Taxation and Fiscal Expenditure in a Growth Model with Endogenous Fertility

Norman Sedgley and Bruce Elmslie

Abstract

Most growth theorists agree that understanding the economics of innovation and technological change is central to understanding why some countries are richer and/or grow faster than other countries. The driving force behind recent developments in endogenous innovation models of growth is a desire to eliminate population scale effects. In the semi endogenous growth model growth becomes proportional to the exogenous population growth rate but invariant to policy. This paper makes population growth endogenous by modeling fertility along the lines of Barro and Becker (*Fertility Choice in a Model of Economic Growth*, 1989) and models an array of government policies to demonstrate how some policies can impact levels and growth rates in a scale free endogenous growth model. In the model government policies are categorized according to whether they have level effects only, level and growth effects, or no impact on levels and/or growth.

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I.   Introduction

The literature on the impact of economic policy variables on the steady state growth rate has been motivated, in part, by the attempt to understand why cross-country growth rates and levels of per capita income differ so much (Lucas, 1988; Rebelo, 1991; Easterly, 1993). Models that allow policy to impact steady state growth rates fit the data better since there are very wide discrepancies in living standards and growth rates that appear to be very persistent. The neoclassical model and semi-endogenous growth models share a common property in that these models typically predict that tax policy impacts levels only, in other words the effects are transitory. We address the question of the long run growth versus level effects of taxation in a relatively simple endogenous growth model that makes fertility choice endogenous. Additionally, the model allows for a reasonably wide variety of policy variables to determine the steady state growth rate. The model we generalize is directly inspired by Jones's semi-endogenous model, but the tax structure of the economy is carefully specified to allow for taxes on wages, assets and consumption, and for general government expenditures that do not enter the utility function or production function as well as government expenditures that support R&D. We find that an increase in the asset tax lowers growth, while an increase in the consumption tax has level effects only and a wage tax has no impact on growth or levels. We can also look at pro family policies that lower the cost of having children.

Recent advances in fully endogenous growth theory (Segerstrom, 1998; Young, 1998; Howitt, 1999; Zeng and Zhang, 2002; Peretto, 2007, 2011) generate growth and level effects from various forms of taxation. Empirically, links have long been established. Early research (Barro, 1991; Kormendi and Meguire 1986; Islam, 1995)

finds, for example, that cross country growth rates are inversely related to the share of government consumption in GDP and growth rates are positively correlated with an index of property rights protection.

Recent empirical work has been directly motivated by endogenous growth theory (Jones, 2002; Sedgley and Elmslie, 2010; Gemmell, Kneller, and Sanz, 2011). While fully acknowledging the limitations of the available time series data, Gemmell, Kneller, and Sanz (2011) show evidence of long run effects of taxation on a panel of OECD countries covering 30 to 35 years. Given the results of Jones (2002) and Sedgley and Elmslie (2010), they also show a remarkably short transition period. While the current paper, along with Peretto (2007 and 2011) and Zeng and Zhang (2002) develop models implying differing growth versus level effects from various forms of taxation (e.g. consumption and asset taxes), the empirical literature has not yet progressed to test for differential effects.[1] In a simpler framework, McGratten (2012) extends a standard neoclassical growth model to allow for a wide range of taxes on property, capital, profits and sales. She finds strong level effects from data during the Great Depression.

Durlauf, Kourtellos, and Ming Tan (2008) show that, in a cross sectional regression framework, macroeconomic policy and regional heterogeneity play a strong role in explaining growth. Romer and Romer (2010) find a strong relationship between changes in taxes measured in terms of taxes as a percentage of GDP and subsequent growth over a three year period. Specifically they estimate that a 1% increase in taxes decreases growth by 3% over the next 3 years. These results are confirmed by Barro and Redlick (2011) who find a one-year tax rate multiplier of -1.1 using a new time series on

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[1] The practical difficulty arising from any empirical test surrounds the issue of differentiating between transitional and steady state effects in the models, Sedgley and Elmslie (2013).

overall average marginal tax rates including federal and state income taxes and the social security (FICA) tax. Both of these results suggest possible growth effects from taxation and strong level effects. Arin et al. (2013), using data on marginal tax rates from the U.S., U.K. and four Scandinavian countries find strong growth effects from increased average marginal rates.

Recent work has pointed to a relationship between government policies and economic growth. This is a major step forward in our understanding of the growth process. The current paper represents an attempt to catch theory up with the developing empirics. Moreover, the model developed here generates new hypotheses to help guide future empirical work. Romer and Romer (2010) suggest that new work should "investigate whether the output consequences of tax changes depend not only on their size, but on their other features as well." (p. 800) Our model makes specific predictions for differential effects of asset, wage, and consumption taxation in terms of growth and level effects that are directly testable.

II. Theoretical Model

The general endogenous innovation framework extended to include endogenous fertility has the tendency to become complicated very quickly. It is our desire to show that adding endogenous demographics can be done in a relatively simple and well accepted framework. The analysis that follows borrows from Becker and Barro (1989) and Barro and Sala-i-Martin (1999) in assuming a linear cost relationship in raising children. Their analysis suggests that the fertility rate is related to the ratio of consumption to the capital stock. An increase in the level of consumption increases child

quality and the demand for children. The increase in the capital stock signals an increase the opportunity cost of having children and lowers fertility.

