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A parametric social security system with skills heterogeneous agents

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Abstract
The purpose of this study is to explore the effects of exogenous social security system parameters on welfare. The set up is an overlapping generations economy, with skills heterogeneity, which distinguishes consumers between high and low skilled. The low-skilled receive an extra supplement pension. The social security system has three exogenous parameters: the benefits, the contributions, and the funding parameter. The author examines and compares the effects of these three exogenous social security parameters, first under inelastic and then under elastic labor supply, on individuals welfare. He finds that when labor supply is inelastic, the parameters affect differently the welfare of the high and the low-skilled, since for the latter, we must also take into account the indirect effects through the supplement pension provision. When labor supply is elastic, the effects of changes in the social security parameters on welfare are the same for both the high and the low skilled, as in the case of inelastic labor supply.

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Keywords Social security; pensions; PAYGO; funded systems; welfare; skills heterogeneity

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1 Introduction

After the demographic explosion of the post war period and the decreasing birth rates in the subsequent decades, many countries were facing the dilemma of whether they should maintain an unfunded defined-benefit pension structure and social security system or undergo a reform, by introducing other financing instruments and alternatives, including the private sector. Increasing longevity and declining fertility rates are leading the way of an ongoing ageing population, making the topic of viability of the social security systems worldwide, a hotspot of research and extensive discussion. Countries which are economically and socially challenged in multiple ways during the last years, call for immediate viable solutions concerning the sustainability of their social security and pension systems. The necessity of immediate solutions, packed with public concerns over low national saving and excessive sovereign debt, has raised an extensive dialogue over the proposed solutions, focusing among others, on whether the system should be reformed toward a more privatized direction.

The shift from an unfunded PAYGO to a partially or fully funded social security system, or the privatization of pension programs, became a major source of economic, academic and political debate (Imrohoroglu, Imrohoroglu and Joines 1998, Miles, 1998, Gonzalez-Eirasa, Niepelt and Zilcha 2008, Kaganovich and Zilcha 2012). Among the first and most influential works on social security was the seminal work of Samuelson (1958), who raised the issue of Pareto efficiency and the necessary conditions in order to implement a social security reform. The conditions for optimality, the properties of an equilibrium distribution, the market structure, the completeness of the capital markets and the presence of uncertainty and risk have stimulated a significant part of research (Croix, 2002). Barbie, Hagedorn and Kaul (2000) examined the problem of dynamic efficiency and Pareto optimality and analyzed the interaction between risk sharing and capital accumulation in an OLG economy with production and uncertainty. Demange and Laroque (2000) are mostly concerned with the comparison of different social security programs within an OLG framework, under the presence of demographic and productivity shocks. Following the same rational, Krueger and Kubler (2002) examine the problem of intergenerational risk-sharing through a social security system, when the financial markets are incomplete and claim that in general equilibrium models of OLG economies, Pareto improving risk-sharing policies are limited. The introduction of a social security system in a PAYGO form might help the current old, but deteriorate the position of later generations. Matsen

and Thogersen (2004) analyse how different public social security systems may provide risk diversification opportunities to households' lifetime income. The authors construct the PAYGO system as a "quasi-asset" and consider particular sources of income risk, namely wage income risk, which reflects technological and demographic shocks. They replicate the optimal size of a PAYGO system and the optimal division between funded and unfunded pension savings, through a portfolio choice mechanism and show how imperfections in the economy can influence the optimal design of the social security system. Diamond and Geanakoplos (2003) examine the effect of social security diversification into private securities. The authors assume heterogeneity in savings, production, assets and taxes, so as to capture the effects of the partial social security privatization in different income levels. The authors do not clearly advocate for a social security diversification into private assets and thus they do not support without hesitation the departure from a purely PAYGO system. They show that young and future savers will undergo a deterioration by a change in the funding of the social security and the subsequent diversification of the pension funds into private bonds and stocks. On the other hand, the current old savers will improve their position. The marginal social benefit to diversification declines as the level of diversification increases, implying that there is an upper bound to the socially optimal level of the social security system privatization. Abel (2001) allows for fixed costs that prevent the households from directly investing in the stock market and investigates the effects of social security diversification, by assuming income heterogeneity, directly related to the agents' productivity. He argues that there can be a real effect in economic decisions, after the transition to a fully funded, defined-contribution system, opposing to the argument that investing part of the social security is a totally neutral rearrangement of the asset holdings in the form of stocks, with no real economic effects.

