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Testing for Unit Roots with Cointegrated Data

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Abstract
This paper demonstrates that unit root tests can suffer from inflated Type I error rates when data are cointegrated. Results from Monte Carlo simulations show that three commonly used unit root tests – the ADF, Phillips–Perron, and DF-GLS tests – frequently overreject the true null of a unit root for at least one of the cointegrated variables. The reason for this overrejection is that unit root tests, designed for random walk data, are often misspecified when data are cointegrated. While the addition of lagged differenced (LD) terms can eliminate the size distortion, this "success" is spurious, driven by collinearity between the lagged dependent variable and the LD explanatory variables. Accordingly, standard diagnostics such as (i) testing for serial correlation in the residuals and (ii) using information criteria to select among different lag specifications are futile. The implication of these results is that researchers should be conservative in the weight they attach to individual unit root tests when determining whether data are cointegrated.

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Keywords Unit root testing; cointegration; DF-GLS test; augmented Dickey–Fuller test; Phillips–Perron test; simulation

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I. INTRODUCTION

When estimating relationships among time series data, it is standard practice to first test for unit roots in the individual series. If the data are integrated, one then moves to testing whether the variables are cointegrated. This paper points out that unit root tests are likely to suffer from size distortions precisely because the data are cointegrated. These size distortions are often substantial.

I illustrate this using a simple autoregressive, distributed lag (ARDL) system of two variables. The ARDL framework has a number of features which make it attractive for modelling dynamic relationships. It allows for interactions between variables, and incorporates both endogeneity and own and cross-lagged effects. These features capture likely behaviors of real economic time series. The ARDL framework can be solved to identify parameter values that cause the two variables to be cointegrated. Furthermore, the ARDL framework is easily transformed to an error correction specification, which facilitates interpretation of dynamic relationships.

TABLE 1 illustrates the problem with size. X and Y are two simulated data series where the parameter values for the data generating process (DGP) have been chosen to ensure that they are cointegrated. Because the series are cointegrated, each of the series must have a unit root. I subject each series to three unit root tests: the augmented Dickey-Fuller test (ADF), the Phillips-Perron test, and the DF-GLS test. 10,000 simulations of sample sizes 100 were conducted. Significance levels were set equal to 0.05. The table reports the associated Type I error rates. All simulations were done using *Stata*, Version 14.[1]

While the ADF and DF-GLS tests produce Type I error rates for X close to 0.05, the Phillips-Perron test produces an error rate over 0.40. For Y, the results are much worse. Type I error rates are 0.206, 1.000, and 0.685 for the ADF, Phillips-Perron, and DF-GLS

[1] All programs used to produce the results for this paper are available from the author.

tests, respectively.[2] The ADF regressions show good diagnostics, with little serial correlation evident in the residuals. As I show below, unit root test results such as these are quite easy to produce with cointegrated data.

I proceed as follows. Section II presents the theory that motivates the simulation work. Section III presents additional Monte Carlo evidence of size distortions for cointegrated data. Section IV provides an explanation for my results. Section V concludes by discussing the implication of these findings for estimation of error correction models.

II. THEORY

Consider the following ARDL(1,1) model.

1)
$$y_t = \beta_{10} + \beta_{12}x_t + \gamma_{11}y_{t-1} + \gamma_{12}x_{t-1} + \varepsilon_{yt} \;, \varepsilon_{yt} \sim NID(0,1) \;,$$
$$x_t = \beta_{20} + \beta_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}x_{t-1} + \varepsilon_{xt} \;, \varepsilon_{xt} \sim NID(0,1) \;,$$

$t = 1,2,\dots,T$. This can be rewritten in VAR form as:

(2) $$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{bmatrix}.$$

where the parameters a_{ij}, *i=1,2, j=1,2,3,4* are each functions of the β and γ terms of Equation (1).

