

## A dynamic theory of economics: What are the market forces?

*Alia Asha Dannenberg, Matti Estola, and Anna Dannenberg*

### Abstract

The main weakness in the neoclassical theory of economics is its static nature. By a static model one cannot explain the observed time paths of economic quantities, like the flows of production of firms, the flows of consumption of consumers, and the prices of goods. The error in the neoclassical framework is that economic units are assumed to be in their optimum state and thus not willing to change their behaviour. Therefore, in neoclassical models a static equilibrium prevails. In this paper, the authors change this assumption so that economic units are assumed to be willing to improve their current state that may not be the optimal one. In this way, one can explain economic dynamics where every economic unit is changing its behaviour towards improving its welfare. The authors define the economic forces acting upon the production of firms, the consumption of consumers, and the prices of goods are changing in time. They show that in this dynamic system, business cycles and bankruptcies of firms emerge in a natural way like in the real world.

(Published in Special Issue [Agent-based modelling and complexity economics](#))

**JEL** D11 D21 C63 C02

**Keywords** Econophysics; Newtonian economics; dynamic economics

### Authors

*Alia Asha Dannenberg*, ✉ Lukema Oy, Joensuu, Finland, [alia.dannenberg@lukema.eu](mailto:alia.dannenberg@lukema.eu)

*Matti Estola*, University of Eastern Finland

*Anna Dannenberg*, University of Helsinki

*Alia Asha Dannenberg thanks the Jenny and Antti Wihuri Foundation and Emil Aaltonen Foundation for the research grants that have enabled her work*

**Citation** Alia Asha Dannenberg, Matti Estola, and Anna Dannenberg (2017). A dynamic theory of economics: What are the market forces? Economics Discussion Papers, No 2017-110, Kiel Institute for the World Economy. <http://www.economics-ejournal.org/economics/discussionpapers/2017-110>

## 1 Introduction

The fundamental weakness in the current macroeconomic theory is the absence of a consistent micro level foundation. Here we present a new microeconomic theory where the macro state of a system is the aggregate of states of the micro units as proposed by Lux and Westerhoff (2009) in the spirit of classical analytical mechanics. Throughout our framework, we define and apply a consistent unit system for economics presented by De Jong (1967), comparable to that of physics.

As e.g. Lux and Westerhoff (2009) state, the neoclassical economic theory is widely known of its inability to model the behaviour of real economic phenomena. The most fundamental shortcoming in the prevailing neo-classical framework, acknowledged e.g. by Mas-Colell et al. (1995), is that it is essentially static in nature, whereas real economic systems are dynamic. There have been attempts to dynamise the neoclassical theory for both consumers by e.g. Ramsey (1928); Cass (1965); Koopmans (1965) and for firms by Evans (1924), but these theories are, according to e.g. Estola (2013), inconsistent with the static neo-classical framework. Within the neoclassical framework, economic units are assumed to be in their optimum state, resulting in that the equations do not cover situations outside the optimum.

This article introduces a dynamic theory of economics, compatible with real economic phenomena. It can be considered as a dynamic extension to the neo-classical framework, including the latter one as a special case with a static setup. Our theory is able to deal with observed dynamic economic phenomena also outside the optimum. Therefore it can be used to simulate economic systems in a realistic way, such as economic crises that the neo-classical framework is unable to forecast or handle according to Lux and Westerhoff (2009). Our theory has been tested with extensive simulations and two empirical evaluations, and it has been found consistent with real data, as shown in Estola and Dannenberg (2012); Estola (2015).