In the current study, I explore the effects of exogenous parameters of the social security system on the welfare of individuals, under the provision of a supplement pension. Social security provision often aims at the reduction of income inequality of the elderly. The provision of a supplement pension meets this purpose, since it acts like a safety net for those that cannot save enough for retirement. Production is taken as given in an overlapping generations economy. It is an economy with no uncertainty. The use of an overlapping generations model captures the intergenerational differences and the distributional aspects of the households in the economy, while addressing more accurately the policy effects on different cohorts of the population.

The emphasis is placed on the provision of the system in a flexible way that enables

the State to choose the parameters according to its policy objectives. Two types of individuals are examined: the low-skilled and the high-skilled. The key parameters for the specification of the skills level include educational background, work experience and other natural or acquired abilities that are considered exogenous. People cannot change their skills type. Individuals are assumed to supply their working time inelastically, a hypothesis that is later relaxed. The State chooses among different types of pension schemes and financing methods. A supplementary pension or supplement is provided to the retired low-skilled, acting as a safety net.

The structure of the paper is as follows. In Section two, I present the microeconomic structure of the model. I assume inelastic labor supply and two distinctive skills types. The optimization problem of the consumers is solved. In Section three, I present the parametric social security system. The system has three exogenous parameters, the benefits rate, the contributions rate and the financing parameter that represents the weight of the funded part of the system. I examine the equilibrium conditions of total contributions and benefits, in order to find the equilibrium supplement, as a function of the exogenous parameters. The parameters of the social security system as policy tools are also discussed. In Section four, I examine the effects of changes of the three exogenous social security parameters on the welfare of the low and the high-skilled. In section five, I introduce labor elasticity in the model and in Section six I examine in what ways a change in the social security parameters can affect welfare under this new assumption. In Section seven, I conclude.

2 Consumers with heterogeneous skills

Consumers exhibit skills heterogeneity. Every individual is assumed to be born with certain skills, by which we mean the human capital which directly affects their productivity. The individuals' skills level is taken to be exogenous. I introduce two types of skills: the low-skilled, denoted by L who exhibit lower productivity, and the high-skilled, denoted by H, who exhibit higher productivity. The type of individuals is denoted by the superscript i, with $\mathsf{i} = \mathsf{L}, \mathsf{H}$. Individuals live for two periods. In the first period they are young and work and in the second period they are old and retired.

It is assumed that there is only one perishable good in a closed economy. All individuals, when young, are supplied with one unit of time, devoted to labor, and one unit of time when old, devoted to leisure. Their wage income depends on their skills' marginal product

of labor that remains constant and, by assumption, exogenous. Since the high-skilled are by default more productive, one hour of work by a high-skilled young will produce more than one hour of work by a low-skilled young. Therefore, skill heterogeneity directly affects labor productivity and thus wage income. Call $w_t^i > 0$ the marginal product of labor, for $i = L, H$. Then, w_t^i is assumed constant over time and therefore across generations. It is $w^H > w^L$. In order to satisfy this inequality, productivity is assumed to be captured by parameter ρ^i, with $i = L, H$. The productivity parameter directly affects wage income and corresponds to the low and high-skilled labor supply respectively. For simplicity, I normalize the productivity parameter of the low-skilled to one, i.e. $\rho^L = 1$. Then, it is $\rho^H = \rho > 1$. Then, the low-skilled wage is equal to $w^L = w$ and the high-skilled wage is $w^H = \rho w$.

Every individual is assumed to contribute a constant over time fraction, $\zeta \in (0, 1)$, of their wage income to the social security system when young. Then, ζ is the contribution rate. Call p^i the individual social security contribution. Then, it is $p^i = \zeta w^i$, for $i = L, H$. When old, the individuals receive a social security pension. The high-skilled receive only the basic pension, whereas the low-skilled receive the basic pension, along with a supplement pension, which acts as a safety net for the low-skilled. Call the basic pension $\bar{p}^i$ and s^i the supplement. Then, it is

$$s^i = \begin{cases} s > 0, \text{ if } i = L \\ 0, \text{ if } i = H \end{cases}$$

Call $\bar{b}^i$ the individual social security benefit or total pension. Then, it is $\bar{b}^i = \bar{p}^i + s^i$, for $i = L, H$. The basic pension is assumed to be a constant fraction, $\gamma \in (0, 1)$, of the first period wage income. Then, γ is the benefit rate. The basic pension is then equal to $\bar{p}^i = \gamma w^i$, for $i = L, H$. We aim to find the equilibrium supplement pension. Two cases are distinguished: a) the case in which the supplement is wage proportional and b) the case in which it is a flat amount, independent of the wage.

