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Metcalfe's law and herding behaviour in the cryptocurrencies market

Daniel Traian Pele and Miruna Mazurencu-Marinescu-Pele

Abstract
In this paper, the authors investigate the statistical properties of some cryptocurrencies by using three layers of analysis: alpha-stable distributions, Metcalfe's law and the bubble behaviour through the LPPL modelling. The results show, in the medium to long-run, the validity of Metcalfe's law (the value of a network is proportional to the square of the number of connected users of the system) for the evaluation of cryptocurrencies; however, in the short-run, the validity of Metcalfe's law for Bitcoin is questionable. As the results showed a potential for herding behaviour, the authors then used LPPL models to capture the behaviour of cryptocurrencies exchange rates during an endogenous bubble and to predict the most probable time of the regime switching. The main conclusion is that Metcalfe's law may be valid in the long-run, however in the short-run, on various data regimes, its validity is highly debatable.

JEL C22 C32 C51 C53 C58 E41 E42 E47 E51 G1 G17
Keywords Cryptocurrency; Bitcoin; CRIX; Log-Periodic Power law; Metcalfe's law; stable distribution; herding

Authors
Daniel Traian Pele, The Bucharest University of Economic Studies, Bucharest, Romania, danpele@ase.ro
Miruna Mazurencu-Marinescu-Pele, The Bucharest University of Economic Studies, Bucharest, Romania

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1. Introduction

After 2008, when the pseudonymous Satoshi Nakamoto developed the Bitcoin (Nakamoto, 2008), an explosion of other cryptocurrencies begun, based on the blockchain technology.

According to one of the major websites dealing with cryptocurrencies[1], at the beginning of September 2018 the total market capitalization was around 180 billion USD, making the cryptocurrencies market extremely desirable within the global assets market.

This new class of assets became interesting not only for traders, but also for the market regulators and academics.

For instance, in 2018, the European Supervisory Authorities for securities, banking and insurance and pensions, released a statement warning, claiming that the "VCs (virtual currencies) such as bitcoin, are subject to extreme price volatility and have shown clear signs of a pricing bubble and consumers buying VCs should be aware that there is a high risk that they will lose a large amount, or even all, of the money invested"[2].

From the academic side, there are a lot of papers dealing with the subject of cryptocurrencies, especially in terms of their statistical properties and the risk modelling. For the purpose of this paper, we will refer only to the most recent papers dealing with three areas regarding the cryptocurrencies market: statistical properties of returns, valuation of cryptocurrencies and log-periodic power laws applied to cryptocurrencies.

Regarding the statistical properties, Hu et al. (2018) carried out a survey dealing with some stylized facts about the cryptocurrencies market, showing that the time series of returns are characterized by large values of kurtosis and volatility.

Zhang et al. (2018) highlighted also some statistical properties of the cryptocurrencies return: the presence of heavy tails, strong volatility clustering, leverage effects and the existence of a power-law correlation between price and volume.

Chen et al. (2017) applied some classical statistical methods (ARIMA, GARCH and EGARCH modelling) to the CRIX indices family, allowing them to observe the volatility clustering phenomenon and the presence of fat tails.

Another analysis of the CRIX index (Chen et al. (2017)) deals with a pricing model of derivatives for CRIX index and Bitcoin options, by using an affine jump diffusion model, SVCJ (Stochastic Volatility with Correlated Jumps) model. An important finding arising from Chen's paper is that the jumps presented in the cryptocurrencies' prices are an essential component.

As for the second area, namely the valuation, there are several papers dealing with the Metcalfe's law, who states that a network's value is proportional to the square of the number of its users.

Peterson (2017) used the Metcalfe's law as a Model for Bitcoin's value, by estimating a model of supply (number of bitcoins) and demand (number of bitcoin wallets) and concluding that the Metcalfe's law is a very good fit for Bitcoin's price.

Wheatley et al. (2018) estimated the Metcalfe's law for Bitcoin, proving the existence of a log-linear relationship between the market capitalization and a proxy for the number of network users (the number of unique addresses).

If the Metcalfe's law is valid for cryptocurrencies, then a significant correlation between the number of users and the market price should be present. If the correlation is also a causality (in one way or another), then there may be room for the occurrence of some herding behaviour: if the market is driven by expected future price increases, then more and more players will enter the market, causing the price to develop a bubble which will end eventually in a crash.

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[1] https://coinmarketcap.com/

[2] https://www.esma.europa.eu/sites/default/files/library/esma50-164-1284_joint_esas_warning_on_virtual_currenciesl.pdf

For example, the Bitcoin market has experienced several crashes during its lifetime, the first one being in 2012, due to a Ponzi fraud involving Bitcoin. Another crash occurred in 2014, when Mt. Gox, a Bitcoin exchange handling over 70% of all Bitcoin transactions worldwide, closed its website and exchange service, and filed for bankruptcy protection from creditors; the value of Bitcoin then dropped by 50 percent in just two days.

The most recent collapse, at the end of 2017, occurred after the intention of the South Korean regulators to shut down the cryptocurrencies exchange market.

As for the third area, LPPL (Log-Periodic Power Law) models are widely used to describe the behaviour of stock prices during an endogenous bubble and to predict the most probable time of the regime switching (see Johansen et al., 2000), as the aggregated behaviour of the investors is reflected in a log-periodic evolution of the trading price before the crash.

As the industry of cryptocurrencies has grown exponentially over the past several years, there are many applications of the LPPL models to the study of this new market.

Malhotra et al. (2013) investigated the evolution of Bitcoin exchange rates in 2013-2014, showing evidence of a super-exponential growth in Bitcoin exchange rates.

MacDonell (2014) used the LPPL model to forecast the Bitcoin price crash that took place on December 4th, 2013, showing how the model can be a valuable tool for detecting bubble behaviour in digital currencies.

Fry (2015) used the LPPL models to test the presence of a bubble in Bitcoin prices before the price crash of December 2013 and they concluded that LPPL models are a valuable tool for understanding the bubble behaviour in digital currencies.

Wheatley et al. (2018) have also used a variant of the LPPL model to estimate the most probable time of the crash for the 2017 Bitcoin bubble.

In this study we are solely focusing on applying three major statistical methods for studying the behaviour of cryptocurrencies market.

First, we are using the alpha-stable distributions to emphasize the heavy-tails property of the distribution of cryptocurrencies daily log-returns.

Second, we employ the generalized Metcalfe's law for the most important cryptocurrency, the Bitcoin, in order to understand the relationship between the Bitcoin's price, Bitcoin's market capitalization and the number of network users, deriving from there the potential for herding behaviour.

Third, we use the LPPL model to fit the bubble dynamics for the Bitcoin and for one major cryptocurrencies index, the CRIX index, showing the value of log-periodic power laws in anticipating the regime switching.

In the light of the findings from the literature, our contribution to the research on the statistical analysis of the cryptocurrencies is mostly empirical.

By using the alpha-stable distributions to emphasize the heavy-tails property of the distribution of cryptocurrencies daily log-returns, our results extend the findings from Zhang et al. (2018), where the presence of heavy tails for cryptocurrencies is highlighted by using the Hill method to estimate the tail index.

By estimating the generalized Metcalfe's law in order to understand the relationship between the Bitcoin's price, our paper extends the results from Wheatley et al. (2018): precisely, we find reverse causality to Metcalfe's law – with price causing users growth.

As a consequence, the potential for herding behaviour is detected, we apply the LPPL models to detect the most probable time of regime switching, in case of the CRIX index and of the Bitcoin.

The main conclusion of the paper is that the Metcalfe's law may be valid in the long-run, however in the short-run, on various data regimes, its validity is highly debatable.

The rest of the paper is organized as follows: Section 2 details the methodology; Section 3 presents the dataset and the empirical results and Section 4 concludes.

2. Methodology

As previously mentioned, the methodology used in this paper has three layers: first, we study the statistical properties of the daily log-returns of the selected cryptocurrencies and we estimate the parameters of alpha-stable distributions, in order to derive their propensity for non-Gaussianity and heavy tails behaviour.

Second, we investigate the validity of the Metcalfe's law for the most popular cryptocurrency, Bitcoin, showing the existence of a potential for herding behaviour.

Third, we apply the Log-Periodic Power Law models (Johansen et al., 2000) to identify the bubble regime in Bitcoin prices and in the evolution of the CRyptocurrency IndeX.

