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Market Dynamics When Participants Rely on Relative Valuation

Sean Lavelle

Abstract
Relative-valuation is a technique whereby financial analysts estimate the value of an asset by comparing it to its peers. The author formalizes the decision-making structure of a relative-valuation strategy and simulate a market defined by its use. He finds that when the distribution of peer valuation-multiples is skewed high or low, the market will tend to equilibrate over or undervalued, respectively. He furthers this analysis by looking at the effect that subjective analyst adjustments of market multiples might have and concludes that they have the potential to be highly destabilizing.

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Keywords Relative valuation; inefficient; EMH; simulation; comparative valuation

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I. Introduction

This paper examines the dynamics of an asset market when participants use relative valuations to make decisions. Relative valuation is the process by which analysts estimate how much an asset is worth by comparing it to its peers. The prevalence of relative valuation as a decision making tool has potentially large implications on the Efficient Market Hypothesis. On the one hand, by using relative valuations, investors and analysts must assume that the market provides correctly valued peer assets on which to base the assessment. On the other hand, if most participants are relying on this assumption, and few people are using fundamental analysis of firm value, it is unclear whether this assumption will remain valid. There is an extensive literature on the accuracy of relative valuation, but absent are explorations into the larger market dynamics that result from its use.

Our theory can be applied in assessing the potential for market-wide departures from efficient valuation and informing analysts about the peril of a preferred technique. Damodaran (2002) estimates that ninety-percent of all equity research valuations use some form of relative valuation. These valuations require less information gathering than, for example, Discounted Cash Flow (DCF) valuations. There are also fewer explicit assumptions that the analyst has to generate. And many papers[1] have found evidence that relative valuation strategies are capable of producing excess returns.

Under the costly information framework posited in Grossman and Stiglitz (1980), there is clearly available justification for a rational investor to utilize relative valuations to make

[1] Zhi and Schaumburg (2011), Mînjina (2009), and Alford (1992)

decisions. This paper will explore whether those individually rational decisions can lead to undesirable market outcomes.

II. The Model (Simple-Relative Valuation)

The internet bubble of the late-90s shows us a possible example of relative valuation leading to negative outcomes. The early market consisted of high-growth firms forging a completely new industry. Implicit in the novelty of the market was an absence of historical data to make strong forecasts of performance. Instead, investors had little on which to base decisions besides relative valuation.

To perform relative valuation, an analyst must find firms that are similar to the firm in question and then identify the factor that accounts for the differing valuations between them. That factor can be anything from EBITDA to numbers of customers to operating income. If the firm in question seems to be undervalued relative to its peers, the analyst recommends his employer buy the firm (and vice-
versa).

The current multiple, M_f, of any given firm is calculated in a straightforward manner with (1), below. P_f is the price of the firm and I_f is the value of the selected indicator.

(1) $$M_f = \frac{P_f}{I_f}$$

To perform a relative valuation, the analyst must ascertain which M_f would imply an appropriate value for the firm. She does this by taking the average M_f of several similar firms, often members of the same sector. For an analysis with N compared firms, the formula for an arithmetic average is described in equation (2a), below. M_c is the average multiple of the compared firms.

(2a)
$$M_c = \frac{\sum_{f=1}^{f=N}(M_f)}{N}$$

While the arithmetic average is a valid way to calculate M_c, Beatty, Riffe and Thompson (1999) established that harmonic averaging yields more accurate predictions of price. (2b) shows M_c calculated via harmonic averaging.

(2b)
$$M_c = \frac{N}{\sum_{f=1}^{f=N}\left(\frac{I_f}{P_f}\right)}$$

For the firm in question, a market participant compares M_c to its current M_f. If M_f is greater than M_c, the trader tries to sell the firm. If M_f is less than M_c, the trader tries to buy the firm. Assuming that enough investors are using the same relative-valuation metrics and so trading similarly, aggregate demand and supply will be significantly impacted. This will move P_f up or down, appropriately. (3) shows the postulated market result from this decision-making framework, where t is time. We denote price movement by C.

(3)
$$M_{f_{t+1}} = \begin{cases} M_{f_t} < M_c: \frac{P_{f_t}+C}{I_{f_t}} \\ M_{f_t} > M_c: \frac{P_{f_t}-C}{I_{f_t}} \\ M_{f_t} = M_c: M_{f_t} \end{cases}$$

C is a function of the proportion of traders basing decisions on relative valuation, *R*, and the proportion of all other traders who make the same decisions as relative value traders, *O*. $C(R,O)$ is proportional to the sum of *R* and *O*: $C(R,O) \propto R + O$.

If every investor were using the same framework and the same objective figures, there would be no buying or selling in time $t+1$, unless C were large enough to make M_f equal to M_c. There would be nobody to take the opposite side of the proposed trades. Given this reality, we can alternatively think of (3) as describing a bidding process, where, when the firm is undervalued, buyers offer P_f+C, and sellers holdout until M_f equals M_c, when they are indifferent between holding or selling the asset.