Another recent approach taken by Galor and Weil (2000) looks at a quantity/quality tradeoff in reproductive choice. Their model is based on the idea that greater future technological change causes families to shift from quantity to quality since the reward to human capital investment is expected to rise. This approach allows the actual transition from rising population growth rates to falling population growth rates to be modeled. The purpose of the current paper is not to predict the point of transition from increasing population growth to decreasing population growth. The reason for our choice of modeling strategy is simplicity. The Becker and Barro model, familiar to and accepted by most economists, allows for a focus on the relationship between capital accumulation, innovation, fertility, and the policy implications of the model as they would most likely apply to a modern R&D driven economy past the turning point of the demographic transition.

Production

Our specification of production is standard and follows a commonly accepted formulation motivated by leading work in the field such as Jones (1995a, 1995b, 1999) and Segerstrom (1998).

(1) $Y_t = C_t + I_t + N_t + G_t = L_t^{1-\alpha} Q_t^{\alpha-1} \int_0^Q A_{i,t} x_{i,t}^\alpha di$,

(2) $x_{i,t} = x_t = \dfrac{K_{i,t}}{A_{i,t}}$

Equation (1) shows that final output, produced under the conditions of perfect competition, is produced using the available labor force, and intermediate goods of

varying quality, A_i. Output can be used for consumption, C, investment in capital, I,

research and development, N, and government spending on final goods and services, G.

We allow $Q = L^\beta$ where Q is the number of intermediate sectors in the economy and

$\beta < 1$. We follow Aghion and Howitt (1998) in assuming that imitation happens as a

consequence of population growth and without the expenditure of additional resources.

We do not impose the condition that $\beta = 1$ and, therefore, the expansion of Q over time

does not eliminate the scale effect on the growth rate. We include the specification of Q

for generality[2]. Equation (2) is a production technology for intermediate goods supplied

under monopolistic competition and demonstrates a fundamental complimentary

relationship between capital accumulation and technology as suggested by Aghion and

Howitt (1998). We agree with the position that the process of capital accumulation and

the process of technological change are best viewed as complementary. The total capital

stock is:

$$(3) \quad K_t = \int_0^Q A_{i,t} x_{,ti} di = x_t \int_0^Q A_{i,t} di$$

We define the average level of productivity to be $A_t = \int_0^Q A_{i,t} di / Q_t$ and the leading edge

technology to be A_{MAX}. Define the productivity adjusted capital stock per sector as

$$(4) \quad \hat{k}_t = K_t / A_t Q_t = x_t \int_0^Q A_{i,t} di / AQ_t = x_{i,t} = x_t$$

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[2] The interested reader could easily explore the model's implications under a Aghion–Howitt type scale free growth model by setting $\beta = 1$ in $Q = L^\beta$ and $\varepsilon = 1$ (discussed below) in the production function for new ideas. Since our model follows Jones and builds a model with the least number of parameter restrictions our model should produce robust results.

Substituting (4) into (1) and suppressing time subscripts gives:

$$(5)\ Y = L^{1-\alpha} Q^{(\alpha-1)} \int_0^Q A_i (\frac{K}{AQ})^\alpha \, di = K^\alpha (AL)^{1-\alpha} \, ,$$

showing that the production technology is Cobb Douglas.

For innovation a Poisson process is specified, with an arrival rate proportional to the level of resources per sector devoted to R&D.

$$(6)\ g_A = \frac{\dot{A}}{A} = \frac{\dot{A}_{MAX}}{A_{MAX}} = \frac{\gamma\lambda}{A_{MAX}^{1-\varepsilon}} \left(\frac{N}{Q} \right)^\phi = \frac{\gamma\lambda}{A_{MAX}^{1-\varepsilon}} \left(\frac{N}{L^\beta} \right)^\phi , \ 0 < \phi \le 1, \ \varepsilon < 1.$$

λ is the Poisson parameter and γ is a research productivity parameter. $0 < \phi < 1$ captures the potential for the "stepping on toes" effect or the possibility of duplicated research effort. $\varepsilon < 1$ captures the balance between the "fishing out" effect where new discoveries become harder to find as the technological frontier advances and the "standing on shoulders" effect where past discoveries aid in the search for new discoveries. The exogenous arrival rate of horizontal innovations, which do not enhance productivity, is

$$\frac{\dot{Q}}{Q} = \beta \frac{\dot{L}}{L} \ .$$

Government

One of the main motivations for this paper is to study the role of a realistic menu of government policies in a scale free growth model with endogenous fertility. We keep the model tractable by making a number of simplifying assumptions. Government spending does not directly impact utility or the production function. The government runs a balanced budget and always satisfies:

(7) $G + T + S_R N_P = \tau_w wL + \tau_a rK + \tau_c C$

Where G is unproductive government spending, T is the value of transfers (assumed lump sum), S_R is an ad valorem subsidy to R&D, N_P is private expenditures on R&D, τ_w is the ad valorem income tax, τ_a is an ad valorem asset tax, and τ_c is an ad valorem consumption tax. To simplify matters further we assume that agents do not expect tax rates to change over time.

Extensions of the model could allow G to enter the production function and/or the utility function. Barro and Sala-I-Martin (1992) show how this can promote welfare and growth. We leave out these potentially important avenues of research to focus on the role of policy when fertility is endogenous in a scale free growth model.

