Foreign capital inflow and its welfare implications in a developing country context

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Abstract

In a small open developing country context, the author considers a three-sector general equilibrium framework and tries to find out the effects of foreign capital inflow on welfare of the country. Comparative-static results show that foreign capital inflow widens the skilled-unskilled wage gap under some reasonable conditions, although it causes an expansion of the foreign enclave and the agricultural sector and contraction of the domestic manufacturing sector. Taking sector specific foreign capital, the author finds that foreign direct investment is beneficial in a small open economy in the absence of tariffs.

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Foreign-capital inflow and its welfare implications in a developing country-context

1. **Introduction**

The impact of foreign-capital inflow on the welfare of a developing economy has always been a subject of immense interest among the policymakers and the academicians. The famous Brecher-Alejandro's (1977) proposition, based on the theoretical works by Bhagawati (1958), Johnson (1967), and Bhagawati (1973), is a benchmark in this context. The work of Brecher-Alejandro (1977) has successfully analysed the effects of foreign-capital inflow on the welfare of an economy. In a two-commodity, two-factor framework, characterized by the presence of full employment, they have shown that, if the relatively capital-intensive sector is protected by tariff and, the income from foreign capital is fully repatriated, then, an increase in foreign-capital inflow would cause a loss in welfare of a country. However, the inflow of foreign capital cannot influence the welfare of a country if the import-competing sector is not protected. This is the famous Brecher-Alejandro's (1977) proposition of the trade theory. The work of Khan (1982) has also thrown light on this in a Harris-Todaro's (1970) framework of urban-unemployment. The result of Khan's work also supports the Brecher-Alejandro's (1977) proposition.

Beladi and Marjit (1992a) have re-examined the Brecher-Alejandro's (1977) proposition in a full-employment model proposed by Heckscher-Ohlin-Samuelson, in the presence of an export-processing-zone (EPZ) that uses the sector-specific foreign capital. According to their work, the inflow of foreign capital obstructs the national welfare if the country imports capital intensive goods while the import-competing sector is protected. Contrarily, if the country imports labour intensive commodity, the expansion of the EPZ will make the country better off.

In an another work, Beladi and Marjit (1992b) have considered a three-sector model in which one sector is EPZ that uses sector-specific foreign capital in a subsystem proposed by Heckscher-Ohlin-Samuelson. In this model, skilled labour force is fully employed and the market of unskilled labour is characterized by the presence of unemployment. This work invents the conditions under which one can find out the negative effect of foreign-capital inflow on welfare of a country. It happens when the foreign capital is not injected directly into the import-competing sector, but it changes the composition of output in other traded sectors through the inter-sectoral reallocation of the existing resources.

The validity of Brecher-Alejandro's (1977) proposition has been presented by the work of Chandra and Khan (1993) in a Harris-Todaro's (1970) framework, where capital is mobile. They consider the presence of a 'flexible wage in urban informal sector', and establish that Brecher-Alejandro's (1977)

proposition remains valid if the average capital-intensities of both the formal and informal sectors of urban area are higher than the capital-intensity of the formal sector of rural area.

Here, we are interested to find out the effects of foreign-capital inflow on the welfare of a developing country. More precisely, our main objective is to examine the impact of foreign-capital inflow on welfare of a developing country in a three-sector general equilibrium setting, the sectors being agriculture, domestic-manufacture and foreign-enclave. Moreover, we consider the domestic-manufacture as an import-competing sector (same as Brecher-Alejandro's (1977) proposition), whereas, other two sectors are considered to be the export sectors.

The paper is organised as follows. Section 2 describes the model. Comparative-static results are discussed in Section 3. Section 4 shows the effects of foreign-capital inflow on welfare of a small open economy. Finally, conclusions are made in Section 5.

2. The Model

We consider a three-sector small open economy in a neoclassical general-equilibrium framework. There are two sector-specific factors, viz., foreign capital, which is specific to foreign enclave and unskilled labour force, which is specific to the agricultural sector. There is no substitutability between domestic capital and foreign capital. There is full-employment in all the factors' markets. Both the products' markets and the factors' markets are assumed to be perfectly competitive and production function in each sector is CES[1] with CRS[2] and follows diminishing marginal productivities of all variable factors. There is free trade of commodities.