From (3), it is apparent that M_f will converge to a fixed M_c iteratively, over time. If included in the calculated M_c are a few firms that have very large, speculative valuations (ie the early-90s dot-com industry), they will pull M_c upward. Since M_f of the firm being traded will converge to this elevated M_c, it will likely become overvalued.

What is less apparent is what will happen if traders are applying relative valuations to all *N* firms at the same time. As all firms' prices change, M_c will change. The amount that M_c will change is defined in (4):

(4) $$\Delta M_c = \frac{\sum_{N=1}^{N=f} \Delta M_f}{N}$$

If the sum of changes in M_f is greater in direction A than the sum of changes in direction B, M_c will move in direction A. Take a scenario where we have five firms, four of which have equal, low M_f's and one of which has a high M_f. All firms will move toward M_c, which will be somewhere between the group of high and low firms. There will be more firms moving up than down, so M_c will usually also move up.[2] While M_c starts nearer the majority of firms, relative valuation creates a situation where a minority of firms drag the overall market valuation in their direction.

As firms' valuations increase and decrease, there may be a stabilization where M_f*'s* for all firms are equal at some middle point. So while the market may not be valued correctly, it will at least not be continuously volatile. The same cannot be said when analysts perform more complex-relative valuations.

III. Relative Valuation with Adjustments

Damodaran (2006) discusses the common practice of adjusting an individual firm's target M_f based on a qualitative perception of growth potential. For example, an analyst might decide that because a firm is expected to announce a new product line, its M_f ought to be ten-percent higher than M_c.

If we denote the adjustment for firm *f* as A_f (which would equal 1.1 in our above example), (5) shows the possible market outcomes in time *t*+1, below. Each trader's estimated adjustment will likely be different, but our market A_f can be thought of as overall market sentiment surrounding a firm.

(5)
$$M_{f_{t+1}} = \begin{cases} \frac{M_{f_t}}{A_f} < M_c : \frac{P_{f_t}+C}{I_{f_t}} \\ \frac{M_{f_t}}{A_f} > M_c : \frac{P_{f_t}-C}{I_{f_t}} \\ \frac{M_{f_t}}{A_f} = M_c : M_{f_t} \end{cases}$$

[2] The only way that this would not be the case is if the M_f of the high firm moved much more quickly than those of the low firms. In our example, the high M_f would have to move 4 times as quickly as the low M_f's in order to leave M_c stable. We have no reason to believe this would often be the case, so for simplification, *C* is the same for all firms in our model.

From the analyst's perspective, this more complex decision structure seems an improvement. They are able to account for more factors than a single I_f. Unfortunately, the change may also induce even more market-wide misvaluation than does simple-relative valuation.

When the analyst adjusts the target M_f by A_f, they typically do not similarly adjust each measured M_f that is being used to calculate M_c. Doing so would negate the search cost advantages of relative valuation and present standardization difficulties.

Let's assume that we start at time t in a condition where M_f and P_f are equal for all firms. With a simple-relative valuation, the market would remain in equilibrium as all M_f's would equal M_c. In a complex-relative valuation, firms with an A_f greater than 1 would increase in value at time t+1.

If analysts ascribed $A_f>1$ to more firms than they did $A_f<1$, M_c would increase in time t+1. If $A_f>1$ firms are again undervalued when compared to the elevated M_c at time t+1, there is potential for this process to eventually pull up the valuations of even firms with $A_f<1$. A runaway market very similar to what was observed in the dotcom bubble at the turn of the century could quickly occur. The easiest way to study the dynamic is with simulation.

IV. Simulations

To extend this analysis beyond the initial starting conditions that we have already discussed, we need to utilize Monte Carlo simulation. Our simulations will start with a set of firms, assigned randomized initial values for P_f and I_f. We will conduct a total of four, ten-thousand iteration simulations. The rules applied to each are displayed in *Table 1*.

TABLE 1—SUMMARY OF SIMULATIONS AND RULES

	Arithmetic Average	**Harmonic Average**
Simple-Relative Valuation	*Simulation 1*: Equations *(2a)* + *(3)*	*Simulation 3*: Equations *(2b)* + *(3)*
Complex-Relative Valuation	*Simulation 2*: Equations *(2a)* + *(4)*	*Simulation 4*: Equations *(2b)* + *(4)*

Our simulations will be set up as follows:

i) 10 firms will be established. This number is small enough so that when the starting conditions are randomly established, the distributions will be sufficiently varied to allow for study of which conditions impact results.

ii) C, the amount P_f changes each iteration, will be assigned a value of 1.

iii) Each firm will be assigned an I_f by sampling a random normal distribution with mean 500 and standard deviation 100. The distribution will be created by the Math.random() function in Javascript and the *Box-Muller transform*.

iv) Each firm's I_f will be multiplied by a different uniformly distributed random variable, M_f, from 0-1 in order to calculate P_f. The random variable will be created by the Math.random() function in Javascript.

v) (for complex-relative valuation simulations) Each firm will be assigned a random A_f sampled from a uniform distribution between .9 and 1.1. The random variable will be created by the Math.random() function in Javascript.

vi) Each I_f will be unchanging throughout the Monte Carlo iteration and we assume that each firm is accurately valued at the beginning of the simulation.