Define $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. The determinant of the matrix $(A - \lambda I) = \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix}$ is the characteristic equation of A, and the values of λ that set this equation equal to zero are the associated characteristic roots, or eigenvalues:

(3) $$\lambda^2 + (-a_{11} - a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0 \;.$$

A necessary condition for y_t and x_t to be CI(1,1) is that the corresponding solutions to (3) be given by $\lambda_1 = 1,\; |\lambda_2| < 1$.

[2] Lag lengths for the Phillips-Perron and DF-GLS tests were chosen using the default options supplied by *Stata*.

The following conditions on a_{11}, a_{12}, a_{21} and a_{22} are sufficient to ensure that $\lambda_1 = 1,\ |\lambda_2| < 1$.[3]

(4a) $0 < a_{22} < 1$

(4b) $0 < a_{12}a_{21} < 1 - a_{22}$

(4c) $a_{11} = 1 - \frac{a_{12}a_{21}}{1-a_{22}}$.

We can work backwards from (4a) – (4c) to obtain β and γ values consistent with $\lambda_1 = 1$, $|\lambda_2| < 1$.

Let β_{12} and β_{21} take any values such that $\beta_{12}\beta_{21} \neq 1$. Then

(5a) $\gamma_{21} = a_{21} - a_{11}\beta_{21}$

(5b) $\gamma_{11} = a_{11}(1 - \beta_{12}\beta_{21}) - \beta_{12}\gamma_{21}$

(5c) $\gamma_{22} = a_{22} - a_{12}\beta_{21}$

(5d) $\gamma_{12} = a_{12}(1 - \beta_{12}\beta_{21}) - \beta_{12}\gamma_{22}$

will produce a_{11}, a_{12}, a_{21} and a_{22} values such that $\lambda_1 = 1,\ |\lambda_2| < 1$.

Equation (2) can be arranged in vector error correction (VEC) model form as:

6)
$$\begin{aligned} \Delta y_t &= a_{10} + \delta_y(y_{t-1} + \theta x_{t-1}) + \epsilon_{yt} \ , \\ \Delta x_t &= a_{20} + \delta_x(y_{t-1} + \theta x_{t-1}) + \epsilon_{xt} \ , \end{aligned}$$

where the coefficients $\theta,\ \delta_y,$ and δ_x, as well as the error terms ϵ_{yt} and ϵ_{xt}, are functions of the a_{ij} terms, *i=1,2, j=1,2,3,4*. This allows the long-run equilibrium relationship between y_t and x_t, represented by the parameter θ; and the speed-of-adjustment parameters δ_y and δ_x, to all be expressed as functions of a_{11}, a_{12}, a_{21} and a_{22}.

7a) $\theta = \frac{a_{22}-1}{a_{21}}$

7b) $\delta_y = -\frac{a_{12}a_{21}}{1-a_{22}}$

[3] These conditions are taken from Enders (2010, page 369). I have made the conditions more restrictive to make sure that the speed of adjustment parameters have the correct sign and size.

7c) $\delta_x = a_{21}$.

The parameter values chosen in this way will ensure that (i) $-1 < \delta_y < 0$, and (ii) $-1 < \delta_x\theta < 0$, so that the VEC model is well-behaved.

III. RESULTS

This section reports results from ten additional cases that highlight the problem with size distortions. The first two columns of TABLE 2 describe the model parameters and time series characteristics associated with the data generating processes (DGPs) for each case. I have chosen cases that cover a wide range of behaviours. The cases are sorted in ascending order of δ_y, the speed of adjustment coefficient for the Y series. δ_y ranges from a low of -0.16 to a high of -0.90. The last column reports the characteristic roots associated with the respective model parameters. In all cases, $\lambda_1 = 1$, $|\lambda_2| < 1$.