Utility Maximization

The model of utility maximization follows the continuous time version of Barro's model developed fully in Barro and Sala-i-Martin (1999). Parents derive utility from consumption, total family size, and the number of children. Each adult inelastically supplies one unit of labor per time period. The problem is to maximize the present value of dynastic utility. The intertemporal utility function is:

$$(8)\ U = \int_0^\infty e^{-\rho t}\,(\Psi \ln L + \ln c + \Im \ln(b-d))dt\,,$$

where $\rho > 0$ is the discount rate, c is per capita consumption, L represents family size, b is the birth rate and d is the mortality rate. $\Psi > 0$ and $\Im > 0$ are elasticities. Utility is maximized subject to the following constraints:

$$(9)\ \dot{k} = (1-\tau_w)w + \pi + (1-\tau_a)rk + t - (b-d)k - Bbk - (1+\tau_c)c\,,$$

$$(10)\ \dot{L} = (b-d)L\,,$$

Where w is the wage rate; π is per capita firm profits; Bbk is a linear cost function for rearing children, such that the present value of expenditures per child is directly proportional to the capital to labor ratio; r is the interest rate, t is per capita transfers, and n is the resources devoted to R&D per capita.

This type of dynamic optimization problem is familiar to economists. The problem has two choice variables, c and b. The state variables are k and L. Denote the Hamiltonian as H. Define the multipliers for each constraint respectively as μ and ν. The first order conditions can be summarized as:

a. $\dfrac{\partial H}{\partial c} = 0$, b. $\dfrac{\partial H}{\partial b} = 0$,

c. $\dot{\mu} = -\dfrac{\partial H}{\partial k}$, d. $\dot{\nu} = -\dfrac{\partial H}{\partial L}$,

First order conditions a and c provide the Euler equation.

(11) $g_c = (1 - \tau_a)r - (\rho + (b - d) + Bb)$.

Equations a, b, d, and $g_L = b - d$ defines the birth rate at each point in time, equal to:

(P) $\dfrac{\dot{L}}{L} = b - d = \dfrac{\Im \rho(c/k)}{\rho \dfrac{(1+B)}{(1+\tau_c)} - \Psi(c/k)}.$

Equation (11) is the familiar Euler equation where $(b - d)$ increases the discount rate; this is due to the diminishing marginal utility of children. Bb is subtracted from r because child rearing costs, Bbk, increase with k. As k rises, the return to capital falls. An asset tax lowers the return to foregoing consumption and through an intertemporal substitution effect raises current consumption and lowers the growth rate of consumption.

As equation (P) shows, the birth rate varies directly with the death rate, and inversely with the cost of child rearing, B. The relationship between c/k and b is direct. Per capita consumption, c, represents an income effect (a higher c improves child quality and raises b) and k represents a substitution effect (a higher k increases the cost of child rearing and lowers b.) Note that to be consistent with optimizing behavior the rate of population growth, $(b-d)$ and, therefore c/k, must be strictly positive (see Jones, 2001). A consumption tax has a direct impact on the fertility rate. Holding the c/k ratio constant an increase in τ_c makes spending resources on child rearing relatively less expensive causing substitution from other types of consumption toward child rearing and raising the birth rate.[3]

Profit Maximization and Innovation

Intermediate goods are supplied to the producers of final goods under the conditions of imperfect competition. Differentiation of equation (1) with respect to x yields the derived demand for an intermediate good. Profits, therefore, are:

(12) $\Pi = \alpha A_i L^{(1-\alpha)(1-\beta)} x_i^{\alpha} - (r+\delta)A_i x_i$,

where δ is the rate of capital depreciation. The first order condition from maximizing equation (12) together with equation (4) implies that the demand for intermediates is symmetric (a fact expressed in equations (2) and (4) and verified here) and:

(13) $r+\delta = \alpha^2(\hat{k}/L^{(1-\beta)})^{\alpha-1}$.

Resources flowing into innovation are determined by equating the marginal benefits of increasing R&D resources with the marginal costs of increasing R&D

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[3] This can be thought of as a sort of child tax credit. In our model the cost of children, bBk, is not subject to the consumption tax rate τ_c. More generous tax credits could be modeled as a reduction in B.

resources. Allow V to represent the present value of innovating. Equations (4), (12), and (13) allow the present value of profits to be expressed as:

$$(14) \quad V = \frac{A_{MAX} L^{(1-\alpha)(1-\beta)} \alpha(1-\alpha)(\hat{k})^{\alpha}}{r + \dfrac{\lambda}{A_{MAX}^{1-\varepsilon}} \left(\dfrac{N}{L^{\beta}}\right)^{\phi}} \; .$$

For an individual investing in research in the leading edge technology resulting in a marginal benefit of λV the marginal cost is $\left(A_{MAX}^{\frac{1-\varepsilon}{\phi}} \right)(1 - S_R)$. Setting marginal benefits equal to marginal costs gives a familiar research arbitrage equation,

$$(15) \quad \left(A_{MAX}^{\frac{1-\varepsilon}{\phi}} \right)(1 - S_R) = \frac{\lambda L^{(1-\alpha)(1-\beta)} \alpha(1-\alpha)(\hat{k})^{\alpha} A_{MAX}}{r + \dfrac{\lambda}{A_{MAX}^{1-\varepsilon}} \left(\dfrac{N}{L^{\beta}}\right)^{\phi}} \; .$$

Equilibrium and analysis

Equations (1) through (15) completely define the dynamics of the model economy in the steady state[4]. Note that the model is no longer a semi-endogenous growth model.

