Do institutions behave rationally in distressed markets?

Hoon Cho, Doojin Ryu, and Sangwook Sung

Abstract

The authors theoretically analyze the efficiency of liquidity flows in stabilizing distressed markets. Their analysis focuses on the incentives for financial institutions; specifically, they focus on arbitrage profit as an incentive and liquidity risk as a disincentive. The authors show that even with a major negative market shock, a financial institution can increase its market investment if it has sufficient funding liquidity. In addition, their model reveals a positive relationship between funding liquidity and liquidity flows. Thus, a distressed market might stabilize more quickly when financial institutions, acting as liquidity providers, have sufficient funding to bear the market's liquidity risk.

Authors

Hoon Cho, College of Business, Korea Advanced Institute of Science and Technology, Seoul, Korea
Doojin Ryu, ✉ College of Economics, Sungkyunkwan University, Seoul, Korea, sharpjin@skku.edu
Sangwook Sung, Samsung Economic Research Institute, Samsung Group, Seoul, Korea

1. Introduction

According to efficient market theory, when market prices temporarily deviate from fundamental values, rational and informed arbitrageurs are expected to trade against the price deviation (Ryu, 2011; Lee *et al.*, 2015); by exploiting such a strategy, arbitrageurs make profits and the market maintains price stability. However, in reality, financial markets consistently experience crashes in which prices deviate substantially from fundamental value. Moreover, arbitrageurs sometimes amplify price deviations, aggravating the crisis.

A number of studies address the failure of arbitrage transactions. Shleifer and Vishny (1997) contend that performance-based compensation leads to myopic institutions. Liu and Longstaff (2004) find that arbitrageurs optimally underinvest in arbitrage opportunities when they are bound to margin constraints on their collateral. Liu and Mello (2011) note that coordination risk among outside financiers limits the arbitrage capabilities of financial institutions. Sudden fund outflows always present a threat to such institutions, occasionally forcing them to unwind existing positions to lower prices. Nofsinger and Sias (1999) report that financial institutions engage in positive feedback, trading more often than individual investors. Meanwhile, De Long *et al.* (1990) analyze noise traders' trend-following strategies and reveal that their speculations destabilize the market.

We theoretically determine an institution's optimal investment strategy in a distressed market state and analyze the conditions under which an institution can effectively perform the role of arbitrageur during market shocks. We assume that financial institution is informed and sophisticated,[1] but there are constraints on funding liquidity and sudden fund withdrawals. In our model, the financial market diverges from fundamental values in the short term but converges to fundamental values in the long term. This context provides arbitrage opportunities, allowing a large institution to trade against temporary price deviations. However, apart from arbitrage profit, it should also consider liquidity risk, which would force it to unwind devalued positions. The optimal investment strategy depends substantially on the overall market situation, which includes the size of the market shock and the institution's funding liquidity. When an institution has sufficient funding liquidity to withstand a market shock, it expands its market investment by exploiting the arbitrage profits. However, when it lacks sufficient funding liquidity, it inevitably reduces the proportion of risky assets in its portfolios, amplifying the negative market shock. This phenomenon is referred to as the flight to quality – the core cause of market crashes.

---

[1] Empirical market microstructure studies support the information superiority of institutional investors (Ahn *et al.*, 2008; Ryu, 2015; Webb *et al.*, 2016; Yang *et al.*, 2017).

2. The Model

We consider a model with two assets, a risky asset and cash, over four time periods (t=0,1,2,3). Two types of investors participate in the risky asset's market, an informed institution and a number of uninformed trend followers, and the market is subject to a negative shock at time 1. The first type of investor, i.e., the institution, is fully rational, with risk-neutral utility; it knows the fundamental value of the risky asset. The institution occasionally trades the risky asset to exploit an arbitrage opportunity, but funding liquidity risk and limited capital prevent it from fully exploiting the profit potential of arbitrage. As a large investor, the institution can move market prices to some extent by changing its own liquidity flows. In equilibrium, an institution optimally maximizes its final asset value by allocating its capital between the risky asset and cash. Trend followers, the second type of investor, have unlimited capital but lack information regarding the fundamental value of the risky asset; hence, they simply follow market trends such that their aggregate demand is proportional to past prices.