TABLE 3 reports more simulation findings demonstrating that the results from TABLE 1 are not isolated outcomes. The ten panels of TABLE 3 correspond to the ten cases of TABLE 2. The top panel reports Monte Carlo results using the parameter values from Case 1:

$$\beta_{10} = 0,\ \beta_{12} = 2, \gamma_{11} = 1.24,\ \gamma_{12} = -1.70,\ \beta_{20} = 0,\ \beta_{21} = 5,\ \gamma_{21} = -4.40, \gamma_{22} = 1.75.$$

The fact that both β_{12} and β_{21} are nonzero implies that if one of the series is I(1), the other must be as well (cf. Equation 1). These values generate a VEC model with long-run equilibrium and speed of adjustment parameters $\theta = 1.25$, $\delta_y = -0.16$, and $\theta\delta_x = -0.25$. This implies that the long-run relationship between Y and X is given by $y_t = -1.25x_t$. A one-unit increase in y_t from its equilibrium value causes the next period's value of Y to decrease by 0.16 units. A one-unit increase in x_t from its equilibrium value causes the next period's value of X to decrease by 0.25 units (=$\delta_x\theta$). As in TABLE 1, the Monte Carlo results are based on 10,000 simulations of sample sizes 100. As a point of comparison,

TABLE 3 also reports the results of unit root tests for a random walk variable, $z_t = z_{t-1} + \varepsilon_{zt}$. $\varepsilon_{zt} \sim NID(0,1)$ The Z column is useful for illustrating the range of deviations that can be expected from sampling error.

The results for the *X* variable demonstrate that the size distortions associated with each of the tests can be quite substantial. The Type I error rates for the ADF, Phillips-Perron, and DF-GLS tests are 53.6, 87.6, and 64.0 percent, respectively. Thus, given sample data from this DGP and applying any of the three tests supplied in *Stata*, a researcher would incorrectly conclude that the *X* variable was stationary over half the time. The results for *Y* also show size distortions, but of a smaller degree. These results are to be compared to those reported for the *Z* variable, which is a pure random walk process. All three unit root tests produce Type I error rates for *Z* that are close to 5 percent.

As is well-known, results from unit root tests can differ substantially depending on the number of lagged differenced (LD) terms included in the unit root specification. *Stata* automatically selects the number of LD terms for the Phillips-Perron and DF-GLS tests. The ADF test requires the user to supply the number of lags. For the ADF tests, I chose lag orders that were sufficient to generate white noise behaviour in the residuals.[4] The last row of the panel reports the results of a Breusch-Godfrey test where the null hypothesis is no serial correlation. The test results for the *X* and *Y* variables are close to the value of 0.05 that one would expect were there no serial correlation. These are virtually identical to those for the random walk *Z* variable which has no serial correlation by construction. Based on these results, a researcher would conclude that the ADF test was correctly specified.

The next nine panels report more unit root test results. I have highlighted the results that show substantial size distortions. In the second panel, unit root tests for both the *X* and *Y* variables reveal Type I error rates well above 5 percent for all three tests. For the ADF test,

[4] Lag lengths for Cases 1 to 10 are 2, 2, 4, 3, 6, 2, 3, 4, 4, and 5, respectively.

Type I error rates are 0.390 and 0.309, respectively. The Breusch-Godfrey test results indicate that sufficient lags have been included in the ADF specification. For the Phillips-Perron and DF-GLS tests, the Type I error rates are 0.770 and 0.655, and 0.492 and 0.394, respectively. The results for the *X* and *Y* variables contrast with the results for the benchmark *Z* variable, which are approximately 5 percent across all three tests. The results from this second panel indicate that a researcher would frequently conclude that both *X* and *Y* were both stationary, and hence not cointegrated.

The highlighted areas in the subsequent panels accumulate further evidence that unit root tests of cointegrated data are frequently characterized by substantial size distortions. An egregious example is Case 6, where the Type I error rates for the *Y* variable are 0.921, 1.000, and 0.931. A researcher would incorrectly classify the order of integration for this variable over 90 percent of the time using any of the three unit root tests provided by *Stata*.