2.1 Consumer's optimization problem

Labor supply is inelastic and thus leisure is not taken into account by consumers in their utility function. There are no other fixed or inherited endowments or any sort of bequests or other transfers made from the old generation to the young and no further taxes are paid to the state. Young individuals save part of their income in the form of capital, with

a risk-free net real rate of return, r, exogenous and constant over time. Then, the gross rate of return is $R = 1 + r$. Call k^i the savings of the young individuals.
Consumers' preferences are represented by a logarithmic, additive utility function of the form $U^i(c_y^i, c_o^i) = \ln c_y^i + b \ln c_o^i$, where $0 < b < 1$ is the time preference parameter. Then, the problem of the individual is to maximize utility over consumption in both periods of life, i.e. over $\{c_y^i, c_o^i\}$. The supplement is assumed to be earnings-related, i.e. proportional to the wage. Call ξ^i the supplement proportion. Then, it is

$$\xi^i = \begin{cases} \xi \in (0,1), \text{ if } i = L \\ 0, \text{ if } i = H \end{cases} \tag{2.1}$$

Wage income, w^i, is divided between first period consumption, c_y^i, savings, k^i, and the social security contribution, p^i. When old, the same individual consumes c_o^i, which is financed by his total pension, b^i, and the proceeds from his savings, Rk^i. Then, the optimal expressions for consumption in both periods and for savings are respectively

$$c_y^i = \frac{[R(1-\zeta) + \gamma + \xi^i]w^i}{(1+b)R} \tag{2.2}$$

$$c_o^i = b\frac{[R(1-\zeta) + \gamma + \xi^i]w^i}{1+b} \tag{2.3}$$

$$k^i = \frac{[(1-\zeta)bR - \gamma - \xi^i]w^i}{(1+b)R} \tag{2.4}$$

Equations 2.2-2.4 describe the optimal solution of the consumer's maximization problem. In the presence of the supplement pension, it is optimal for the low-skilled to be saving less for retirement than they would without it. Thus, the supplement creates a disincentive for the low-skilled to save when they are young.

3 The social security system

In this section, the social security system is developed. The young and old individuals interact and the flows of the social security system affect both generations. The State is assumed to act as a social planner and has zero consumption or government spending. The role of the government is restricted to the administration of the social security system. We make the following assumptions. The total number of young and old consumers in

period t equals N_t. Denote by $N_{y,t}$ and $N_{o,t}$, the number of young and old respectively in period t. We assume that the population of the old individuals at time t are the young individuals of time $t-1$, i.e. nobody dies before reaching the old age. Moreover, the population of the young born every period grows at a constant growth rate, $n \in (0,1)$. Let $x \in (0,1)$ be a time-invariant proportion that represents the low-skilled young born every period, as a fraction of the overall young population. Then, $1-x$ is the proportion of the high-skilled born in every period. Then, the population of the low and the high-skilled young individuals respectively in period t is $N^L_{y,t} = xN_{y,t}$ and $N^H_{y,t} = (1-x)N_{y,t}$. For every period, it holds that $N_{y,t} = N^L_{y,t} + N^H_{y,t}$ and $N_{o,t} = N^L_{o,t} + N^H_{o,t}$.
The state collects the social security contributions from all young individuals and distributes the benefits to the retirees. There are no individual retirement accounts and the social security contributions are mandatory. Call the sum of total contributions TC. Total contributions represent the financing resources of the social security system in each period. After collecting them, the state decides how they will be distributed. It is assumed that a fraction, $\beta \in [0,1]$, of total contributions is immediately redistributed to finance the benefits of the old of the same period. The rest of total contributions, $1-\beta$, is invested in a social security fund, in order to finance the benefits of the old of the next period. Call "unfunded" the immediately redistributed amount of total contributions and "funded" the remaining amount invested in the fund. Let the unfunded total contributions be denoted by TU and the funded contributions by TF. Then, the unfunded part of total contributions is the PAYGO component of the social security system. Thus, every period, the total benefits of the old are financed by the unfunded part of total contributions and the proceeds of the funded part of total contributions of the previous period. Let total benefits be denoted by TB. Then, for every period, the following relations hold

$$TC_t = TU_t + TF_t \tag{3.1}$$

$$TB_t = TU_t + RTF_{t-1} \tag{3.2}$$

where

$$TU_t = \beta TC_t \tag{3.3}$$

$$TF_{t-1} = (1-\beta)TC_{t-1} \tag{3.4}$$

Let the total basic pensions be denoted by $\bar{P}$ and total subsidies by S. Then, it is

$$TB_t = \bar{P}_t + S_t \tag{3.5}$$

and

$$TU_t + RTF_{t-1} = \bar{P}_t + S_t, \forall t \tag{3.6}$$

or equally,

$$\bar{P}_t + S_t = \beta TC_t + R(1-\beta)TC_{t-1} \tag{3.7}$$

Equation 3.7 is the basic financing constraint of the social security system. It shows that the total basic pensions and the total supplement pensions of any period must be financed by the PAYGO part of total contributions of the same period, plus the proceeds from the funded part of total contributions of the previous period. Summing over all young of period t, we have the following expression for total contributions

