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Cost-reduction Innovation under Mixed Economy

Pu-yan Nie and Yong-cong Yang

Abstract

Industries with mixed oligopoly are exceedingly popular all over the world, especially in developing countries, such as China. This paper highlights the innovation strategies of mixed duopoly with a (semi-) public firm and another private firm, and the effects of mixed oligopoly on innovation are captured. Firstly, the (semi-) public firm innovates more and produces more than the private firm. Secondly, the degree of the public ownership stimulates the output and innovation. Finally, the price difference and the price dispersion all increase with the degree of the public ownership under independent goods.

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Keywords Innovation; industrial organization; mixed duopoly; game theory

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1 Introduction

Many industries all over the world own mixed oligopoly structures, such as the telecommunications in China, the airport industry both in USA and in China (Nie, 2014) and so on. In developing countries, many industries fall into the mixed oligopoly community. All these facts manifest that the mixed oligopoly has significant effects on the economy of developing countries and it is important to capture the mixed economy in detail.

There are many papers in industrial organization community to address the mixed oligopoly (Matsumura, 1998; White, 2002; Donder and Roemer, 2009; Barcena-Ruiz, 2007; Lu, 2006; Yang and Nie, 2015; Nie, 2016). The existed theoretic literature about mixed oligopoly focuses on two aspects. Some papers focus on the effects of mixed oligopoly on firms' strategies and the others highlight the effects of other factors on the mixed oligopoly. In practice, some special industries with mixed oligopoly receive extensive attention.

About the effects of mixed oligopoly on firms' strategies, including endogenous timing, cooperative innovation and so on, Barcena-Ruiz (2007) discussed the endogenous timing under mixed oligopoly. Giannakas and Fulton (2005) examined the cooperative innovation under mixed oligopoly in the agricultural industry. Matsumura (1998) addressed the privatization in a mixed oligopoly. Molto et al. (2006) investigated the R&D policies under mixed oligopoly. Li and Zhang (2011) addressed the location of mixed oligopoly. Nie (2014) recently addressed the effects of capacity constraints on mixed economy. Chen, Yang, Wang and Wu (2014)

discussed the effects of mixed oligopoly on technology license.

About the effects of other factors on mixed oligopoly, Donder and Roemer (2009) discussed the firms' endogenous objectives affect the mixed oligopoly equilibrium. When foreign competitors are introduced, Lu (2006) gave the endogenous timing, which is different from Barcena-Ruiz's conclusions (2007). Lu and Poddar (2006) remarked the effects of the uncertain demand on the equilibrium under mixed oligopoly. White (2002) identified the effects of political manipulation in a mixed oligopoly. Gil-Molto, Poyago-Theotoky and Zikos (2011) identified the subsidies of R&D under mixed oligopoly.

In practice, Giannakas and Fulton (2005) discussed the agricultural industry with mixed oligopoly. Annaki (2011) investigated the effects of the regulated price on the hospital industry in Germany. Barcena-Ruiz (2012) captured the privatization if the public firm is as efficient as private firms.

In summary, the extant literature, except the interesting papers of Molto et al. (2006), Ishibashi and Matsumura (2006), Heywood and Ye (2009), and, Giannakas and Fulton (2005), cares rarely about the effects of the public degree in mixed oligopoly on innovation. This paper continues to develop the innovation under mixed economy. This paper describes the effects of the public ownership on the cost-reduction innovative investment and outputs. Compared with Molto et al. (2006) and Giannakas and Fulton (2005), the product substitutability is introduced, while R&D subsidies are neglected, and, Giannakas and Fulton (2005) stressed cooperative innovation in the agricultural industry.

This paper is organized as follows: The model, based on the mixed duopoly innovation with a (semi-) state-owned firms and a private one, is given in Section 2. The model is analyzed in Section 3. the innovation and outputs of two firms are captured. The effects of the degree of the public ownership for the public firm are described. Some remarks are given in the final section.