## 2 Firm and production

In building our theory, let us begin from the basics. Let the profit  $\Pi$  with unit  $\text{€}/\text{time}$  of a multi-product firm under perfect competition be

$$\Pi = \sum_k^K P_k \dot{Q}_k - C(\dot{\mathbf{Q}}), k = 1, \dots, K, \quad (1)$$

where  $\dot{Q}_k$  is the flow of production of good  $k$  with unit *piece<sub>k</sub>/time*,  $P_k$  the price of product  $k$  with unit *€/piece<sub>k</sub>*, and  $C(\dot{\mathbf{Q}})$  with unit *€/time* the costs at production flow vector  $\dot{\mathbf{Q}}$ . Now, one can solve the optimum conditions as

$$\frac{\partial \Pi}{\partial \dot{Q}_k} = P_k - \frac{\partial C(\dot{\mathbf{Q}})}{\partial \dot{Q}_k} = 0, \quad (2)$$

where  $\partial C(\dot{\mathbf{Q}})/\partial \dot{Q}_k$  with unit *€/piece<sub>k</sub>* denotes marginal costs.

This is the neoclassical optimum that corresponds to the “zero-force” situation in Newtonian mechanics: a body does not change its state of motion unless there is a force acting upon it. If the flows of production of a firm yield the maximum profit, the firm does not want to change them (compare with Newton’s first law). But what if  $P_k - \partial C(\dot{\mathbf{Q}})/\partial \dot{Q}_k \neq 0$ ? The firm is not in its optimum and, according to Fisher (1983); Varian (2006), should either increase or decrease its flow of production of good  $k$  to gain higher profit. For a profit-seeking firm, there is a “force” driving it to adjust its flow of production. In the neoclassical theory, these economic forces have been acknowledged but never defined exactly which, according to Mirowski (1989), has led to the static framework.

**Theorem 1:** Quantity  $F_{\dot{Q}_k} = P_k - \partial C(\dot{\mathbf{Q}})/\partial \dot{Q}_k$  is the economic force acting upon the production of good  $k$  of the firm. The unit of economic force is the same as the unit of price, i.e., *€/piece<sub>k</sub>*.

**Theorem 2:**  $F_{\dot{Q}_k} = m_{\dot{Q}_k} \ddot{Q}_k$ . Economic force  $F_{\dot{Q}_k}$  causes either positive or negative acceleration on production.

This is similar to Newton’s second law.  $Q_k$  is the accumulated production of good  $k$ ,  $\ddot{Q}_k$  the acceleration of accumulated production, and  $m_{\dot{Q}_k}$  the inertia of production<sup>1</sup> (it takes time to speed up or wind down production). The unit of inertia is *€×time<sup>2</sup>/piece<sub>k</sub><sup>2</sup>*.

The work done by the economic force acting upon production can be calculated like the work of physical force, see Estola and Dannenberg (2016):

$$\begin{aligned} \Delta W &= \int \mathbf{F}_{\dot{\mathbf{Q}}} \cdot d\mathbf{Q} = \sum_k m_{\dot{Q}_k} \int \ddot{Q}_k dQ_k \\ &= \sum_k m_{\dot{Q}_k} \int \frac{d\dot{Q}_k}{dt} dQ_k = \sum_k m_{\dot{Q}_k} \int \frac{dQ_k}{dt} d\dot{Q}_k \\ &= \sum_k \frac{1}{2} m_{\dot{Q}_k} \dot{Q}_{k,\text{final}}^2 - \sum_k \frac{1}{2} m_{\dot{Q}_k} \dot{Q}_{k,\text{initial}}^2. \end{aligned} \quad (3)$$

<sup>1</sup> We are assuming that inertias of production and consumption are time-independent. Time-dependent masses are possible, though would complicate the equations.

The equation for kinetic terms suggests that kinetic energy of production exists in the production process. The unit of economic kinetic energy and work is €, see e.g., Dragulescu and Yakovenko (2000); Kusmartsev (2011). On the other hand, work  $\Delta W_k$  is

$$\Delta W_k = \int_{Q_{k,i}}^{Q_{k,f}} F_{\dot{Q}_k} dQ_k = \int_{t_i}^{t_f} \left( P_k \dot{Q}_k - \dot{Q}_k \frac{\partial C}{\partial \dot{Q}_k} \right) dt = \int_{t_i}^{t_f} \dot{Q}_k \left( P_k - \frac{\partial C}{\partial \dot{Q}_k} \right) dt, (4)$$