$$TC_t = (1+n)N_{y,t-1}[x + (1-x)\rho]\zeta w \tag{3.8}$$

with

$$TC_{t-1} = N_{y,t-1}[x + (1-x)\rho]\zeta w \tag{3.9}$$

Respectively, summing over all old individuals of period t, we have the following expression for total basic pensions

$$\bar{P}_t = N_{y,t-1}[x + (1-x)\rho]\gamma w \tag{3.10}$$

Finally, summing over only the low-skilled old of period t, we have the following expression for total subsidies

$$S = N_{y,t-1} x s \tag{3.11}$$

We derive the equilibrium expression for the supplement ratio, ξ, which is

$$\xi = \frac{[x + (1-x)\rho]}{x}\{[\beta(1+n) + R(1-\beta)]\zeta - \gamma\} \tag{3.12}$$

with the supplement being equal to $s = \xi w$.
The equilibrium supplement is then determined by the social security parameters $\{\gamma, \zeta, \beta\}$ and by the exogenous parameters $\{x, \rho, w, r\}$.

Since the supplement is positive, it follows that

$$\gamma < [\beta(1+n) + R(1-\beta)]\zeta \tag{3.13}$$

The supplement, as a function of the exogenous social security parameters $\{\beta, \gamma, \zeta\}$ is a function of the exogenous financing parameter β, such that $\frac{\partial s}{\partial \beta} = \frac{(n-r)\zeta w[x+(1-x)\rho]}{x}$, with a) $\frac{\partial s}{\partial \beta} = 0 \iff n = r$, b) $\frac{\partial s}{\partial \beta} > 0 \iff n > r$ and c) $\frac{\partial s}{\partial \beta} < 0 \iff n < r$. When the interest rate and the population growth rate are equal, the funding method of the system does not affect the supplement. However, it depends on how much can be raised through funding or through population growth. It follows that the interest rate and the population growth rate act complementary with respect to the funding of the system, expressed by the parameter β. The unfunded part of the system is expressed by the term $\beta(1+n)$ and the rest, the funded part is expressed by the complementary part of the gross interest rate, $(1-\beta)(1+r)$. This is why an increase in the financing parameter β has a neutral effect on the supplement, when the population growth rate and the interest rate are equal. In this case the funded part, which is actually savings at rate r and the unfunded part, i.e. funding from a population that grows at rate n, have the same effect on the supplement, when n and r are equal. If the population growth rate is higher than the interest rate, then, in order to increase the supplement, we should have a higher unfunded part of the system, i.e. β should be higher. In the opposite case, when the population growth rate is lower than the interest rate, then, in order to increase the supplement, it is better to have a higher funded part, i.e. $1-\beta$ should be higher.
Moreover, the supplement is a negative function of the exogenous benefits parameter γ. The higher the benefits of the basic pension distributed to all, the lower will be the supplement given to the low-skilled. The supplement is also a positive function of the exogenous contributions parameter ζ, implying that the higher the benefits of the basic pension, the lower will be the supplement for the low-skilled.

3.1 The parameters of the social security system as policy tools

The values of the social security policy parameters represent different pension schemes and reflect the state's adopted policy. For example, if we have no funded part, then the social security system is pure PAYGO. Below, we present the definitions for the cases in which the social security system is "Unfunded" or "Pure PAYGO", "Fully funded" and "Partially

funded", "Universal" or "Means-tested" and "Defined-benefit" or "Defined-contribution".

DEFINITION 3.1 *The social security system is pure PAYGO (or unfunded/ redistributive), when all contributions are redistributed to the old of the same period. It is the case in which the funding parameter equals the unity. In the opposite case, when all contributions are invested and distributed to the old of the next period, it is* $\beta = 0$ *and the social security system is fully funded. For* $\beta \in (0, 1)$*, as it is in our model, the system is partially funded.*

DEFINITION 3.2 *The social security system is "Universal", when there is no supplement pension. In this case the State provides only the basic pension to all individuals. The social security system is "means-tested" for* $s > 0$*, in the sense that the provision of the supplement consists an extra payment for the low-skilled, due to their limited income.*

DEFINITION 3.3 *The social security system is "Defined-benefit", when the benefits parameter,* γ *is pre-defined by the State and the contributions parameter,* ζ *is based on* γ*. Contributions are adjusted to provide a certain level of benefits. On the contrary, the social security system is "Defined-contribution", when the contributions parameter,* ζ *is pre-defined by the State and the benefits parameter,* γ *is determined by* ζ*. Benefits depend on contributions in this case.*

The flows of the social security system are depicted diagrammatically below, in Figure 1. In the next section, we derive the conditions for individuals' welfare optimization, after taking into account the equilibrium supplement. We show how a change in the social security parameters, $\{\beta, \zeta, \gamma\}$, can affect the utilities of the low and the high-skilled.