The economy consists of three sectors, viz., the manufacturing sector (M), the foreign enclave (T) and the agricultural sector (A). S and L stand for skilled and unskilled labour-endowment respectively. K and F stand, respectively, for stock of domestic capital and inflow of foreign capital. Three sectors use different combinations of factor-inputs for their production, like, sector M uses S and K, sector T uses S and F, and, sector A uses L and K. Here, we assume that S is perfectly mobile within M and T; and, that of K is perfectly mobile between A and M within the domestic enclave. We take agricultural product as the numeraire, its price being equal to unity.

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[1]Constant Elasticity of Substitution
[2]Constant Returns to Scale

Let a_{ij} be the amount of ith factor required to produce one unit of commodity j, X_j be the level of jth commodity produced, P_j^* be the global price per unit of commodity j, Θ_{ij} be the share of expenditure on factor i to the total cost of producing one unit of commodity j, σ_j be the elasticity of substitution for commodity j, λ_{ij} be the proportion of ith factor employed in j^{th} sector, and, D_j be the demand for commodity j, where, $i = K, L, S, F$ and $j = M, A, T$. Let w and w_S stand for wage rates of unskilled and skilled labours respectively, while r and r_F stand for rental rates of domestic and foreign capital respectively. We take $U(.)$ as the social welfare function, where, U_j is the marginal social utility of commodity j. To describe the model, we have the following main equations:

$$a_{SM}w_S + a_{KM}r = P_M^* \tag{1}$$

$$a_{LA}w + a_{KA}r = 1 \tag{2}$$

(Agricultural product is taken as the numeraire. Hence, $P_A^* = 1$)

$$a_{ST}w_S + a_{FT}r_F = P_T^* \tag{3}$$

All these three equations imply the competitive zero-profit pricing system.

From the conditions of full-employment at all relevant factors' markets, we have the following four equations:

$$a_{SM}X_M + a_{ST}X_T = S \tag{4}$$

$$a_{LA}X_A = L \tag{5}$$

$$a_{KA}X_A + a_{KM}X_M = K \tag{6}$$

$$a_{KF}X_T = F \tag{7}$$

In the present model, the endogenous variables to be determined are $w, r, w_S, r_F, X_M, X_A,$ and X_T. P_T^*, P_A^* and P_M^* are exogenously given world's prices and the assumption of small open economy implies that the country is a price-taker. We now describe how does the model work.

Given, P_M^*, $w_S = w_S(r)$, $w_S' < 0$ (equation 1). On the other hand, given, P_T, $r_F = r_F(w_S)$, $r_F' < 0$ (equation 3); so, $r_F = v_1(r)$, $v_1' > 0$. Again, it is given that, $P_A = 1$, $r = r(w)$, $\frac{dr}{dw} < 0$ (equation 2). So, $w_S = w_S[r(w)]$ with $\frac{dw_S}{dw} = \frac{dw_S}{dr}\frac{dr}{dw}$, where $\frac{dw_S}{dw} > 0$. Hence, the system is not decomposable. Now, equations 4 and 7 give:

$$a_{SM}X_M + \frac{a_{ST}}{a_{FT}}F = S \qquad\qquad (4.1)$$

It is known that, $a_{ST} = a_{ST}(w_S, r_F)$ and $a_{FT} = a_{FT}(w_S, r_F)$. It is also known that, w_S is a diminishing function of r, and r_F is an increasing function of r. Hence, $\frac{a_{ST}}{a_{FT}} = g_1(r)$, $g_1^I > 0$. This is because, a rise in rental rate of domestic capital implies a rise in rental rate of the foreign capital and a consequent decline in the wage rate of the skilled labour. Therefore, the foreign enclave has a tendency to substitute the foreign capital by skilled labour which causes a rise in labour-capital ratio in the foreign enclave.

On the other hand, we know, $a_{SM} = a_{SM}(w_S, r)$. As w_S is negatively related to r, so, $a_{SM} = a_{SM}(r)$ with $a_{SM}^I > 0$. This can be explained in a way that, when the rental rate of domestic capital is increased, the wage rate of skilled labour declines, and hence, the domestic manufacturing sector has the tendency to substitute capital by labour. Equation 4.1 can be expressed then as:

$$g_1(r)F + a_{SM}(r)X_M = S \qquad\qquad (4.1.1)$$

This equation shows a unique relationship between r and X_M, that maintains the equilibrium in the skilled labour market. The relationship is depicted by a negatively sloped SS curve (Figure 1) with $\frac{dr}{dX_M} < 0$ (see Appendix 1). The intuitive explanation behind this can be easily understood. Given skilled labour supply, a rise in the production of domestic manufacturing good will be followed by an excess demand situation for skilled labour. Consequently, there will be a rise in the wage rate of skilled labour reducing rental rate of domestic capital to maintain equilibrium condition. As a result, the manufacturers will be induced to substitute skilled labour by cheaper domestic capital. This process will be continued unless and until the equilibrium will be restored.