where  $\int_{t_i}^{t_f} P_k \dot{Q}_k dt$  are the revenues from sales, and the term  $\int_{t_i}^{t_f} \dot{Q}_k \partial C / \partial \dot{Q}_k dt$  represents costs within the time interval  $\Delta t = t_f - t_i$  if unit cost equals marginal cost. If  $P_k > \partial C / \partial \dot{Q}_k$ , force  $F_{\dot{Q}_k}$  does work to change the kinetic state of production, increasing the flow of production (note that  $\dot{Q}_k \geq 0$ ). If  $P_k < \partial C / \partial \dot{Q}_k$ , production does work against the force  $F_{\dot{Q}_k}$ .

### 3 Consumer and consumption

For a consumer, the corresponding theorems are the following:

**Theorem 3:** There exists a force acting upon the consumption of a consumer of good  $k$ :  $F_{\dot{X}_k} = \partial H(\dot{\mathbf{X}}) / \partial \dot{X}_k - P_k$ .

This is similar to Theorem 1.  $H(\dot{\mathbf{X}})$  with unit €/time is the willingness to pay of a consumer for the consumption flow  $\dot{\mathbf{X}}$ , and  $\partial H(\dot{\mathbf{X}}) / \partial \dot{X}_k$  is the marginal willingness to pay that corresponds to the marginal costs of a firm, see Dannenberg and Estola (2017). The consumer surplus (measured in unit €/time, similarly to the profit of a firm) is  $\phi = H(\dot{\mathbf{X}}) - \sum_k^K P_k \dot{X}_k$ , and its optimum corresponds to the zero force acting upon consumption, i.e.,  $F_{\dot{X}_k} = 0 \Leftrightarrow \partial H(\dot{\mathbf{X}}) / \partial \dot{X}_k = P_k$ . Consumers' marginal willingness to pay for a good can be measured, e.g., by making consumer surveys, as proposed by Cameron and James (1987).

**Theorem 4:** The force acting upon consumption causes either positive or negative acceleration in consumption, i.e.,  $F_{\dot{X}_k} = m_{\dot{X}_k} \ddot{X}_k$ .

This is similar to Theorem 2. The consumption of good  $k$  has kinetic energy  $\frac{1}{2} m_{\dot{X}_k} \dot{X}_k^2$ . Since according to e.g., Dannenberg and Estola (2017) the theories of a firm and a consumer are symmetrical and have the same mathematical form, similar work effects for force  $F_{\dot{X}_k}$  are obtained for consumer dynamics.

### 4 Market mechanism as a spring system

Let us next consider a simple physical problem: two masses, say  $m_X$  and  $m_Q$ , are attached to each other with a spring with spring constant  $k$ , length  $L$  and rest length



**Figure 1:** A simple spring system.

zero. The masses have initial velocities  $\dot{X}_0$  and  $\dot{Q}_0$  towards the direction of the symmetry axis. Moreover,  $m_X$  is drawn by force  $F_X$  and there is dragging force  $-F_Q$  affecting  $m_Q$ . Figure 1 illustrates the setup. The forces are, according to Hooke's law,

$$m_Q \ddot{Q} = F_k - F_Q \quad (5)$$

$$m_X \ddot{X} = F_X - F_k \quad (6)$$

$$F_k = kL = k(X - Q). \quad (7)$$

Now one can calculate the time evolution of the system.

As Theorems 1 and 3 state, the form of economic forces is similar to the spring force. Most notably, price  $P$  resembles the harmonic force  $F_k$  that in economics connects production with consumption. We observe the following similarities: (i)  $F_k \sim P$ . (ii)  $F_Q \sim \partial C / \partial \dot{Q}$ , and  $F_X \sim \partial H / \partial \dot{X}$ . Marginal costs and marginal willingness to pay are external forces. (iii) The time derivative of Eq. (7) yields the law of demand and supply  $\dot{P} = k(\dot{X} - \dot{Q})$  that relates price changes to excess demand or supply, as shown e.g. by Samuelson (1941, 1942). Forces  $\partial C / \partial \dot{Q}$  and  $\partial H / \partial \dot{X}$  depend on the corresponding velocities. (iv) Natural constraints are  $\dot{X} \geq 0$ ,  $\dot{Q} \geq 0$ , and  $P \geq 0$ , because it is impossible to produce or consume negative amounts of goods or pay negative prices.

