# Dynamic Pricing with Reference Price Dependence 

## Régis Chenavaz


#### Abstract

A firm usually sets the selling price of a product by taking into account consumers' reference price. A behavioral pricing scheme integrating reference effects would suggest that the higher the reference price, the higher the firm can set the price. In this paper, the author investigates this intuition by accounting for reference dependence in an optimal control framework. Results show that the dynamics of price originates from the sensitivity of customers to reference price. Contrary to intuition, price dynamics is not systematically associated to the evolution of the reference price, but derives from competing effects related to the dynamics of the reference price.


JEL C61 M21 D03
Keywords Dynamic pricing; behavioral pricing; reference dependence; optimal control
Authors
Régis Chenavaz, Kedge Business School, Aix-Marseille School of Economics, CNRS \& EHESS, France, regis.chenavaz@kedgebs.com

Citation Régis Chenavaz (2016). Dynamic Pricing with Reference Price Dependence. Economics Discussion Papers, No 2016-20, Kiel Institute for the World Economy. http://www.economics-ejournal.org/economics/ discussionpapers/2016-20

```
conomics Discussion Paper
```


## 1 Introduction

Standard economics, investigating optimal pricing strategies, assumes a rational consumer for whom the selling price is the sole relevant variable related to price. Behavioral economics, understanding descriptive consumer elements, also integrates a reference price in the decision process of the consumer. A reference price is an internal personal benchmark against which the customer compares a selling price (Kalyanaram and Winer, 1995; Mazumdar et al., 2005); a selling price above the reference price looks large and reduces demand, whereas a selling price below the reference price seems low and stimulates demand (Sorger, 1988; Putler, 1992; Kopalle et al., 1996; Fibich et al., 2003; Zhang et al., 2013; Xue et al., 2014; Zhang et al., 2014). This paper accounts for consumer behavior by integrating reference prices, and it analyzes the dynamic pricing policy in this context.

In this article, I study the determinants of a dynamic pricing policy for a monopolistic firm, when descriptive aspects of consumer behavior are considered. The analysis integrates the main behavioral element of decision making, namely reference dependence (see the surveys of Kalyanaram and Winer 1995; Mazumdar et al. 2005). The analysis also considers heterogeneous consumers for which the demand depends on selling price and reference price. The reference price, a psychological variable internal to the consumer, is formally operationalized by the past observed prices, following a long tradition (Sorger, 1988; Kopalle et al., 1996; Fibich et al., 2003; Popescu and Wu, 2007; Zhang et al., 2013; Xu and Liu, 2014; Zhang et al., 2014). The literature studying the behavioral element as a driver of dynamic pricing thus informs this research.

This article belongs to the formal behavioral literature on dynamic pricing in which demand evolves adaptively on the basis of the firm's past prices. The first attempt to formalize reference price effects originates in Sorger (1988) and is followed by Kopalle et al. (1996). Fibich et al. (2003) show the advantages of continuous time formulation of reference effects. Popescu and Wu (2007) provide the first analysis with a general demand function, and establish structural results. Nasiry and Popescu (2011) study more precisely peak-end anchoring. Recent applications of reference prices include the newsvendor problem (Nagarajan and Shechter, 2013; Long and Nasiry, 2014), deteriorating items (Xue et al., 2014), supply chain (Zhang et al., 2013; Xu and Liu, 2014; Zhang et al., 2014), and inventory systems (Li et al., 2014; Lu et al., 2014). A common point of the aforementioned research is the characterization of the intertemporal equilibrium of the selling price, which is of theoretical interest since product life cycles are supposed infinite or relatively long.

Of practical interest though, the life cycle of most products is finite and relatively short. Thus within a few years period, the intertemporal equilibrium is unlikely to arise in a managerial situation, but the dynamics of price plays a major
role worth studying through the optimal path. In contrast to previous research mainly focusing on the intertemporal equilibrium, this article is primarily interested in characterizing the explicit dynamics of the selling price along the optimal path. In the study and following the seminal method of Popescu and Wu (2007), I use a general demand function to establish results linked to the sole properties of the demand function, and independent from any parameter specification. Further and in the vein of the pioneering approach of Fibich et al. (2003), I take advantage of continuous time formulation. Based on optimal control, the simpler modeling enables the characterization of qualitative properties of the optimal path of pricing policy.