The expression of equation 5 gives, $X_A = \frac{L}{a_{LA}}$, and substituting this value of X_A in equation 6, one gets:

$$\frac{a_{KA}}{a_{LA}}L + a_{KM}X_M = K \qquad\qquad (6.1)$$

So, this gives us two relationships, say, $a_{KA} = a_{KA}(w, r)$ and $a_{LA} = a_{LA}(w, r)$. Now, $w = w(r)$ $with$ $w' < 0$. So, $\frac{a_{KA}}{a_{LA}} = g_2(r)$, $g_2' < 0$. As rental rate is increased, the wage rate of unskilled labour falls. The producer has a tendency to substitute capital by unskilled labour in agriculture. Therefore, the capital-labour ratio will decline. By analogous reason it follows that, $a_{KM} = a_{KM}(r)$, with $a_{KM}^I < 0$. Hence, from equation 6.1,

$$Lg_2(r) + a_{KM}(r)X_M = K \qquad\qquad (6.1.1)$$

Equation (6.1.1) shows the one to one correspondence between rental rate of domestic capital and level of output of domestic manufacturing sector that keeps the domestic capital market in equilibrium. The relationship between the two is positive ($i.e,\ \frac{dr}{dX_M} > 0$) and is depicted by KK curve in Figure 1 (see Appendix 2). This can be explicated by following explanation. With constant supply of domestic capital, whenever there is an increase in the production of manufacturing goods, there will be a higher demand for capital. That will cause an excess demand situation in the domestic capital market leading to a rise in the rental rate of domestic capital. The equilibrium level of $r\ and\ X_M$ are determined by the intersection of KK and SS curves. Once these two unknowns are solved, one can solve the entire system.

Figure 1
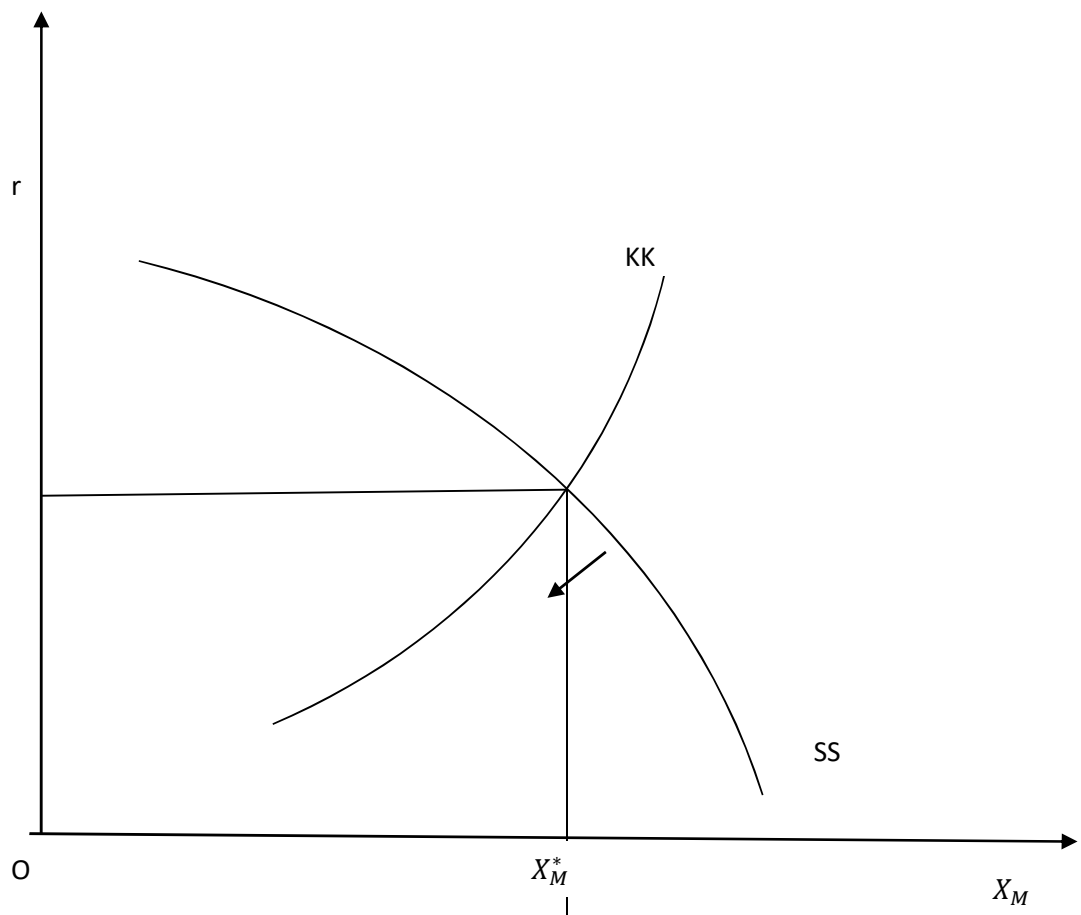
3. Inflow of Foreign Capital: Comparative Statics