4 Welfare optimization under inelastic labor supply

We next derive the optimal welfare of the low and the high-skilled. We use the equilibrium supplement as defined by equation 3.12. The individual's intertemporal utility function becomes

$$U^i(c_y^i, c_o^i) = (1 + b) \ln[R(1 - \zeta) + \gamma + \xi^i] + \Delta^i \tag{4.1}$$

where $\Delta^i = (1 + b) \ln w^i - \ln[(1 + b)R] + b \ln(\frac{b}{1+b})$ consists of parameters other than the social security parameters and has been isolated, for the comparative statics we want to perform are with respect to the social security parameters.

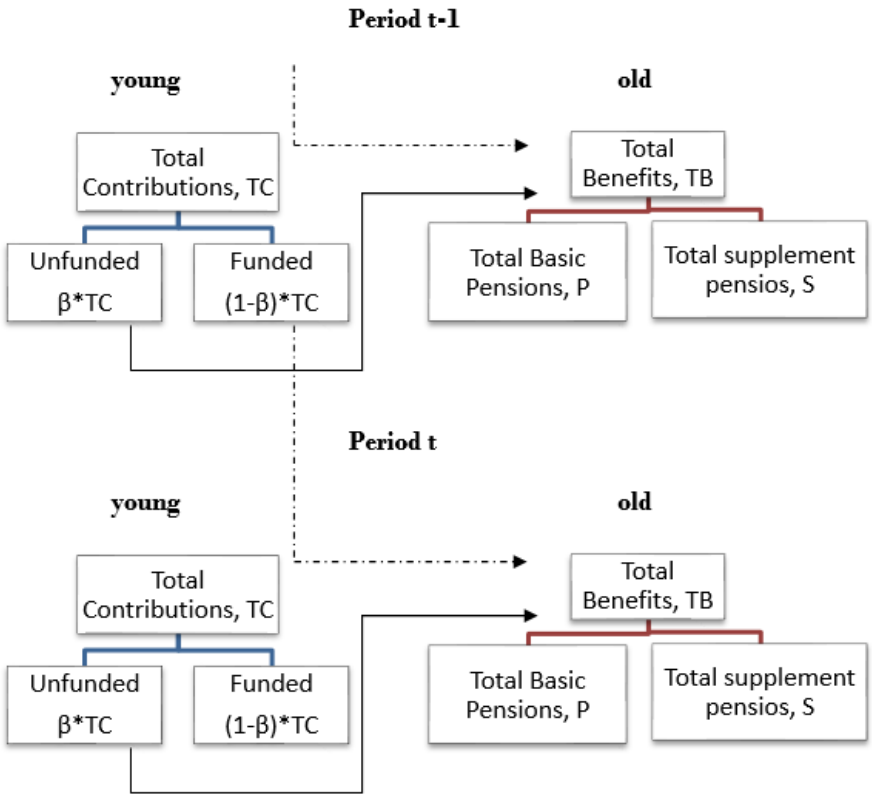

Figure 1: Flows of the social security system

For the high and the low-skilled, it is respectively

$$U^H = (1+b)\ln[R(1-\zeta)+\gamma] + \Delta^H \tag{4.2}$$

$$U^L = (1+b)\ln\{[R(1-\zeta)+\gamma]x + [x+(1-x)\rho]\{[\beta(1+n)+R(1-\beta)]\zeta\} - \gamma\} + (\Delta^L)' \tag{4.3}$$

with $\Delta^H = (1+b)\ln(w\rho) - \ln[(1+b)R] + b\ln(\frac{b}{1+b})$ and $\Delta^L = (1+b)\ln w - \ln[(1+b)R] + b\ln(\frac{b}{1+b}) - (1+b)\ln(\frac{1}{x})$. Equations 4.2-4.3 are the optimal utilities of the high and the low-skilled respectively, as functions of the exogenous parameters $\{b, r, w, x, \rho, n\gamma, \zeta, \beta\}$.