It is now a fully endogenous growth model where $g_A = \dfrac{\phi(1-\beta)}{(1-\varepsilon-\phi)} \left[\dfrac{\Im \rho \dfrac{c}{k}(.)}{\rho \dfrac{(1+B)}{(1+\tau_c)} - \psi \dfrac{c}{k}(.)} \right]$

and $\dfrac{c}{k}(.)$ expresses the ratio of consumption to capital as a function of policy variables

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[4] We focus on a positive analysis of policy effects on growth rates. In terms of social welfare the decentralized equilibrium will generally differ from the equilibrium chosen by a social planner. Our model includes all of the externalities discussed in Jones and Williams (2000). They outline four externalities where two externalities lead to underinvestment in R&D in a decentralized equilibrium and two externalities lead to over investment in R&D in a decentralized equilibrium. After calibrating the model they find that the economy is likely to under invest in R&D. Our model includes one additional externality associated with the marginal social benefit of raising children that is not incorporated in the private utility function (see Jones 2003). This externality reinforces the Jones and Williams conclusion that society is likely to under invest in R&D. In our model a Pareto optimal equilibrium can be obtained with a lump sum tax and an R&D subsidy or a direct subsidy to child production. Since the welfare implications are standard we leave a full exploration to the reader.

and parameters. Equation (5) specifies constant returns to scale in rival inputs and increasing returns in rival inputs and knowledge in goods production. These properties together with decreasing returns in knowledge production, equation (6), and a linear differential equation describing population growth in equation (P) guarantee the existence of a positive stable steady state growth rate. Our focus is on the steady state.   To understand the nature of equilibrium we derive a graphical representation of the determination of the steady state growth rate similar to that found in Aghion and Howitt (1998). Begin by expressing the model in terms of the average product of capital,

$Z = \tilde{k}^{\alpha-1}$ (where $\tilde{k} = K / AL$) and the ratio c / k. A system of graphical representations of the equilibrium is derived, so that taken together, it allows for an easy exposition of comparative steady state analysis under alternative value of policy parameters.

Proceed by deriving two unique relationships between Z and g_A. In the steady state $g_A = g_c$. Using the Euler equation this implies $r = \dfrac{g_A + \rho + (b-d) + Bb}{(1-\tau_a)}$. Combine this equation with equation (13) and note that $\hat{k} = \tilde{k}L/Q$:

(Z) $g_A = (1-\tau_a)\alpha^2 Z - (\rho + (1-\tau_a)\delta - d) - (1+B)b$.

This Z equation implies a positive relationship between Z and g_A since the partial derivative is $\dfrac{\partial g_A}{\partial Z} = (1-\tau_a)\alpha^2$, is greater than zero. This positive relationship is interpreted as the impact of a change in g_A on the Z. An increase in g_A increases the growth rate of consumption via the Euler equation. This implies an increase in the rate of interest and an increase in the rental cost of capital. The demand for capital falls. A lower level of capital implies that the average product of capital is greater.

The second relationship between Z and g_A is derived from the research arbitrage equation. Equation (15) is rewritten

as $r + g_A / \gamma (A_{MAX}^{\frac{\phi}{\phi}})(1 - S_R) = \lambda \alpha (1-\alpha) L^{(1-\alpha)(1-\beta)} \hat{k}^\alpha A_{MAX}$. Since $\tilde{k}^\alpha = Z^{\frac{\alpha}{\alpha-1}}$ equation (13) can be used to express the arbitrage equation as:

(R) $\quad g_A = \dfrac{\gamma \lambda \alpha (1-\alpha) L^{1-\beta} A_{MAX}^{1-\frac{1-\varepsilon}{\phi}}}{(1-S_R)} Z^{\frac{\alpha}{\alpha-1}} - \gamma \alpha^2 Z + \gamma \delta$

Where $L^{1-\beta} A_{MAX}^{1-\frac{1-\varepsilon}{\phi}}$ is constant in the steady state.[5]

The R equation defines a negative relationship between Z and g_A as seen from the

partial derivative, $\dfrac{\partial g_A}{\partial Z} = \dfrac{\alpha}{\alpha-1} \dfrac{\gamma \lambda \alpha (1-\alpha) L^{1-\beta} A_{MAX}^{1-\frac{1-\varepsilon}{\phi}}}{(1-S_R)} Z^{\frac{-1}{\alpha-1}} - \gamma \alpha^2$, is less than zero. The

intuition is, again, straight forward. A higher Z implies a lower capital stock. This

directly raises the rental rate of capital and diminishes the present value of an innovation

(see equation (14)). With the marginal benefits of raising R&D resources falling relative

to the marginal costs of raising R&D resources, resources to R&D are lowered and g_A is

lowered.

The model is now expressed in the form of the three equations: (Z), (R), and (P).

It is convient to re-write the (R) and (Z) equations in two alternative forms to use together

to understand the properties of balanced growth. One transformation, labeled (R') and

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[5] Log differentiating $L^{1-\beta} A_{MAX}^{1-\frac{1-\varepsilon}{\phi}}$ with respect to time gives

$(1 - \dfrac{1-\varepsilon}{\phi} g_A) + (1-\beta) g_L = -(1-\beta) g_L + (1-\beta) g_L = 0$ where we use the result that, according to

equation (6), it must be true in a steady state that $g_A = \dfrac{\phi(1-\beta)}{(1-\varepsilon-\phi)} g_L$.

(Z') expresses relationships between the fertility rate, b, and the average product of capital, Z. The second transformation, labeled (R'') and (Z'') provides for a unique relationship between Z and c/k on a balanced growth path.

(R') and (Z') are derived by noting that, according to equation (6), the steady state growth rate must satisfy:

(G) $g_A = \dfrac{\phi(1-\beta)}{1-\varepsilon-\phi} g_L$ where it is assumed that $\varepsilon + \phi < 1$.