It turns out that the size distortions for the ADF test can be eliminated by adding sufficient lagged differenced (LD) terms to the ADF specification. However, knowing the correct number of LD terms to add is impossible in practice. Two common methods for determining the number of LD terms are (i) testing the residuals for serial correlation; and (ii) using information criteria such as the AIC and SIC to select the LD specification with the lowest AIC/SIC value (Harris, 1992).

TABLE 4 reports the results of an analysis where these two methods are employed to determine the appropriate number of LD terms to add to the ADF specification. The *X* and *Y* data for TABLE 4 are generated using the DGP for Case 1. As before, the *Z* data are pure random walk data and are included as a benchmark. One to ten LD terms are successively added to the ADF specification. Breusch-Godfrey tests for each LD specification are reported in the top panel of TABLE 4. Average AIC and SIC values for each LD specification are reported in the subsequent two panels.

TABLE 4 is designed to address this thought experiment: based on the results from the first three panels, how many LD terms would a researcher think is the "correct" number of terms to add? For example, when LAGS = 1, the null hypothesis of no serial correlation is rejected approximately 6.0 and 5.7 percent of the time for the *X* and *Y* series. For LAGS = 2, rejection rates are 5.6 and 5.5 percent.[5] The average AIC values for the *X* and *Y* series when LAGS = 1 are 168.96 and 41.51. These successively increase as additional LD terms are added. Likewise, the average SIC values for the *X* and *Y* series achieve their minimum when LAGS = 1. Using the diagnostics from these three panels, a researcher might conclude that the "correct" number of LD terms to add was 1 or 2.

The fourth panel of TABLE 4 reports the ADF Type I error rates for each LD specification. When LAGS = 1, the Type I error rates for the *X* and *Y* variables are 0.734 and 0.148. When LAGS = 2, they are 0.546 and 0.106, respectively. In other words, using commonly accepted methods for determining the appropriate number of LD terms, a researcher would likely conclude that one, or at most two, LD terms was sufficient to control for serial correlation in the ADF specification. Either strategy would result in the researcher concluding that the *X* variable was stationary over half the time. In fact, it would take ten or more LD terms to reduce the size of the ADF test to 5 percent. The diagnostic tests are unable to identify the appropriate number of LD terms.

Similar results are obtained for the remaining cases (the Appendix reports the results of following the same procedure for Case 2). In all cases, the information criteria select a single LD term. Tests for serial correlation generally indicate that more than one LD term should be included, but not so many as to eliminate the size distortion. The next section explains that this inability of the diagnostic tests is because the apparent "success" from adding LD terms to the ADF specification is spurious.

[5] In practice, a researcher only has a single test for serial correlation to go on, so that it is likely that that he/she would find a single LD term to be sufficient in this case.

IV. DISCUSSION

The explanation for the poor size performance of unit root tests with cointegrated data can be linked to how critical values are determined in the Dickey-Fuller framework. Given Monte Carlo simulation of the random walk process,

$z_t = z_{t-1} + \varepsilon_t$,

repeated OLS estimation of the DF specification below

$\Delta z_t = \gamma + \rho z_{t-1} + error_{zt}$.

produces an empirical distribution of t values, $t = \frac{\hat{\rho}}{s.e.(\hat{\rho})}$, associated with testing the null hypothesis: $H_0: \rho = 0$, which is true given the random walk process.

In my simulations of the ARDL(1,1) data, the DGP is given by

$y_t = \beta_{10} + \beta_{12} x_t + \gamma_{11} y_{t-1} + \gamma_{12} x_{t-1} + \varepsilon_{yt}$,

$x_t = \beta_{20} + \beta_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} x_{t-1} + \varepsilon_{xt}$,

and the corresponding differenced specifications are given by

$\Delta y_t = \beta_{10} + \rho_y y_{t-1} + error_{yt}$

$\Delta x_t = \beta_{20} + \rho_x x_{t-1} + error_{xt}$,

where $\rho_y = \delta_y$, $\rho_x = \delta_x \theta$, $error_{yt} = \delta_y \theta x_{t-1} + \epsilon_{yt}$, and $error_{xt} = \delta_x y_{t-1} + \epsilon_{xt}$ (cf. Equation 6). For cointegrated data, either $\delta_y < 0$, or $\delta_x \theta < 0$, or both. This implies that the unit roots are misspecified, because they test $H_0: \rho_y, \rho_x = 0$. While this is appropriate for random walk data, it is not correct when the data are cointegrated.