2 The Model

Consider an industry of two producers, in which one is a (semi-) public firm and the other is private. The (semi-) public firm maximizes its profits adding the weighted consumer surplus, while the private firm maximizes its profits. We establish a two-stage model of the substitutability product with duopoly innovation. Denote two producers to be A and B , where the firm A is a public firm and the firm B is private. At the first stage, two firms simultaneously choose innovation investment I_A and I_B . For $i \in \{A, B\}$, launching innovative investment I_i , $\frac{1}{2}I_i^2$ represents the costs incurred by innovation, which is similar to that in Sacco and Schmutzler (2011), and Chen and Nie (2014). At the second stage, firms simultaneously compete in quantity, which is a Cournot competition model.

Demand. For $i \in \{A, B\}$, p_i is the price, and the quantity of production is q_i . Denote $p = (p_A, p_B)$ and $q = (q_A, q_B)$. The utility function of representative consumers is outlined by

$$u(p, q) = a(q_A + q_B) - \frac{1}{2}[(q_A)^2 + (q_B)^2] - p_A q_A - p_B q_B - g q_A q_B. \quad (1)$$

where $a > 0$ is a constant and $g \in [0, 1]$. $a > 0$ means the total market size, and

$g \in [0,1]$ indicates the degree of substitutability. $g = 0$ stands for that two goods are independent, and $g = 1$ manifests perfect substitutes (Nie and Chen, 2012). The inverse demand function, which is the same as that in Liu, Wang and Yang (2012), is given as follows:

$$p_i = a - q_i - gq_j, \quad (2)$$

$i, j \in \{A, B\}$ and $i \neq j$. Note that the inverse demand function (2) is directly induced by the above utility function. The consumer surplus is correspondingly given by

$$CS = \frac{1}{2}(q_A^2 + q_B^2) + gq_Aq_B. \quad (3)$$

Producers. For $i, j \in \{A, B\}$ and $i \neq j$, given the constant $c_i > 0$, the price p_i , the outputs q_i and the innovative investment to reduce the costs $I_i < c_i$, the profit functions of two producers are listed as follows:

$$p_i = p_i q_i - (c_i - I_i) q_i - \frac{1}{2} I_i^2, \quad (4)$$

The term $p_i q_i$ means the revenue of the firm i . $c_i > 0$ indicates the marginal costs of the firm $i \in \{A, B\}$ without innovation. To simplify, we assume that $c_A = c_B = c_0$.

Or the marginal costs of these two firms are identical without innovation. The firm A highlights profits pulsing weighted consumer surplus $\Pi_A = p_A + \frac{t}{2}(q_A^2 + q_B^2 - 2gq_Aq_B)$, where $t \geq 0$ stands for the degree of public ownership.

The timing of game is as following: At the first stage, two firms determine the innovative investment. At the second stage, two firms compete in quantity. The following assumption is launched to guarantee the existence of the unique solution for the above system.

$$\text{Assumption} \left| \begin{array}{cc} (4-2t-g^2+g^2t)^2 - (8-4t+g^2t) & 2(2-t)g \\ (2-t)(2g-gt) & (4-2t-g^2+g^2t)^2 - 2(2-t)^2 \end{array} \right| > 0,$$

and $8-4t+g^2t < (4-2t-g^2+g^2t)^2$.

Apparently, the above assumption is met under $g=0$. When g is close to 1, it is very difficult to satisfy the above assumption. In practice, in an industry with perfect substitutability product, there almost do not exist a public firm and a private one simultaneously. Moreover, $8-4t+g^2t < (4-2t-g^2+g^2t)^2$ indicates the relationship $(4-2t-g^2+g^2t)^2 - 2(2-t)^2 > 0$.

3 Main Results

As a (semi-) public firm, the firm A aims to maximize its profits plus the weighted consumer surplus (CS). The firm A attacks the following problem.

$$\text{Max}_{I_A, q_A} \Pi_A = p_A + \frac{t}{2}(q_A^2 + q_B^2 - 2gq_Aq_B) = (a - q_A - gq_B - c_A + I_A)q_A + \frac{t}{2}(q_A^2 + q_B^2 + 2gq_Aq_B) - \frac{1}{2}I_A^2, \quad (5)$$

where $1 \geq t \geq 0$, which describes the degree of the public ownership. $t=1$ implies that the firm A is a complete public one. $t=0$ indicates the firm A is private. Larger t indicates that the firm A is more public ownership.