In this article, the results support three claims. First, the price elasticity of demand is higher than one in each instance in which customers are sensitive to reference price. Second, selling price dynamics is decomposed between four competing effects, two linked to references price dynamics and two tied to anchoring adjustment. Third, the dynamics of price are not systematically associated to the dynamics of the reference price, and the conditions of association are provided. By integrating descriptive aspects of customer behavior, this paper offers a better understanding of a successful firm pricing policy. A firm ignoring future implications of the behavioral element on its pricing policy will charge inadequately for its products, and lose profit.

## 2 Modeling Framework

I model the behavior of a firm in a monopoly situation. The horizon of the firm $T$ is finite and the time $t \in[0, T]$ is continuous. I describe here how consumers decide to purchase a product on the basis of selling price and reference price.

### 2.1 Reference Price

A reference price $r(t)$ is an anchor (or benchmark) against which customers compare the current selling price $p(t)$ (Kalyanaram and Winer, 1995; Mazumdar et al., 2005). In most research, consumers build the current reference price as the continuous weighted average of the past selling prices (Winer, 1986; Sorger, 1988; Fibich et al., 2003; Popescu and Wu, 2007; Aflaki and Popescu, 2013; Xue et al., 2014; Zhang et al., 2014). For an exponentially decaying function with $\beta$ being the continuous speed adjustment parameter (also known as the forgetting rate or memory parameter) and $r_{0}$ being the initial reference price at $t=0$, we have

$$
\begin{equation*}
r(t)=e^{-\beta t}\left(r_{0}+\beta \int_{0}^{t} e^{\beta s} p(s) d s\right), \quad \beta, t \geqslant 0 . \tag{1}
\end{equation*}
$$

```
conomics Discussion Paper
```

Differentiate (1) with respect to time $t$ yields the dynamic of the reference price:

$$
\begin{equation*}
\frac{d r(t)}{d t}=\beta(p(t)-r(t)), \tag{2}
\end{equation*}
$$

with the initial reference price condition $r(0)=r_{0}$.
Equation (2) states that the impact of the price $p(t)$ on the adaptation of the reference price $r(t)$ increases with the adjustment parameter $\beta$. In the singular case $\beta=0$, the price does not affect the reference price adaptation, and the reference price remains constant.

### 2.2 Demand Formulation

Part of the existing literature that formally models reference effects (Sorger, 1988; Kopalle et al., 1996; Fibich et al., 2003; Zhang et al., 2013; Xue et al., 2014; Zhang et al., 2014) considers a linear demand function of the price and the reference price such as

$$
\begin{equation*}
D=a-\delta p(t)-\gamma(p(t)-r(t)), \quad a, \delta, \gamma>0 . \tag{3}
\end{equation*}
$$

In this paper, in the vein of Popescu and Wu (2007), Nasiry and Popescu (2011), and Aflaki and Popescu (2013) I generalize the usual linear demand function (3). I study a general reference-dependent demand function $D>0$, which accounts for nonlinearities and dynamics in response to variations in the reference price:

$$
\begin{equation*}
D=D(p(t), r(t)) . \tag{4}
\end{equation*}
$$

For tractability, the function $D$ is assumed twice continuously differentiable as in Chenavaz $(2011,2012)$. The demand decreases (stricly) with price and increases (weakly) with reference price. Moreover, demand decreases (weakly) with price even more with a higher reference price. Assumptions of strict and weak impacts are made for technical convenience without loss of generality. Where there is no confusion, I omit now the function parameters to simplify the presentation. Formally the conditions write

$$
\begin{equation*}
\frac{\partial D}{\partial p}<0, \frac{\partial D}{\partial r} \geqslant 0, \frac{\partial^{2} D}{\partial p \partial r} \leqslant 0 . \tag{5}
\end{equation*}
$$

