoclassical model is that it ignores the creation of new jobs by technological progress. Following the task-based approach developed by Zeira (1998) and Acemoglu (2018a-f), we propose a more general model. Assuming that the interval of task i is [N-1, N]. A higher N means that the development of automation creates new demand for employment and thus new tasks, and the higher the N is, the more complicated the task is. N-1 gets larger as N becomes larger, indicating the disappearance of old tasks. The economic tuition is that the development of automation creates new jobs while eliminates some others. Define the production function of enterprise as follow,

(16) $$Y = K^{\alpha}\left[\int_{N-1}^{I}(A_p(i)p(i))^{\frac{\sigma-1}{\sigma}}di + \int_{I}^{N}(A_l(i)l(i))^{\frac{\sigma-1}{\sigma}}di\right]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

where, $p(i), l(i)$ are the capital and labor invested in the task i, A_p, A_l are capital-augmenting and the labor-augmenting technology respectively, and I indicates the frontier of artificial intelligence possibilities, a higher I means more advanced artificial intelligence. It is assumed that artificial intelligence capital works in a smaller interval, while workers work in a large one, because the low-skilled workers are more easily to be replaced by artificial intelligence capital, and finally workers mainly engage in more complex tasks, such as creative ones. Define

$$Z = \int_{N-1}^{I}(A_p(i)p(i))^{\frac{\sigma-1}{\sigma}}di + \int_{I}^{N}(A_l(i)l(i))^{\frac{\sigma-1}{\sigma}}di.$$

The enterprise's profit is

(17) $$\pi = Y - R_K K - R_p\int_{N-1}^{I}pdi - \omega\int_{I}^{N}ldi.$$

In order to maximize profit, wages are

(18) $$\omega = (1-\alpha)l^{-\frac{1}{\sigma}}A_l^{\frac{\sigma-1}{\sigma}}K^{\alpha}Z^{\frac{1-\sigma\alpha}{\sigma-1}},$$

the rent of traditional capital is

(19) $$R_K = \alpha Z^{\frac{\sigma(1-\alpha)}{\sigma-1}}K^{\alpha-1},$$

the rent of artificial intelligence capital is

(20) $$R_p = (1-\alpha)p^{-\frac{1}{\sigma}}A_p^{\frac{\sigma-1}{\sigma}}K^{\alpha}Z^{\frac{1-\sigma\alpha}{\sigma-1}},$$

Apply the no-arbitrage condition $R_K = R_p$,

(21) $$K = \frac{\alpha}{1-\alpha}p^{\frac{1}{\sigma}}A_p^{\frac{1-\sigma}{\sigma}}Z.$$

Substitute the Eq. (21) into the Eq. (18) and (19),

(22) $$\omega = \alpha^{\alpha}(1-\alpha)^{1-\alpha}l^{-\frac{1}{\sigma}}p^{\frac{\alpha}{\sigma}}A_l^{\frac{\sigma-1}{\sigma}}A_p^{\frac{(1-\sigma)\alpha}{\sigma}}Z^{\frac{1-\alpha}{\sigma-1}},$$

(23) $$R = \alpha^{\alpha}(1-\alpha)^{1-\alpha}p^{\frac{\alpha-1}{\sigma}}A_p^{\frac{(\sigma-1)(1-\alpha)}{\sigma}}Z^{\frac{1-\alpha}{\sigma-1}}.$$

In terms of the new production function, Proposition 1, 2 and 3 are still hold. We study the impact of the creation of new tasks on wages, capital prices, labor share, and capital share in the following part.

Proposition 4 (i) When N gets larger, that is, artificial intelligence creates new tasks, wages and capital prices go up.
Proposition 4 (ii) When N gets larger, the share of labor and the share of artificial intelligence capital go down, and the share of traditional capital remains at α .
Proposition 4 (iii) When I gets larger, the share of labor decreases while the share of artificial intelligence capital increases, and the share of traditional capital remains at α .
All proofs are contained in the appendix.

The economic intuition of Proposition 4 is that the development of artificial intelligence creates new tasks and widens the work interval of labor. The new tasks create new demand for labor and hence raises wages and the share of labor. In addition, a higher I means more jobs are replaced by artificial intelligence. Consequently, the work interval of labor narrows, or say, less jobs are available to humans, lowering the the share of labor. In all, if technology advances create new tasks, the positive effect of new tasks can offset the negative effect of alternative artificial intelligence on labor.