We now consider that the government adopts the policy of partial liberalization, i.e., a certain amount of foreign capital is allowed to enter into the economy in the form of direct investment. This dosage of

foreign capital is directly injected in the foreign enclave. Here, we examine the impact of foreign-capital inflow on the endogenous variables.

3.1 Foreign-Capital Inflow and Skilled-Unskilled Wage Gap

As the entire F (foreign investment) is injected into sector T (foreign enclave), the demand for S (skilled labour) and so w_S (skilled wage) will rise in this sector. As S is perfectly mobile within sectors M (domestic manufacturing) and T, hence, S will start to move from sector M to sector T. This process is continued unless and until the wage rate in sector M rises to match to that of in sector T. Due to this movement, the labour-capital ratio declines in sector M causing a decline in marginal productivity of K (domestic capital) followed by a decline in r. In a similar manner, as K is perfectly mobile between two sectors, M and A (agriculture), the decline in r will induce the owners of K to transfer their capital from sector M to sector A by inducing a decline in unskilled labour-capital ratio in sector A. This results in a decline in the marginal productivity of K in sector A. This movement of K is continued unless equality is attained in the rental rates of K in both the sectors. At the same time, the rise in K-L ratio causes a rise in the marginal productivity of L (unskilled labour) in sector A. It will be reflected through a rise in w in sector A. Last, but not the least, the inflow of foreign capital causes an excess supply situation in the foreign capital market, thereby declining r_F (rental rate of foreign capital). Therefore, we can see that foreign-capital inflow causes an increase in both w_S and w. Hence, it can be concluded that partial investment liberalization makes the work force better off and the domestic capital owners worse off.

The impact of foreign-capital inflow on r and X_M can be derived from equations (4.1.1) and (6.1.1). By differentiating equation (4.1.1): [3]

$$Fg_1^I dr + X_M a_{SM}^I dr + a_{SM} dX_M = -g_1 dF \qquad (4.1.2)$$

Equivalently, by differentiating 6.1.1:

$$Lg_2^I dr + X_M a_{KM}^I dr + a_{KM} dX_M = 0 \qquad (6.1.2)$$

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[3] Here, we consider $dF > 0$ as foreign-investment liberalization causes a rise in inflow of foreign capital.

So, $\frac{dr}{dF} < 0$, and $\frac{\hat{r}}{\hat{F}} < 0$ (see Appendix 3).

Let us see the impact of investment-liberalization on the gap between w and w_S.

From equation 1, we see that, $\widehat{w}_S = -\frac{\theta_{KM}}{\theta_{SM}}\hat{r}$ (see Appendix 3). As $\theta_{ij} > 0$ and $\hat{r} < 0$, then $\widehat{w}_S > 0$. Similarly, $\widehat{w} = -\frac{\theta_{KA}}{\theta_{LA}}\hat{r}$ (see Appendix 3). So, $\widehat{w} > 0$ and $\widehat{w}_S - \widehat{w} = (\frac{\theta_{KA}}{\theta_{LA}} - \frac{\theta_{KM}}{\theta_{SM}})\hat{r}$. Now, $\frac{\theta_{KA}}{\theta_{LA}} < \frac{\theta_{KM}}{\theta_{SM}}$ and $\hat{r} < 0$[4]. So, $\widehat{w}_S - \widehat{w} > 0$ and $\frac{\widehat{w}_S - \widehat{w}}{\hat{F}} > 0$.

We summarize the result in the form of the following proposition.

Proposition 1: Foreign-capital inflow in a small open economy widens the skilled-unskilled wage gap under some reasonable conditions.

The widening of wage gap can be explained as follows. Because of foreign-capital inflow, the extra investment is injected into sector T. This rise in foreign-capital stock increases w_S in the form of increased marginal productivity. This is a direct effect. The rise in w_S causes a decline in r in sector M. This is followed by a transfer of domestic capital from sector M to sector A. Consequently, the marginal productivity of unskilled labour rises causing a rise in wage rate. This is the indirect effect of foreign investment which will be dominated by that of direct effect.