For simplicity, we normalize the market supply at time 1 to unity and express market factors such as the market value, funding liquidity, and market shock relative to the price at time 1. At time 0, before the negative shock distresses the market, the price of the risky asset is equal to its fundamental value, and the (normalized) fundamental return is denoted R. At t=1, capital inflow f (henceforth, funding liquidity) is provided to the informed institution; simultaneously, a negative shock hits the market and reduces the market return by s. After observing the impact of the shock, the institution invests capital μ henceforth, market liquidity) in the risky asset and holds the remaining amount c, or f–μ, in cash, where $\mu \in [0, f]$. The institution will not receive additional capital inflows any more. Because the supply of the risky asset is normalized to unity at time 1, the market clearing condition becomes,[2]

$$1 = R - s + \mu, \tag{1}$$

where $R > 1$. At $t = 2$, the trend followers respond to past market conditions: between time 0 and time 1, the market price of the risky asset changes from R to one, and between time 1 and time 2, the market price is assumed to decline to α/R due to the behavior of trend followers, where $\alpha > 1$. In addition, the price declines at time 2 because the negative market shock causes a fund outflow θ, where θ is assumed to follow a uniform distribution over the interval $[0, f]$.

In our framework, liquidity risk is expressed as the possibility that cash holdings do not meet the fund outflow θ; liquidity risk is the main reason that the institutions cannot pursue a pure arbitrage strategy. When θ is less than cash holdings c, the institution covers fund outflows with cash and the liquidity risk does not devalue the market price of the risky asset. However, when θ is larger than c,

---

[2] We also assume that the market price at time 1 is unity.

the institution should liquidate its risky asset by an amount equal to the shortfall $\theta - c$ at the lower market price. Moreover, when the fund outflow θ exceeds the institution's total asset value v at time 2, it must sell all of its risky assets, and the market price will reach its lowest level. Therefore, from time 1 to time 2, the short-term return r depends on the size of the funding liquidity shock θ such that

$$r(\theta) = \begin{cases} \dfrac{\alpha}{R} & 0 \leq \theta < c \\ \dfrac{\alpha}{R} - (\theta - c) & c \leq \theta < v \\ \dfrac{\alpha}{R} - \mu r(\theta) & v \leq \theta \leq f \end{cases} \tag{2}$$

where $v = \mu r + c$ and $0 < r < 1$. At $t = 3$, all market participants become informed, and the market price converges to its fundamental value. The institution completely liquidates its risky asset and realizes its profit. Its long-term market return is R, that is,

$$R = 1 + s - \mu \tag{3}$$

Notably, in Equations (2) and (3), both the institution's short-term return from time 1 to time 2 and its long-term return from time 1 to time 3 are endogenously determined by its investment decision at $t = 1$. When it reduces market liquidity μ, its long-term return R increases in Equation (3), though its profit declines. This means both lower market liquidity and a higher long-term return on the risky asset. Furthermore, the smaller value of μ reduces the institution's liquidity risk. Therefore, when choosing its optimal market liquidity μ, the institution considers the effect of its market liquidity on both its long-term return and its liquidity risk.

The institution's final asset value, denoted V, also depends on θ as follows:

$$V(\theta) = \begin{cases} \mu R + (c - \theta) & 0 \leq \theta < c \\ \mu R - (\theta - c)\dfrac{R}{r(\theta)} & c \leq \theta < v \\ 0 & v \leq \theta \leq f \end{cases} \tag{4}$$

When θ is less than c, the institution can reimburse all fund outflows from cash, and the market return is μR. If θ is greater than c, the institution should liquidate the risky asset by as much as the shortfall amount, reducing the asset value by $(\theta - c)\frac{R}{r}$. When θ exceeds v, the institution must sell all of its holdings. Thus, liquidity risk drives an institution to hold more cash, even if there is a good arbitrage opportunity in the market. This is called the limit to arbitrage for liquidity risk.

3. Equilibrium

This section derives the optimal investment strategy of a risk-neutral institution and analyzes how both the market shock and funding liquidity affect market stability.