**Theorem 5:** Price  $P$  is a harmonic force that connects the flows of production and consumption.

Recall Hooke's law and Theorems 1 and 3. Price is an external force for individual consumers and firms, because they all participate on determining the "right" price.

**Theorem 6:** For each action, there is equal reaction in opposite direction.

The law of mutual forces of action and reaction (Newton's third law) holds in economics as well. The sum of forces of a closed system is zero. However, most economic systems are open, as is our simulation example. The whole real global economy is naturally a closed system.

## 5 Simulated economic crises

Now, one can construct an arbitrarily large system consisting of  $i = 1 \dots I$  firms,  $j = 1 \dots J$  consumers and  $k = 1 \dots K$  goods. The equations governing the dynamics are

$$m_{\dot{Q}_{i,k}} \ddot{Q}_{i,k} = P_k - \frac{\partial C_i(\dot{Q})}{\partial \dot{Q}_{i,k}}, \quad (8)$$

$$m_{\dot{X}_{j,k}} \ddot{X}_{j,k} = \frac{\partial H_j(\dot{X})}{\partial \dot{X}_{j,k}} - P_k, \quad (9)$$

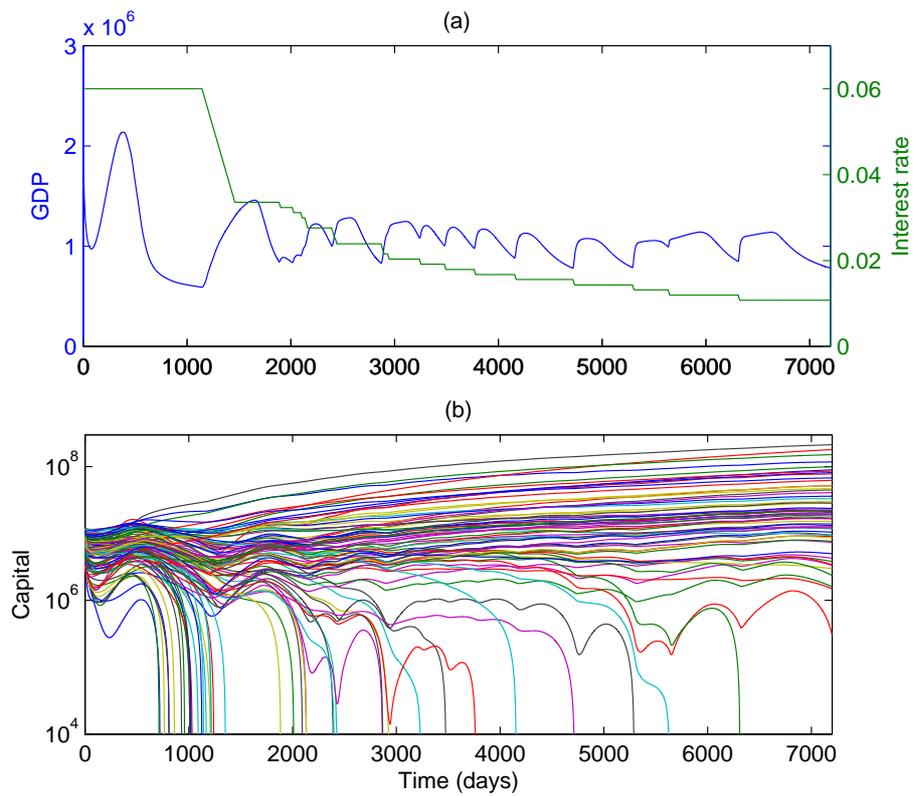
$$k P_k^{-1} \dot{P}_k = \sum_j \dot{X}_{j,k} - \sum_i \dot{Q}_{i,k}. \quad (10)$$