2.1. Stable distributions

In order to characterize the tail behaviour of the cryptocurrencies we fit the distribution of daily log-returns through the alpha-stable approach. A random variable X follows an alpha-stable distribution with parameters $(\alpha, \beta, \gamma, \delta)$ (Nolan, 2011) if exists $\gamma>0$, $\delta \in$ R, such as X and $\gamma Z+\delta$ have the same distribution, where Z is a random variable with the characteristic function:

$$\phi(t) = \mathbf{E}[e^{itZ}] = \begin{cases} \exp(-|t|^{\alpha}[1 - i\beta \tan(\frac{\pi\alpha}{2})sign(t)]), \alpha \neq 1 \\ \exp(-|t|[1 + i\beta t\frac{2}{\pi} sign(t)(\ln(|t|)]), \alpha = 1 \end{cases}. \quad (1)$$

A random variable X follows an alpha-stable distribution $S(\alpha, \beta, \gamma, \delta; 0)$ if its characteristic function has the form (Nolan, 2011):

$$\varphi(t) = \mathbf{E}[e^{itX}] = \begin{cases} \exp(-\gamma^{\alpha}|t|^{\alpha}[1 + i\beta \tan(\frac{\pi\alpha}{2})sign(t)(|\gamma t|^{1-\alpha} - 1)] + i\delta t), \ \alpha \neq 1 \\ \exp(-\gamma|t|[1 + i\beta t\frac{2}{\pi} sign(t)(\ln(|\gamma t|)] + i\delta t), \ \alpha = 1 \end{cases}. \quad (2)$$

In the above notations $\alpha \in (0.2]$ is the stability index, controlling for probability in the tails (for Gaussian distribution $\alpha = 2$), $\beta \in [-1,1]$ is the skewness parameter, $\gamma \in (0, \infty)$ is the scale parameter and $\delta \in$ R is the location parameter.

The tail behaviour of the stable distributions is driven by the values of stability index α: small values are associated to higher probabilities in the tails of the distribution.

In this paper we are using a regression-based method for estimating the parameters of an alpha-stable distribution (Kogon and Williams, 1998). This method is implemented as a SAS macro in Pele (2014) and can be used to obtain estimates for the parameters of stable distributions (see the Appendix A).

2.2. *Metcalfe's law*

In the 1980s, Robert Metcalfe, the co-inventor of Ethernet, stated what was called later the Metcalfe's law (Gilder 1993): the value of a network is proportional to the square of the size of the number of connected users.

Metcalfe's law was validated in various contexts, by using social network data: Zhang et al. (2015) proved the validity of the law for Facebook and Tencent (Chinese social network). Other researchers (Madureira et al., 2013, Van Hove, 2014, 2016, Metcalfe, 2013) have shown the validity of the law, mostly regarding internet networks.

Peterson (2017) showed that the Metcalfe's law can be used to explain the evolution of the Bitcoin transaction price, by using factors relating to supply (number of bitcoins) and demand (number of wallets).

In this paper we are using the Metcalfe's law version from Wheatley et al. (2018):

$$C_t = e^{\alpha} u_t^{\beta} \tag{3}$$

where:

- C_t is the Bitcoin's market capitalization at time t;
- u_t is the Bitcoin's number of unique addresses at time t.

If the Metcalfe's law is valid for Bitcoin, then the coefficient *β=2;* in this paper we are testing the equation (3) over the entire sample and by using a rolling window approach.

In addition to the classical form of the Metcalfe's law, we are testing the hypothesis that the Bitcoin's price itself is driven by the Bitcoin's network size, showing potential for some herding behaviour.

Also, by using cointegration analysis and Granger causality, we infer that the expected price increase is a driver for more investors to join the Bitcoin network, which may lead in the end to a super-exponential price growth, due to the herding behaviour of investors.

2.3. *Log-periodic power laws (LPPL)*

According to the field theory (Goldenfeld, 1992), an imitative process can be described through its hazard rate *h(t):* $\frac{dh}{dt} = Ch^{\delta}$, where *C>0*, and *δ+1>1* is the average number of interactions.

Then $h(t) = \left(\frac{h_0}{t_c - t}\right)^{\alpha}$, with $\alpha = \frac{1}{\delta - 1}$ and t_c being the critical time, so the price dynamics prior to the crash should be $\ln\frac{p(t)}{p(0)} = k\int_{t_0}^{t} h(u)du$.

As the crash probability should be compensated by larger price changes, prior to the stock market crash (Blanchard, 1979), the hazard rate could be expressed via the Ising model: $h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha}\cos\left[\omega\ln(t_c - t) + \psi'\right]$.

Thus, the trading price before the crash follows a log-periodic power law (Johansen et al., 2000):

$$E\left[\log p(t)\right] = A + B\left(t_c - t\right)^{\beta}\{1 + C\cos[\omega\ln\left(t_c - t\right)^{\beta} + \phi]\}\,, \tag{4}$$

where *p(t)* is the price at moment *t*, t_c is the critical time (the most probable moment of the crash), and $\beta, B_0, B_1, \omega, \phi$ are the parameters of the model which give its log-periodic feature.

In order to have a proper specification of the model, there are several constrains applied to the parameters: *A>0*, *B<0*, $C \neq 0, |C| < 1, 0 < \beta < 1, \omega \in (0, \infty)$ and $\phi \in [0, 2\pi]$.

In this paper, we are using the LPPL models to test the propensity for herding behaviour in the case of Bitcoin and the CRIX Index, following the methodology used in Fantazzini et al. (2016), who applied the LPPL modelling to Bitcoin exchange rates, finding evidence of explosive behaviour in the bitcoin-USD exchange rates during August – October 2012 and November, 2013 – February, 2014.

3. Empirical results

3.1. Dataset

The dataset presented in the table below consists of daily cryptocurrency data (transaction count, on-chain transaction volume, value of created coins, price, market capitalization and exchange volume)[3]. One market index was also used for the analysis: Cryptocurrency Index[4] as a reference for the cryptocurrencies market (Trimborn and Härdle, 2018).

Table 1. Description of the dataset

No.	Symbol	Cryptocurrency/ Index	Number of daily observations	Start date	End date
1	ANT	Aragon	502	5/19/2017	10/2/2018
2	BTC	Bitcoin	1983	4/29/2013	10/2/2018
3	DASH	Dash	1691	2/15/2014	10/2/2018
4	DCR	Decred	965	2/11/2016	10/2/2018
5	DGB	Digibyte	1699	2/7/2014	10/2/2018
6	DOGE	Dogecoin	1752	12/16/2013	10/2/2018
7	ETC	Ethereum Classic	800	7/25/2016	10/2/2018
8	ETH	Ethereum	1152	8/8/2015	10/2/2018
9	GNO	Gnosis	519	5/2/2017	10/2/2018
10	GNT	Golem	683	11/19/2016	10/2/2018
11	GOLD	GoldCoin	2122	12/11/2012	10/2/2018
12	ICN	Iconomi	732	10/1/2016	10/2/2018
13	LSK	Lisk	909	4/7/2016	10/2/2018
14	LTC	Litecoin	1983	4/29/2013	10/2/2018
15	MAID	MaidSafeCoin	1618	4/29/2014	10/2/2018
16	NEO	NEO	753	9/10/2016	10/2/2018
17	PIVX	PIVX	962	2/14/2016	10/2/2018
18	REP	Augur	1071	10/28/2015	10/2/2018
19	USDT	Theter	590	2/20/2017	10/2/2018
20	VTC	Vertcoin	1716	1/21/2014	10/2/2018
21	WAVES	Waves	846	6/3/2016	9/26/2018
22	XEM	NEM	1280	4/2/2015	10/2/2018
23	XLM	Stellar	1519	8/6/2014	10/2/2018
24	XMR	Monero	1595	5/22/2014	10/2/2018
25	XRP	Ripple	1835	8/5/2013	8/13/2018
26	XVG	Verge	1438	10/26/2014	10/2/2018
27	ZEC	ZCash	703	10/30/2016	10/2/2018
28	**CRIX**	**CRyptocurrency IndeX**	**1524**	**8/1/2014**	**10/2/2018**

[3] The source for these data is https://coinmarketcap.com .

[4] The CRyptocurrency IndeX is a benchmark for the crypto market. The CRIX is realtime computed by the Ladislaus von Bortkiewicz Chair of Statistics at Humboldt University Berlin, Germany.

The dataset used in this paper deals only with cryptocurrencies for which at least 2 years of daily transaction data (at least 500 daily observations) were available at the moment of the data collection (October 2nd, 2018). For the purpose of data analysis, the statistical software SAS 9.3 was used.

3.2. Estimating the parameters of an alpha - stable distribution for cryptocurrencies daily log-returns

In order to fit the stable-distribution to the selected time series of daily log-returns $r_t = \log(P_t) - \log(P_{t-1})$, a SAS macro (Pele 2014) was applied and the results are presented in Table 2.