The general demand (4) together with the conditions (5) place little restriction on the way (selling) price and reference price affect demand. Indeed, the price effect $\partial D / \partial p<0$ and the reference effect $\partial D / \partial r \geqslant 0$ are in line with the usual linear demand (3). Further, the cross effect $\partial D^{2} / \partial p \partial r \leqslant 0$ enriches the linear demand (3) where it vanishes. (Note that the cross effect is null for the class of demand function additively separable in the selling price and the reference price, but it is active for the class of demand function multiplicatively separable, as for say $D=(a-\delta p) e^{-\gamma(p-r)}$.)

## 3 Dynamic Pricing

I model a monopolist firm in an optimal control framework. Table 1 gives the notations used throughout the article.

Table 1: Notation

| $T$ | $=$ fixed terminal time of the planning horizon, |
| :--- | :--- |
| $\rho$ | = interest rate, |
| $p(t)$ | $=$ selling price at time $t$ (decision variable), |
| $r(t)$ | $=$ reference price at time $t$ (state variable), |
| $\beta$ | $=$ adjustment speed of the reference price, |
| $d r(t) / d t$ | $=\beta(p-r)=$ reference price dynamics at time $t$, |
| $\lambda(t)$ | $=$ current-value adjoint variable at time $t$, |
| $D(p, r)$ | $=$ current demand, |
| $\pi(p, r)$ | $=p D(p, r)=$ current profit, |
| $H(p, r, \lambda)$ | $=$ current-value Hamiltonian. |

The current profit $\pi>0$ function is

$$
\begin{equation*}
\pi=p(t) D(p(t), r(t)) . \tag{6}
\end{equation*}
$$

Because I seek interior solutions (assuming that they exist), the function $\pi$ is assumed strictly concave in $p$. This assumption is common in research using a general demand function (Popescu and Wu, 2007; Nasiry and Popescu, 2011; Aflaki and Popescu, 2013).

The firm maximizes the discounted profit by finding the optimal pricing strategy that accounts for the reference price dynamics. When charging the price $p$ at each time $t$, the firm trades off current demand with future demand. Indeed, a higher selling price reduces the current demand but it increase the reference price, and thus expands future demand. In line with Popescu and Wu (2007) and Nasiry and Popescu (2011), (6) assumes away the production cost for simplicity, but all the results hold with positive production cost. With the discount rate $\rho \geqslant 0$, the problem of the firm is

$$
\begin{aligned}
& \max _{p(s) \geqslant 0, s \in[0, T]} \int_{0}^{T} e^{-\rho t} \pi(t) d t, \\
& \text { subject to } \frac{d r(t)}{d t}=\beta(p(t)-r(t)), \text { with } r(0)=r_{0} .
\end{aligned}
$$

The intertemporal profit maximization problem is solved with the necessary and sufficient optimality conditions of Pontryagin's maximum principle. On this
basis, the current-value Hamiltonian $H$ formed with the shadow price $\lambda(t)$ (or current-value adjoint variable) writes

$$
\begin{equation*}
H(p, r, \lambda)=p D(p, r)+\lambda \beta(p-r) . \tag{7}
\end{equation*}
$$

The maximum principle imposes the dynamic of $\lambda$

$$
\begin{align*}
\frac{d \lambda}{d t} & =\rho \lambda-\frac{\partial H}{\partial r} \\
& =(\rho+\beta) \lambda-p \frac{\partial D}{\partial r} \tag{8}
\end{align*}
$$

with the transversality condition $\lambda(T)=0$.
The value of $\lambda(t)$, the value of a marginal increase in the reference price $r(t)$ at time $t$, is then obtained through integrating (8) with the transversality condition $\lambda(T)=0$, which yields ${ }^{1}$

$$
\begin{equation*}
\lambda(t)=\int_{t}^{T} e^{-(\rho+\beta)(s-t)} p \frac{\partial D}{\partial r} d s \tag{9}
\end{equation*}
$$

From (9) the shadow price $\lambda$ is positive over the planning horizon. Formally, $\lambda(t) \geqslant 0 \forall t \in[0, T]$. In addition, the higher the reference effect $\partial D / \partial r$, the larger $\lambda$ is. But if the reference effect does not play out, that is $\partial D / \partial r=0$, then $\lambda$ is null.