With $g_L = b - d$ equation (G) is used with equations (Z) and (R) to express a relationship between b and Z.

(Z') $b = \dfrac{(1-\varepsilon-\phi)\alpha^2(1-\tau_a)}{\phi(1-\beta)+(1-\varepsilon-\phi)(1+B)} Z - \dfrac{(1-\varepsilon-\phi)}{\phi(1-\beta)+(1-\varepsilon-\phi)(1+B)}[\rho - d + (1-\tau_a)\delta] +$

$\dfrac{\phi(1-\beta)}{\phi(1-\beta)+(1-\varepsilon-\phi)(1+B)} d$

(R') $b = \dfrac{(1-\varepsilon-\phi)\gamma\lambda A_{MAX}^{\frac{1-\varepsilon}{\phi}}}{(1-\beta)\phi} \dfrac{L^{1-\beta}\alpha(1-\alpha)}{(1-S_R)} Z^{\frac{\alpha}{\alpha-1}} - \dfrac{(1-\varepsilon-\phi)\gamma\alpha^2}{(1-\beta)\phi} Z + \dfrac{(1-\varepsilon-\phi)\gamma\delta}{(1-\beta)\phi} + d$

(Z') defines a positive relationship between b and Z and (R') defines an inverse relationship between b and Z. The intuition for the slopes is the same as the intuition for the slopes of (R) and (Z) recognizing that b and g_A are positively related. The (R') and (Z') equations are represented in the top left graph in Figure 1.

(R'') and (Z'') are derived by taking the equation (P) for population growth and substituting out b in (R') and (Z'). Given the positive relationship between b and the ratio c/k it should be clear that (Z'') defines a positive relationship between Z and

c/k while (R'') implies a negative or inverse relationship between Z and c/k. Defining

the right hand side of (Z') as F and the right hand side of (R') as G we have:

(Z'') $$c/k = \frac{\rho \frac{(1+B)}{(1+\tau_C)}(F(Z,\tau_a,B)-d)}{\Im\rho + \psi(F(Z,\tau_a,B)-d)}$$

(R'') $$c/k = \frac{\rho \frac{(1+B)}{(1+\tau_C)}(G(Z,S_R)-d)}{\Im\rho + \psi(G(Z,S_R)-d)}$$

(Z'') and (R'') appear in the bottom graph in Figure 1. Equation (G) appears in the right

hand graph in Figure 1.

Consider what is gained from this extension of the endogenous innovation model.

This is a model of fully endogenous growth that is consistent with the lack of empirical

evidence of scale effects. Furthermore, it reintroduces policy variable effectiveness in

levels and growth rates. Any policy variation, therefore, has a much greater potential to

explain cross country differences in growth rates and levels of per capita income. The

latter must be the case since some policy variations have growth rate effects and these

differences compound into greater and greater differences in levels over time. This only

strengthens this approach to modeling endogenous growth since the model can more

easily explain the empirical regularity of great variance in levels and growth rates across

economies at a point in time and within some countries over time.

This model considers a number of policy variables in a fully endogenous new

growth model. The government collects taxes from wage income, consumption

expenditures, and asset income. The government uses these funds to support R&D

subsidies, lump sum transfers to households, and unproductive government spending. τ_w

does not appear in any of the equilibrium equations. Because labor supply is inelastic τ_w does not influence equilibrium. This is probably a reasonable approximation for most industrialized economies given the empirical evidence that individual labor supply is very inelastic. τ_a appears in the Euler equation. An increase in τ_a decreases the return of assets and lifts current consumption: lowering the growth rate of consumption intertemporally.   The equation describing the birth rate includes the tax rate τ_c. This is a novel result given that agents don't expect the consumption tax rate to change over time. If fertility where exogenous then τ_c would not impact the economies balanced growth path (Barro and Sala-i-Martin 1999). The research subsidy appears in the research arbitrage equation with an increase in S_R promoting private R&D spending.

The general equilibrium effects of different policies is easily understood using the (R'), (Z'), (R''), (Z'') and (G) equations.   These equations are summarized together here.

(Z') $b = F(\overset{+}{Z}, \overset{-}{\tau_a}, \overset{-}{B})$

(R') $b = G(\overset{-}{Z}, \overset{+}{S_R})$

(Z'') $\dfrac{c}{k} = \dfrac{\dfrac{\rho(1+B)}{(1+\tau_c)}(F(\overset{+}{Z}, \overset{-}{\tau_a}, \overset{-}{B}) - d)}{(\Im\rho + \psi(F(\overset{+}{Z}, \overset{-}{\tau_a}, \overset{-}{B}) - d))}$

(R'') $\dfrac{c}{k} = \dfrac{\dfrac{\rho(1+B)}{(1+\tau_c)}(G(\overset{-}{Z}, \overset{+}{S_R}) - d)}{(\Im\rho + \psi(G(\overset{-}{Z}, \overset{+}{S_R}) - d))}$

(G) $g_A = \dfrac{\phi(1-\beta)}{1-\varepsilon-\phi} g_L$

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The signs over the policy variables refer to the partial derivative of the function with respect to the associated variable. The signs of the partial derivatives are easily derived by referring to the (R) and (Z) equations. Refer to Figure 1.   Consider two economies that differ only in the level of the consumption tax. Since τ_c does not appear in (Z'), (R') or (G) the two economies share the same fertility rate and rate of economic growth.