Once the time t=0 market conditions are established, our simulations will deterministically iterate through the rules that we established for either simple or complex-relative valuations. Our simulations will first calculate the initial market-wide M_c. *Simulations 1 and 2* will utilize (2a) to calculate M_c, while *Simulations 3 and 4* will utilize (2b). Each firm's measured M_f (or M_f divided by A_f for *Simulations 2 and 4*) will then be compared to an M_c that does not include itself.

P_f will then change appropriately, based on the results of the comparison. After all firms have been evaluated using the appropriate relative valuation method, M_c will be recalculated and *Equations 3 and 4* will applied again. This process will repeat 350 times per Monte Carlo iteration, which allows each iteration to develop into a steady state in all four simulations.
To establish clarity around the inner workings of the simulation, we will explore vignettes from *Simulations 1 and 2*.

A. *Vignette 1 (Simple-Relative Valuation)*

Vignette 1, from *Simulation 1*, utilizes simple-relative valuation with arithmetic averaging. Its initial set of 10 M_f's has a positive Fisher Skew of 0.711. Figure 1 shows the change in every M_f and the market M_c, over time.

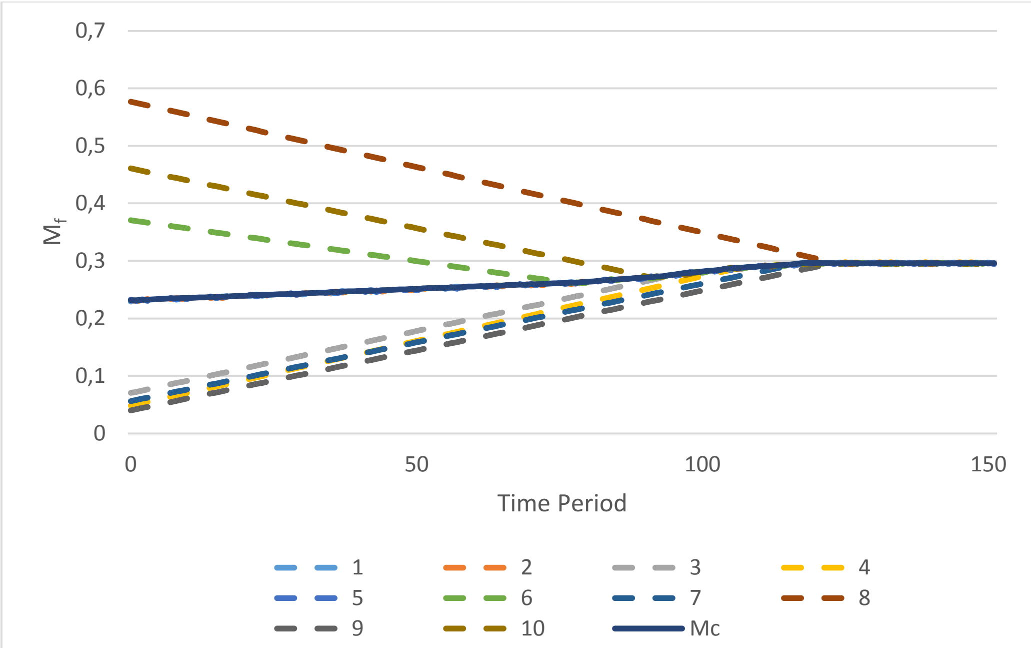

FIGURE 1. SIMPLE-RELATIVE VALUATION: M_F OVER TIME

At the beginning of this simulation, there are 3 firms with M_f above the solid M_c line, and 4 firms with M_f below M_c. The remaining 3 are approximately equal to M_c. Every M_f steadily converges to the M_c line. Since there are more upwardly changing M_f's than downwardly, M_c moves upwards until equilibrium. And since we assumed that all firms were appropriately valued in time $t=0$, every firm is now misvalued at the end of the simulation. However, what is arguably most important from is whether the market as a whole is misvalued. And Figure 2 shows clearly that it is overvalued.