The law of demand and supply in Eq. (10) is familiar from the neo-classical theory, e.g. Samuelson (1941, 1942). Equations governing production (8) and consumption (9) do not exist in the neo-classical theory, but they are fundamental in the dynamic theory of economics, as shown by Estola and Hokkanen (2008); Estola (2017). The effects of production and consumption on prices cannot be treated separately. The neoclassical optimum is obtained by setting all masses  $m_{\dot{X} \vee \dot{Q}} \rightarrow 0$  and  $1/k_P \rightarrow 0$ .

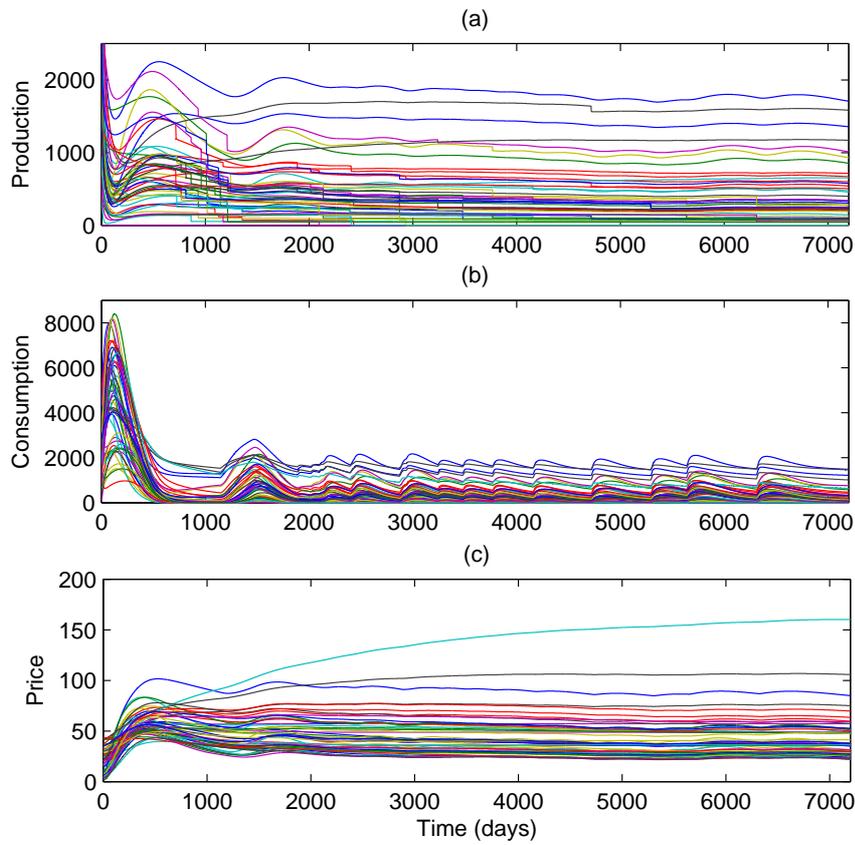
For simulating economic crises, we use a standard cost function  $C_i(\dot{Q}) = A_i + \sum_k (B_{i,k} \dot{Q}_{i,k} + D_{i,k} \dot{Q}_{i,k}^2)$  for firm  $i$ , where  $A_i$  represents fixed costs and  $B_{i,k}, D_{i,k}$  are the constants of variable costs. Each firm produces three randomly chosen products. Moreover, we use utility function  $U_j = u_j \sum_k (1 - \exp(-E_{j,k} \dot{X}_{j,k} / \dot{X}_{j,k}^0))$  that obeys positive and decreasing marginal utility, see e.g. Varian (2006). Our utility function has some advantages over the usual logarithmic utility functions used by e.g. Varian (2006), most notably that utility can never be infinite. Our utility-money conversion factor  $\eta_j = F_j (1 - \exp(-G_j M_j / \langle M \rangle))$  obeys the law of decreasing marginal utility of money presented by e.g. Bernoulli (1738); von Neumann and Morgenstern (1953). Constants  $E_{j,k}, \dot{X}_{j,k}^0$  describe which goods consumer  $j$  prefers. Constants  $F_j, G_j$  describe whether consumer  $j$  prefers spending or saving.  $M_j = M_j^0 + \rho_j r W_j$  is the income that consists of labour income  $M_j^0$  and interest earnings or payments on wealth  $r W_j$ .  $\langle M \rangle$  is the average wage income of all consumers. Capital gains or losses  $r W_j$  depend on whether the net worth  $W_j$  of consumer  $j$  is positive or negative.  $r$  is the interest rate (see Dannenberg and Estola (2017) for more details; for simplicity, the same rate is assumed for interest income and costs).  $\rho_j$  is a consumer specific factor that magnifies the wealth effect if  $\rho > 1$  or dilutes it if  $\rho < 1$ .<sup>2</sup>