3.1 Optimal Asset Allocation

Following the model setup above, the negative market shock at time 1 creates an arbitrage opportunity that offers the institution an incentive to increase its market investment against price divergence. However, the institution also has an incentive to reduce its market investment because of the fear of liquidity risk. If the market shock is strong enough, the institution may face the decline in asset value and the resulting fund outflows. Therefore, this liquidity risk forces the institution to hold more cash.

The optimal strategy of the risk-neutral institution is to maximize the expected final asset value, which is

$$E[V(\mu)] = \frac{1}{f}\int_0^f V(\theta)\, d\theta = \frac{1}{f}\int_0^c (\mu R + (c - \theta))\, d\theta + \frac{1}{f}\int_c^v \left(\mu R - (\theta - c)\frac{R}{r(\theta)}\right) d\theta \qquad (5)$$

In this equilibrium, the optimal market investment μ^* is endogenously determined by the exogenous variables, the size of the market shock s and funding liquidity f. Therefore, at time 1, the institution determines μ^* such that

$$\mu^*(s, f) = arg\max_{\mu} E[V(\mu)] \qquad (6)$$

The first-order condition of the optimization problem is given by

$$\frac{\partial E[V]}{\partial \mu} = \frac{1}{f}\left(3\mu^2 - (1 + 2f + 2s)\mu + sf + \frac{\mu\alpha}{1+\mu}\right) = 0 \qquad (7)$$

In Equation (7), f and s are alternatives, being symmetric and mutually interchangeable. In fact, f and s have similar natures, as s offers the institution an arbitrage opportunity and f offers the ability to pursue it. To solve the optimization problem in Equation (6), the market investment μ should also satisfy the boundary conditions; first, both short sales and borrowing are prohibited; second, the fundamental return is greater than one; third, the short-term return is less than one. In Equation (2), the highest short-term return is α/R, and the range of $\alpha/R < 1$ includes the range of $1 < R$. So the third condition is sufficient for the second condition. Therefore, the boundary conditions are expressed as

$$\begin{cases} 0 \le \mu \le f \\ 1 < \alpha < R \end{cases} \tag{8}$$

With the optimal investment allocation and the boundary conditions in Equations (6) and (8), the institution invests μ^* in the risky asset and holds $f - \mu^*$ in cash at time 1. At time 2, for the optimal investment μ^*, the short-term market return r^* is determined as a function of θ; at time 3, the fundamental return R^* is realized as $1 + s - \mu^*$.

Figure 1 illustrates variations in the expected final asset value $E[V]$ with respect to market investment μ for $\alpha = 2$, $f = 3$, and $s = 5$. In this example, the optimal market investment is 1.20, and thus the institution holds 1.80 in cash. The long-term market return would be 4.80. When μ is less than 1.20, the incentive to obtain arbitrage profits is greater than the incentive to hold cash against liquidity risk. In contrast, when μ is greater than 1.20, the incentive to hold cash against liquidity risk is greater.

3.2 Market Stability

One of our study's main goals is to determine the conditions under which an institution invests against market shocks. The optimal investment decision depends on the relative extent of arbitrage profit and liquidity risk. When arbitrage profit dominates liquidity risk, the institution increases its market investment and the market becomes more stable, whereas when liquidity risk dominates arbitrage profit, the institution prefers to hold cash and the market becomes less stable.

Proposition 1. *If $\alpha \le \left(1 + \frac{f}{2}\right)^2$, the institution increases its optimal market investment μ^* as the size s of the market shock increases. However, if $\alpha > \left(1 + \frac{f}{2}\right)^2$, the institution reduces its optimal market investment μ^* as the size s of the market shock increases; in other words, the institution amplifies the market shock.*

Proposition 1 implies that if $\alpha \le \left(1 + \frac{f}{2}\right)^2$, the arbitrage profit incentive dominates the liquidity risk disincentive. This condition is represented by the inequality $\alpha / \left(1 + \frac{f}{2}\right) \le 1 + \frac{f}{2}$. Since f denotes budget limitations, an institution with a higher f can exploit the arbitrage opportunities created by the market shock. Thus, the right-hand side $1 + \frac{f}{2}$ represents the incentive for market investment in anticipation of long-term arbitrage profit. Similarly, the left-hand side $\alpha / \left(1 + \frac{f}{2}\right)$ represents the disincentive of market investment due to the liquidity risk from the short-term price drop. Therefore, we can infer from Proposition 1 that funding liquidity f contributes to risk-bearing capacity, and when