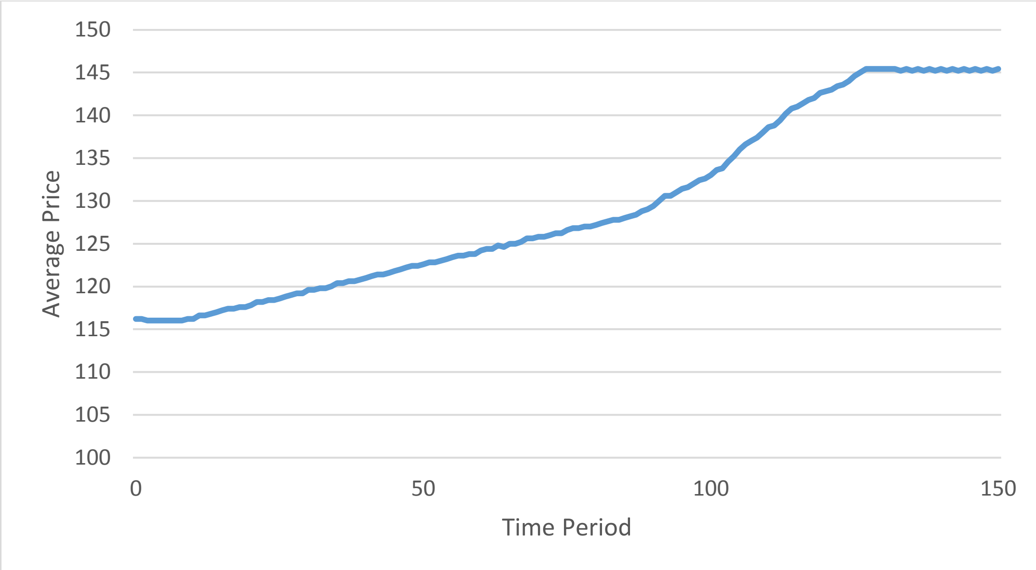

FIGURE 2. SIMPLE-RELATIVE VALUATION: AVERAGE PRICE OVER TIME

We should note that while our market does appear to be about 25 percent overvalued, it eventually reaches an equilibrium at time *t=127*. *Vignette 2*, which utilizes complex-relative valuation, will not share a similar result.

B. *Vignette 2 (Complex-Relative Valuation)*

In *Vignette 2*, the skew of our initial set of M_f's is negative at -1.159. Given the theory that we have already built, we should expect this to tend to pull M_c and the market value downwards. However, eight out of ten A_f values are greater than 1, which suggests a likely increase in value. To see how these competing factors play out, we must look to Figure 3:

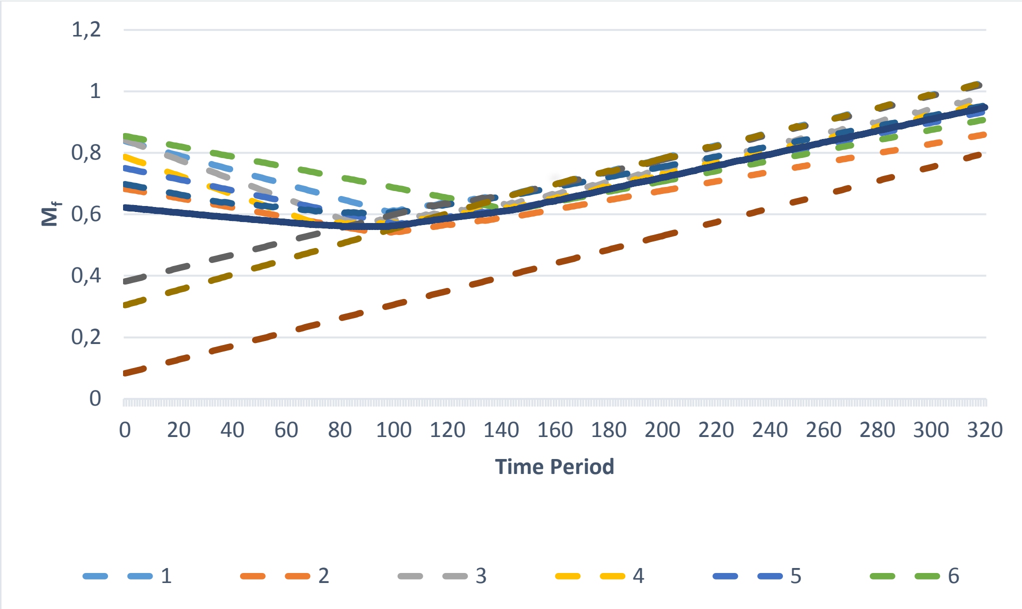

FIGURE 3. COMPLEX-RELATIVE VALUATION M_F OVER TIME

The solid line that starts generally in the middle is M_c. We can see that seven M_f's start above M_c, while only three start below, yielding our negative skew. At the outset, this negative skew leads toward a steady decline in M_c. As each firm begins to coalesce around an equilibrium, though, the effect from our A_f's begins to dominate the process. After this point, M_c and every M_f begin a climb that will continue *ad infinitum*.

The key difference between these two vignettes (and between simple-relative valuation and complex-relative valuation) is the nature of their equilibria. A market with analysts conducting simple-relative valuations, initiating in disequilibrium, will quickly move toward a steady equilibrium characterized by unchanging M_f's and P_f's, even if those values diverge from the firms' true values. A market where analysts perform complex-relative valuations will move toward that stable equilibrium, but once it is nearly reached, the adjustments that analysts proscribe will dominate the process until we reach an equilibrium characterized by values changing in a constant manner. Both processes can easily lead to inaccurate valuations, but complex-relative valuation seems more capable of producing wildly inaccurate prices.