The current value Hamiltonian $H$ obtained in (7) sums the current and future profit, and measures the instantaneous total profit of the firm at any time $t$. The firm maximizes the intertemporal profit $H$ if and only if the following necessary and sufficient first- and second-order conditions hold ${ }^{2}$

$$
\begin{align*}
& \frac{\partial H}{\partial p}=0 \Longrightarrow D+p \frac{\partial D}{\partial p}+\beta \lambda=0  \tag{10a}\\
& \frac{\partial^{2} H}{\partial p^{2}}<0 \Longrightarrow-2 \frac{\partial D}{\partial p}-p \frac{\partial^{2} D}{\partial p^{2}}>0 \tag{10b}
\end{align*}
$$

Divide (10a) by $D$ and rearrange gives

$$
\begin{equation*}
-\frac{\partial D}{\partial p} \frac{p}{D}=1+\frac{\beta \lambda}{D}, \tag{11}
\end{equation*}
$$

with $\lambda$ given by (9).
The pricing rule (11) states that it is optimal for the firm to set a price such that the price elasticity of demand $-(\partial D / \partial p) /(p / D)$ is higher than one (recall

[^0]$\beta, \lambda \geqslant 0$ and $D>0)$ : that is, if the price increases by $1 \%$, the demand decreases by more than $1 \%$. A higher speed of adjustment $\beta$ and a higher shadow price of reference price $\lambda$ are associated to a higher price elasticity of demand. If the reference price does not adapt $(\beta=0)$ or the reference price effect is inactive ( $\partial D / \partial r=0$ implies $\lambda=0$ from (9)), then the price elasticity of demand is unitary. The managerial implications are straightforward. The firm operating only at a specific time-static case, because it ignores lasting reference price effects, charges such that the elasticity of demand equals one. In contrast, the firm operating over the whole life cycle of a product-dynamic case, by considering lasting reference price effects, charges such that the elasticity of demand is larger than one.

The first-order condition on price (10a) must hold over the whole planning horizon, that is for all $t \in[0, T]$. Differentiate (10a) with respect to time $t$ gives the decomposition ${ }^{3}$

$$
\begin{equation*}
\frac{d p}{d t}(\underbrace{-2 \frac{\partial D}{\partial p}-p \frac{\partial^{2} D}{\partial p^{2}}}_{+})=\frac{d r}{d t}(\underbrace{\frac{\partial D}{\partial r}}_{+} \underbrace{+p \frac{\partial^{2} D}{\partial p \partial r}}_{-})+\beta(\underbrace{(\rho+\beta) \lambda}_{+} \underbrace{-p \frac{\partial D}{\partial r}}_{-}), \tag{12}
\end{equation*}
$$

where $\lambda$ is given in (9).
The rule of dynamic pricing (12) quantifies the dynamics of selling price $p$ over time. It represents the first decomposition of the effects tied to the structural properties of the demand function affecting price dynamics along the optimal path. Previous research like Popescu and Wu (2007), Nasiry and Popescu (2011), and Aflaki and Popescu (2013), discuss in great detail the intertemporal equilibrium but not the explicit dynamic of the selling price along the optimal path leading to this equilibrium. In practice, however, the selling period is finite and relatively shortsome years or months. Therefore, analyzing price dynamics along the optimal path highlights managerial practice.