Table 2. Parameters of the estimated alpha - stable distributions

No.	Symbol	α	95% half-width	β	95% half-width	δ	95% half-width	ϒ	95% half-width
1	ANT	1.825	0.063	-0.066	0.015	0.081	0.029	0.047	0.037
2	BTC	1.468	0.100	0.169	0.033	0.211	0.025	0.017	0.074
3	DASH	1.494	0.073	-0.391	0.023	-0.195	0.142	0.030	0.053
4	DCR	1.645	0.084	-0.528	0.158	-0.107	0.397	0.038	0.055
5	DGB	1.620	0.056	-0.245	0.040	0.064	0.103	0.046	0.037
6	DOGE	1.306	0.087	-0.338	0.050	-0.714	0.462	0.024	0.073
7	ETC	1.501	0.089	-0.459	0.054	-0.164	0.255	0.031	0.064
8	ETH	1.589	0.099	-0.457	0.077	0.081	0.273	0.030	0.067
9	GNO	1.733	0.060	-0.030	0.065	-0.077	0.105	0.045	0.037
10	GNT	1.772	0.061	-0.167	0.074	0.439	0.133	0.049	0.037
11	GOLD	1.543	0.089	0.080	0.054	-0.048	0.100	0.003	0.063
12	ICN	1.669	0.060	-0.167	0.085	0.204	0.164	0.052	0.039
13	LSK	1.361	0.025	-0.302	0.068	-0.099	0.220	0.042	0.020
14	LTC	1.336	0.081	-0.202	0.039	-0.384	0.252	0.020	0.066
15	MAID	1.789	0.048	0.028	0.031	-0.079	0.043	0.038	0.029
16	NEO	1.525	0.050	-0.417	0.045	0.058	0.146	0.045	0.035
17	PIVX	1.630	0.076	-0.242	0.038	0.062	0.109	0.054	0.050
18	REP	1.573	0.055	-0.125	0.045	0.076	0.109	0.037	0.037
19	USDT	0.509	0.306	0.111	0.126	-0.003	0.034	0.001	0.680
20	VTC	1.549	0.043	-0.325	0.030	-0.078	0.097	0.045	0.030
21	WAVES	1.716	0.053	-0.015	0.027	0.128	0.046	0.042	0.033
22	XEM	1.631	0.048	-0.208	0.053	0.053	0.117	0.040	0.032
23	XLM	1.515	0.072	-0.279	0.055	-0.072	0.180	0.032	0.052
24	XMR	1.701	0.068	-0.169	0.007	0.167	0.034	0.036	0.043
25	XRP	1.323	0.072	-0.306	0.023	-0.534	0.243	0.023	0.059
26	XVG	1.551	0.105	-0.177	0.076	-0.082	0.226	0.073	0.074
27	ZEC	1.575	0.037	-0.082	0.037	0.107	0.077	0.039	0.025
28	**CRIX Index**	**1.490**	**0.109**	**0.254**	**0.103**	**0.427**	**0.169**	**0.015**	**0.080**

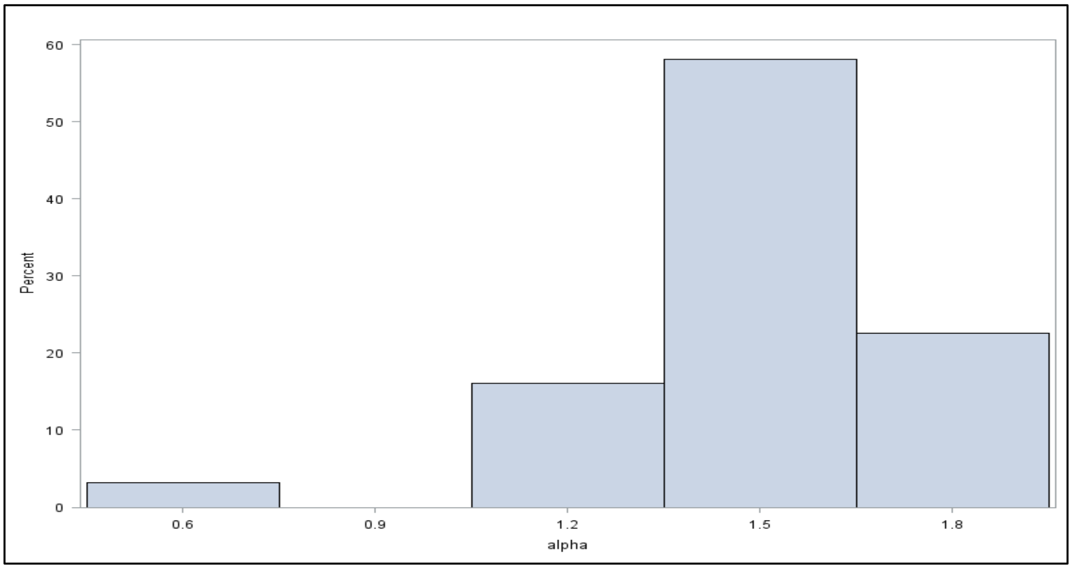


Figure 1. Distribution of the stability index α for log-returns distribution of the selected assets

As depicted in Table 2 and Figure 1, in most of the cases, all the analysed cryptocurrencies exhibit large departures from normality, the values of the stability index α being significantly lower than 2, the value that corresponds to the Gaussian distribution.

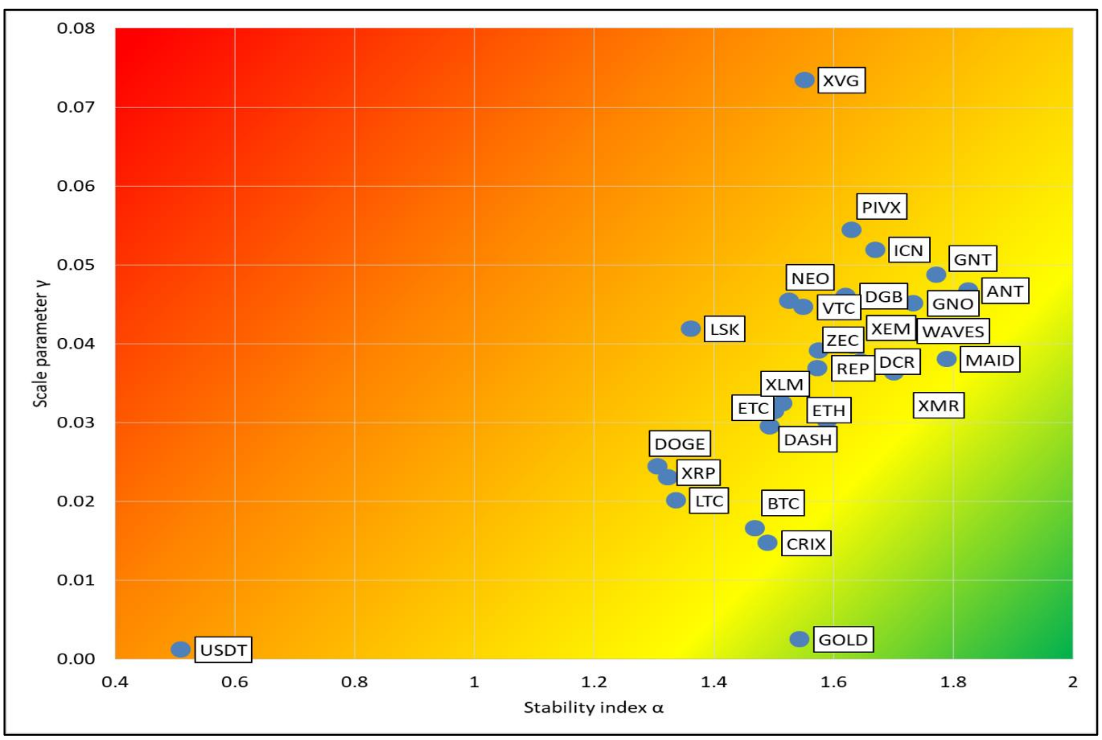
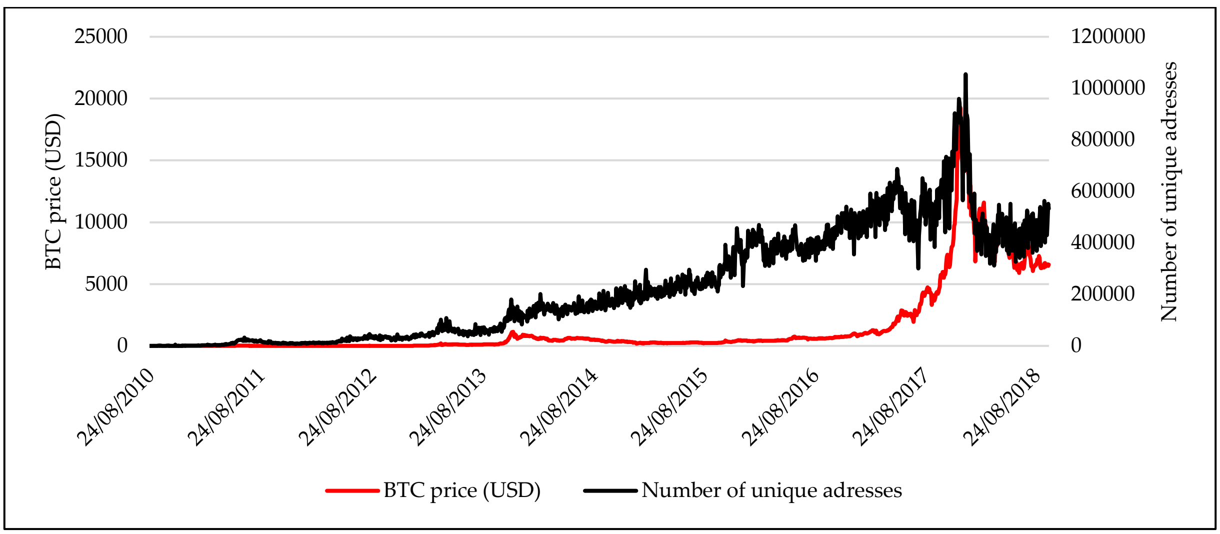