V. Statistical Models and Hypotheses

The fundamental questions we are trying to answer are which conditions lead to misvaluation of a market and which determine the direction of the misvaluation. To that end, we calculate two key variables from our Monte Carlo simulations to test as independent variables in linear regression: directional change in market valuation (MV_d), and absolute change in market valuation (MV_a). We define MV_d as the natural log of the average price at the final time period divided by the average price at $t=1$. We define MV_a as the absolute value of MV_d. (6) defines MV_d where we simulate 350 time periods.

(6)
$$MV_d = \ln\left(\frac{\sum_{f=1}^{f=N} P_{f_{350}}}{\sum_{f=1}^{f=N} P_{f_1}}\right)$$

We hypothesize that the key determinants of these two independent variables will be the standard deviation, Fisher skewness, the absolute value of Fisher skewness, and the Fisher kurtosis of the initial set of M_f's. For complex-relative valuation, we expect the proportion of A_f's will values over 1 will be very impactful. And for all four simulations, we will test to see whether the correlation between the initial set of P_f's and I_f's makes a difference. Table 2 lists all variables along with descriptions.

TABLE 2—LIST OF VARIABLES

Variable Symbol	Independent/ Dependent	Description
MV_d	Independent	The natural log of the average price at the end of the simulation divided by the average price at the beginning of the simulation
MV_a	Independent	The absolute value of MV_d
Stdev	Dependent	The standard deviation of the initial set of M_f's
Skew	Dependent	The Fisher Skewness of the initial set of M_f's
$Skew_a$	Dependent	The absolute value of *Skew*
Kurt	Dependent	The Fisher Kurtosis of the initial set of M_f's
AOO	Dependent	The proportion of A_f's over 1
AOO_a	Dependent	The absolute value of *AOO* minus .5
Corr	Dependent	The correlation between the initial P_f's and I_f's

A. Hypotheses: Simple-Relative Valuation

The two regression we will perform on results from both *Simulations 1 and 3* are below, where α is the intercept and ε_i is the error term:

Regression 1: $MV_d = \alpha + \beta_1 * Skew + \varepsilon_i$

Regression 2: $MV_a = \alpha + \beta_1 * Skew_a + \beta_2 * Kurt + \beta_3 * Corr + \beta_4 * Stdev + \varepsilon_i$

We expect *Regression 1* to indicate a positive value for β_1. The key expectation from our theory is that when the distribution is skewed in a particular direction, relative valuations will move toward that same direction.

We expect *Regression 2* to indicate positive values for β_1 for the same reason as in *Regression 1*. We expect β_2 to be positive because a distribution with a larger mass in one section and then a smaller, but further out tail, will produce both large Kurtosis values and large changes in M_f, as we saw in Vignette 1. β_3 should be negative because as indicators become better predictors of prices, prices ought to be more accurate. We should expect β_4 to be positive because a higher *Stdev* should indicate a wider distribution with larger resulting changes in M_c.

B. Hypotheses: Complex-Relative Valuation

We will perform two more regressions each on data from *Simulations 2 and 4*:

Regression 3: $MV_d = \alpha + \beta_1 * Skew + \beta_2 * AOO + \varepsilon_i$

Regression 4: $MV_a = \alpha + \beta_1 * Skew_a + \beta_2 * Kurt + \beta_3 * Corr + \beta_4 * Stdev + \beta_5 * AOO_a + \varepsilon_i$

We expect *Regression 3* to indicate the same, positive value for β_1 as in *Regression 1*. β_2 will likely be positive because firms with elevated adjustments should pull M_c up. More firms with elevated adjustments should strengthen the effect.

For *Regression 4,* we expect the same values for all coefficients as in *Regression 2,* except for β_5. β_5 will likely be positive for the same reasons we expect β_3 to be positive in *Regression 3*.

These expectations are summarized in *Table 3*.

TABLE 3—VARIABLE COEFFICIENT EXPECTATIONS

Variables	*Regression 1 + 3*	*Regression 2 + 4*
Stdev	-	Positive
Skew	Positive	-
$Skew_a$	-	Positive
Kurt	-	Positive
AOO	Positive (*Regression 3* only)	-
AOO_a	-	Positive (*Regression 4* only)
Corr	-	Negative

C. Hypotheses: Arithmetic vs Harmonic

We will qualitatively assess the difference in outcomes between arithmetic and harmonic simulations. Liu et al (2002) and several other papers indicate that we should expect more accurate valuations for harmonic simulations. This should translate to a lower MV_a with harmonic simulations versus arithmetic simulations.