V. CONCLUSION

This paper demonstrates that unit root tests can suffer from inflated Type I error rates when data are cointegrated. This should be of interest to researchers who are interested in estimating relationships between nonstationary variables. Standard procedure calls for testing variables for unit roots before proceeding to the estimation of error correction models.

The results of this study demonstrate that the very fact that the data are cointegrated can render unit root tests unreliable. This suggests that researchers should be conservative in the weight they attach to individual unit root tests, opting for a more holistic approach when determining whether data are cointegrated.

REFERENCES

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TABLE 1
Example of Unit Root Test Results Using Cointegrated Data

UNIT ROOT TEST	***X***	***Y***
ADF	0.051	0.206
Phillips-Perron	0.410	1.000
DF-GLS	0.087	0.685

NOTE: Values in the table are Type I error rates associated with the null hypothesis that the data series have a unit root. The underlying DGP is the ARDL framework represented by Equation 1 in the text, with the following parameter values:

$\beta_{10} = 0, \beta_{12} = 3, \gamma_{11} = 1,\ \gamma_{12} = -3,\ \beta_{20} = 0,\ \beta_{21} = -1,\ \gamma_{21} = -0.2, \gamma_{22} = 0.2$

The corresponding long-run equilibrium and speed of adjustment parameters (see Equations 7a)-7c) are given by: $\theta = -0.1, \delta_y = -0.5, \delta_x = 1, \theta\delta_x = -0.1$.

TABLE 2
DESCRIPTION OF CASES

CASE	***MODEL PARAMETERS (1)***	***TIME SERIES CHARACTERISTICS (2)***	***CHARACTERISTIC ROOTS (3)***
1	$\beta_{12} = 2.00, \gamma_{11} = 1.24,\ \gamma_{12} = -1.70,$ $\beta_{21} = 5.00,\ \gamma_{21} = -4.40, \gamma_{22} = 1.75$	$\delta_y = -0.16, \delta_x = -0.20, \theta\delta_x = -0.25, \theta = 1.25$	$\lambda_1 = 1, \lambda_2 = 0.59$
2	$\beta_{12} = 2.00, \gamma_{11} = 1.80,\ \gamma_{12} = -1.60,$ $\beta_{21} = 5.00,\ \gamma_{21} = -4.50, \gamma_{22} = 1.25$	$\delta_y = -0.20, \delta_x = -0.50, \theta\delta_x = -0.25, \theta = 0.50$	$\lambda_1 = 1, \lambda_2 = 0.55$
3	$\beta_{12} = 2.00, \gamma_{11} = 0.35,\ \gamma_{12} = 0.60,$ $\beta_{21} = 3.00,\ \gamma_{21} = -2.05, \gamma_{22} = -2.80$	$\delta_y = -0.25, \delta_x = 0.20, \theta\delta_x = -0.80, \theta = -4.00$	$\lambda_1 = 1, \lambda_2 = -0.05$
4	$\beta_{12} = 1.00, \gamma_{11} = 1.45,\ \gamma_{12} = -0.90,$ $\beta_{21} = 5.00,\ \gamma_{21} = -3.65, \gamma_{22} = 1.30$	$\delta_y = -0.45, \delta_x = -0.90, \theta\delta_x = -0.20, \theta = 0.22$	$\lambda_1 = 1, \lambda_2 = 0.35$
5	$\beta_{12} = 0.50, \gamma_{11} = 0.72,\ \gamma_{12} = -0.85,$ $\beta_{21} = 1.00,\ \gamma_{21} = -0.92, \gamma_{22} = 1.10$	$\delta_y = -0.48, \delta_x = -0.40, \theta\delta_x = -0.50, \theta = 1.25$	$\lambda_1 = 1, \lambda_2 = 0.02$
6	$\beta_{12} = 3.00, \gamma_{11} = 1.00,\ \gamma_{12} = -3.00,$ $\beta_{21} = -1.00,\ \gamma_{21} = -0.20, \gamma_{22} = 0.20$	$\delta_y = -0.50, \delta_x = -1.00, \theta\delta_x = -0.10, \theta = 0.10$	$\lambda_1 = 1, \lambda_2 = 0.40$
7	$\beta_{12} = 2.00, \gamma_{11} = 2.20,\ \gamma_{12} = -1.80,$ $\beta_{21} = 5.00,\ \gamma_{21} = -2.90, \gamma_{22} = 1.35$	$\delta_y = -0.60, \delta_x = -0.90, \theta\delta_x = -0.15, \theta = 0.17$	$\lambda_1 = 1, \lambda_2 = 0.25$
8	$\beta_{12} = 0.80, \gamma_{11} = 0.72,\ \gamma_{12} = -1.08,$ $\beta_{21} = 0.60,\ \gamma_{21} = -0.64, \gamma_{22} = 0.96$	$\delta_y = -0.60, \delta_x = -0.40, \theta\delta_x = -0.40, \theta = 1.00$	$\lambda_1 = 1, \lambda_2 = 0$