Figure 2. Heatmap of scale parameter γ versus stability index α for selected assets

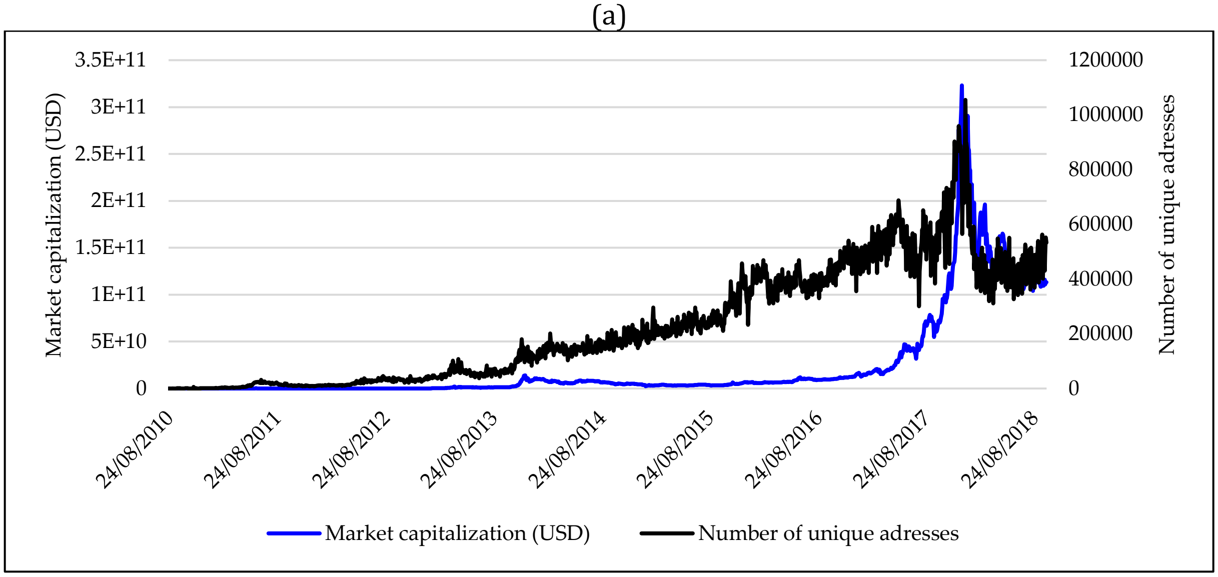
Figure 2 shows the correlation between the scale parameters γ (the equivalent of the volatility in the classical approach) and the stability index α, controlling for the tail probability. Based on this correspondence, we are able to cluster the selected cryptocurrencies based on their propensity to heavy-tails and the likelihood of high volatility. For example, the cryptocurrency Theter (USDT) has the lowest stability index α (large departure from normality), but the scale parameter is low, so USDT is placed in the orange zone. The closest to the normal distribution is Aragon (ANT), yet his scale parameter is around the sample average, so it is placed in the yellow zone.

3.3. Metcalfe's law for Bitcoin

In order to evaluate the applicability of the Metcalfe's law for cryptocurrencies, we limit the research to the most known and traded cryptocurrency, the Bitcoin, also due to the availability of transaction and network data[5].



(a)



(b)

Figure 3. (a) Bitcoin average price (USD) vs. Number of unique Bitcoin addresses used per day. (b) Bitcoin market capitalization (USD) vs. Number of unique addresses.

[5] https://www.blockchain.com

As stated in the original formulation of the Metcalfe's law, the value of the network should be proportional to the squared number of network users; however, in the case of cryptocurrencies, the actual number of users is unknown and we need to use a proxy for it, i.e. the number of unique addresses.

Unique addresses in the Bitcoin ecosystem are the payment addresses that have a non-zero balance; this metric can be used as a proxy for the number of network users, although we cannot assume that the number of users is equal to the number of unique addresses. The number of unique addresses is not constant over time: when fees are high, investors leave their cryptocurrencies in multiple addresses, because a consolidation into a single address will require a high cost. When fees are low, investors can consolidate their funds into a single address.

As the Bitcoin network grows, the number of unique addresses will also grow over time, but when the market is going down, less unique addresses are in use as the number of transactions decreases, as seen in Figure 3.

3.3.1. The Metcalfe's law for the entire data sample

In this section, we are estimating the generalized Metcalfe's law, which is a log-linearization of the equation (3):

$$\log C_t = \alpha + \beta \log u_t + \varepsilon_t . \quad (5)$$

where:

- C_t is the Bitcoin's market capitalization at time *t*;
- u_t is the number of unique Bitcoin addresses used at time *t*.

The estimation results for the equation (5) are shown below, using daily data for the period 2010/08/24 – 2018/10/05.

Table 3. Estimation results for the equation (5)

Parameter	**Estimated value**
α	1.856***
	(0.146)
β	1.696***
	(0.013)
R^2_{adj}	0.924

Note: *** denotes statistical significance at 99% confidence level; standard errors in ().

Although the slope of the equation (5) is β=1.696, below the theoretical value of 2, the model has a high explanatory power ($R^2_{adj} = 0.924$), supporting the validity of the Metcalfe's law for Bitcoin.

The log-linear relationship between the Bitcoin's market capitalization and the number of unique Bitcoin addresses used is illustrated in the figure below.

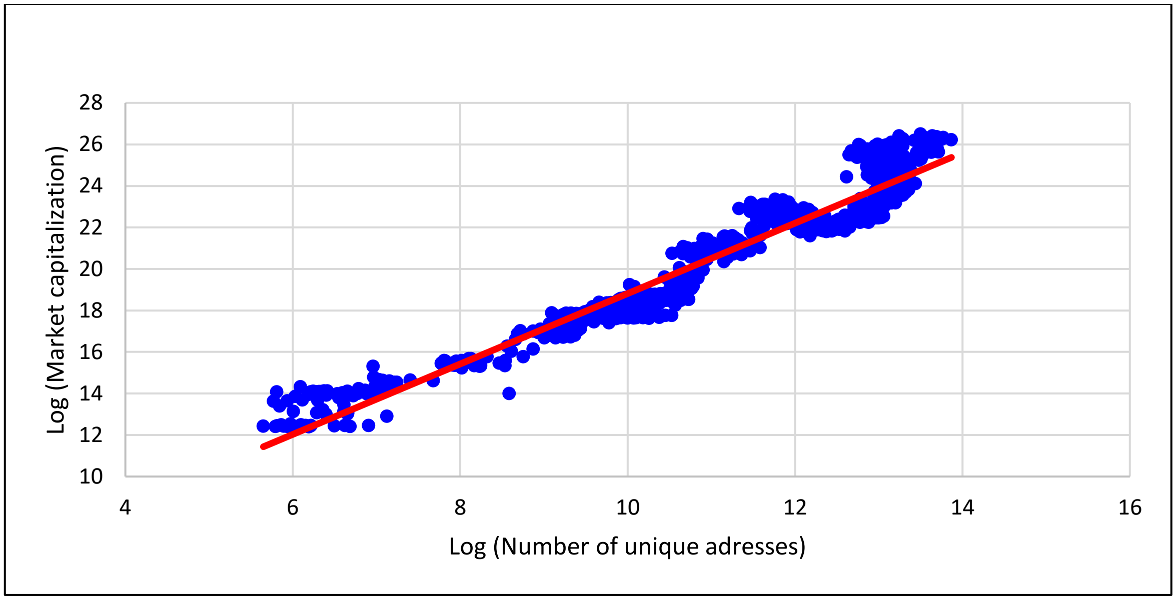


Figure 4. Generalised Metcalfe's law for Bitcoin's market capitalization, for the entire data sample

From the validity of the Metcalfe's law for Bitcoin one can infer the existence of a possible herding effect: as an increase of the number of users is reflected in an increase of the market capitalization, this may be explained by the fact that there is a mimetic effect among users, making the price to have an ascending trend.

One insight into this direction can be found by estimating the generalized Metcalfe's law for the Bitcoin's price:

$$\log P_t = \alpha + \beta \log u_t + \varepsilon_t \,. \tag{6}$$

The estimation results of the equation (6) are shown below.

Table 4. Estimation results for the equation (6)

Parameter	Estimated value
α	-12.040***
	(0.143)
β	1.489***
	(0.012)
R^2_{adj}	0.906

Note: *** denotes statistical significance at 99% confidence level; standard errors in ().
The sample covers the period 2010/08/24 – 2018/10/05.

The results of the estimation show that there is strong log-linear relationship between the Bitcoin's market price and the number of unique addresses, as a proxy for the number of Bitcoin's network users (see also Figure 5); moreover, the price increase may be a direct effect of the increasing network size, through a possible mimetic behaviour.