6

a strong negative market shock is anticipated, an institution that has sufficient funding liquidity expands its market portion to capture arbitrage profit despite the fear of short-term risk. If $\alpha > \left(1 + \frac{f}{2}\right)^2$, the amplification of the market shock happens. As described above, if the institution's risk-bearing capacity is not sufficient, it cannot withstand the short-term risk driven by trend followers. The strong market shock therefore leads the institution to prefer cash to risky assets, and the price diverges further.

Figure 2 depicts the change in the optimal investment value given changes in the size of the market shock when $\alpha = 2$. To compare the patterns according to funding liquidity, we draw several curves to represent various values of f. Among the optimal market investments satisfying Equation (7), we exclude values that violate boundary condition (8). Therefore, the left portions of the curves have been removed from the figure. As Proposition 1 states, μ^* is positively related to s when f is greater than $2(\sqrt{2} - 1)$. However, when f is less than $2(\sqrt{2} - 1)$, μ^* has a negative relationship with s.

In reality, when a market is caught in an illiquidity trap, policymakers agree to give public funds to institutions to reinforce their risk-bearing capacities against price drops, which corresponds to an increase in f. Therefore, we contend that the optimal investment decision varies with funding liquidity f. As mentioned above, funding liquidity f encourages an institution to exploit arbitrage profits; thus, an institution with sufficient liquidity can bear short-term risk, and as expected, the institution expands its market investment.

Proposition 2. *As more funding flows to an institution, it increases optimal market investment* μ^*.

Proposition 2 verifies the positive relationship between funding liquidity and market liquidity flows: market liquidity increases as funding liquidity grows. This assertion supports bailout during a financial crisis because a new liquidity to institutions promotes liquidity inflows to the market. Figure 3 clearly displays the positive relationship between f and μ^*. In Figure 3, μ^* is monotonically increasing with an increase in f, and this pattern is independent of the size s of the market shock. In Figure 2, at the point where $f = 2(\sqrt{2} - 1)$, the curves intersect, and hence their order reverses because, as stated in Proposition 1, the sign of $\frac{\partial \mu^*}{\partial s}$ changes at this point.

To help understand the relationships in Propositions 1 and 2, we illustrate the surface of optimal investments as a function of f and s. In this example, α is assumed to equal two. Some parts of the surface have been removed because they violate boundary condition (8). The surface increases with f. The institution's market liquidity does not generally increase much when s is relatively small because of the unattractive arbitrage profit incentive, but it increases in the range of a relatively large s. In addition, as discussed in Equation (7), Figure 4 depicts the mutual symmetry of f and s.

4. Conclusion

In sum, we theoretically analyze the optimal investment decisions of an institution in a distressed market. Our theoretical approach focuses on the incentives of the institution as a major liquidity provider. Because it has sufficient market power to change the market price, its role in recovering market stability is too important to ignore. However, given both limited funding liquidity and uncertain funding outflow, the institution enjoys only limited ability to pursue arbitrage opportunities.

Even given a massive negative market shock, an institution can increase its market investment as long as it has sufficient funding liquidity. Therefore, abundant funding liquidity is an important component in developing a market structure that is resilient to negative market shocks. In addition, the greater the liquidity flow into an institution, the greater the liquidity flow into the market. Thus, a distressed market might recover its stability more quickly when institutions, as liquidity providers, have sufficient funding liquidity to bear liquidity risk in the market.