3.2 Foreign-Capital Inflow and Expansion of Three Sectors

Now, we consider the effect of rise in F within different sectors. Equations 4.1.1 and 6.1.1 show that in determination of X_M and the rental rate of capital in sector M, the amount of foreign capital plays a parametric role. In the SS curve (Figure 1), F plays the role of a parameter. If there is a change in demand for capital, the SS curve will shift. This is because, change in foreign capital causes a parametric change in demand for foreign capital. In fact, a rise in F is followed by a leftward shift of SS curve. The reason is that, increase in F calls for larger amount of skilled labour in sector T. The skilled labour then will be transferred from sector M to sector T and consequently, the volume X_M will fall. This leads to, $\frac{dX_M}{dF} < 0$. In other words, $\frac{\hat{X}_M}{\hat{F}} < 0$ (see appendix 4). In figure 1, the increase in F is shown by a leftward shift of the SS curve and the equilibrium point moves leftward. But volume of X_T increases, i.e., $\frac{\hat{X}_T}{\hat{F}} >$

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[4]Though sector A and sector M cannot be compared in terms of relative factor-intensity as they are using different types of workers, the empirical studies have found that normally, a sector that uses S, commonly uses more capital per unit of worker than that normally a sector using L does.

0 (see appendix 5). It has already been found that an increase in w_S in sector T leads to a transfer of skilled labour from sector M to sector T. Therefore, sector M releases capital, which is transferred to sector A, and so, X_A increases (i.e., $\hat{X}_A > 0$, or, $\frac{\hat{X}_A}{\hat{F}} > 0$).

We summarize these results in the form of proposition 2 as follows.

Proposition 2: Foreign-capital inflow causes an expansion of the foreign enclave and the agricultural sector, though it reduces the output of domestic manufacturing sector.

The reasons behind this proposition can be given as follows. Investment liberalization causes a rise in the capital stock in sector T and transfer of some of the skilled labours from sector M to sector T. Consequently, sector T expands and there is a transfer of domestic capital from sector M to sector A. Hence, sector A also expands with a contraction in sector M.

4. Foreign-Capital Inflow and Welfare in Absence of Tariff

To find out the impact of foreign-capital inflow on the social welfare of the economy with imposition of no tariff, we take the social welfare function of the following form:

$$U = U(D_M, D_T, D_A) \qquad (8)$$

$$\text{So, } dU = \frac{\partial U}{\partial D_M} dD_M + \frac{\partial U}{\partial D_T} dD_T + \frac{\partial U}{\partial D_A} dD_A \qquad (8.1)$$

Then, the marginal utility of product j ($j = M, T, A$), $U_j = \frac{\partial U}{\partial D_j}$.

$$\text{Alternatively: } dU = U_M dD_M + U_T dD_T + U_A dD_A \qquad (8.2)$$

$$\text{and, } \frac{dU}{U_A} = \frac{U_M}{U_A} dD_M + \frac{U_T}{U_A} dD_T + dD_A \qquad (8.3)$$

The utility maximizing behaviour implies that, $\frac{U_M}{U_A} = \frac{P_M^*}{P_A} = P_M$ (as, $P_A = 1$). Similarly, we can say that, $\frac{U_T}{U_A} = P_T^*$. Let $d\Omega$ denotes the change in utility in terms of agricultural product. So, $\frac{dU}{U_A}$ (i.e., $d\Omega$) is the change in welfare in terms of agricultural product. Putting these values in equation (8.3) one gets:

$$d\Omega = P_M^* dD_M + P_T^* dD_T + dD_A \qquad (8.4)$$

$$= P_M^* dX_M + P_T^* dX_T + dX_A - r_F dF - F dr_F \text{ }^5$$

$$= P_M^* dX_M + P_T^* dX_T + dX_A - r_F dF - F dr_F \tag{8.5}$$

$$= S dw_S + K dr + L dw + r_F dF + F dr_F - r_F dF - F dr_F {}^6$$

$$= S dw_S + K dr + L dw \tag{8.6}^7$$

And, $\frac{d\Omega}{dF} = S \frac{dw_S}{dF} + K \frac{dr}{dF} + L \frac{dw}{dF}$

$$= \frac{dr}{dF} S \left(\frac{K}{S} - \frac{a_{KM}}{a_{SM}} \right) + L \frac{dw}{dr} \frac{dr}{dF} \text{ (see appendix 6)}$$