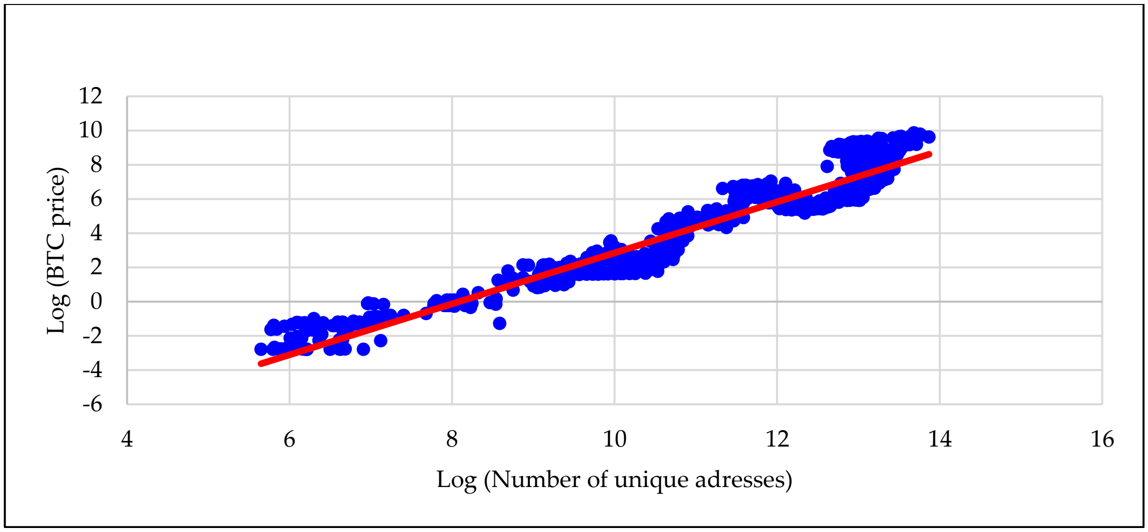


Figure 5. Generalised Metcalfe's law for Bitcoin's price

3.3.2. The Metcalfe's law on rolling windows

The validity of the Metcalfe's law for Bitcoin's market capitalization is questionable: as shown below, due to the different regimes, one will obtain very different parameter estimates by fitting the model on different sub-windows of the data.

For w the length of a rolling window, we estimated the following model:

$$\log C_t = \alpha_k + \beta_k \log u_t + \varepsilon_t \quad , \tag{7}$$

where $t \in \{k+1, \ldots, k+w\}$, $k \in \{0, \ldots, T-w+1\}$ and T the number of observations if the sample. The results are presented in the succession of graphs below.

Figure 6 depicts the values of the Adjusted R-squared for various rolling windows; there are periods when the explanatory power of the model is close to, or higher than 90%, but there also situations when the number of network users has no explanatory power on the market capitalization of Bitcoin.

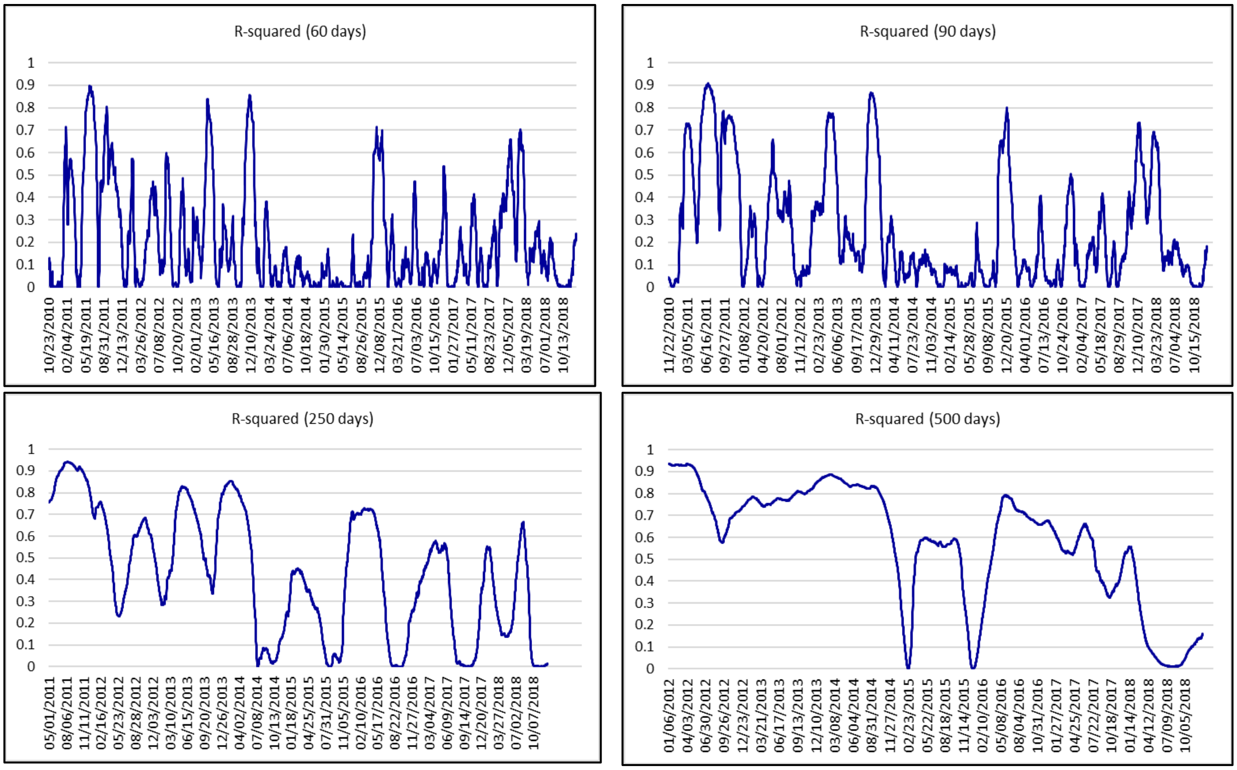


Figure 6. Adjusted R-squared for the Metcalfe's law (equation 5), estimated on rolling windows of 60, 90, 250 and 500 trading days

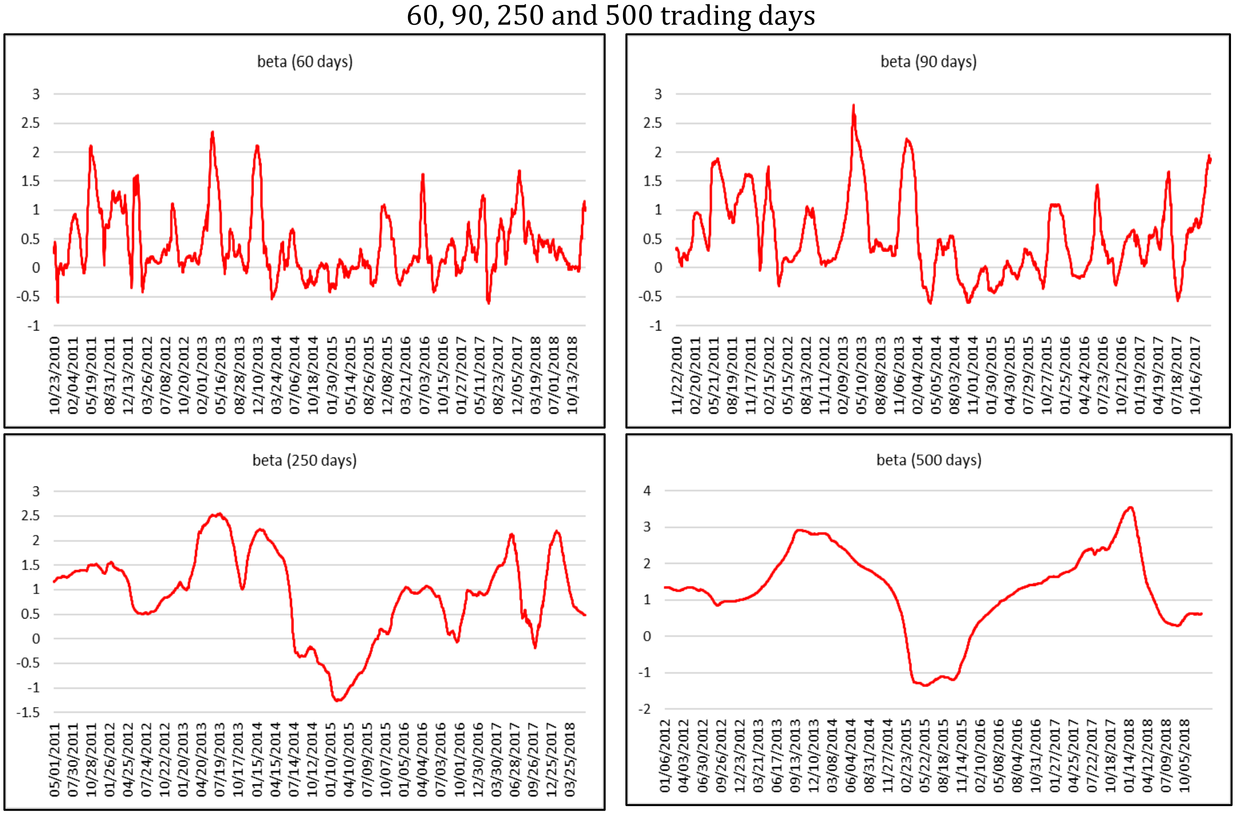


Figure 7. The beta coefficient for the Metcalfe's law (equation 5), estimated on rolling windows of 60, 90, 250 and 500 trading days

Figure 7 shows the estimated values of the coefficient from the equation (5), which, according to the classical formulation of the Metcalfe's law, should be equal to 2. However, there is a huge volatility in the evolution of this coefficient, its average values being significantly lower than 1, for the rolling windows of 60, 90 and 250 days and significantly lower than 2 for the 500 days rolling window.