The firm B , as a private one, maximizes the corresponding profits

$$\text{Max}_{I_B, q_B} p_B = (a - q_B - gq_A)q_B - (c_B - I_B)q_B - \frac{1}{2}I_B^2. \quad (6)$$

Here we consider the above model by backward induction. The second stage is first addressed. The first stage is then discussed.

Apparently, (5) is concave in q_A and (6) is concave in q_B . The solution of (5) and (6) is uniquely determined by their first optimal conditions. We analyze the above

model by backward induction approaches. Firstly, the second stage is discussed and then the first stage is analyzed.

At the second stage, we have the following first order optimal conditions.

$$\frac{\partial \Pi_A}{\partial q_A} = (a - 2q_A - gq_B - c_A + I_A) + t(q_A + gq_B) = 0, \quad (7)$$

$$\frac{\partial p_B}{\partial q_B} = (a - 2q_B - gq_A) - (c_B - I_B) = 0. \quad (8)$$

(7) and (8) jointly imply that the equilibrium at the second stage is

$$q_A^* = \frac{2(a - c_A + I_A) - g(1-t)(a - c_B + I_B)}{4 - 2t - g^2 + g^2t}, \quad q_B^* = \frac{(2-t)(a - c_B + I_B) - g(a - c_A + I_A)}{4 - 2t - g^2 + g^2t}. \quad (9)$$

Here we address the first stage. From (5),(6) and (9), we have

$$\text{Max}_{I_A} \Pi_A = q_A^2 - \frac{t}{2}q_A^2 + \frac{t}{2}q_B^2 - \frac{1}{2}I_A^2,$$

$$\text{Max}_{I_B} p_B = q_B^2 - \frac{1}{2}I_B^2.$$

The first optimal conditions of the above maximum problem are

$$\frac{\partial \Pi_A}{\partial I_A} = \frac{2(a - c_A + I_A) - g(1-t)(a - c_B + I_B)}{4 - 2t - g^2 + g^2t} (2-t) \frac{2}{4 - 2t - g^2 + g^2t} - \frac{(2-t)(a - c_B + I_B) - g(a - c_A + I_A)}{4 - 2t - g^2 + g^2t} \frac{gt}{4 - 2t - g^2 + g^2t} - I_A = 0$$

$$\text{and } \frac{\partial p_B}{\partial I_B} = 2q_B^* \frac{\partial q_B^*}{\partial I_B} - I_B = 2 \frac{(2-t)(a - c_B + I_B) - g(a - c_A + I_A)}{4 - 2t - g^2 + g^2t} \frac{(2-t)}{4 - 2t - g^2 + g^2t} - I_B = 0.$$

$$\text{Assumption } \left| \begin{array}{cc} (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) & 2(2-t)g \\ (2-t)(2g - gt) & (4 - 2t - g^2 + g^2t)^2 - 2(2-t)^2 \end{array} \right| > 0 \text{ and}$$

$8 - 4t + g^2t < (4 - 2t - g^2 + g^2t)^2$ in Section 2 imply that the above problems are all

concave. The first order optimal conditions are restated as

$$(a - c_A + I_A)(8 - 4t + g^2t) - (a - c_B + I_B)[g(1-t)(4 - 2t) + (2-t)gt] - (4 - 2t - g^2 + g^2t)^2 I_A = 0$$

$$\text{and } 2(2-t)[(2-t)(a - c_B + I_B) - g(a - c_A + I_A)] - (4 - 2t - g^2 + g^2t)^2 I_B = 0.$$

Or,

$$f_1 = -(a - c_A)(8 - 4t + g^2t) + (a - c_B)g(2 - t)(2 - t) + [(4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t)]I_A + I_Bg(2 - t)(2 - t) = 0$$

and

$$f_2 = -2(2 - t)[(2 - t)(a - c_B) - g(a - c_A)] + [(4 - 2t - g^2 + g^2t)^2 - 2(2 - t)^2]I_B + 2(2 - t)gI_A = 0.$$