In reality, the development of technology gives birth to new jobs such as couriers. Although alternative artificial intelligence may cause some negative consequences, the development of technology will certainly create new demand for employment and change the labor structure. For example, the rise of the industrial revolution resulted in the flow of rural surplus labor to the manufacturing industry and the economic growth. However, the change in labor structure cannot be completed in one day, and the process must be long and painful. Therefore, the education system should be adjusted to help people adapt to the changes more efficiently.

3 Discussion on growth rate

We assume that labor L is a constant and $P \to +\infty$ to discuss the equilibrium path of long-term economic growth. The following three kinds of production function are used to analyze the growth rates of per capita output, per capita traditional capital, and per capita artificial intelligence capital respectively.

(i) The production function is given as $Y = K^{\alpha}(L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma(1-\alpha)}{\sigma-1}}$. L is a constant, $P \to +\infty$, thus

(24) $$Y \approx K^{\alpha} P^{1-\alpha} .$$

Suppose the saving rate is s, ρ of savings is invested in traditional capital, $1-\rho$ is invested in artificial intelligence capital, and the capital depreciation rate is δ . Thus,

(25) $$\dot{K} = \rho s Y - \delta K ,$$

(26) $$\dot{P} = (1-\rho) s Y - \delta P .$$

Define $g_m = \frac{\dot{m}}{m}$, where $m = Y, y, K, k, P, p$. Since capital has no differences, the growth rates of traditional capital and artificial intelligence capital must be the same, $g_K = g_P$, or $\frac{\dot{K}}{K} = \frac{\dot{P}}{P}$.

Substitute into the Eq. (25) and (26),

(27) $$\rho = \frac{K}{K+P}.$$

Since L is a constant and $P \to +\infty$, simply $K = \frac{\alpha}{1-\alpha} Z P^{\frac{1}{\sigma}}$ to

(28) $$K = \frac{\alpha}{1-\alpha} P.$$

Substitute the Eq. (28) into the Eq. (27), we have $\rho = \alpha$, thus

(29) $$g_K = g_P = as - \delta,$$

where $a = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

Given the Eq. (24), we have

(30) $$g_Y = \alpha g_K + (1-\alpha) g_P = as - \delta.$$

Define the population growth rate as n, per capita output as $y = \frac{Y}{L}$, per capita traditional capital as $k = \frac{K}{L}$, and per capita artificial intelligence capital as $p = \frac{P}{L}$.

The Eq. (29) and (30) imply that

$$g_y = g_k = g_p = as - (n+\delta).$$

In the long-run balanced growth path (referred as BGP), the wage and capital price are

$$\omega = aL^{-\frac{1}{\sigma}} Z^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}} \approx aL^{-\frac{1}{\sigma}} P^{\frac{1}{\sigma}},$$

$$R = aZ^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}} \approx a,$$

where $a = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.

(ii) If capital is invested to improve artificial intelligence capital-augmenting technology, the production function is as follow,

$$Y = K^{\alpha}[L^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}},$$

Similarly, we have

(31) $$Y \approx A_P^{1-\alpha} K^{\alpha} P^{1-\alpha},$$

Apply the no-arbitrage condition and obtain $K = \frac{\alpha}{1-\alpha} P$, since $\frac{\dot{K}}{K} = \frac{\dot{P}}{P}$, we have

(32) $$g_K = g_P = asA_P^{1-\alpha} - \delta .$$

Define the growth rate of artificial intelligence as $g_{A_P} = g$, the Eq. (13) implies that

(33) $$g_Y = (1-\alpha)g + \alpha g_K + (1-\alpha)g_P = (1-\alpha)g + asA_P^{1-\alpha} - \delta ,$$

thus

(34) $$g_k = g_p = asA_P^{1-\alpha} - (n+\delta) ,$$

(35) $$g_y = (1-\alpha)g + asA_P^{1-\alpha} - (n+\delta) .$$

In the BGP,

(36) $$\omega = aA_P^{\frac{(1-\sigma)\alpha}{\sigma}} L^{-\frac{1}{\sigma}} Z_P^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}} \approx aA_P^{\frac{1-\sigma\alpha}{\sigma}} L^{-\frac{1}{\sigma}} P^{\frac{1}{\sigma}}$$

(37) $$R = aA_P^{\frac{(1-\sigma)(\alpha-1)}{\sigma}} Z_P^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}} \approx aA_P^{1-\alpha}$$

(iii) If capital is invested to improve labor-augmenting technology, the production function is as follow,

$$Y = K^{\alpha}[(A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}}$$

Since L is a constant and $P \to +\infty$, $Y \approx K^{\alpha} P^{1-\alpha}$ .Similarly, we have

(38) $$g_y = g_k = g_p = as - (n+\delta) .$$

In the BGP, the wage and capital price are

(39) $$\omega = aA_L^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} Z^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha}{\sigma}} \approx aA_L^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} P ,$$

(40) $$R = aZ_L^{\frac{1-\alpha}{\sigma-1}} P^{\frac{\alpha-1}{\sigma}} \approx a .$$

The above analysis leads to Proposition 5.