4.1 Changes in the funding parameter

The exogenous funding parameter β does not affect the welfare of the high-skilled. The way the social security system is distributing its contributions, i.e either it is PAYGO or funded, has a neutral effect on the welfare of the high-skilled. This is because, since there are no other distortions or uncertainties in the economy, a redistributive social security

policy for those receiving only the basic pension will be equivalent to a funded social security policy.
However, parameter β affects non monotonically the utility of the low-skilled. We have,

$$\frac{\partial U^L}{\partial \beta} = \frac{\zeta(1+b)[x+(1-x)\rho](n-r)}{[R(1-\zeta)+\gamma]x+[x+(1-x)\rho]\{[\beta(1+n)+R(1-\beta)]\zeta-\gamma\}} \tag{4.4}$$

Both the numerator and the denominator of equation 4.4 are positive quantities. Therefore, the optimal welfare of the low-skilled is not affected by parameter β, when the net rate of return on capital is equal to the population growth rate. On the other hand, their welfare is positively affected by the funding parameter when the net rate of return on capital is lower than the population growth rate and negatively when the net rate of return on capital is higher than the population growth rate. The effect of the funding parameter of the social security system, β, on the welfare of the low-skilled when the supplement is wage related is non monotonic and its effects passes through the channel of the supplement.

4.2 Changes in the benefits parameter

The exogenous parameter γ has opposite effects on the utilities of the high and the low-skilled. It positively affects the utility of the high-skilled, since

$$\frac{\partial U^H}{\partial \gamma} = \frac{1+b}{R(1-\zeta)+\gamma}$$

The positive effect of γ on the welfare of the high-skilled passes through the channel of the basic pension. The benefits parameter positively affects the basic pension, therefore, when increased, the basic pension of the high-skilled is increased.
However, the parameter γ negatively affects the utility of the low-skilled

$$\frac{\partial U^L}{\partial \gamma} = \frac{-(1+b)(1-x)\rho}{[R(1-\zeta)+\gamma]x+[x+(1-x)\rho]\{[\beta(1+n)+R(1-\beta)]\zeta-\gamma\}} \tag{4.5}$$

Parameter γ, although it has a positive effect on the basic pension, it negatively affects the supplement, creating a substitution effect between the basic pension and the supplement pension. This is because the higher the basic pension, the less will be left over to be distributed as supplement pensions. The magnitude of the negative effect of a change of

γ on the supplement is greater than its positive effect on the basic pension. Thus, the overall result of a change in γ on the welfare of the low-skilled is negative. Therefore, when increased, the basic pension of the low-skilled is increased, but the supplement decreased and the total effect on their welfare is negative.

4.3 Changes in the contributions parameter

The contributions parameter ζ has a negative effect on the utility of the high-skilled, by increasing their pension contribution. It is

$$\frac{\partial U^H}{\partial \zeta} = \frac{-R(1+b)}{R(1-\zeta)+\gamma} \tag{4.6}$$

However, for the low-skilled, it is

$$\frac{\partial U^L}{\partial \zeta} = \frac{(1+b)\{[\beta(1+n)+R(1-\beta)][x+(1-x)\rho]-xR\}}{[R(1-\zeta)+\gamma]x+[x+(1-x)\rho]\{[\beta(1+n)+R(1-\beta)]\zeta-\gamma\}} \tag{4.7}$$

The effect of ζ on the utility of the low-skilled depends on the sign of the term $\{[\beta(1+n)+R(1-\beta)][x+(1-x)\rho]-xR\}$. Ceteris paribus, an increase of ζ has a non monotonic effect on the utility of the low-skilled. Although it is the contributions parameter, ζ has an ambiguous effect on the welfare of low-skilled, because it also positively affects their supplement. Thus, although ζ can decrease the welfare of the low-skilled, since it increases their pension contribution, it also increases their welfare, through the supplement. The net effect depends on the sign of the term $[\beta(1+n)+R(1-\beta)][x+(1-x)\rho]-xR$.
In the case the supplement is a flat amount, the individual's optimal welfare are exactly the same as in the case the supplement is wage proportional when labor supply is inelastic. This holds for both the high and the low skilled. Therefore, the same results apply in both cases. We next examine if and how the effects of a change in the social security parameters are differentiated for both the welfare of the low and the high skilled, when labor supply is elastic, for both cases the supplement is a flat amount or wage related.

5 Introduction of elastic labor supply

Labor supply is now considered elastic and leisure becomes a choice variable in the utility function. Labor supply is measured by hours of work. For simplicity, we normalize total

time of an individual in the first period to unity. Thus, individuals are supplied with one unit of time when young, which they divide between labor and leisure, and one unit of time when old, which are assumed to devote to leisure. All other basic characteristics remain the same.

Let l^i be the labor supplied by skills type i, where $i = L, H$, and e^i the time devoted to leisure. Then, in every period, it is $l^i + e^i = 1$. Call W^i the *elastic labor income*. Then, W^i is equal to $W^i = l^i w^i$, $p^i = \zeta W^i$ is the contribution of an individual to the social security system, $\bar{p}^i = \gamma W^i$ is the basic pension and $\bar{b}^i = \gamma W^i + s^i$ is the total benefit.