By $c_A = c_B = c_0$, we therefore have

$$I_A^* = (a - c_0) \frac{\begin{vmatrix} (8 - 4t + g^2t) - (2 - t)(2g - gt) & 2(2 - t)g \\ 2(2 - t)[(2 - t) - g] & (4 - 2t - g^2 + g^2t)^2 - 2(2 - t)^2 \end{vmatrix}}{\begin{vmatrix} (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) & 2(2 - t)g \\ (2 - t)(2g - gt) & (4 - 2t - g^2 + g^2t)^2 - 2(2 - t)^2 \end{vmatrix}}, \quad (10)$$

$$I_B^* = (a - c_0) \frac{\begin{vmatrix} (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) & (8 - 4t + g^2t) - (2 - t)(2g - gt) \\ (2 - t)(2g - gt) & 2(2 - t)[(2 - t) - g] \end{vmatrix}}{\begin{vmatrix} (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) & 2(2 - t)g \\ (2 - t)(2g - gt) & (4 - 2t - g^2 + g^2t)^2 - 2(2 - t)^2 \end{vmatrix}}. \quad (11)$$

Compared (10) and (11), we have

$$I_A^* - I_B^* = (a - c_0) \frac{\begin{vmatrix} (8 - 4t + g^2t) - (2 - t)(2g - gt) & 2(2 - t)g + (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) \\ 2(2 - t)[(2 - t) - g] & (4 - 2t - g^2 + g^2t)^2 - (2 - t)(2g - gt) \end{vmatrix}}{\begin{vmatrix} (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) & 2(2 - t)g \\ (2 - t)(2g - gt) & (4 - 2t - g^2 + g^2t)^2 - 2(2 - t)^2 \end{vmatrix}}.$$

Since $(8 - 4t + g^2t) - (2 - t)(2g - gt) - 2(2 - t)[(2 - t) - g] = g^2t + (2 - t)gt + 2(2 - t)t > 0$,

the following formulation holds.

$$\begin{aligned} & \left| \begin{vmatrix} (8 - 4t + g^2t) - (2 - t)(2g - gt) & 2(2 - t)g + (4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t) \\ 2(2 - t)[(2 - t) - g] & (4 - 2t - g^2 + g^2t)^2 - 2(2 - t)^2 + (2 - t)(2g - gt) \end{vmatrix} \right| \\ &= \left| \begin{vmatrix} (8 - 4t + g^2t) - (2 - t)(2g - gt) & (4 - 2t - g^2 + g^2t)^2 + (2 - t)gt \\ 2(2 - t)[(2 - t) - g] & (4 - 2t - g^2 + g^2t)^2 + (2 - t)gt \end{vmatrix} \right| \geq 0. \end{aligned}$$

We therefore attain $I_A^* \geq I_B^*$. Substituting (10) and (11) into (9), the corresponding outputs are

$$q_A^* = (a - c_0) \frac{4 - 2t - g^2 + g^2 t}{4 - 2t} \left\{ \frac{\left| \begin{array}{cc} (8 - 4t + g^2 t) - (2 - t)(2g - gt) & 2(2 - t)g \\ 2(2 - t)[(2 - t) - g] & (4 - 2t - g^2 + g^2 t)^2 - 2(2 - t)^2 \end{array} \right|}{\left| \begin{array}{cc} (4 - 2t - g^2 + g^2 t)^2 - (8 - 4t + g^2 t) & 2(2 - t)g \\ (2 - t)(2g - gt) & [(4 - 2t - g^2 + g^2 t)^2 - 2(2 - t)^2] \end{array} \right|} \right. \\ \left. + \frac{gt}{2} \frac{\left| \begin{array}{cc} (4 - 2t - g^2 + g^2 t)^2 - (8 - 4t + g^2 t) & (8 - 4t + g^2 t) - (2 - t)(2g - gt) \\ 2g - gt & 2[(2 - t) - g] \end{array} \right|}{\left| \begin{array}{cc} (4 - 2t - g^2 + g^2 t)^2 - (8 - 4t + g^2 t) & 2(2 - t)g \\ (2 - t)(2g - gt) & (4 - 2t - g^2 + g^2 t)^2 - 2(2 - t)^2 \end{array} \right|} \right\}, \quad (12)$$

$$q_B^* = \frac{(a - c_0)(4 - 2t - g^2 + g^2 t)}{4 - 2t} \frac{\left| \begin{array}{cc} (4 - 2t - g^2 + g^2 t)^2 - (8 - 4t + g^2 t) & (8 - 4t + g^2 t) - (2 - t)(2g - gt) \\ (2 - t)(2g - gt) & 2(2 - t)[(2 - t) - g] \end{array} \right|}{\left| \begin{array}{cc} (4 - 2t - g^2 + g^2 t)^2 - (8 - 4t + g^2 t) & 2(2 - t)g \\ (2 - t)(2g - gt) & (4 - 2t - g^2 + g^2 t)^2 - 2(2 - t)^2 \end{array} \right|}. \quad (13)$$

$I_A^* \geq I_B^*$ implies $q_A^* \geq q_B^*$.