Here, it can be noted that the manufacturing sector is relatively capital intensive than the agricultural sector. This implies that the ratio of domestic capital to skilled labour, required to produce one unit of manufacturing sector, is higher than the ratio of domestic capital to skilled labour. In other words, $\frac{K}{S} - \frac{a_{KM}}{a_{SM}} < 0$. Again, $\frac{dr}{dF} < 0$, and so, $\frac{dr}{dF} S \left(\frac{K}{S} - \frac{a_{KM}}{a_{SM}} \right) > 0$. On the other hand, as, $\frac{dw}{dr} < 0$, so, $L \frac{dw}{dr} \frac{dr}{dF} > 0$, and finally, $\frac{d\Omega}{dF} > 0$.

The following proposition is immediately derived from the above result.

Proposition 3: Foreign-capital inflow in a small open economy makes it better off in terms of social welfare in the absence of tariff.

5. **Concluding Remarks**

The main objective of the present exercise is to develop a model to find out the impact of foreign-capital inflow on welfare of a small open economy, with agriculture, domestic manufacture and foreign enclave being the three sectors of the economy. Foreign capital is specific to only foreign enclave. The work finds that though the inflow of foreign capital increases the skilled-unskilled wage-gap and declines the rental rate of domestic capital, the entire labour force becomes better off in the absence of any type of

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[5]As the budget constraint of the economy, given that foreign capital is fully repatriated, gives: $P_M^* D_M + P_T^* D_T + D_A = P_M^* X_M + P_T^* X_T + X_A - r_F F$. Then by differentiating both sides: $P_M^* dD_M + P_T^* dD_T + dD_A = P_M^* dX_M + P_T^* dX_T + dX_A - r_F dF - F dr_F$.

[6] $P_M^* X_M + P_T^* X_T + X_A = w_S S + rK + wL + r_F F$ (as national product at market price is equal to the national product at factor cost). Hence, $P_M^* dX_M + P_T^* dX_T + dX_A = Sd_S + Kdr + Ldw + r_F dF + F dr_F.$

[7] It may be noted that the change in social welfare in terms of agricultural product is nothing but the change in net national product at factor cost (no change has taken place in the endowments of domestically owned factors of production).

market distorting policies adopted by the state. It implies that, in a small open economy, foreign direct investment is beneficial when the market is allowed to work freely. This is a departure from the traditional model of Brecher-Alejandro (1977) which says that, in a free market system, the introduction of investment liberalization leaves the level of national welfare unchanged in the absence of tariff. Here, we see that, the instillation of the sector-specific foreign capital makes the country better off in the absence of tariff, whereas, the work of Beladi and Marjit (1992a) has shown that Brecher-Alejandro's (1977) proposition remains valid when sector-specific foreign capital is introduced. Our model shows that an economy, experiencing an expansion of export sector and a contraction of import-competing sector, in fact enjoys a welfare gain. The result is something new in the context of existing literature in this field.

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Appendix 1

From equation 4.1.1, we have,

$$g_1(r)F + a_{SM}(r)X_M = S$$

So, $dr(Fg_1^I + X_M a_{SM}^I) = -a_{SM}(r)dX_M$ (By differentiating totally)

$$\text{So,} \frac{dr}{dX_M} = -\frac{a_{SM}(r)}{(Fg_1^I + X_M a_{SM}^I)}$$

As, $a_{SM}(r) > 0$, $a_{SM}^I > 0$ *and* $g_1^I > 0$, so, $\frac{dr}{dX_M} < 0$ (SS curve is negatively sloped).

Similarly, from equation 6.1.1, we have,

$$Lg_2(r) + a_{KM}(r)X_M = K$$

Total differentiation of this expression gives,

$$dr[Lg_2^I(r) + X_M a_{KM}^I] = -a_{KM}(r)dX_M$$

$$\text{So,} \frac{dr}{dX_M} = \frac{-a_{KM}(r)}{Lg_2^I(r) + X_M a_{KM}^I}$$

*A*s, $a_{KM}, L, X_M > 0$, and, a_{KM}^I, $g_2^I < 0$, so, $\frac{dr}{dX_M} > 0$ (KK curve is positively sloped).