VI. Results

All four simulations produced results in line with our expectations, with a few notable exceptions. Chief among those exceptions was the effect of Kurtosis on MV_a. Before we can adequately explain the unexpected results, we need to take a look at the distribution of MV_d's. For our simple, arithmetic simulation (*Simulation 1*), we saw a fairly normal, though flat, distribution of MV_d's. It had a mean of 0.0063, standard deviation of 0.1419, and kurtosis of 0.6907. *Figure 4* shows the distribution of $\mathrm{MV_d}$'s for *Simulation 1*.

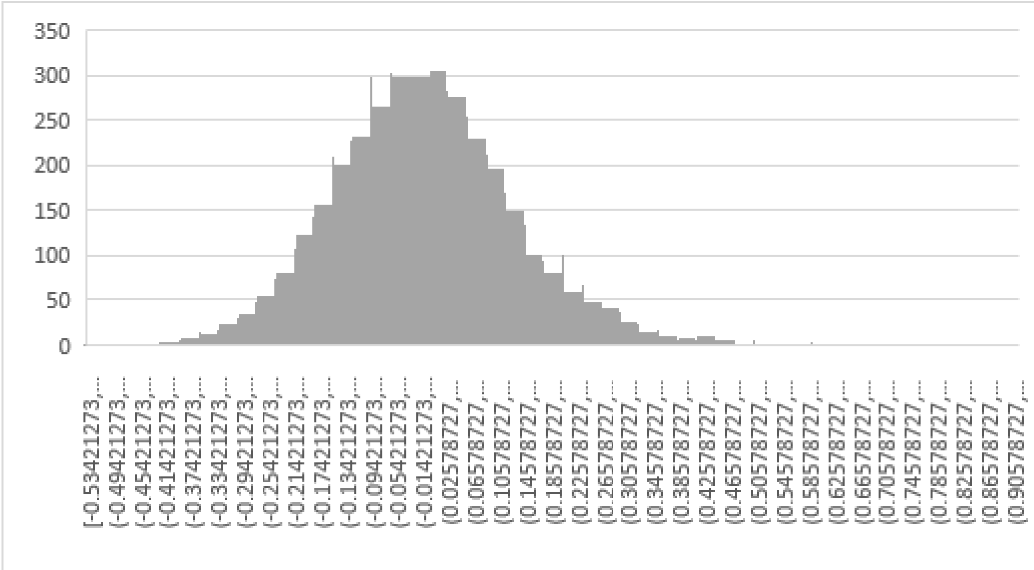

FIGURE 4. DISTRIBUTION OF SIMULATION 1 MV_D'S (AFTER 350 TIME PERIODS)

Taken in the context of *Vignette 1*, this result makes sense. There was no runaway increase in M_c that would lead us to expect a tail-heavy distribution. And all coefficients for both regressions were in line with our expectations. Our results for *Regression 1*, *Simulation 1* are summarized in *Table 4*.

TABLE 4—REGRESSION 1, SIMULATION 1 (INDEPENDENT VARIABLE IS MVD)

Constant	*.00564*
	(.00077)
Skew	*.29541*
	(.00191)
R^2	*.7062*
Observations	*10,000*

Standard Errors are reported in parentheses.

All variables are significant at the 99% level

Skew is positively correlated with MV_d, as expected. The results for *Regression 2*, *Simulation 1* are summarized in *Table 5*, which are also all in line with expectations.

TABLE 5—REGRESSION 2, SIMULATION 1 (INDEPENDENT VARIABLE IS MV_A)

Constant	*-0.06637*
	(0.00661)
$Skew_a$	*0.21593*
	(0.00378)
Kurt	*0.02096*
	(0.00230)
Corr	*-0.02494*
	(0.02055)
Stdev	*0.28571*
	(0.01557)
R^2	*.4745*
Observations	*10,000*

Standard Errors are reported in parentheses.

All variables are significant at the 99% level.

The coefficient for *Kurt* is positive, as expected. A distribution with a clear center of mass that chases an outlier, will have a higher kurtosis and MV_a.

In the interest of brevity, we will not include the regressions for *Simulation 2* or *Simulation 4,* as they are not significantly different from those of *Simulation 1* or *Simulation 3,* respectively. The results for *Simulation 3*, though, were far different than those of *Simulation 1*. Our distribution of MV_d's at iteration 350 of *Simulation 3* was extremely tail heavy. For *Simulation 1*, we used MV_d to display the distribution, which takes the natural log of average price at the end of the simulation divided by the average price at the beginning. For *Simulation 3,* it is easier to visualize the result with the unlogged version, as shown in *Figure 5*.