CASE	***MODEL PARAMETERS (1)***	***TIME SERIES CHARACTERISTICS (2)***	***CHARACTERISTIC ROOTS (3)***
9	$\beta_{12} = 0.80, \gamma_{11} = 0.52,\ \gamma_{12} = -1.16,$ $\beta_{21} = 0.60,\ \gamma_{21} = -0.52, \gamma_{22} = 1.06$	$\delta_y = -0.80, \delta_x = -0.40, \theta\delta_x = -0.30, \theta = 0.75$	$\lambda_1 = 1, \lambda_2 = -0.1$
10	$\beta_{12} = 2.00, \gamma_{11} = 1.90,\ \gamma_{12} = -1.90,$ $\beta_{21} = 5.00,\ \gamma_{21} = -1.40, \gamma_{22} = 1.40$	$\delta_y = -0.90, \delta_x = -0.90, \theta\delta_x = -0.10, \theta = 0.11$	$\lambda_1 = 1, \lambda_2 = 0$

NOTE: Each of the cases above is based on the ARDL framework of Equation (1), with associated parameter values given in Column (1). The values of the characteristics δ_y, δ_x, and θ reported in Column (2) are, respectively, the values of the speed of adjustment parameters and the long-run relationship parameter between *Y* and *X* in the error correction models of Equation (6) that correspond to the values of the model parameters for that case. $\theta\delta_x$ identifies the systematic change in Δx_t corresponding to a one-unit change in x_t. Necessary conditions for the series to be well-behaved are (i) $-1 < \delta_y < 0$, and (ii) $-1 < \delta_x\theta < 0$. The last column reports the values of the characteristic roots in the VAR specification of Equation (2) for that case. A necessary condition for the series to be cointegrated is that $\lambda_1 = 1$, $|\lambda_2| < 1$.