Appendix

Proof of Proposition 1

The differential equation $\frac{d}{ds}\left(\frac{\partial E[V(\mu^*)]}{\partial \mu}\right)$ is given as $\frac{d}{ds}\left(\left[\frac{\partial E[V(\mu)]}{\partial \mu}\right]_{\mu=\mu^*}\right) = \left[\frac{\partial^2 E[V(\mu)]}{\partial \mu^2}\right]_{\mu=\mu^*} \cdot \frac{\partial \mu^*}{\partial s} +$

$\left[\frac{\partial^2 E[V(\mu)]}{\partial s \partial \mu}\right]_{\mu=\mu^*}$. From the first-order condition in Equation (7), we know that $\left[\frac{\partial E[V(\mu)]}{\partial \mu}\right]_{\mu=\mu^*} = 0$; thus,

$\frac{\partial \mu^*}{\partial s} = -\frac{\left[\frac{\partial^2 E[V(\mu)]}{\partial s \partial \mu}\right]_{\mu=\mu^*}}{\left[\frac{\partial^2 E[V(\mu)]}{\partial \mu^2}\right]_{\mu=\mu^*}}$. As the second-order condition ensures that $\left[\frac{\partial^2 E[V(\mu)]}{\partial \mu^2}\right]_{\mu=\mu^*} < 0$, the sign of $\frac{\partial \mu^*}{\partial s}$

is same as the sign of $\left[\frac{\partial^2 E[V(\mu)]}{\partial s \partial \mu}\right]_{\mu=\mu^*}$. A simple calculation shows that $\left[\frac{\partial^2 E[V(\mu)]}{\partial s \partial \mu}\right]_{\mu=\mu^*} = \frac{1}{f}(-2\mu^* + f)$.

To investigate the conditions under which the sign of $\frac{\partial \mu^*}{\partial s}$ changes, we use $\mu = \frac{f}{2}$ in $\frac{\partial E[V(\mu)]}{\partial \mu}$. Then,

$\frac{\partial E[V(f/2)]}{\partial \mu} = \frac{1}{2(1+f/2)}\left(\alpha - \left(1 + \frac{f}{2}\right)^2\right)$, which implies that $\frac{\partial \mu^*}{\partial s} \geq 0$ when $\alpha \leq \left(1 + \frac{f}{2}\right)^2$ and $\frac{\partial \mu^*}{\partial s} < 0$

otherwise.

Proof of Proposition 2

As in the above proof, the sign of $\frac{\partial \mu^*}{\partial f}$ is the same as the sign of $\left[\frac{\partial^2 E[V(\mu)]}{\partial f \partial \mu}\right]_{\mu=\mu^*}$. Because

$\left[\frac{\partial^2 E[V(\mu)]}{\partial f \partial \mu}\right]_{\mu=\mu^*} = \frac{1}{f}(-2\mu^* + s)$, $\frac{\partial \mu^*}{\partial f}$ is negative if $\mu^* > \frac{s}{2}$ and positive otherwise. Let us substitute

$\mu^* = \frac{s}{2}$ in $\frac{\partial E[V(\mu)]}{\partial \mu}$. Then $\frac{\partial E[V(s/2)]}{\partial \mu}$ becomes $\frac{s}{2f(1+s/2)}\left(\alpha - \left(1 + \frac{s}{2}\right)^2\right)$, which means that $\frac{\partial \mu^*}{\partial f} \geq 0$

when $\alpha \leq \left(1 + \frac{s}{2}\right)^2$ and $\frac{\partial \mu^*}{\partial f} < 0$ otherwise. From boundary condition (8), $\alpha < R = 1 + s - \mu^* <$

$1 + s + \left(\frac{s}{2}\right)^2 = \left(1 + \frac{s}{2}\right)^2$. Therefore, $\frac{\partial \mu^*}{\partial f} > 0$ is always valid.

References

Ahn, H-.J., Kang, J., Ryu, D., 2008. Informed trading in the index option market: The case of KOSPI 200 options. Journal of Futures Markets 28 (12), 1118-1146.

De Long, J.B., Shleifer, A., Summers, L.H., Waldmann, R.J., 1990. Noise trader risk in financial markets. Journal of Political Economy 98 (4), 703-738.

Lee, J., Kang, J., Ryu, D., 2015. Common deviation and regime-dependent dynamics in the index derivatives markets. Pacific-basin Finance Journal 33, 1-22.

Liu, J., Longstaff, F.A., 2004. Losing money on arbitrage: Optimal dynamic portfolio choice in markets with arbitrage opportunities. Review of Financial Studies 17 (3), 611-641.