Proposition 5 (i) Given the production function as $Y = K^{\alpha}(L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma(1-\alpha)}{\sigma-1}}$ or $Y = K^{\alpha}[(A_L L)^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}}$, where L is a constant and $P \to +\infty$, economic growth reaches a steady state equilibrium in the long run, and all the growth rates of per capita output, per capita traditional capital, and per capita artificial intelligence capital are $as - (n+\delta)$. A higher saving rate is associated with a higher economic growth rate, while a higher population growth rate or depreciation rate results in slower economic growth. In equilibrium, wage increases with artificial intelligence capital, while capital price remains unchanged.

Proposition 5 (ii) Given the production function as $Y = K^{\alpha}[L^{\frac{\sigma-1}{\sigma}} + (A_P P)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma(1-\alpha)}{\sigma-1}}$, where L is a constant and $P \to +\infty$, the development of artificial intelligence capital-augmenting technology leads to sustainable economic growth, and all the growth rates of per capita output, per capita traditional capital, and per capita artificial intelligence capital increase together. A higher saving rate is associated with a higher economic growth rate, while a higher population growth rate or depreciation rate results in slower economic growth. In equilibrium, wage increases with artificial intelligence capital, while capital price increases with technological progress.

The economic intuition of Proposition 5 is that the development of artificial intelligence benefits the whole society. Artificial intelligence not only improves economic growth, but also increases wages.

Proposition 6 Without the consideration of unemployment, in order to balance between economic growth and workers' welfare, it is beneficial to develop complementary artificial intelligence first. When artificial intelligence capital is complementary to labor, investing capital in improving the artificial intelligence capital-augmenting technology can both keep sustainable economic growth and increase wages, which is more efficient that investing in promoting labor-augmenting technology.

4 Conclusion and prospects

On the basis of previous research, we propose a general framework of the macroeconomic consequences of artificial intelligence. We assume that all workers are homogenous and fully employed, and focus on the effects of two kinds of artificial intelligence on wages, capital prices, labor share, capital share and economic growth. We study two issues, one is about the effect of artificial intelligence capital on the economy, and the other relates to the effect of technological progress on the economy.

In terms of the effect of artificial intelligence capital on the economy, our model shows that more artificial intelligence capital results in cheaper capital price and higher wage. If artificial intelligence capital is complementary to labor, artificial intelligence capital increases the share of labor. However, if artificial intelligence capital and labor substitute each other, humans compete with machines, artificial intelligence capital makes more jobs replaced by machines, and the share of labor drops.

In addition, to improve the technology of artificial intelligence capital-augmenting or labor-augmenting production is another concern of this model. If artificial intelligence capital is complementary to labor, the Baumol's cost disease will be incurred, which means that the increase in one sector's productivity leads to a further decline in its share. Under this assumption, the technological advances of artificial intelligence capital-augmenting provide assistance to humans, reduce workload and improve wages. The improvement of labor-augmenting technology decreases the work completed by automation, more work is allocated by humans and wages drop. However, if artificial intelligence capital and labor substitute each other, humans will compete with machines. Therefore, the technological advances of artificial intelligence must make more workers lose their job, lower wages and the share of labor. The improvement of labor-augmenting technology can gain workers competitive advantages and more jobs, thus promoting both wages and the share of labor.

Lastly, we extend the task-based model to explore the effects of the creation of new tasks on wages and the share of labor. It is found that the creation of new tasks can always raise wages and the share of labor.

Furthermore, we examine the long-term economic growth rate. Without the consideration of exogenous technology, the development of any kind of artificial intelligence can lead to a steady state equilibrium of economic growth in the long run, and all growth rates of per capita output, per capita traditional capital, and per capita artificial intelligence capital are the same positive level. With exogenous technology considered, the development of labor-augmenting technology can lead to a steady state equilibrium of economic growth in the long run, while the development of artificial intelligence can achieve sustainable economic growth.

In fact, further analysis can be made by extending our general model.