This analysis shows that there is clear pattern of inconsistency over time, questioning the validity of the Metcalfe's law for Bitcoin, when considering different sub-windows of the data.

3.3.3. Granger causality and cointegration between the Bitcoin's price and the network size

Granger causality and cointegration analysis has been previously applied to analyse the correlation of the Bitcoin price to macroeconomic indicators (see, for example, Zhu et al., 2017).

Going deeper with the analysis, we have also performed a Granger causality test in order to detect the existence of the causal links between the Bitcoin's price and the number of unique addresses. We consider the two-dimensional vector $Y_t = (\ln P_t , \ln u_t)'$, where P_t is the Bitcoin's price and u_t is the number of unique addresses.

As shown in Table 5, these time series are nonstationary (according to the Augmented Dickey Fuller – ADF test) and integrated of order one.

Table 5. ADF test results at 99% confidence level

Variables	**Prob.**	**Conclusion**
log_p	0.386	non-stationarity
log_u	0.041	non-stationarity
Δ(log_p)	0.000	stationarity
Δ(log_u)	0.000	stationarity

Table 6 presents the result of Johansen test; as we can see, there are two cointegration equations at the significance level of 0.05. Thus, we can draw a conclusion that there exists a long-term dynamic equilibrium between the Bitcoin price and the Bitcoin's network size.

Table 6. Johansen cointegration test results

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(s)	**Eigenvalue**	**Trace Statistic**	**0.05 Critical value**	**Prob.[**]**
None[a]	0.011	38.549	20.262	0.000
At most 1[a]	0.004	10.621	9.165	0.026

Note: Trace test indicates 2 cointegrating equations at the 0.05 level; [a] denotes rejection of the hypothesis at the 0.05 level; [**] MacKinnon-Haug-Michelis (1999) *p*-values.

As these time series are not stationary and both of them are integrated of order one, in order to test for Granger causality, the Toda-Yamamoto (1995) procedure was applied, by using the steps below:

a. Test the two time-series to determine their order of integration.
b. Let the m=1 the maximum order of integration for the group of the two time-series.
c. Estimate a VAR model in level.
d. Determine the appropriate maximum lag length (p) for the variables in the VAR, using the AIC, criterion.
e. Check and correct for serial correlation in the residuals.
f. Test for cointegration of the two time-series.

g. Estimate the VAR(p+m) model and test the Granger causality using the Block Exogeneity Wald Test.

Table 7. VAR Granger Causality/Block Exogeneity Wald Tests

Included observations: 1468			
Dependent variable: LOG_P			
Excluded	Chi-sq	df	Prob.
LOG_U	13.343	5	0.020
All	13.343	5	0.020
Dependent variable: LOG_U			
Excluded	Chi-sq	df	Prob.
LOG_P	41.171	5	0.000
All	41.171	5	0.000

Note: the optimum number of lags (15) was chosen based on the lag length criteria from VAR specification.

Based on the Granger causality tests, one can deduce the existence of a unidirectional causal relationship from the Bitcoin's prices to the size of the network, expressed as the number of unique addresses.

The temporal dependency can be captured via a Vector Autoregressive (VAR (p)) model, of the following form: $Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \varepsilon_t$, where $Y_t = (\ln P_t , \ln u_t)'$.

Table 8. VAR (5) estimates

	LOG_U	LOG_P
LOG_U(-1)	0.528***	0.003
	(-0.020)	(-0.007)
LOG_U(-2)	0.086***	0.004
	(-0.022)	(-0.008)
LOG_U(-3)	0.089***	-0.009
	(-0.022)	(-0.008)
LOG_U(-4)	0.106***	-0.013
	(-0.022)	(-0.008)
LOG_U(-5)	0.191***	0.016***
	(-0.020)	(-0.007)
LOG_P(-1)	0.212***	1.071***
	(-0.052)	(-0.020)
LOG_P(-2)	-0.003	-0.100***
	(-0.076)	(-0.029)
LOG_P(-3)	-0.153***	-0.011
	(-0.076)	(-0.029)
LOG_P(-4)	0.053	0.058
	(-0.076)	(-0.029)
LOG_P(-5)	-0.110***	-0.019
	(-0.052)	(-0.020)
Adj. R-squared	0.989	0.999

Note: *** denotes significance at 99% confidence level; standard errors in ().

One can note from the above table with the VAR estimation results that the past realizations of the Bitcoin's price can be used to forecast the future realizations of the network size. For example, if at time *t-1* the Bitcoin's price increases by 1%, then at time *t* one can expect a 0.212% increase of the number of unique addresses.

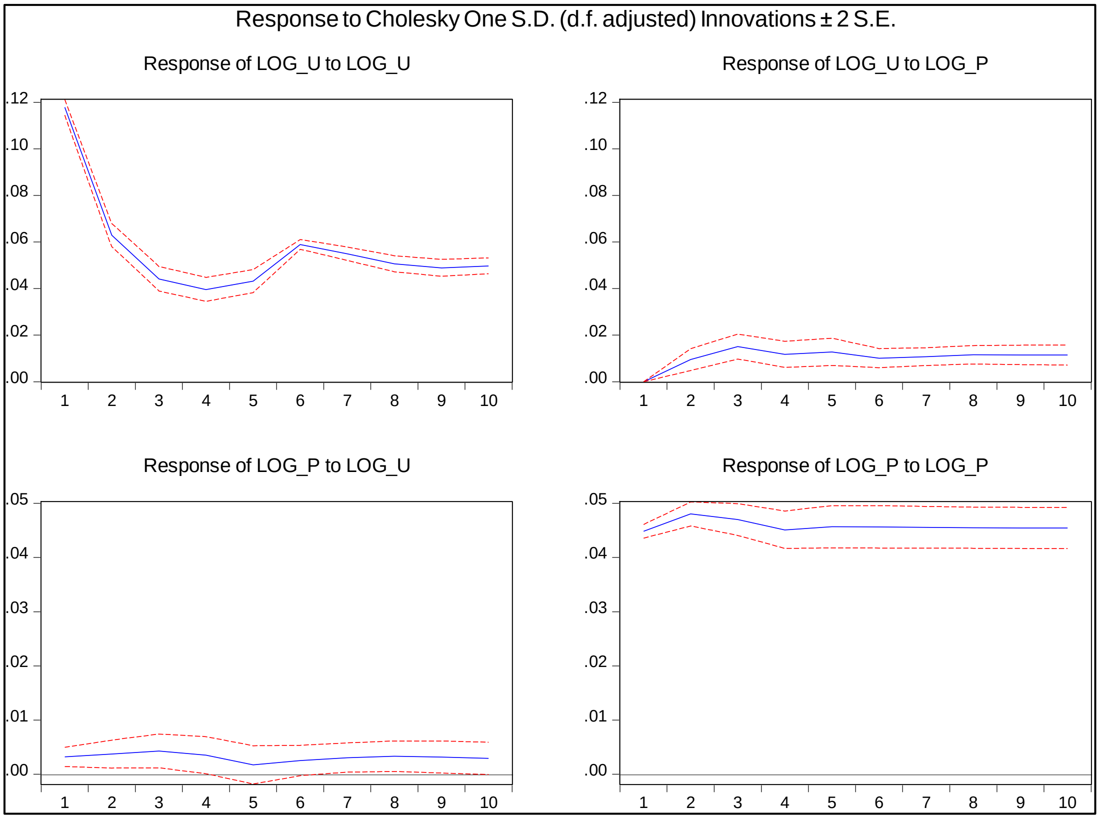


Figure 8. Impulse Response Function for the estimated VAR model

Moreover, the behaviour of the impulse response function offers an indication that a shock from the Bitcoin's price have a positive effect on the network size, and the effect is permanent and significantly different from zero (see Figure 8).

One can infer from this analysis that the expected price increase is a driver for more investors to join the Bitcoin network, which may lead in the end to a super-exponential price growth, due to a herding behaviour of investors.

3.4. LPPL models

In order to capture the bubble regime and to estimate the most probable time of the crash, the algorithm from Pele (2012), using price gyrations and peak detection was applied.

3.4.1. Numerical results for Bitcoin

In case of Bitcoin, the regime swithcing was recorded in December 2017, the exchange rate hitting a local maxima on December 19th, 2017. The initial sample for fitting LPPL model in the case of Bitcoin for predicting the phase transition from December 2017 was 1 Jan 2016 – 30 Nov 2017 (700 daily observations).