Liu, X., Mello, A.S., 2011. The fragile capital structure of hedge funds and the limits to arbitrage. Journal of Financial Economics 102 (3), 491-506.

Nofsinger, J.R., Sias, R.W., 1999. Herding and feedback trading by institutional and individual investors. Journal of Finance 54 (6), 2263-2295.

Ryu, D., 2011. Intraday price formation and bid-ask spread components: A new approach using a cross-market model. Journal of Futures Markets 31 (12), 1142-1169.

Ryu, D., 2015. The information content of trades: An analysis of KOSPI 200 index derivatives. Journal of Futures Markets 35 (3), 201-221.

Shleifer, A., Vishny, R.W., 1997. The limits of arbitrage. Journal of Finance 52 (1), 35-55.

Yang, H., Choi, H-.S., Ryu, D., 2017. Option market characteristics and price monotonicity violations. Journal of Futures Markets, 37 (5), 473-498.

Webb, R.I., Ryu, D., Ryu, D., Han, J., 2016. The price impact of futures trades and their intraday seasonality. Emerging Markets Review 26, 80-98.
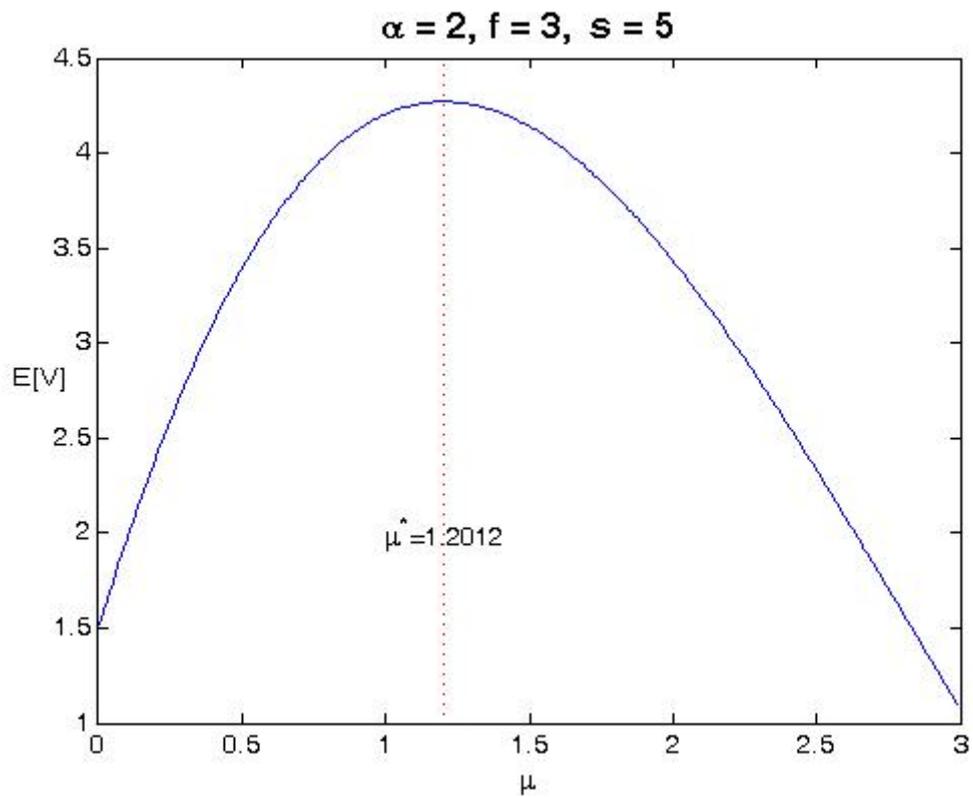
Fig. 1. Change in the expected asset value with respect to market investment