TABLE 3
More Examples of Unit Root Test Results Using Cointegrated Data

CASE	*TEST*	*X*	*Y*	*Z*
1	***ADF***	0.536	0.100	0.055
	Phillips-Perron	0.876	0.203	0.062
	DF-GLS	0.640	0.121	0.045
	BREUSCH-GODFREY TEST:	0.056	0.058	0.059
2	***ADF***	0.390	0.309	0.055
	Phillips-Perron	0.770	0.655	0.062
	DF-GLS	0.492	0.394	0.045
	BREUSCH-GODFREY TEST	0.059	0.057	0.059
3	***ADF***	0.113	0.048	0.049
	Phillips-Perron	0.970	0.029	0.062
	DF-GLS	0.429	0.066	0.045
	BREUSCH-GODFREY TEST	0.056	0.063	0.062
4	***ADF***	0.200	0.403	0.053
	Phillips-Perron	0.787	0.963	0.062
	DF-GLS	0.419	0.680	0.045
	BREUSCH-GODFREY TEST	0.063	0.069	0.057
5	***ADF***	0.159	0.106	0.045
	Phillips-Perron	1.000	1.000	0.062
	DF-GLS	0.756	0.639	0.045
	BREUSCH-GODFREY TEST	0.053	0.063	0.063
6	***ADF***	0.051	0.921	0.055
	Phillips-Perron	0.024	1.000	0.062
	DF-GLS	0.038	0.931	0.045
	BREUSCH-GODFREY TEST	0.061	0.056	0.059
7	***ADF***	0.083	0.578	0.053
	Phillips-Perron	0.452	0.998	0.062
	DF-GLS	0.170	0.808	0.045
	BREUSCH-GODFREY TEST	0.057	0.066	0.057
8	***ADF***	0.266	0.405	0.049
	Phillips-Perron	0.999	1.000	0.062
	DF-GLS	0.677	0.782	0.045
	BREUSCH-GODFREY TEST	0.062	0.062	0.062

CASE	***TEST***	***X***	***Y***	***Z***
9	***ADF***	0.087	0.311	0.049
	Phillips-Perron	0.952	1.000	0.062
	DF-GLS	0.354	0.696	0.045
	BREUSCH-GODFREY TEST	0.055	0.061	0.062
10	***ADF***	0.050	0.347	0.048
	Phillips-Perron	0.316	1.000	0.062
	DF-GLS	0.085	0.826	0.045
	BREUSCH-GODFREY TEST	0.059	0.063	0.060

NOTE: The values in the table are the rejection rates of the respective null hypothesis. For the unit root tests (*ADF, Phillips-Perron,* and *DF-GLS)*, the null hypothesis is that the series has a unit root. For the Breusch-Godfrey tests, the null hypothesis is that the residuals associated with the *ADF* test are not serially correlated.

TABLE 4
The Effect of Adding Lagged Differenced Terms to the Dickey-Fuller Unit Root Regression Equation: Case 1

	X	*Y*	*Z*
BREUSCH-GODFREY TESTS:			
LAGS = 1	0.060	0.057	0.057
LAGS = 2	0.056	0.055	0.063
LAGS = 3	0.062	0.056	0.060
LAGS = 4	0.056	0.054	0.059
LAGS = 5	0.059	0.059	0.060
LAGS = 6	0.060	0.060	0.062
LAGS = 7	0.059	0.061	0.061
LAGS = 8	0.067	0.058	0.063
LAGS = 9	0.056	0.064	0.065
LAGS = 10	0.060	0.061	0.064
AIC VALUES:			
LAGS = 1	168.96	41.51	279.67
LAGS = 2	169.93	42.39	280.44
LAGS = 3	170.91	43.55	281.41
LAGS = 4	171.84	44.49	282.39
LAGS = 5	172.91	45.38	283.40
LAGS = 6	173.65	46.40	284.03
LAGS = 7	174.65	47.31	284.93
LAGS = 8	175.51	48.31	285.79
LAGS = 9	176.73	49.25	286.81
LAGS = 10	177.49	50.05	287.48
SIC VALUES:			
LAGS = 1	179.34	51.89	290.05
LAGS = 2	182.90	55.37	293.42
LAGS = 3	186.48	59.12	296.98
LAGS = 4	190.00	62.66	300.56
LAGS = 5	193.67	66.14	304.16
LAGS = 6	197.01	69.75	307.39
LAGS = 7	200.60	73.26	310.88
LAGS = 8	204.06	76.86	314.33
LAGS = 9	207.87	80.39	317.95
LAGS = 10	211.23	83.78	321.22

TABLE 4 (continued)