(1) Unemployment. If most of the jobs are replaced by alternative artificial intelligence, what kind of actions the government can take to address the problem of mass unemployment. One solution is to tax robots and subsidize the unemployed. However, taxation will certainly result in the loss of efficiency and foreign transfer of artificial intelligence capital. Moreover, over subsidizing may cause more people stay at home while they can go out for work. How to make reasonable and effective policy is still a worthy concern.

(2) Labor market polarization. If technological progress decreases the share of medium-skilled workers and increases the shares of both high- and low- skilled ones, the polarization of labor market occurs. It is possible to discuss the possibility of artificial intelligence replacing medium-skilled workers and the changes in their wages and the share of labor.

(3) Aging of population. The aging problem is becoming more and more serious in most developed countries and some developing countries. Both the decreasing birth rate and increasing average life span exacerbate the labor shortage. Whether the development of artificial intelligence can solve this problem to keep economic growth and social welfare is another interesting topic.

(4) Endogenous artificial intelligence. We assume that the technology of artificial intelligence is exogenous in this model. Therefore, another question is what the effects of artificial intelligence on economic variables are if this assumption is relaxed.

(5) Education. Most countries in the world are facing a dilemma that a lot of college graduates cannot find a job while some emerging industries are starved for talents such as data analysts and AI developers. How to reform the education system to cope with the shock from artificial intelligence deserves attention.

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Appendix:

Proof of Proposition 1:

(i) by (6)

$$\ln\omega = \ln\alpha(L^{-\frac{1}{\sigma}} +)\frac{1-\alpha}{\sigma-1}\ln Z + \frac{\alpha}{\sigma}\ln E$$

thereby

$$\frac{\partial(\ln\omega)}{\partial P} = \frac{1}{\sigma ZP}[P^{\frac{\sigma-1}{\sigma}} + \alpha L^{\frac{\sigma-1}{\sigma}}] > 0$$

Similarly

$$\frac{\partial(\ln R)}{\partial P} = -\frac{1-\alpha}{\sigma ZP}L^{\frac{\sigma-1}{\sigma}} < 0$$

（ii）The traditional capital share is

$$\alpha_K = \frac{RK}{Y} = \alpha$$

The labor share is

$$\alpha_L = \frac{\omega L}{Y} = (1-\alpha)\frac{L^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}}$$

$$\alpha_P = \frac{RP}{Y} = (1-\alpha)\frac{P^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}} + P^{\frac{\sigma-1}{\sigma}}}$$

thereby

When $0<\sigma<1$， $\frac{\partial(\alpha_L)}{\partial P} > 0, \frac{\partial(\alpha_P)}{\partial P} < 0$； when $\sigma > 1$， $\frac{\partial(\alpha_L)}{\partial P} < 0, \frac{\partial(\alpha_P)}{\partial P} > 0$.

When $\sigma = 1$， α_L 、 α_P 、 α_K are all fixed values.

Proof of Proposition 2:

Proof: (i) by (10)

$$\frac{\partial(\ln\omega)}{\partial A_P}=\frac{L^{\frac{\sigma-1}{\sigma}}}{\sigma Z_P A_P}[(1-\sigma)\alpha+(1-\sigma\alpha)(\frac{A_P P}{L})^{\frac{\sigma-1}{\sigma}}]$$

thereby

when $0<\sigma<1$, $\frac{\partial(\ln\omega)}{\partial A_P}>0$; when $\sigma>\frac{1}{\alpha}>1$, $\frac{\partial(\ln\omega)}{\partial A_P}<0$。

(ii) by (11)

$$\frac{\partial(\ln R)}{\partial A_P}=\frac{(1-\alpha)L^{\frac{\sigma-1}{\sigma}}}{\sigma Z_P A_P}[\sigma(\frac{A_P P}{L})^{\frac{\sigma-1}{\sigma}}+(\sigma-1)]$$

When $\sigma>1$, $\frac{\partial(\ln R)}{\partial A_P}>0$.

(iii) by (8), (10), (11)

$$\alpha_L=\frac{\omega L}{Y}=(1-\alpha)\frac{L^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}}+(A_P P)^{\frac{\sigma-1}{\sigma}}},\alpha_P=\frac{RP}{Y}=(1-\alpha)\frac{(A_P P)^{\frac{\sigma-1}{\sigma}}}{L^{\frac{\sigma-1}{\sigma}}+(A_P P)^{\frac{\sigma-1}{\sigma}}}$$

When $0<\sigma<1$, $\frac{\partial(\alpha_L)}{\partial A_P}>0,\frac{\partial(\alpha_P)}{\partial A_P}<0$;when $\sigma>1$, $\frac{\partial(\alpha_L)}{\partial A_P}<0,\frac{\partial(\alpha_P)}{\partial A_P}>0$.