The above analysis is summarized as follows:

Proposition 1 $I_A^* \geq I_B^*$, $q_A^* \geq q_B^*$ and $p_A^* \leq p_B^*$.

Remarks: $q_A^* \geq q_B^*$ and (2) indicate $p_A^* \leq p_B^*$. Taking the consumer surplus into account, the (semi-) public firm launches more innovation and produces more than the private firm. The public ownership has stimulating effects on both the innovation and outputs.

According to the above formulations, we further have $\frac{I_B^*}{q_B^*} = \frac{4 - 2t}{4 - 2t - g^2 + g^2 t}$ and

$\frac{I_A^*}{q_A^*} < \frac{4 - 2t}{4 - 2t - g^2 + g^2 t}$. The ratio between the innovation and the outputs of the (semi-) public firm is lower than that of the private one. This interesting conclusion comes

from the properties of the (semi-) public firm to maximize its profits pulsing the weighted consumer surplus. The public ownership has more effects on the output than

the innovation.

Moreover, under the special case of $g=0$, (10) and (11) imply $\frac{I_A^*}{I_B^*} = \frac{2}{2-t}$ and

$\frac{q_A^*}{q_B^*} = \frac{2}{2-t}$. Another special situation is $t=0$. This is a symmetric Cournot

competition. We have $I_A^* = I_B^*$ and $q_A^* = q_B^*$.

The equilibrium is further discussed. From (10)-(13), we have the following conclusions

Proposition 2 The equilibrium innovation of both the (semi-) public firm and the private one increases with the degree of the public ownership.

Proof: See in appendix. ■

Remarks: About innovative investment, the degree of the public ownership stimulates the innovation of two firms. As a special situation, $\frac{\partial I_B}{\partial t} = 0$ if $g=0$. This implies that the innovative investment of the private firm has no relation to the degree of the public ownership if two firms' products are completely independent.

Proposition 3 The equilibrium outputs of both the (semi-) public firm and the private one increase with the degree of the public ownership. The equilibrium price decreases with the degree of the public ownership.

Proof: Similar to the proof of Proposition, the conclusion is achieved. ■

Remarks: The degree of the public ownership also has stimulating effects on the outputs of two firms. Similarly, we also have $\frac{\partial q_B}{\partial t} = 0$ when $g = 0$. The degree of the public ownership stimulates the innovation and outputs of the firms, which yields the above conclusions of Proposition 2 and 3. Moreover, the price decreases with the degree of the public ownership and the consumer surplus is improved with higher degree of the public ownership.

We here address the price difference ($\Delta p = p_B - p_A$) and the price dispersion ($h = \frac{p_B - p_A}{p_B + p_A}$) under $g = 0$. These definitions are the same as those in Samuelson and

Zhang (1992) or Nie and Chen (2012). (2) indicates $\Delta p = p_B - p_A = q_A - q_B$ and

$$h = \frac{p_B - p_A}{p_B + p_A} = \frac{q_A - q_B}{2a - (q_B + q_A)} = \frac{2a - 2q_B}{2a - (q_B + q_A)} - 1. \text{ Under } g = 0, \frac{\partial q_B}{\partial t} = 0 \text{ and } \frac{\partial q_A}{\partial t} > 0$$

imply $\frac{\partial(\Delta p)}{\partial t} > 0$ and $\frac{\partial h}{\partial t} > 0$. We therefore have

Proposition 4 The firm-size difference, the price difference and the price dispersion all increase with the degree of the public ownership under independent goods.

Remarks: Under independent goods, since the degree of the public ownership has no effects on the innovation and outputs on private firm, the degree of public ownership increase the firm-size difference, the price difference and the price dispersion because the innovation has stimulating effects on the public firms' outputs.