The tax rate τ_c does appear in the (R'') and (Z'') equations. The economy with the higher tax rate, economy two($\tau_c(1) < \tau_c(2)$), has (R'') and (Z'') equations that lie below the (R'') and (Z'') schedules of the economy with a lower τ_c. In equilibrium G(.)=F(.) which is enough to establish that the equilibrium for the two economies exists at the same value of Z but different values of the c/k ratio.

The consumption tax has a level effect but no growth effect. This result is similar to the results for all policy variables in a semi-endogenous growth model when population growth is exogenous. Higher consumption taxes are considered permanent and have strong permanent income effects on consumption. This lowers the c/k ratio by just enough to offset the direct impact of the higher tax rate on fertility in equation (P). The net result is a lower level of per capita consumption at all points in time with no effects in first differences.

Figure 2 demonstrates the difference between two economies that differ only in the size of the R&D subsidy. Economy two has a higher subsidy ($S_R(1) < S_R(2)$). The economy with the higher R&D subsidy, S_R, has an (R') and (R'') schedule that are to

the right of the (R') and (R'') of the economy with the lower R&D subsidy[6].   The

economy with the higher R&D subsidy experiences a higher steady state growth rate.

Thus the level and growth rate must be higher for the economy with the higher R&D

subsidy. Increased innovation boosts the average product of capital and the c/k ratio.

The increase in the c/k increases the population growth rate and the long run rate of

growth.

While the result that an R&D subsidy boosts innovation is familiar and seems

reasonable, Jones's (2003) comes to the opposite conclusion after endogenizing the

fertility rate. As outlined earlier, our paper assumes an inelastic supply of labor in the

output sector of the economy, which is enough to overturn Jones's conclusion that

subsidies will lower growth rates in an endogenous fertility framework.   Jones ignores

any explicit goods cost of child rearing, assumes no substitution between labor and

leisure, and assumes that only labor is used in the production of new ideas. [7] A research

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[6] The direction of the shift in (R') and (Z') is very easy to verify. To establish the direction of shift in (R'')

and (Z'') note that from R'' $\dfrac{\partial c/k}{\partial G} = \dfrac{\dfrac{\Im\rho^2(1+B)}{(1+\tau_c)}}{(\Im\rho+\psi(G-d))^2} > 0$ and from Z''

$\dfrac{\partial c/k}{\partial F} = \dfrac{\dfrac{\Im\rho^2(1+B)}{(1+\tau_c)}}{(\Im\rho+\psi(F-d))^2} > 0$. These results follow from noting that the fertility rate is a number

between zero and one and F=G=b in the steady state. Therefore the direction of shift in the (R'') and (Z'')
is in the same direction as the shifts in (R') and (Z') for all policy variables with the exception of τ_c and
B.

[7] A truly integrated model would specify a household's choice between labor, leisure, and child rearing
where the three are distinct activities.   Furthermore it would allow capital to be used in knowledge
production as in the present paper. This generality would come at a cost in tractability, but now the birth
rate would be a function of the time cost of raising children, as in Jones, and the goods cost, as in the
present model. A subsidy would raise the wage rate in the formal sector and this would tend to lower the
birth rate as in Jones, but this effect would be offset by an increase in the c/k ratio and a quality to goods
cost ratio, which would raise the birth rate as in this model. The net effect is likely to be ambiguous and
would depend on the elasticity of the birth rate with respect to the goods cost relative to the elasticity of
labor supply relative to the wage rate.

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subsidy then increases the opportunity cost of spending time having children and workers take time away from child rearing and spend more time in R&D. This lowers the population growth rate, and, subsequently the steady state growth rate in his model.

Figure 3 compares two economies where the asset tax rate, τ_a, for economy one is lower than the asset tax rate in economy two($\tau_a(1)< \tau_a(2)$). The (Z') and (Z'') schedules are unambiguously lower for the economy with the higher tax rate. The higher tax rate lowers the effective rate of return on assets and, through standard intertemporal income and substitution effects, causes a rise in current consumption and the growth rate of consumption falls. In the steady state $g_A = g_c$. In other words the higher tax causes a lower c/k ratio consistent with lower fertility and lower long run growth.

The last policy experiment comes from a change in the cost of having children, B. This is viewed as a policy since governments can promote larger families in a number of important ways. Providing quality public education, medical insurance covering pregnancy and subsequent expenses[8], tax breaks for firms who provide on site day care, and tax credits related to the number of children are all examples of government policies that lower the cost of rearing children. These varied policies can be investigated by comparing the steady states of two economies where the second economy has a higher cost of rearing children(B(1)<B(2)). Refer to Figure 4. In the center graph only (Z') is different, and is clearly lower for economy two. The higher B lowers the choice of the number of children to have in economy two, lowers b, for any value of the average product of capital along the (Z') schedule implying a (Z') that is lower and to the right of

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[8] This may also impact growth through an exogenous decrease in d, which Barro and Sala-i-Martin (1999) argue can be interpreted as a child mortality rate in the infinite time horizon setup used here. Allowing for this effect would only strengthen the argument that improved health care for mothers and children increases the long run growth rate.

the (Z') schedule for economy one. This causes b to be lower in the right hand graph as economy two operates at a point to the left of economy one along the (G) schedule. In the bottom graph things are only slightly more complex. Since F(.)=G(.) in the steady state the direct impact of a higher B is an equal move up in both the (Z'') and (R'') schedules in a manner similar to our analysis of a consumption tax variation. Stopping here would ignore the indirect impact of a higher B on Z'', which partially offsets the higher placement of the Z'' schedule for economy two. The net effect is a higher average product of capital and c/k ratio for economy two[9]. The higher c/k ratio is not enough to offset the increase in B in the (P) equation and economy two experiences a lower fertility rate and a lower growth rate in the right hand graph. Policies that promote a lower cost of having children have level and growth effects on the steady state equilibrium.