Note. The domain of market investment is from zero to three. The exogenous factors are fixed at $\alpha = 2$, $f = 3$, $s = 5$. The optimal market investment is 1.2012.
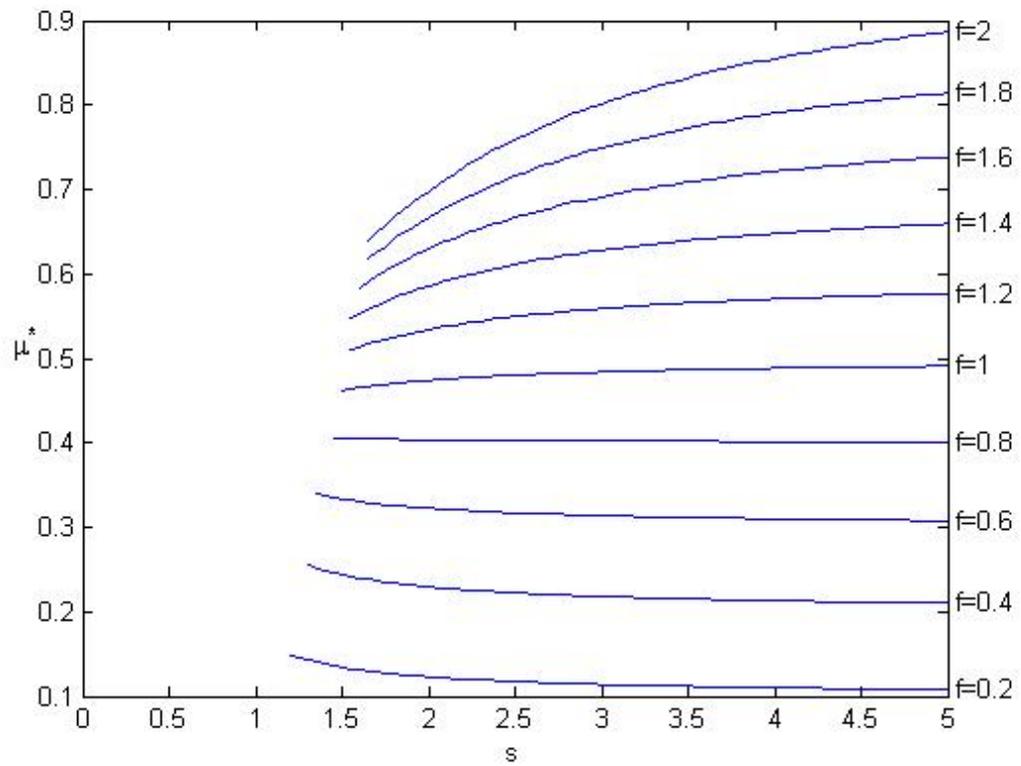
Fig. 2. Optimal market investment as a function of the size of the market shock

Note. The domain of the size of the market shock is from zero to five. Each curve indicates the function for a different value of f, from 0.2 to two. Values that violate boundary conditions have been removed. Here, $\alpha=2$.
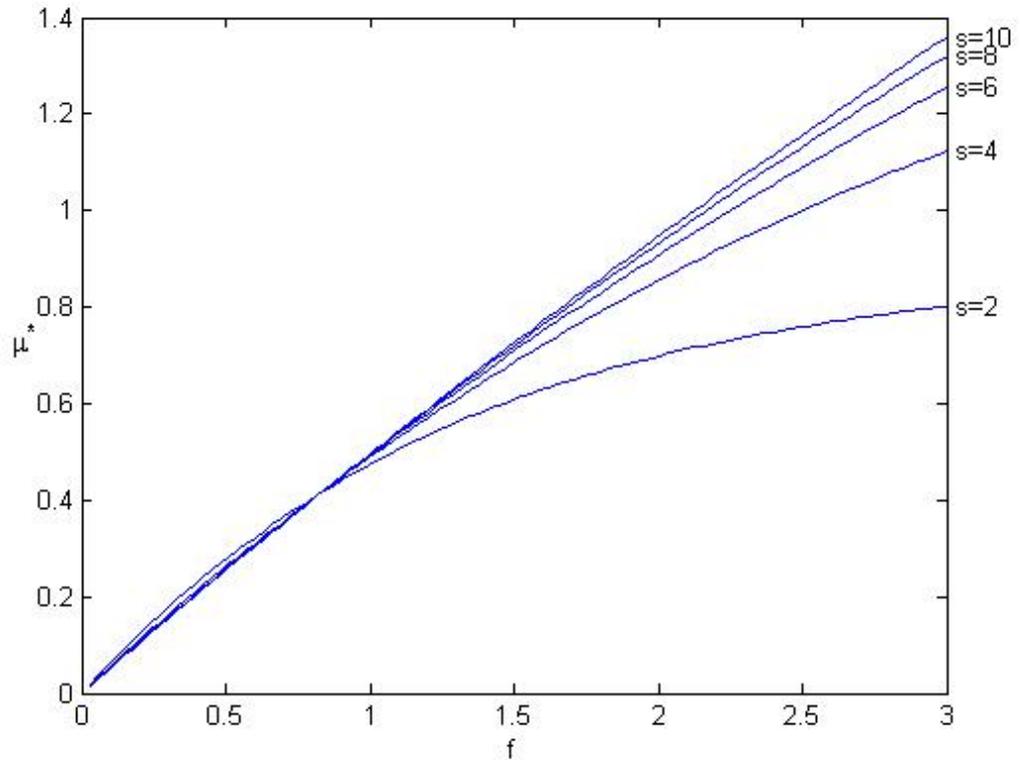
Fig. 3. Optimal market investment as a function of funding liquidity