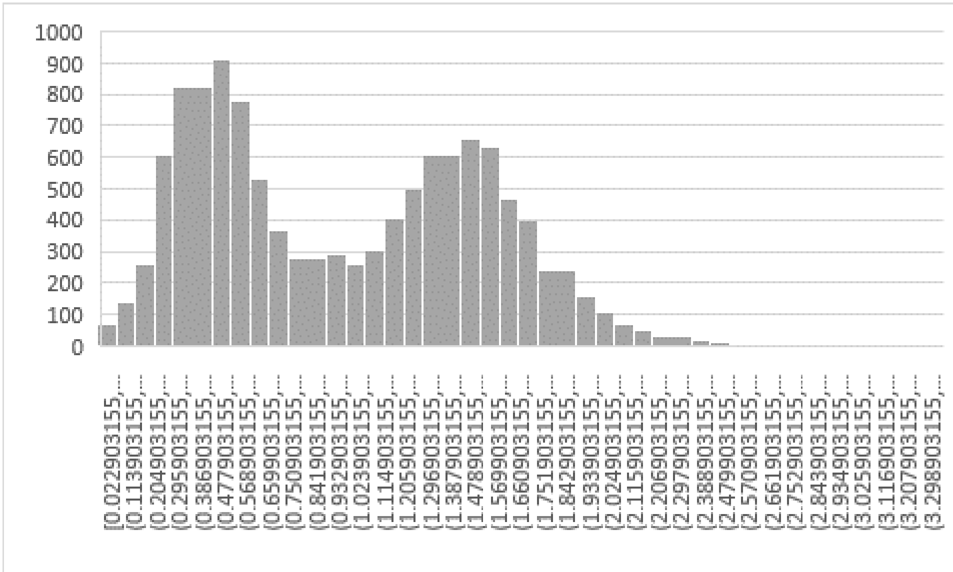

FIGURE 5. DISTRIBUTION OF SIMULATION 3 UNLOGGED MV_D'S (AFTER 350 TIME PERIODS)

The two-tailed nature of the distribution suggests that our conclusions from observing *Vignette 2* are generally correct. Complex-relative valuation leads to extreme valuations that will move in one direction for as long as they are allowed to continue. As A_f's across the market move to one direction, this effect becomes stronger. We can see this in the results from *Simulation 3*'s *Regression 3* and *4* in *Table 6*.

TABLE 6—REGRESSION 3, SIMULATION 3 (INDEPENDENT VARIABLE IS MV_D)

Constant	*-1.58278*
	(0.01501)
Skew	*0.25776*
	(0.01127)
AOO	*2.89873*
	(0.02854)
R^2	*.5184*
Observations	*10,000*

Standard Errors are reported in parentheses.

All variables are significant at the 99% level

The skew still mattered, as in *Simulation 1*, but the percentage of firm's with A_f greater than 1 was by far the most predictive factor in which direction MV_d moved, as indicated by t-values. This result was not surprising. What was surprising was the result from *Regression 4 (see Table 7)*:

TABLE 7—REGRESSION 4, SIMULATION 3 (INDEPENDENT VARIABLE IS MV_A)

Constant	*1.6069*
	(0.03574)
AOO_a	*0.87907*
	(.03394)
$Skew_a$	*0.12499*
	(0.01967)
Kurt	*-0.16522*
	(0.01227)
Corr	*-0.17558*
	(0.01079)
Stdev	*-3.11256*
	(0.08390)
R^2	*.1814*
Observations	*10,000*

Standard Errors are reported in parentheses.

All variables are significant at the 99% level.

Kurtosis and Standard Deviation have negative coefficients where we expected positive ones. *Figure 6* shows Kurtosis plotted against MV_a.

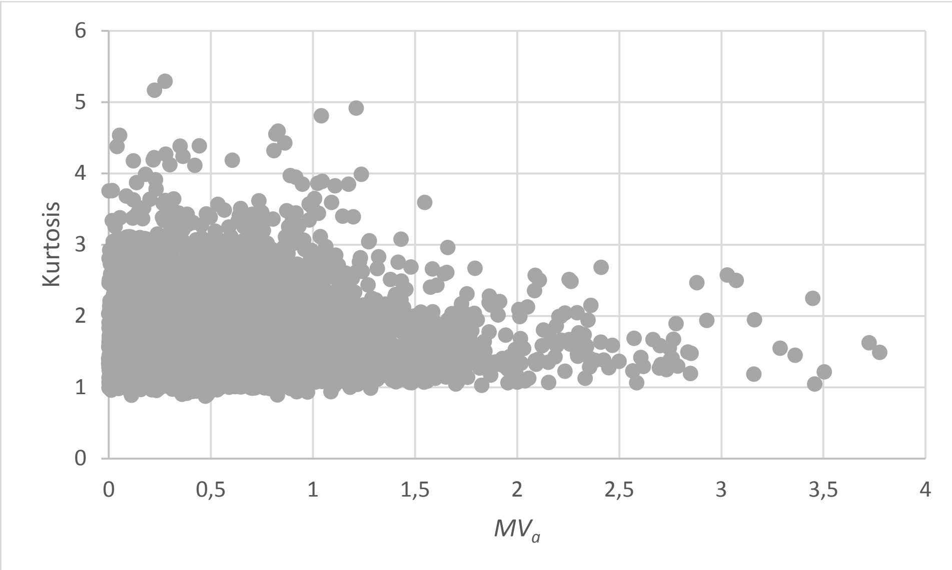

FIGURE 6. SIMULATION 3: KURTOSIS VS MV_A (AFTER 350 TIME PERIODS)

The inverse relationship between MV_a and Kurtosis is apparent at the observable extremes in *Figure 6*. But the straight line nature of the plot suggests that higher values for Kurtosis are just delaying the time period where A_f's dominate M_c movement. In other words, as Kurtosis increases, M_c moves further (as we saw in *Regression 1*) and for longer. But that movement becomes insignificant once the complex-relative valuation specific dynamics begin to drive price movement.