4. Concluding Remarks

This paper addresses the innovation under mixed duopoly and substitutability products. This article identifies that the (semi-) public firm launches more innovation and outputs. The degree of the public ownership has stimulating effects on the innovation and outputs of two firms. The price difference is also identified. This article shows that the price difference and the price dispersion all increase with the degree of the public ownership under independent goods ($g = 0$). In general cases, it is difficult to discuss the price difference and price dispersion.

Some further researching issues arise. This paper neglects governmental subsidies, which is discussed by Yang (2014) in profit-maximizing firms, and we will consider it in future. Moreover, this paper does not address the transportation costs (Nie, 2013) and it is interesting to consider the effects of transportation on the innovation investment.

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Appendix

Proof of Proposition 2

According to

$$f_1 = -(a - c_0)[8 - 4t + g^2t - g(2-t)(2-t)] + [(4 - 2t - g^2 + g^2t)^2 - (8 - 4t + g^2t)]I_A + I_B g(2-t)(2-t) = 0$$

and $f_2 = [(4 - 2t - g^2 + g^2t)^2 - 2(2-t)^2]I_B + (4 - 2t)gI_A - (a - c_0)(4 - 2t)(2 - t - g) = 0$, we

$$\text{have } \frac{\partial f_1}{\partial t} = (a - c_0)(4 - g^2 - 4g + 2tg) - [(4 - 2t - g^2 + g^2t)(4 - 2g^2) - 4 + g^2]I_A - I_B(4g - 2tg)$$

$$\text{and } \frac{\partial f_2}{\partial t} = (a - c_0)2(4 - 2t - g) + [-(4 - 2t - g^2 + g^2t)(4 - 2g^2) + 4(2 - t)]I_B - 2gI_A.$$

Namely, $\frac{\partial f_1}{\partial t} < \frac{\partial f_2}{\partial t} < 0$. By the implicit function theorem, we have

$$\frac{\partial I_A}{\partial t} = - \frac{\begin{vmatrix} \frac{\partial f_1}{\partial t} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial t} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}} \quad \text{and} \quad \frac{\partial I_B}{\partial t} = - \frac{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial t} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial t} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}}.$$

Because $\frac{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}} > 0$, we care about the signs of $\frac{\begin{vmatrix} \frac{\partial f_1}{\partial t} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial t} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}}$ and $\frac{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial t} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial t} \end{vmatrix}}$.

$$\frac{\begin{vmatrix} \frac{\partial f_1}{\partial t} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial t} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}}{\begin{vmatrix} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial I_B} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial I_B} \end{vmatrix}} = [(4 - 2t - g^2 + g^2t)^2 - 2(2-t)^2] \times$$

$$\{(a - c_0)(4 - g^2 - 4g + 2tg) - [(4 - 2t - g^2 + g^2t)(4 - 2g^2) - 4 + g^2]I_A - I_B(4g - 2tg)\}$$

$$- g(2-t)^2 \{(a - c_0)2(4 - 2t - g) + [-(4 - 2t - g^2 + g^2t)(4 - 2g^2) + 4(2 - t)]I_B - 2gI_A\}$$

$$\leq 0.$$

Moreover, we have

$$\begin{aligned} & \left| \begin{array}{cc} \frac{\partial f_1}{\partial I_A} & \frac{\partial f_1}{\partial t} \\ \frac{\partial f_2}{\partial I_A} & \frac{\partial f_2}{\partial t} \end{array} \right| = \frac{\partial f_1}{\partial I_A} \frac{\partial f_2}{\partial t} - \frac{\partial f_2}{\partial I_A} \frac{\partial f_1}{\partial t} = [(4-2t-g^2+g^2t)^2 - (8-4t+g^2t)] \times \\ & \{(a-c_0)2(4-2t-g) + [-(4-2t-g^2+g^2t)(4-2g^2) + 4(2-t)]I_B - 2gI_A\} - \\ & (4-2t)g\{(a-c_0)(4-g^2-4g+2tg) - [(4-2t-g^2+g^2t)(4-2g^2) - 4+g^2]I_A - I_B(4g-2tg)\} \leq 0. \end{aligned}$$

Therefore, we have the relationship $\frac{\partial I_A}{\partial t} \geq 0$ and $\frac{\partial I_B}{\partial t} \geq 0$. Conclusions are achieved

and the proof is complete. ■

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