Appendix 2

Equation 4.1.2 gives,

$$Fg_1^I dr + X_M a_{SM}^I dr + a_{SM}dX_M = -g_1 dF$$
$$\text{So, } Kg_2^I dr + X_M a_{KM}^I dr + a_{KM}dX_M = 0 \text{ (by total differentiation)}$$

In vector matrix form we can write it as,

$$\begin{bmatrix} Fg_1^I + X_M a_{SM}^I & a_{SM} \\ Kg_2^I + X_M a_{KM}^I & a_{KM} \end{bmatrix} \begin{bmatrix} dr \\ dX_M \end{bmatrix} = \begin{bmatrix} -g_1 dF \\ 0 \end{bmatrix}$$

Applying Cramer's rule (as $\Delta = 0$) we find,

$$dr = \frac{1}{\Delta} \begin{vmatrix} -g_1 dF & a_{SM} \\ 0 & a_{KM} \end{vmatrix} = \frac{1}{\Delta}(-g_1 a_{KM} dF)$$

As, g_1, a_{KM} and $dF > 0$, hence $dr < 0$, so, $\frac{dr}{r} < 0$ ($\hat{r} < 0$). Now, $\frac{dr}{dF} = -\frac{g_1}{\Delta} a_{KM}$, so, $\frac{dr}{dF} < 0$, or, $\frac{\hat{r}}{\hat{F}} < 0$.

Appendix 3

Equation 6.1.1 shows,

$$a_{SM} w_S + a_{KM} r = P_M.$$

Differentiating totally, $a_{SM} dw_S + a_{KM} dr + w_S da_{SM} + r da_{KM} = 0$.

Now the least-cost-factor-combination for production of M requires,

$$MRTS_{SK}^M = -\frac{w_S}{r};$$

$$\text{or, } \frac{da_{KM}}{da_{SM}} = -\frac{w_S}{r};$$

$$\text{or, } w_S da_{SM} + r da_{KM} = 0.$$

Substituting this value in equation 1.1, one gets,

$$0 = a_{SM} dw_S + a_{KM} dr$$

$$= \frac{dw_S}{w_S} \frac{a_{SM} w_S}{P_M} + \frac{dr}{r} \frac{a_{KM} r}{P_M}$$

$$= \theta_{SM} \widehat{w}_S + \theta_{KM} \hat{r} \text{ (dividing both sides by } P_M)$$

$$\text{So, } \widehat{w}_S = -\frac{\theta_{KM}}{\theta_{SM}} \hat{r}$$

As $\theta_{ij} > 0$ and $\hat{r} < 0$, we can say, $\widehat{w}_S > 0$.

Now, for sector A, we can write, $\widehat{w} = -\frac{\theta_{KM}}{\theta_{LM}}\hat{r}$. Similarly, using the least-cost-factor-combination from sector T, we have, $\hat{r}_F = -\frac{\theta_{KM}}{\theta_{LM}}\widehat{w}_S$. As $\widehat{w}_S > 0$ and $\frac{\theta_{ST}}{\theta_{FT}} > 0$, we can say that, $\hat{r}_F < 0$.

Appendix 4

Equilibrium condition (equation 4) in the unskilled labour market implies, $a_{LA}X_A = L$. So, $\hat{a}_{LA} + \hat{X}_A = 0$. Alternatively, $\hat{X}_A = -\hat{a}_{LA}$ (as there is no change in labour endowment of the country). Now, the elasticity of substitution in sector A can be expressed as, $\sigma_A = \frac{\hat{a}_{KA} - \hat{a}_{LA}}{\widehat{w} - \hat{r}}$. In other words, $\hat{a}_{KA} - \hat{a}_{LA} = \sigma_A(\widehat{w} - \hat{r})$. On the other hand, the efficiency in production gives,

$$\frac{da_{KA}}{da_{LA}} = -\frac{w}{r}$$

So, $0 = wda_{LA} + rda_{KA}$

$$= \frac{wa_{LA}}{P_A}\frac{da_{LA}}{a_{LA}} + \frac{ra_{KA}}{P_A}\frac{da_{KA}}{a_{KA}}$$
$$= \theta_{LA}\hat{a}_{LA} + \theta_{KA}\hat{a}_{KA}$$
Or, $\hat{a}_{KA} = -\frac{\theta_{LA}}{\theta_{KA}}\hat{a}_{LA}$

Putting the value of $\hat{a}_{KA}$ in the expression of σ_A gives,

$$\sigma_A(\widehat{w} - \hat{r}) = -\hat{a}_{LA}\left(1 + \frac{\theta_{LA}}{\theta_{KA}}\right)$$

$$= -\hat{a}_{LA}\left(\frac{\theta_{LA} + \theta_{KA}}{\theta_{KA}}\right)$$

So, $\hat{a}_{LA} = -\theta_{KA}\sigma_A(\widehat{w} - \hat{r})$ (as $\theta_{LA} + \theta_{KA} = 1$)

We found earlier, $\widehat{w} > 0$, $\hat{r} < 0$, $\theta_{KA} > 0$ and $\sigma_A > 0$, implying $\hat{a}_{LA} < 0$ (as $\hat{X}_A = -\hat{a}_{LA}$). Hence, we can say that, $\widehat{X_A} > 0$, so, $\widehat{X_A}\big/\hat{F} > 0$.