Starting from the last observation in the initial sample, we extended the sample using a rolling window with fixed lower limit, so we estimated at every step the LPPL model for $t \in [1,T+k], k=1...17$:

$$E[\ln p(t)] = A_k + B_k (t_{c;k} - t)^{\beta} \{1 + C_k \cos[\omega_k \ln(t_{c;k} - t)^{\beta_k} + \varphi_k]\}. \quad (7)$$

The best results of the LPPL models, in terms of minimzing the RMSE, are given below.

Table 9. The best fit for Bitcoin's LPPL model

	Model 1	**Model 3**	**Model 3**
Orbs	711	701	706
A	9.768	9.328	9.489
B	-0.161	-0.08	-0.104
C	-0.062	0.085	0.076
t_c	0.494	0.588	0.552
β	3.863	3.472	3.588
ω	-10361.29	-9103.55	-5960.18
ϕ	6.28	5.585	4.83
Start date	01-Jan-2016	01-Jan-2016	01-Jan-2016
End date	11-Dec-2017	01-Dec-2017	06-Dec-2017
RMSE	0.148	0.152	0.157
AdjRSq	0.975	0.971	0.97
Date of crash	**12-Dec-2017**	**02-Dec-2017**	**07-Dec-2017**

As a result of the estimation, three models were kept, with the best Root Minimum Squared Error (RMSE). The model with the minimum RMSE anticipated on December 11th 2017 an imminent crash for the next day, as seen in Figure 9.

The other two selected models offer close predictions, for December 2nd 2017 and December 7th 2017.

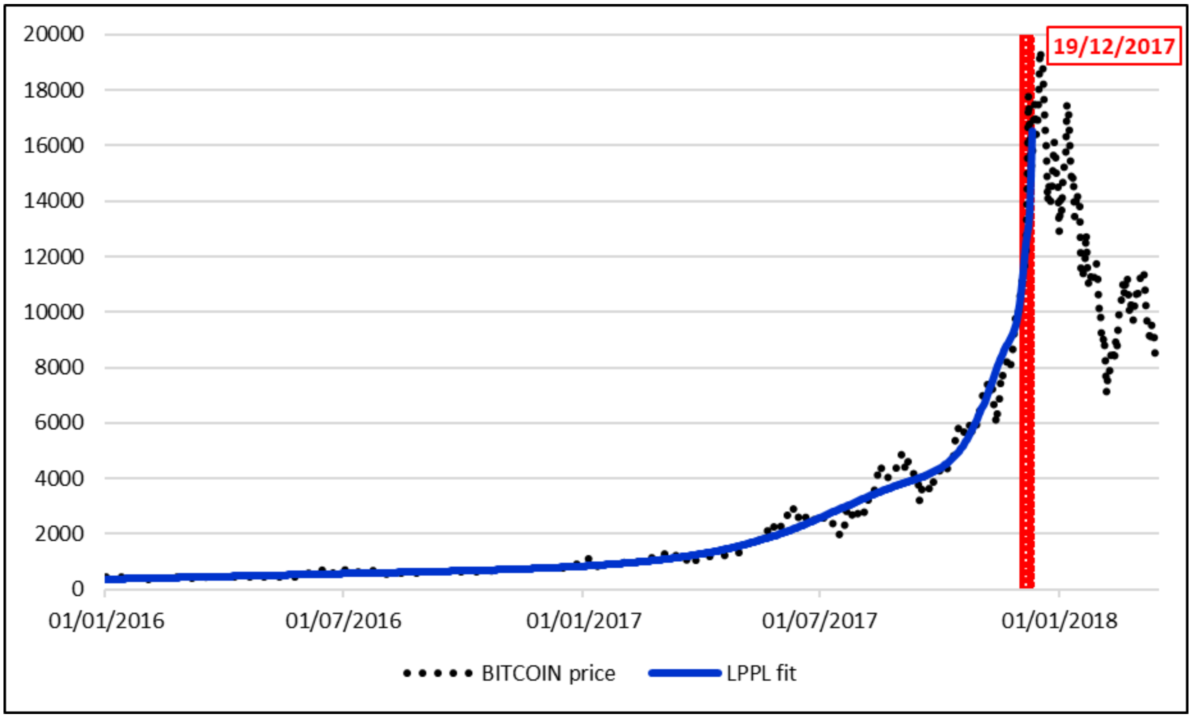


Figure 9. LPPL fit for BTC (model with the minimum RMSE)

3.2. Numerical results for the CRIX Index

The local maxima for the CRIX index was recorded on January 7th 2018, this being the moment of the regime switching.

The initial sample for fitting LPPL model in the case of the CRIX index for predicting the phase transition from January 2018 was 1 Jan 2016 – 15 Dec 2017 (716 daily observations).

Starting from the last observation in the initial sample, we extended the sample by using a rolling window with fixed lower limit, so we estimated at every step the LPPL model for $t \in [1,T+k], k=1...20$:

$$E[\ln p(t)] = A_k + B_k (t_{c;k} - t)^{\beta} \{1 + C_k \cos[\omega_k \ln(t_{c;k} - t)^{\beta_k} + \varphi_k]\}. \qquad (8)$$

The evolution of the CRIX index is depicted in the figure below.

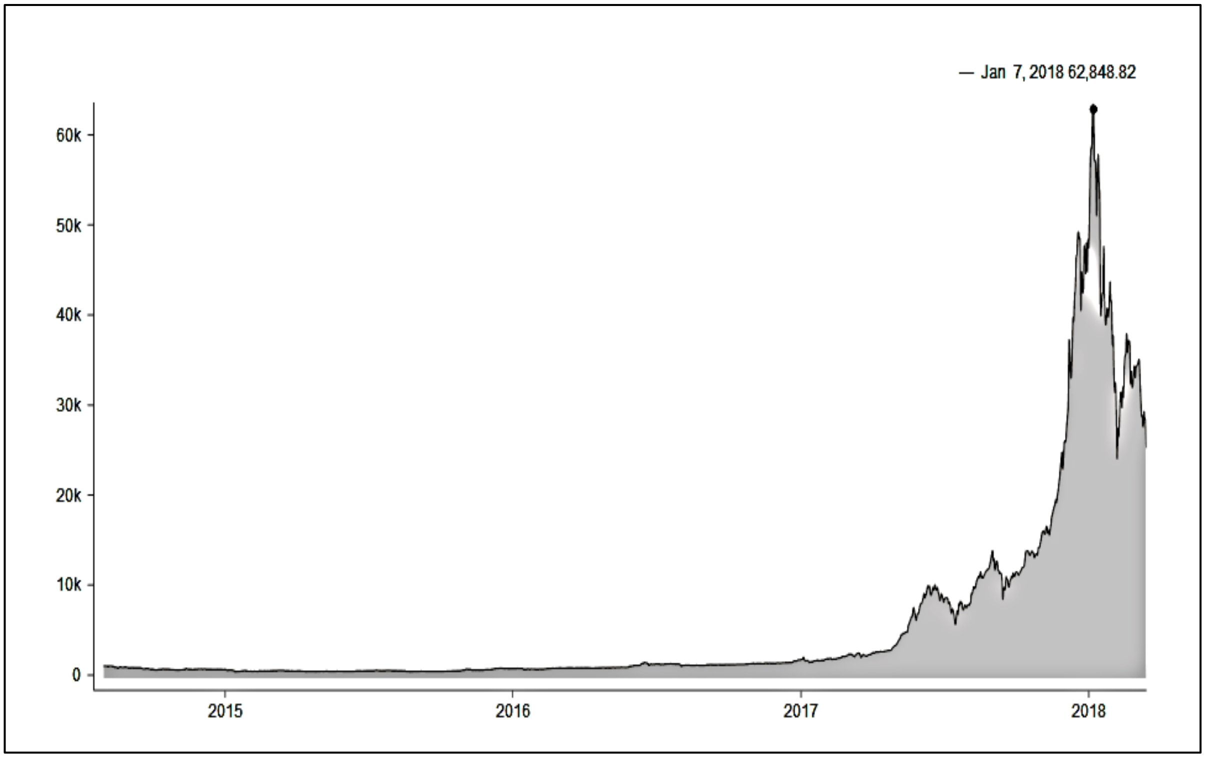


Figure 10. The CRIX Index

The best results of the LPPL models, in terms of minimzing the RMSE, are given in the table below.

Table 10. The best fit for CRIX's LPPL model

	Model 1	Model 3	Model 3
Obs	729	732	727
A	12.393	12.373	12.383
B	-0.627	-0.603	-0.631
C	-0.007	-0.008	0.006
t_c	737	739	736
β	0.344	0.349	0.342
ω	-10361.3	-9103.55	-5960.18
ϕ	67211.29	58656.45	38870.71
Start date	01-Jan-2016	01-Jan-2016	01-Jan-2016
End date	30-Dec-2017	02-Jan-2018	28-Dec-2017
RMSE	0.2406	0.2407	0.2408
AdjRSq	0.9578	0.9587	0.9571
Date of crash	**07-Jan-2018**	**10-Jan-2018**	**06-Jan-2018**

The best fit for the CRIX index was given by the model estimated for the period January 1st 2016 – December 30th 2017, for which the estimated critical time was exactly the date of local maximum, January 7th 2018 (see Figure 11).