Furthering this theory is *Regression 5*, a regression of Kurtosis and Standard Deviation against a new variable, "Time to Equilibrium." *TTE* measures the number of time periods in a simple-relative valuation simulation it takes until the equilibrium value is reached to within one-percent.

Regression 5: $TTE = \alpha + \beta_1 * Kurt + \beta_2 * Stdev + \varepsilon_i$

The results of *Regression 5* are displayed in *Table 8.*

TABLE 8—REGRESSION 5, SIMULATION 1.1 (INDEPENDENT VARIABLE IS TTE)

Constant	*-51.36907*
	(5.92618)
Kurt	*31.41932*
	(1.57016)
Stdev	*494.8026*
	(15.13696)
R^2	*.0996*
Observations	*10,000*

Standard Errors are reported in parentheses.

All variables are significant at the 99% level

Kurtosis and Standard Deviation are both positively correlated with the length of time it takes until an equilibrium is reached. This result further supports our explanation for the negative coefficients on Kurtosis and Standard Deviation in *Regression 4*.

All OLS regressions contained significant levels of heteroscedasticity. To account for this, we reran all regressions re-specified as robust regressions, yielding no significant changes in p-values.

The other surprising result was the performance of harmonic vs arithmetic averaging (see *Table 9*). We found that arithmetic averaging leads to either equal or lesser amounts of misvaluation than harmonic averaging. Our two-sample t-test fails to reject the null hypothesis that arithmetic and harmonic averaging results are different in simple-relative valuation. But for complex-relative valuation, arithmetic averaging results in higher MV_d's and lower MV_a's. The lower MV_a for arithmetic averaging seems to contradict the literature's consensus that harmonic averaging provides more accurate estimations of value. One potential explanation is that there is simply more use of harmonic averaging in the market. Most studies measure the accuracy of the technique by comparing its predictions to market prices. If market prices are determined by harmonic averaging, then we should expect harmonic averaging to more accurately predict them.

TABLE 9—ARITHMETIC VS HARMONIC AVERAGING

Difference in Mean Results for Arithmetic vs Harmonic Averaging		MV_d		
		Mean	Standard Deviation	P-Value*
Simple Simulation	Arithmetic	0.0063	0.1420	0.1754
	Harmonic	0.0036	0.1398	
Complex Simulation	Arithmetic	-0.1309	0.6507	0.0000
	Harmonic	-0.2133	0.6758	
		MV_a		
		Mean	Standard Deviation	P-Value*
Simple Simulation	Arithmetic	0.1108	0.0890	0.2415
	Harmonic	0.1093	0.0872	
Complex Simulation	Arithmetic	0.5498	0.3718	0.0000
	Harmonic	0.5777	0.4104	

**P-Values are from independent t-test that a difference in means exists*

VII. Implications

Our theory and simulation provide evidence that the relative valuation techniques employed by financial professionals are potentially flawed when used by a large proportion of market participants. Depending on the distribution of valuation multiples present in a market sector, we might expect that market to become over or undervalued. It should be understood, though, that we have not explored the dynamics of a market where use of relative valuation methods that take into account many variables, instead of just one, are prevalent. Nor have we tested to see what happens when firms experience heterogeneous (or proportional) volatilities. These are important topics for future research, along with empirically testing our theory with real-world and experimental data.

Another important area of research to be explored is how relative-valuation fits into the evolutionary finance literature as surveyed by Evstigneev, Hens and Schenk-Hoppe (2009). No paper has formalized an agent-based simulation with traders pursuing relative-valuation as a strategy, and given its ubiquity in analyst reports, it is likely a strategy pursued by many.

New industries, spawned by technological innovation, seem likely to be particularly vulnerable to overvaluation resulting from relative valuation. The more poorly that an indicator correlates with true value, the more likely it is that market prices will diverge from true value. In a new industry, it seems probable that analysts will be more likely to choose poor indicators of value. And a few highly performing firms will establish a high market multiple for the mass of underperforming firms. When these underperforming firms rise in price to match the market multiple, the multiple will increase further, and a bubble may result. If the theory established in this paper is confirmed empirically, policymakers and market participants will need to take the distribution of key market multiples into account when determining whether a bubble is present or not. Likewise, if a market contains a few firms whose price falls dramatically, the rest may follow the lowered market multiple down.

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