Appendix 5

From equation 5 we have, $a_{SM}X_M + a_{ST}X_T = S$.

So, $\frac{a_{SM}X_M}{S}\frac{dX_M}{X_M} + \frac{a_{SM}X_M}{S}\frac{da_{SM}}{a_{SM}} + \frac{a_{ST}X_T}{S}\frac{dX_T}{X_T} + \frac{a_{ST}X_T}{S}\frac{da_{ST}}{a_{ST}} = 0$ (differentiating and dividing both sides by S)

So, $\lambda_{SM}\hat{X}_M + \lambda_{SM}\hat{a}_{SM} + \lambda_{ST}\hat{X}_T + \lambda_{ST}\hat{a}_{ST} = 0$

Or, $\lambda_{ST}\hat{X}_T = -\lambda_{SM}\hat{X}_M - \lambda_{ST}\hat{a}_{ST} - \lambda_{SM}\hat{a}_{SM}$

The elasticity of substitution for sector M is,

$$\sigma_M = \frac{\hat{a}_{KM} - \hat{a}_{SM}}{\hat{w}_S - \hat{r}}$$

Or, $\hat{a}_{KM} - \hat{a}_{SM} = \sigma_M(\hat{w}_S - \hat{r})$

For the least-cost-factor-combination we require,

$$\theta_{SM}\hat{a}_{SM} + \theta_{KM}\hat{a}_{KM} = 0$$

Or, $\hat{a}_{KM} = -\hat{a}_{SM}\frac{\theta_{SM}}{\theta_{KM}}$

Substituting the value in the expression of σ_M we get,

$$-\left(1 + \frac{\theta_{SM}}{\theta_{KM}}\right)\hat{a}_{SM} = \sigma_M(\hat{w}_S - \hat{r})$$

Or, $\hat{a}_{SM} = -\theta_{KM}(\hat{w}_S - \hat{r})$

Similarly, from the least-cost-factor-combination of sector T, we find,

$$\hat{a}_{ST} = -\theta_{ST}\sigma_T(\hat{w}_S - \hat{r}_F)$$

So, $\lambda_{ST}\hat{X}_T = -\lambda_{SM}\hat{X}_M + \lambda_{SM}\theta_{KM}(\hat{w}_S - \hat{r}) + \lambda_{ST}\theta_{ST}\sigma_T(\hat{w}_S - \hat{r}_F)$ (Substituting the values of $\hat{a}_{SM}$ and $\hat{a}_{SM}$).

As, $\theta_{ij}, \lambda_{ij}$ and $\sigma_j > 0$; $(\hat{w}_S - \hat{r})$, $(\hat{w}_S - \hat{r}) > 0$ and $\hat{X}_M < 0$, hence, we can say that, $\hat{X}_T > 0$ and so, $\frac{\hat{X}_T}{\hat{F}} > 0$.

Appendix 6

Equation 8.6 gives $d\Omega = Sdw_S + Kdr + Ldw$. Dividing both sides by dF, one gets,

$$\frac{d\Omega}{dF} = S\frac{dw_S}{dF} + L\frac{dw}{dF} + K\frac{dr}{dF}$$

$$= S\frac{dw_S}{dr}\frac{dr}{dF} + K\frac{dr}{dF} + L\frac{dw}{dr}\frac{dr}{dF}$$

$$= S\frac{dr}{dF}\left(\frac{dw_S}{dr} + \frac{K}{S}\right) + L\frac{dw}{dr}\frac{dr}{dF}$$

Now,

$$a_{SM}dw_S + a_{KM}dr = 0 \text{ (from Appendix 3)}$$

$$\text{So, } \frac{dw_S}{dr} = -\frac{a_{KM}}{a_{SM}}$$

$$\Rightarrow \frac{d\Omega}{dF} = S\frac{dr}{dF}\left(\frac{K}{S} - \frac{a_{KM}}{a_{SM}}\right) + L\frac{dw}{dr}\frac{dr}{dF}.$$