Proof of Proposition 3:
From (14), (15)

$$\frac{\partial(\ln\omega)}{\partial A_L}=\frac{1}{\sigma Z_L A_L}[(\sigma-\alpha)(A_L L)^{\frac{\sigma-1}{\sigma}}+(\sigma-1)P^{\frac{\sigma-1}{\sigma}}]$$

$$\frac{\partial(\ln R)}{\partial A_L}=\frac{1-\alpha}{\sigma Z_L}A_L^{-\frac{1}{\sigma}}L^{\frac{\sigma-1}{\sigma}}>0$$

thereby

when $0<\sigma<\alpha$, $\frac{\partial(\ln\omega)}{\partial A_L}<0$; when $\sigma>1$, $\frac{\partial(\ln\omega)}{\partial A_L}>0$.

(ii) by (11), (12), (13)

$$\alpha_L=\frac{\omega L}{Y}=(1-\alpha)\frac{(A_L L)^{\frac{\sigma-1}{\sigma}}}{(A_L L)^{\frac{\sigma-1}{\sigma}}+P^{\frac{\sigma-1}{\sigma}}}$$

$$\alpha_P=\frac{RP}{Y}=(1-\alpha)\frac{P^{\frac{\sigma-1}{\sigma}}}{(A_L L)^{\frac{\sigma-1}{\sigma}}+P^{\frac{\sigma-1}{\sigma}}}$$

When $0<\sigma<1$, $\frac{\partial(\alpha_L)}{\partial A_L}<0,\frac{\partial(\alpha_P)}{\partial A_L}>0$;when $\sigma>1$, $\frac{\partial(\alpha_L)}{\partial A_L}>0,\frac{\partial(\alpha_P)}{\partial A_L}<0$.

Proof of Proposition 4:

Proof: (i) by (31), (32)

$$\frac{\partial(\ln\omega)}{\partial N}=\frac{\partial(\ln R)}{\partial N}=\frac{1-\alpha}{(\sigma-1)Z}[N^{\frac{\sigma-1}{\sigma}}-(N-1)^{\frac{\sigma-1}{\sigma}}]>0$$

(ii) $\alpha_L=\frac{\omega\int_I^N ldi}{Y}=(1-\alpha)(N-I)l^{\frac{\sigma-1}{\sigma}}A_l^{\frac{\sigma-1}{\sigma}}Z^{-1},\alpha_P=\frac{R\int_{N-1}^I pdi}{Y}=(1-\alpha)p^{\frac{\sigma-1}{\sigma}}A_p^{\frac{\sigma-1}{\sigma}}Z^{-1}$

$$\alpha_K=\frac{RK}{Y}=\alpha\text{，且}\,\alpha_L+\alpha_P+\alpha_K=1.$$

$$\frac{\partial(\ln\alpha_L)}{\partial N}=\frac{1}{N-1}-\frac{1}{Z}[N^{\frac{\sigma-1}{\sigma}}-(N-1)^{\frac{\sigma-1}{\sigma}}]$$

$$\frac{\partial(\ln\alpha_P)}{\partial N}=-\frac{1}{I-N+1}-\frac{1}{Z}[N^{\frac{\sigma-1}{\sigma}}-(N-1)^{\frac{\sigma-1}{\sigma}}]$$

When $0<\sigma<1$， $N^{\frac{\sigma-1}{\sigma}}<(N-1)^{\frac{\sigma-1}{\sigma}}$， so $\frac{\partial(\ln\alpha_L)}{\partial N}>0$， That is, as it grows larger, the labor share will increase, the share of artificial intelligence capital must decrease, so $\frac{\partial(\ln\alpha_P)}{\partial N}<0$.When $\sigma>1$， $N^{\frac{\sigma-1}{\sigma}}>(N-1)^{\frac{\sigma-1}{\sigma}}$， so $\frac{\partial(\ln\alpha_P)}{\partial N}<0$， That is, as N becomes larger, the share of artificial intelligence capital will decrease, the labor share will increase, so $\frac{\partial(\ln\alpha_P)}{\partial N}>0$.

（iii） $\frac{\partial(\ln\alpha_L)}{\partial I}=-\frac{1}{N-1}<0$， $\frac{\partial(\ln\alpha_P)}{\partial I}=\frac{1}{I-N+1}>0$