Note. The domain of the funding amount is from zero to three. Each curve indicates the function for a different s value, from two to 10. Here, $\alpha=2$.
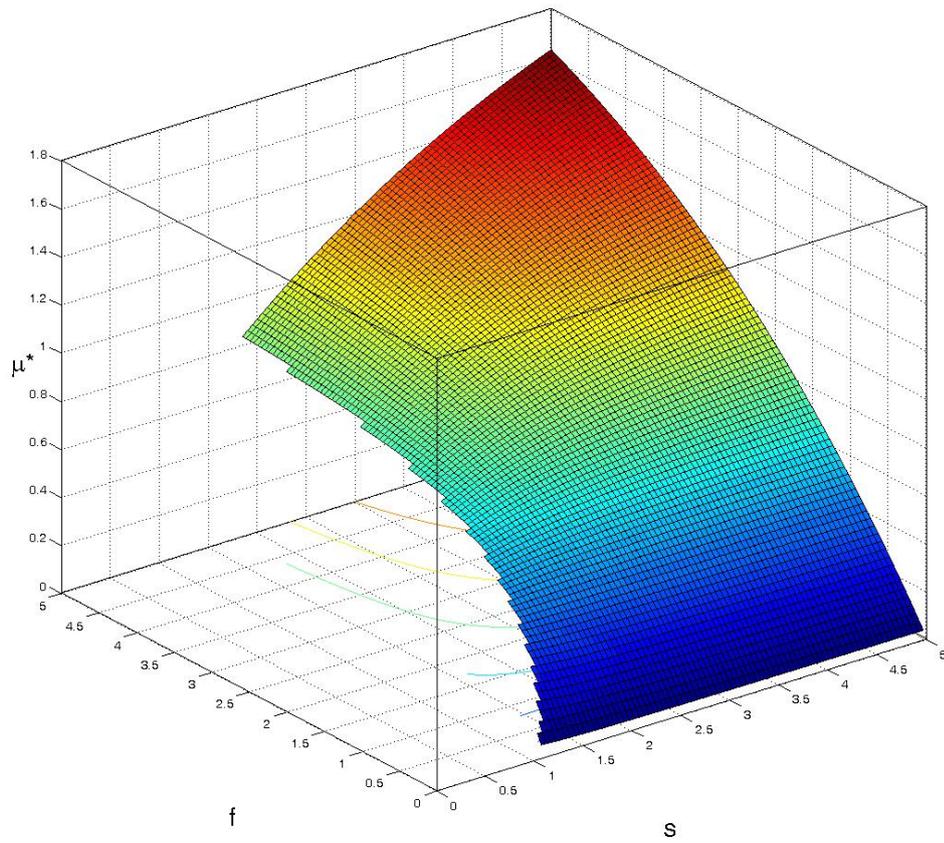
Fig. 4. Optimal market investment as a function of the size of the market shock and of funding liquidity

Note. The domains of s and f are from zero to five. Values that violate the boundary conditions have been removed. Here, $\alpha=2$.

Economics

The Open-Access, Open-Assessment E-Journal

Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:

The Editor