5.1 Consumer's optimization problem when labor supply is elastic

Elastic labor income, W^i, is divided between first period consumption, c_y^i, savings, k^i, and the social security contribution, p^i. Consumers' preferences are represented by a time separable additive, logarithmic utility function of the form $U^i(c_y^i, c_o^i, e^i) = \ln c_y^i + \ln e^i + b \ln c_o^i$, with $0 < b < 1$. The problem of the individual is to maximize utility over consumption in both periods and over leisure in the first period.

The supplement is again proportional to the wage, in ratio ξ^i. For $p^i = \zeta W^i$, $\bar{p}^i = \gamma W^i$, $s^i = \xi^i W^i$ and $W^i = (1 - e^i) w^i$, the optimal expressions for consumption in both periods of life, leisure, labor supply and savings are

$$c_y^i = \frac{[R(1-\zeta) + \gamma + \xi^i] w^i}{(2+b)R} \tag{5.1}$$

$$c_o^i = b \frac{[R(1-\zeta) + \gamma + \xi^i] w^i}{2+b} \tag{5.2}$$

$$e^i = \frac{1}{2+b} \tag{5.3}$$

$$l^i = \frac{1+b}{2+b} \tag{5.4}$$

$$k^i = \frac{[(1-\zeta)bR - \gamma - \xi^i] w^i}{(2+b)R} \tag{5.5}$$

Equations 5.1-5.5 describe the optimal solutions of the consumer's maximization problem. The optimal labor supply and leisure of the low and the high-skilled are the same when the supplement is a proportion of labor income. Leisure is thus a constant for both the

low and the high-skilled and depends on the time preference parameter, b. Therefore, the utility derived from leisure is not wage affected or equally, the wage does not affect labor decisions for both the low and the high skilled and thus their leisure choices. The negative relation between ξ and the optimal private savings of the low-skilled still holds. In the presence of a proportional to income supplement pension, the low-skilled find it optimal to save less for retirement than they would without it. The supplement creates a disincentive for the low-skilled to save when they are young, regardless of whether they can adjust labor supply or not.

5.2 Social security system when labor supply is elastic

We distinguish total labor supply, which is the sum of time units of the low and high-skilled individuals devoted to labor, from *total effective labor supply*. The latter is the labor input measured in effective units of time, in the sense that one unit of labor of type i enhances individual labor supply by its productivity, ρ^i, with $\rho^L = 1$ and $\rho^H = \rho > 1$. Let L_t^i be the total labor supply per type of skills when individuals are young. Then, L_t^i equals $L_t^i = \sum_{i=1}^{N_{y,t}^i} l^i, \forall i = L, H$. Call $L_{e,t}^i$ is the total effective labor supply of type i. It is

$$L_{e,t}^i = \sum_{i=1}^{N_{y,t}^i} \rho^i l^i \tag{5.6}$$

Total effective labor supply, $L_{e,t}$, is then equal to

$$L_{e,t} = N_{y,t}[\chi l^L + (1-\chi)\rho l^H] \tag{5.7}$$

The basic social security identity of the inelastic labor supply case remains the same. It is

$$\bar{P}_t + S_t = \beta TC_t + R(1-\beta)TC_{t-1} \tag{5.8}$$

with total contributions equal to $TC_t = (1+n)N_{y,t-1}[\chi l^H + (1-\chi)l^L]\zeta w$, total basic pensions $\bar{P}_t = N_{y,t-1}[\chi l^H + (1-\chi)l^L]\gamma w$ and total supplement pensions $S = N_{y,t-1}\chi\xi w$. Observe that all terms that involved total labor supply in the case of inelastic labor supply, are now expressed in terms of total effective labor supply.

Then, from equation 5.8, we have the following expression for the equilibrium supplement

pension

$$\xi = \frac{L_{e,t-1}}{\chi l^{L} N_{y,t-1}}\{[\beta(1+n) + R(1-\beta)]\zeta - \gamma\} \tag{5.9}$$

Equation 5.9 for the equilibrium supplement ratio in the elastic labor supply case differs from that of the inelastic labor supply in that now, the supplement is expressed in terms of the total effective labor supply. The equilibrium supplement pension is determined by social security parameters $\{\gamma, \zeta, \beta\}$, but also by labor supply.

6 Welfare optimization under elastic labor supply

We next derive the optimal welfare of the high and the low-skilled and examine their welfare implications, from changes in the social security parameters.