The consumer spending problem is an open optimisation problem with soft boundaries, similar to the optimisation problem of a firm as shown by

<sup>2</sup> Corresponding units of our parameters and new functions are:  $[u_j \wedge U_j] = \text{util}/\text{time}$ ;  $[\eta_j \wedge F_j] = \text{€}/\text{util}$ ;  $[M_j \wedge M_j^0] = \text{€}/\text{time}$ ;  $[W_j] = \text{€}$ ;  $[r] = 1/\text{time}$ ;  $E_{j,k}, G_j$  and  $\rho_j$  are pure numbers.



**Figure 2:** (a) GDP and central bank interest rate of the simulated economy. Lowering interest rate temporarily increases GDP. (b) Capitals of firms. If the capital of a firm decreases to zero, the firm is declared into bankruptcy. The amount of bankruptcies affects the interest rate adjusted by central bank.



**Figure 3:** (a) Production, (b) consumption and (c) price of goods in the simulated economy. A business cycle of 300–500 days is clearly visible in consumption. Production and prices follow consumption, but they are slower to react to changes in the market.

Dannenberg and Estola (2017). Consumers may use credit or spend only a part of their income, depending on their marginal willingness to pay and the prices of goods. The marginal willingness to pay  $\partial H_j(\dot{X})/\partial \dot{X}_{j,k}$  is calculated from the willingness to pay function  $H_j(\dot{X}) \equiv \eta_j(M_j)U_j(\dot{X})$ . “Masses”  $m_{\dot{X}_{j,k}}$ ,  $m_{\dot{Q}_{i,k}}$  are the inertias of consumption and production, and  $k_{P_k}$  is the spring constant of price  $P_k$ .

With initial conditions  $\dot{X}_{j,k}^0$ ,  $\dot{Q}_{i,k}^0$ , and  $P_k^0$ , one can solve the dynamics of the system by numerical simulations. For simplicity, we have set the initial wealth of consumer  $j$  to  $W_j^0 = 0$ , and the firms have randomly distributed initial capitals. The system is open because income and interest earnings come from outside the system and costs and interest payments go outside the system; it is possible, though, to make the system closed. Unlike many economic models, bankruptcies are allowed: in our first approximation model, a firm that loses all of its capital is declared into bankruptcy and taken instantly out of the market by setting its production to zero. Moreover, there is an exogenous “central bank” that sets the interest rate  $r$  by economic conditions, basically based on the bankruptcy rate of firms.

In our simulations, we have created an economy of 100 firms, 50 goods and 5000 consumers and simulated it for 7200 days. Figures 2 and 3 illustrate the results. The most notable result is that from the complexity of many interacting agents, business cycles naturally emerge. The duration of a business cycle depends on the initial conditions; with our initial values, it varies roughly between 300 and 500 days. The business cycles are most clearly visible in the Figures representing GDP (2a) and consumption (3b) of the economy. Notice that we have assumed no technical progress in production which would propel economic growth in time, according to e.g. Estola (2001).