By clearly decomposing the effects at play, rule (12) clarifies price dynamics when customer behavior is considered. The second-order condition (10b) insures that, on the left-hand side of (12), the second factor $\left(-2 \partial D / \partial p-\partial^{2} D / \partial p^{2}\right)$ is positive. On the right-hand side, the first term refers to the impact of the reference price dynamics and the second term to the impact of the adjustment speed. Both impacts, resulting from two competing effects, are ambiguous.

Rule (12) stipulates that if the firm discount profits $(\rho>0)$, it does not continuously set higher prices leading to continuously higher demand. Further, if the firm does not discount profits ( $\rho=0$ ), the increase in price over time is even lower (by $\beta^{2} \lambda$ ). The rationale is that when future profits count less, the firm has less incentives to increase reference price with higher selling price. A deeper

[^1]understanding of the dynamics of price requires studying two cases according to the value of the speed of adjustment $\beta$.

- In the case $\beta=0$ for which there is no reference price adjustment, there is no price dynamic. In fact, from (2) $d r / d t=0$, and from (12) $d p / d t=0$ : the reference price and the selling price are constant over the planning horizon. In particular, for all $t \in[0, T]$ we have $r(t)=r_{0}$ from (1) and $p(t)=p$ such that $-(\partial D / \partial p) /(p / D)=1$ from (11). The case $\beta=0$ thus matches the static monopolist case, for which $\lambda=0$. Interestingly, effective adjustment of the reference price is thus a condition for the dynamics of price.
- In the case $\beta>0$ below which the reference price adjusts over time, the dynamics of price is effective and depends on both the reference price dynamics and adjustment speed impacts. The total reference dynamic impact induced by a reference price increase over time is positively related to the direct reference effect $\partial D / \partial r$ and negatively associated to the indirect reference effect $p \partial D^{2} /(\partial p \partial r)$. The total adjustment impact augments with the investment in reference price $(\rho+\beta) \lambda$ and declines with the deterioration of the reference effect $p \partial D / \partial r$. The case $\beta>0$ thus yields the conditions for a dynamic pricing policy.


## 4 Discussion and Conclusion

Contrary to intuition, a higher reference price for the consumer does not always imply a higher selling price. In other words, reference price dynamics does not imitate selling price dynamics. Instead, the final impact of an increased reference price on the selling price is the sum of four competing effects. The rule of dynamic pricing (12) provides the first explicit decomposition of the effects at play along the optimal path. The first two effects are related to reference price dynamics. Thus a higher reference price develops the demand, enabling a higher selling price (positive effect). The demand, however, would be even higher with a lower selling price (negative effect). The last two effects are associated with reference price adjustment. On one side, a faster adjustment encourages the firm to increase the price, thereby augmenting the reference price and developing demand in the future (positive effect). On the other side, a faster adjustment also means lower memory of past prices, and the interest for a higher reference price will decline more quickly (negative effect).

The present article establishes the conditions under which the selling price is positively associated to the reference price. Consequently, the results undermine the idea that the optimal price monotonically decreases with the reference price or
with the adjustment speed. The dynamic pricing rule (12) enables deriving drivers to pricing policies in managerial situations. This pricing rule can thus characterize both skimming and penetration pricing policies, and also a mix of these policies yielding $U$ - and inverted $U$-shaped pricing curves.

To conclude, I provide a qualitative analysis of the optimal path of the dynamic pricing policy resulting from a general demand function. This structural approach captures with little restriction consumer behavioral effects related to reference dependence. Any firm that misunderstands these general effects loses profit by charging inadequately for its product. This work provides thus a deeper understanding of dynamic pricing policies when customers are subject to reference effects in the spirit of prospect theory. Such theoretical implications, in turn, call for empirical support in future research.