6.1 High-skilled welfare optimization

The high-skilled intertemporal utility function, after replacing for the optimal values of consumption and leisure becomes

$$U^{H} = (1+b)\ln[R(1-\zeta) + \gamma] + E^{H} \tag{6.1}$$

where

$$E^{H} = (1+b)\ln(w\rho) - \ln[R(2+b)^{2}] + b\ln(\frac{b}{2+b}) \tag{6.2}$$

Apart from the term E^{H}, which incorporates all other exogenous parameters, except from the social security parameters, the optimal welfare expression of the high skilled remains the same under elastic labor supply. Therefore, the following remark can be made.

REMARK 6.1 *Changes in the social security parameters, $\{\gamma, \zeta, \beta\}$, under elastic labor supply, will have the same effect on the welfare of the high-skilled as under inelastic labor supply.*

From this we can infer that policy changes of the exogenous social security parameters, $\{\beta, \gamma, \zeta\}$, will not differently affect the welfare of the high skilled if labor supply is elastic. This is because the effect of a change on the social security parameters in the case of the high skilled goes through the consumption channel only and not through leisure.

6.2 Low-skilled welfare optimization

The low-skilled intertemporal utility function under the equilibrium supplement is

$$U^L = (1+b)\ln\{[R(1-\zeta)+\gamma]x+[x+(1-x)\rho]\{[\beta(1+n)+R(1-\beta)]\zeta\}-\gamma\}+(E^L)' \quad (6.3)$$

where

$$E^L = (1+b)\ln w - \ln[(1+b)^2R] + b\ln(\frac{b}{2+b}) - (1+b)\ln(\frac{1}{x}) \quad (6.4)$$

Observe that, when the supplement is proportional to the wage, we obtain the same functional form for the maximized welfare of the low-skilled, as we did in the inelastic labor supply case. The difference between elastic and inelastic labor supply case lies in the term E^L, given by equation 6.4, which slightly differs when compared to the respective term in inelastic labor supply case, Δ^L. Then, the following Remark can be made.

REMARK 6.2 *Changes in the exogenous social security parameters, $\{\gamma, \zeta, \beta\}$, will have the same effect on the utility of the low-skilled, under inelastic or elastic labor supply, when the supplement is defined as a proportion of the wage. Then, labor elasticity does not change the effects of the social security parameters on their welfare. Policy changes through the social security parameters cannot be undone by low-skilled consumers through changes in labor supply.*

7 Concluding remarks

This paper develops a model of parametric social security system in an overlapping generations economy, where individuals are distinguished according to their skills. The social security parameters represent different pension schemes and reflect the state's adopted policy. Pensions are financed by the PAYGO part of total contributions and the proceeds from the funded part of the invested total contributions of the previous period.
Under inelastic labor supply and a wage related supplement, it is shown that the funded parameter does not affect the welfare of the high-skilled. However, it affects non monotonically the utility of the low-skilled. In particular, we derive a neutrality result for the welfare of the low-skilled as well, when the net rate of return on capital is equal to the population growth rate. Therefore, the way the social security system is distributing the contributions, either by redistributing them in the same period (PAYGO/unfunded), or by investing them and distribute them in the next period (funded), affects the supplement

and thus it is non monotonic for the welfare of the low-skilled. The parameter that reflects the benefits rate has an opposite effect on the utilities of the high and the low-skilled. It positively affects the utility of the high-skilled and negatively the low-skilled. The contributions parameter has a negative effect on the utility of the high-skilled, but affects non monotonically the low-skilled.

The different effects of the exogenous social security parameters on the welfare of the high and the low skilled is explained by the fact that, for the latter, the parameters also affect the equilibrium supplement and thus indirectly their utility. When this secondary effect prevails, compared to the direct effect, then it can alter the results compared to the effects on the welfare of the high-skilled. This secondary effect due to the supplement is not present in the welfare of the high-skilled.

When labor supply is elastic, the effect of a change in any of the three exogenous social security parameters is the same for both the high-skilled and the low-skilled. Thus labor elasticity does not affect the welfare of neither the high nor the low skilled after a policy change in one of the three social security parameters.

The outcome in the various examined sub cases suggests that the social security policy decisions are strongly related to what the policy makers aim to achieve. The objective of the policy makers determines the outcome and the specifications of the structure of the social security system. These specifications must take into account skills heterogeneity which usually implies different working capacities, different income and retirement living conditions. Heterogeneity in skills thus also implies heterogeneous effects of the basic social security parameters on the various population cohorts. Moreover, the rate at which a population grows is significant in the design of the social security system and in its funding dimension. In a simple economy like this, with no uncertainty, the funding of the social security system affects more the people who are less equipped. When the population growth rate is low, as it is usually in western countries, and especially if it is lower when compared to the interest rate, then the chances of hurting more the less equipped people by remaining in an unfunded system are increasing.

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