The central bank actions suppress natural business cycles in production and prices. Fig. 2 shows that each time the interest rate is lowered, GDP temporarily increases, but it quickly returns to its original level or even lower, demanding a new intervention. While production and prices have greater inertia than consumption, the central bank interventions prevent them from getting low enough for the next healthy expansion phase of the economy. Instead, they are supported by lower interest rate that allows consumers more credit. The interest rate policy is effective in saving firms from bankruptcy (see Fig. 2), but it results in overindebtedness of consumers.

The whole economy is thus highly dependent on central bank actions. In the end of our simulation, the interest rate is lowered to  $\sim 1\%$ /year, and there is not much ammo left in the central bank’s interest rate arsenal. (The real global economy passed this mark a couple of years ago, causing central banks to adopt extraordinary measures such as “quantitative easing”.) In our model, central bank actions seem to improve the short term economic situation but ultimately result in economic crises.

Our title asks: “what are the market forces?”. Our analysis gives a well defined answer: the market forces are marginal costs, marginal willingness to pay and price.

## References

- D. Bernoulli, Exposition of a new theory on the measurement of risk (1738); Translated by Dr. Louise Sommer, *Econometrica* 22(1), 22–36 (1954).
- T. A. Cameron and M. D. James, *Journal of Marketing Research* 24(11), 389–395 (1987).
- D. Cass, *Review of Economic Studies* 32, 233–240 (1965).
- A. A. Dannenberg and M. Estola, *Willingness to pay in the theory of a consumer* (submitted).
- F. De Jong, *Dimensional Analysis for Economists* (North Holland, Amsterdam, 1967).
- A. A. Dragulescu and V. M. Yakovenko, *European Physical Journal B* 17, 723–729 (2000).
- M. Estola, *Advances in Complex Systems* 2001.
- M. Estola, *Hyperion International Journal of Econophysics & New Economy* 6(1), 7–20 (2013).
- M. Estola, *Hyperion International Journal of Econophysics & New Economy* 8(1), 7–22 (2015).
- M. Estola, *Newtonian Microeconomics: A Dynamic Extension to Neoclassical Micro Theory* Palgrave Macmillan, 2017.
- M. Estola and A. A. Dannenberg, *Physica A* 391(24), 6519–6527 (2012).
- M. Estola and A. A. Dannenberg, *Newtonian and Lagrangian Mechanics of a Production System*, *Hyperion International Journal of Econophysics New Economy* 9(2), 7–26 (2016).
- M. Estola and V.-M. Hokkanen, *Consumer, Firm, and Price Dynamics: An Econophysics Approach. Modeling by Economic Forces* (VDM Verlag Dr. Müller, Saarbrücken, 2008).
- G. C. Evans, *American Mathematical Monthly* 31(2), 77–83 (1924).

- F. M. Fisher, *Disequilibrium Foundations of Equilibrium Economics* (Cambridge University Press, 1983).
- T. C. Koopmans, On the concept of optimal economic growth (Cowles Foundation Discussion Papers from Cowles Foundation for Research in Economics **163**, Yale University 1965).
- F. V. Kusmartsev, *Physics Letters A* **375**(6), 966–973 (2011).
- T. Lux and F. Westerhoff, *Nature Physics* **5**, 2–3 (2009).
- A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory* (Oxford University Press, 1995), p. 620.
- P. Mirowski, *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics* (Cambridge University Press, 1989).
- J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, 3rd ed. (Princeton University Press, Princeton, 1953).
- F. P. Ramsey, *Economic Journal* **38**, 543–559 (1928); Reprinted in: *Readings in the Modern Theory of Economic Growth*, edited by J. E. Stiglitz and H. Uzawa (MIT Press, Cambridge, Mass. 1969).
- P. A. Samuelson, *Econometrica* **9**(2), 97–120 (1941).
- P. A. Samuelson, *Econometrica* **10**(1), 1–25 (1942).
- H. Varian, *International Microeconomics*, 7th ed. (W. W. Norton, New York, 2006), pp. 92, 327.

Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:

<http://www.economics-ejournal.org/economics/discussionpapers/2017-110>

The Editor