## A Appendix

## A. 1 Proof of Equation (9)

Recall that the dynamic of $\lambda$ writes in (8)

$$
\frac{d \lambda(t)}{d t}=(\rho+\beta) \lambda(t)-p \frac{\partial D}{\partial r}, \text { with } \lambda(T)=0 .
$$

Consider the integrating factor $e^{-(\rho+\beta) t}$, such that

$$
\frac{d \lambda(t) e^{-(\rho+\beta) t}}{d t}=e^{-(\rho+\beta) t}\left(\frac{d \lambda(t)}{d t}-(\rho+\beta) \lambda(t)\right) .
$$

Since $\frac{d \lambda(t)}{d t}-(\rho+\beta) \lambda(t)=-p \frac{\partial D}{\partial r}$, then

$$
\begin{aligned}
& \frac{d \lambda(t) e^{-(\rho+\beta) t}}{d t}=e^{-(\rho+\beta) t}\left(-p \frac{\partial D}{\partial r}\right), \\
& \int_{t}^{T} d \lambda(s) e^{-(\rho+\beta) s}=\int_{t}^{T} e^{-(\rho+\beta) s}\left(-p \frac{\partial D}{\partial r}\right) d s, \\
& \lambda(T) e^{-(\rho+\beta) T}-\lambda(t) e^{-(\rho+\beta) t}=\int_{t}^{T} e^{-(\rho+\beta) s}\left(-p \frac{\partial D}{\partial r}\right) d s .
\end{aligned}
$$

Substitute the transversality condition $\lambda(T)=0$ yields

$$
\lambda(t)=\int_{t}^{T} e^{-(\rho+\beta)(s-t)}\left(p \frac{\partial D}{\partial r}\right) d s,
$$

which completes the proof.

## conomics Discussion Paper

## A. 2 Proof of Equation (10b)

The first-order condition with respect to $p$ (10a) writes

$$
\frac{\partial H}{\partial p}=0 \Longrightarrow D+p \frac{\partial D}{\partial p}+\beta \lambda=0
$$

The second-order condition with respect to $p$ imposes

$$
\begin{aligned}
\frac{\partial^{2} H}{\partial p^{2}}<0 & \Longrightarrow \frac{\partial D}{\partial p}+\frac{\partial D}{\partial p}+p \frac{\partial^{2} D}{\partial p^{2}}<0 \\
& \Longrightarrow-2 \frac{\partial D}{\partial p}-p \frac{\partial^{2} D}{\partial p^{2}}>0
\end{aligned}
$$

which completes the proof.

## A. 3 Proof of Equation (12)

The first-order condition with respect to $p(10 \mathrm{a})$ states

$$
\frac{\partial H}{\partial p}=0 \Longrightarrow D+p \frac{\partial D}{\partial p}+\beta \lambda=0 .
$$

Derivate this condition with respect to $t$ :

$$
\begin{aligned}
& \frac{d}{d t}\left(D+p \frac{\partial D}{\partial p}+\beta \lambda\right)=0 \\
& \frac{\partial D}{\partial p} \frac{d p}{d t}+\frac{\partial D}{\partial r} \frac{d r}{d t}+\frac{d p}{d t} \frac{\partial D}{\partial p}+p\left(\frac{\partial^{2} D}{\partial p^{2}} \frac{d p}{d t}+\frac{\partial^{2} D}{\partial p \partial r} \frac{d r}{d t}\right)+\beta \frac{d \lambda}{d t}=0
\end{aligned}
$$

Substitute the dynamics of $\lambda$ from (8) gives

$$
\begin{aligned}
& \frac{\partial D}{\partial p} \frac{d p}{d t}+\frac{\partial D}{\partial r} \frac{d r}{d t}+\frac{d p}{d t} \frac{\partial D}{\partial p}+p\left(\frac{\partial^{2} D}{\partial p^{2}} \frac{d p}{d t}+\frac{\partial^{2} D}{\partial p \partial r} \frac{d r}{d t}\right)+\beta\left((\rho+\beta) \lambda-p \frac{\partial D}{\partial r}\right)=0, \\
& \frac{d p}{d t}\left(-2 \frac{\partial D}{\partial p}-p \frac{\partial^{2} D}{\partial p^{2}}\right)=\frac{d r}{d t}\left(\frac{\partial D}{\partial r}+p \frac{\partial^{2} D}{\partial p \partial r}\right)+\beta\left((\rho+\beta) \lambda-p \frac{\partial D}{\partial r}\right),
\end{aligned}
$$

which completes the proof.