	X	***Y***	***Z***
ADF UNIT ROOT TESTS:			
LAGS = 1	0.734	0.148	0.054
LAGS = 2	0.546	0.106	0.050
LAGS = 3	0.405	0.085	0.055
LAGS = 4	0.291	0.068	0.054
LAGS = 5	0.222	0.063	0.047
LAGS = 6	0.166	0.054	0.048
LAGS = 7	0.143	0.054	0.043
LAGS = 8	0.116	0.046	0.042
LAGS = 9	0.092	0.051	0.045
LAGS = 10	0.082	0.046	0.044

NOTE: The values in the top panel ("Breusch-Godfrey Tests") are the rejection rates associated with the null hypothesis of no serial correlation for alternative specifications of lagged differenced (LD) terms in the ADF specification. The values in the next two panels ("AIC Values" and "SIC Values") are the average information criteria values associated with the respective LD specifications. The number of observations are held constant across the different specifications. The values in the bottom panel ("ADF Unit Root Tests") are the Type I error rates associated with the null hypothesis of a unit root using the ADF test with the designated number of lagged, differenced terms.

APPENDIX
The Effect of Adding Lagged Differenced Terms to the Dickey-Fuller Unit Root Regression Equation: Case 2

	X	*Y*	*Z*
BREUSCH-GODFREY TESTS:			
LAGS = 1	0.069	0.069	0.057
LAGS = 2	0.060	0.060	0.063
LAGS = 3	0.063	0.059	0.060
LAGS = 4	0.056	0.057	0.059
LAGS = 5	0.062	0.056	0.060
LAGS = 6	0.059	0.058	0.062
LAGS = 7	0.059	0.061	0.061
LAGS = 8	0.065	0.056	0.063
LAGS = 9	0.060	0.060	0.065
LAGS = 10	0.063	0.064	0.064
AIC VALUES:			
LAGS = 1	179.18	14.35	279.67
LAGS = 2	179.93	15.04	280.44
LAGS = 3	181.00	16.15	281.41
LAGS = 4	181.89	16.95	282.39
LAGS = 5	182.89	17.86	283.40
LAGS = 6	183.62	18.93	284.03
LAGS = 7	184.67	19.96	284.93
LAGS = 8	185.51	20.94	285.79
LAGS = 9	186.71	21.91	286.81
LAGS = 10	187.58	22.78	287.48
SIC VALUES:			
LAGS = 1	189.56	24.73	290.05
LAGS = 2	192.90	28.02	293.42
LAGS = 3	196.57	31.72	296.98
LAGS = 4	200.06	35.12	300.56
LAGS = 5	203.65	38.62	304.16
LAGS = 6	206.97	42.28	307.39
LAGS = 7	210.62	45.91	310.88
LAGS = 8	214.06	49.49	314.33
LAGS = 9	217.85	53.05	317.95
LAGS = 10	221.32	56.52	321.22

APPENDIX (continued)

	X	*Y*	*Z*
ADF UNIT ROOT TESTS:			
LAGS = 1	0.592	0.477	0.054
LAGS = 2	0.411	0.324	0.050
LAGS = 3	0.287	0.218	0.055
LAGS = 4	0.202	0.163	0.054
LAGS = 5	0.145	0.123	0.047
LAGS = 6	0.113	0.094	0.048
LAGS = 7	0.098	0.083	0.043
LAGS = 8	0.079	0.065	0.042
LAGS = 9	0.067	0.056	0.045
LAGS = 10	0.059	0.052	0.044

NOTE: Values in the top panel ("Breusch-Godfrey Tests") are the rejection rates associated with the null hypothesis of no serial correlation. Values in the next two panels ("AIC Values" and "SIC Values") are the average information criteria values associated with the respective lag specifications. The number of observations are held constant across the different specifications. The values in the bottom panel ("ADF Unit Root Tests") are the Type I error rates associated with the null hypothesis of a unit root using the ADF test with the designated number of lagged, differenced terms.