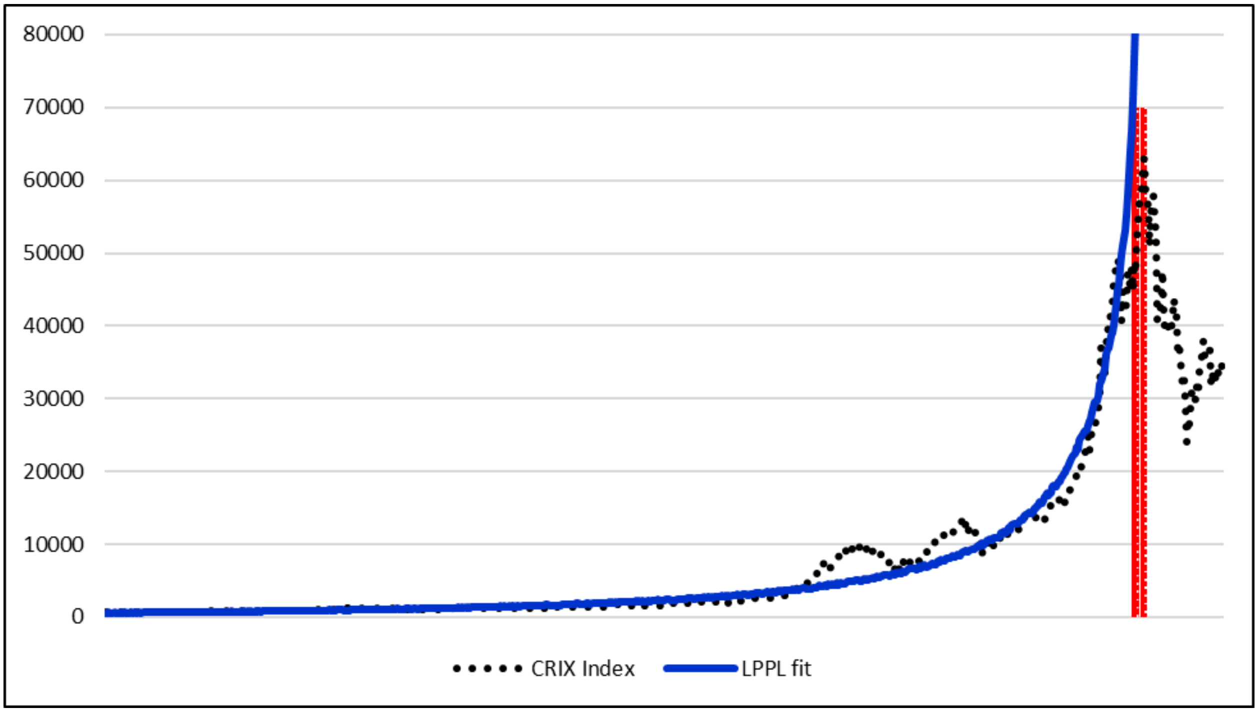


Figure 11. LPPL fit for the CRIX Index (the model with the minimum RMSE)

Conclusions

Our paper deals with a new class of assets, digital currencies or cryptocurrencies, from the point of view of their statistical properties and the herding behaviour.

One of the main findings is that daily cryptocurrencies log-returns exhibits large departures from normality, leaving room for high uncertainty levels, as shown the estimated stability indexes of stable distributions.

Moreover, by analysing Bitcoin related data, we prove, in the medium to long-run, the validity of the Metcalfe's law (the value of a network is proportional to the square of the number of connected users of the system) for the evaluation of cryptocurrencies; however, in the short-run, the validity of the Metcalfe's law for Bitcoin is questionable.

This analysis shows that there is clear pattern of inconsistency over time, questioning the validity of the Metcalfe's law for Bitcoin, when considering different sub-windows of the data.

As there is a strong correlation between the size of the network and the market price of cryptocurrencies, this may be a sign for a mimetic behaviour of investors, who enter the market driven by high expected currency rates, which may lead the market into a super-exponential bubble regime.

LPPL models could be useful in estimating the most probable time of the regime switching for an endogenous cryptocurrency bubble.

By analysing the behaviour of the Bitcoin's price and the CRIX index, we have proven that LPPL models can be a useful tool in recognizing and mapping out the behaviour of a developing bubble.

This is a validation of the predictive power of LPPL models in detecting the imitative behaviour of investors in the cryptocurrencies market, our results being useful both from a theoretical point of view and from a business perspective.

At the same time, this validation of the LPPL models is another argument questioning the universal validity of the Metcalfe's law for Bitcoin, when modelling the relationship between the price and the number of network users.

The econometric method to determine causality is based on cointegration, which requires that the residuals between the linear regression of the log-price onto the log number of users should be stationary (integrated of order 0) – i.e., controlling for the long run Metcalfe law equilibrium, the relative movement of the two series cointegrated series will be stationary.

This stationarity assumption contradicts/excludes the existence of LPPL bubbles as well as different market regimes observed within the price series, studied in part 3 – which are radically non-stationary.

The main conclusion is that the Metcalfe's law may be valid in the long-run, however in the short-run, on various data regimes, its validity is highly debatable.

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Appendix A – Estimating the parameters of an alpha-stable distribution

A.1. Estimating the parameters of an alpha-stable distribution using McCulloch method

McCulloch method (1986) involves the following steps for estimating the parameters of a $S(\alpha,\beta,\gamma,\delta;0)$ random variable:

- estimate α and β, using the quintiles of the empirical distribution (for more details, see Racheva-Iotova, 2010);

- define $v_\alpha = \frac{x_{0.95}-x_{0.05}}{x_{0.75}-x_{0.25}}$ and $v_\beta = \frac{x_{0.95}+x_{0.05}-2x_{0.25}}{x_{0.95}-x_{0.05}}$, where x_p is the *p*-quintile of the empirical distribution, having thus $v_\alpha = \phi_1(\alpha,\beta)$ and $v_\beta = \phi_2(\alpha,\beta)$ or, by inversion, $\alpha = \psi_1(v_a, v_\beta)$ and $\beta = \psi_2(v_a, v_\beta)$.

More, $\alpha = \psi_1(v_a, v_\beta) = \psi_1(v_a, -v_\beta)$ and $\beta = \psi_2(v_a, v_\beta) = -\psi_2(v_a, -v_\beta)$.

The functions $\psi_1(\cdot)$ and $\psi_2(\cdot)$ are tabulated for different values of v_a and v_b, so the estimates of α and β can be obtained using a bi-linear interpolation.

In a quite similar manner, the location parameter δ and the scale parameter γ can be estimated using the corresponding tabulated functions and the previous estimations for α and β.

The code used in this paper for estimating the parameters of an alpha-stable distribution using McCulloch method can be found as the quantlet ***mc_culloch*** on the website www.quantlet.de.

A.2. Estimating parameters of an alpha-stable distribution using the Kogon-Williams method

In order to estimate the parameters of a stable distribution in parameterisation S1, the following algorithm can be applied (following Kogon and Williams, 1998 and Pele, 2014):

Step 1. Use the initial estimates $\alpha_0, \beta_0, \gamma_0, \delta_0$ from McCulloch method and normalize the sample: $x_j \to \frac{x_j - \delta_0}{\gamma_0}$;

Step 2. Estimate the regression model $y_k = b + \alpha_1 w_k + \varepsilon_k$, with $k = 0,..,9$, $y_k = \ln[-\mathrm{Re}[\ln(\hat{\phi}(u_k))]]$, $w_k = \ln|u_k|$, $u_k = 0.1 + 0.1k$, $k = 0,..,9$, and $\hat{\phi}(\cdot)$ is the empirical characteristic function of the normalized sample. If $\hat{b}$ and $\hat{\alpha}_1$ are the estimates of the regression model, then the estimate of the scale parameter is $\hat{\gamma}_1 = \exp(\hat{b}/\hat{\alpha})$.

Step 3. Estimate the regression model $z_k = \delta_{11} + \beta_1 v_k + \eta_k$, with $k = 0,..,9$, $z_k = \mathrm{Im}[\ln(\hat{\phi}(u_k))]$, $w_k = \hat{\gamma}_1 u_k (|\hat{\gamma}_1 u_k|^{\hat{\alpha}_1 - 1} - 1)\tan(\pi\hat{\alpha}_1/2)$, $u_k = 0.1 + 0.1k$, $k = 0,..,9$.

Step 4. The final estimates are the following: $(\alpha_1, \beta_1, \gamma_1, \delta_1) = (\hat{\alpha}_1, \hat{\beta}_1, \hat{\gamma}_1, \hat{\delta}_{11} - \hat{\gamma}_1\hat{\beta}_1 \tan(\pi\hat{\alpha}_1/2))$.

The code used in this paper for estimating the parameters of an alpha-stable distribution using Kogon-Williams method can be found as the quantlet ***stab_reg_kw*** on the website www.quantlet.de.

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