III Conclusion

It is our desire to show that adding endogenous demographics can be done in a relatively simple and well accepted framework. This paper develops a fully endogenous non-scale growth model by allowing families to make choices about family size, thereby making the fertility rate endogenous. In order to allow for fertility choice we follow the well accepted approach developed by Barro and Becker (1989). Government is included in a more realistic formulation compared to the previous literature. This paper fills two long standing gaps in the growth literature. First, a model is developed that is scale free and does not rely on the equality between the growth of sectors and the growth of population, but reintroduces a policy impact on the steady state growth rate. Second, the

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[9] This does not imply that steady state per capita consumption is higher in economy 2. In fact c and k are lower in economy two. Comparing one and two the difference in the capital labor ratio is relatively larger than is the difference in per capita consumption causing a higher c/k ratio in economy two.

model incorporates a more realistic government sector and can be used to understand a wide array of government spending and taxation policies.

In our model government policies can be categorized according to whether they have level effects only, level and growth effects, or no impact on levels and/or growth. This makes a generalized Jones model a more powerful tool for understanding long standing differences in growth rates and income levels across countries at a point in time and within some countries over time. Wage taxes are found not to impact levels or growth rates because we assume that labor supply is inelastic. This result may seem counter intuitive and inconsistent with the available empirical findings of Romer and Romer (2010) and Barro and Redlick (2011) who find strong level effects from changes in average marginal tax rates. However, Barro and Redlick persuasively argue that these results are coming mostly from labor-leisure substitution effects from a change in income. The mechanism that we are modeling of an inelastic labor supply regarding fertility decisions is independent of the consideration of the elasticity with regard to income. Thus, we find that there are no additional empirically relevant growth or level effects from wage taxation arising from changes in fertility decisions that the traditional focus of steady-state growth models.

Consumption taxes have a level effect but no growth effect. This result is novel from a neoclassical perspective but is consistent with the endogenous growth perspective of Zeng and Zhang (2002) and Peretto (2007). In a neoclassical setting with exogenous population growth the consumption tax would have no impact on levels or growth rates. This result is the direct consequence of making population growth endogenous and does

not depend on the specification of technological change. Our model predicts that an asset tax discourages the long run growth rate.

This paper also predicts that a research subsidy promotes long run growth. This result is in contrast to Jones (2003) who finds that a subsidy to research lowers long run growth in a new growth model with endogenous fertility. The assumption of inelastic labor supply in the output producing sector of the economy is enough to change the direction of the impact of a subsidy on steady state growth.

We also find level and growth effects for policies that directly lower the cost of having children. We argue that these policies can include providing quality public education, medical insurance covering pregnancy and subsequent expenses, tax breaks for firms who provide on site day care, and tax credits related to the number of children. These are all examples of government policies that lower the cost of rearing children. Certainly there are many more examples of related government policies that influence both levels of per capita income and long run growth rates.