## References

Aflaki, S., and Popescu, I. (2013). Managing retention in service relationships. Management Science, 60(2): 415-433.

```
Conomics Discussion Paper
```

Chenavaz, R. (2011). Dynamic pricing rule and R\&D. Economics Bulletin, 31(3): 2229-2236.

Chenavaz, R. (2012). Dynamic pricing, product and process innovation. European Journal of Operational Research, 222(3): 553-557.

Fibich, G., Gavious, A., and Lowengart, O. (2003). Explicit solutions of optimization models and differential games with nonsmooth (asymmetric) reference-price effects. Operations Research, 51(5): 721-734.

Kalyanaram, G., and Winer, R. S. (1995). Empirical generalizations from reference price research. Marketing Science, 14(3_supplement): G161-G169.

Kopalle, P. K., Rao, A. G., and Assuncao, J. L. (1996). Asymmetric reference price effects and dynamic pricing policies. Marketing Science, 15(1): 60-85.

Li, S., Zhang, J., and Tang, W. (2014). Joint dynamic pricing and inventory control policy for a stochastic inventory system with perishable products. International Journal of Production Research, (ahead-of-print): 1-14.

Long, X., and Nasiry, J. (2014). Prospect theory explains newsvendor behavior: The role of reference points. Management Science Forthcoming.

Lu, L., Zhang, J., and Tang, W. (2014). Optimal dynamic pricing and replenishment policy for perishable items with inventory-level-dependent demand. International Journal of Systems Science, (ahead-of-print): 1-15.

Mazumdar, T., Raj, S., and Sinha, I. (2005). Reference price research: Review and propositions. Journal of Marketing, 69(4): 84-102.

Nagarajan, M., and Shechter, S. (2013). Prospect theory and the newsvendor problem. Management Science, 60(4): 1057-1062.

Nasiry, J., and Popescu, I. (2011). Dynamic pricing with loss-averse consumers and peak-end anchoring. Operations research, 59(6): 1361-1368.

Popescu, I., and Wu, Y. (2007). Dynamic pricing strategies with reference effects. Operations Research, 55(3): 413-429.

Putler, D. S. (1992). Incorporating reference price effects into a theory of consumer choice. Marketing Science, 11(3): 287-309.

Sorger, G. (1988). Reference price formation and optimal pricing strategies. In G. Feichtinger (Ed.), Optimal Control Theory and Economic Analysis 3, pages 97-120. Elsevier.

Winer, R. S. (1986). A reference price model of brand choice for frequently purchased products. Journal of Consumer Research, 13(2): 250-256.

Xu, J., and Liu, N. (2014). Research on closed loop supply chain with reference price effect. Journal of Intelligent Manufacturing, Forthcoming, pages 1-14.

Xue, M., Tang, W., and Zhang, J. (2014). Optimal dynamic pricing for deteriorating items with reference-price effects. International Journal of Systems Science Forthcoming.

Zhang, J., Gou, Q., Liang, L., and Huang, Z. (2013). Supply chain coordination through cooperative advertising with reference price effect. Omega, 41(2): 345-353.

Zhang, J., Kevin Chiang, W.-y., and Liang, L. (2014). Strategic pricing with reference effects in a competitive supply chain. Omega, 44: 126-135.

The Open-Access, Open-Assessment E-Journal

Please note:

You are most sincerely encouraged to participate in the open assessment of this discussion paper. You can do so by either recommending the paper or by posting your comments.

Please go to:
http://www.economics-ejournal.org/economics/discussionpapers/2016-20

The Editor


[^0]:    ${ }^{1}$ The proof of (9) is in Appendix A. 1
    2 The proof of (10b) is in Appendix A.2.

[^1]:    3 The proof of (12) is in Appendix A.3.