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Figure 1. A Consumption Tax and the Steady State
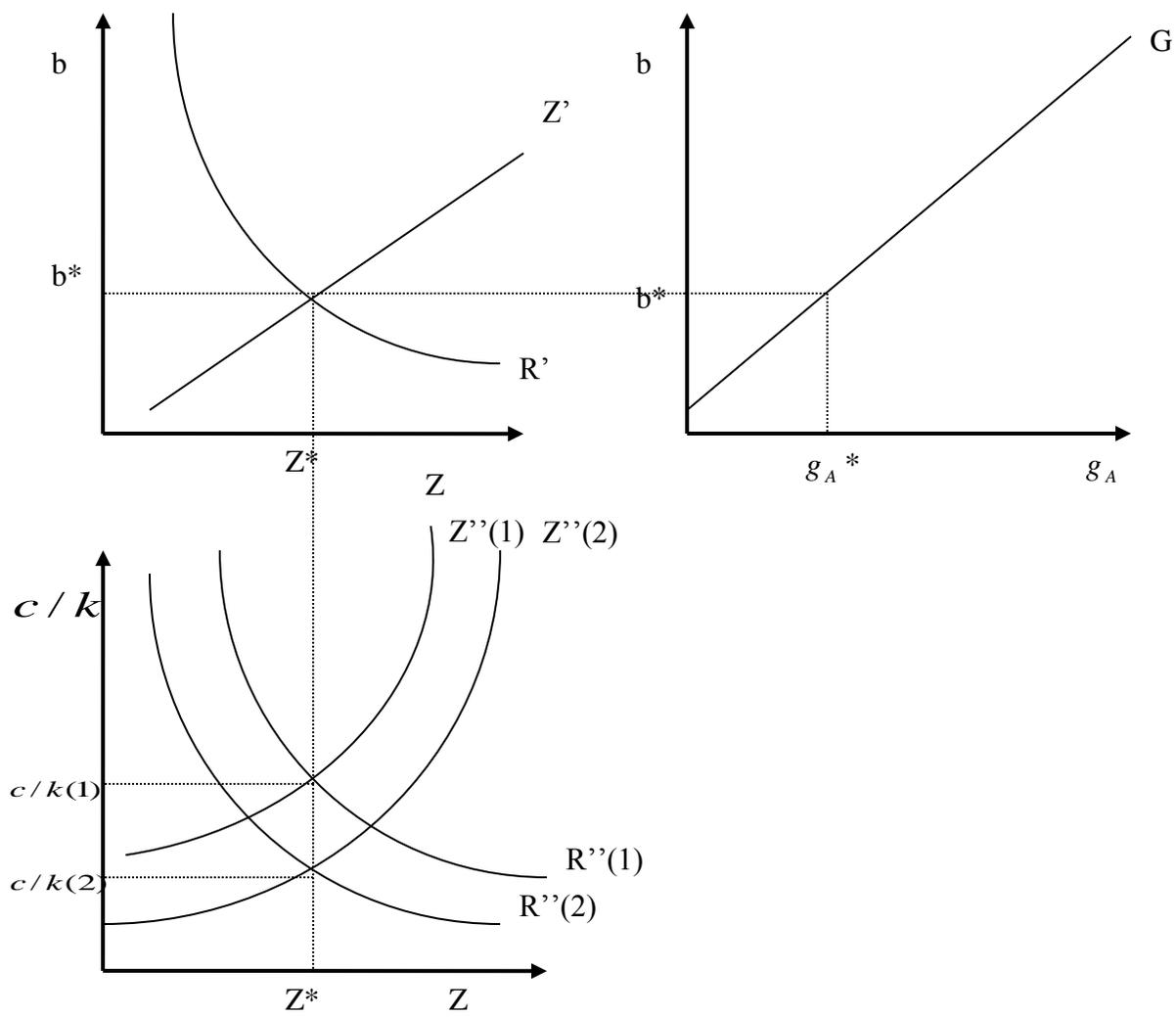
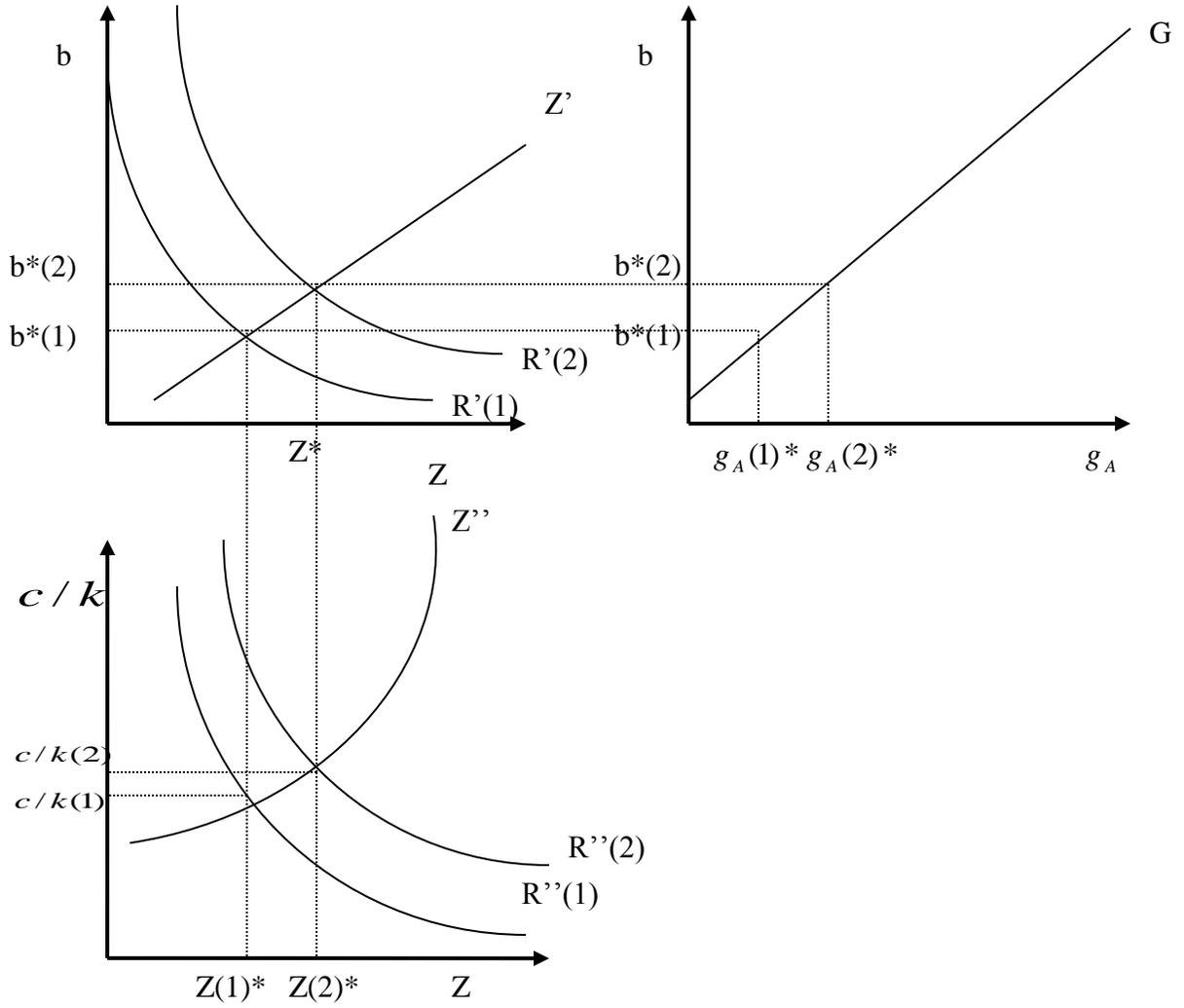Figure 2. A Research Subsidy and the Steady State
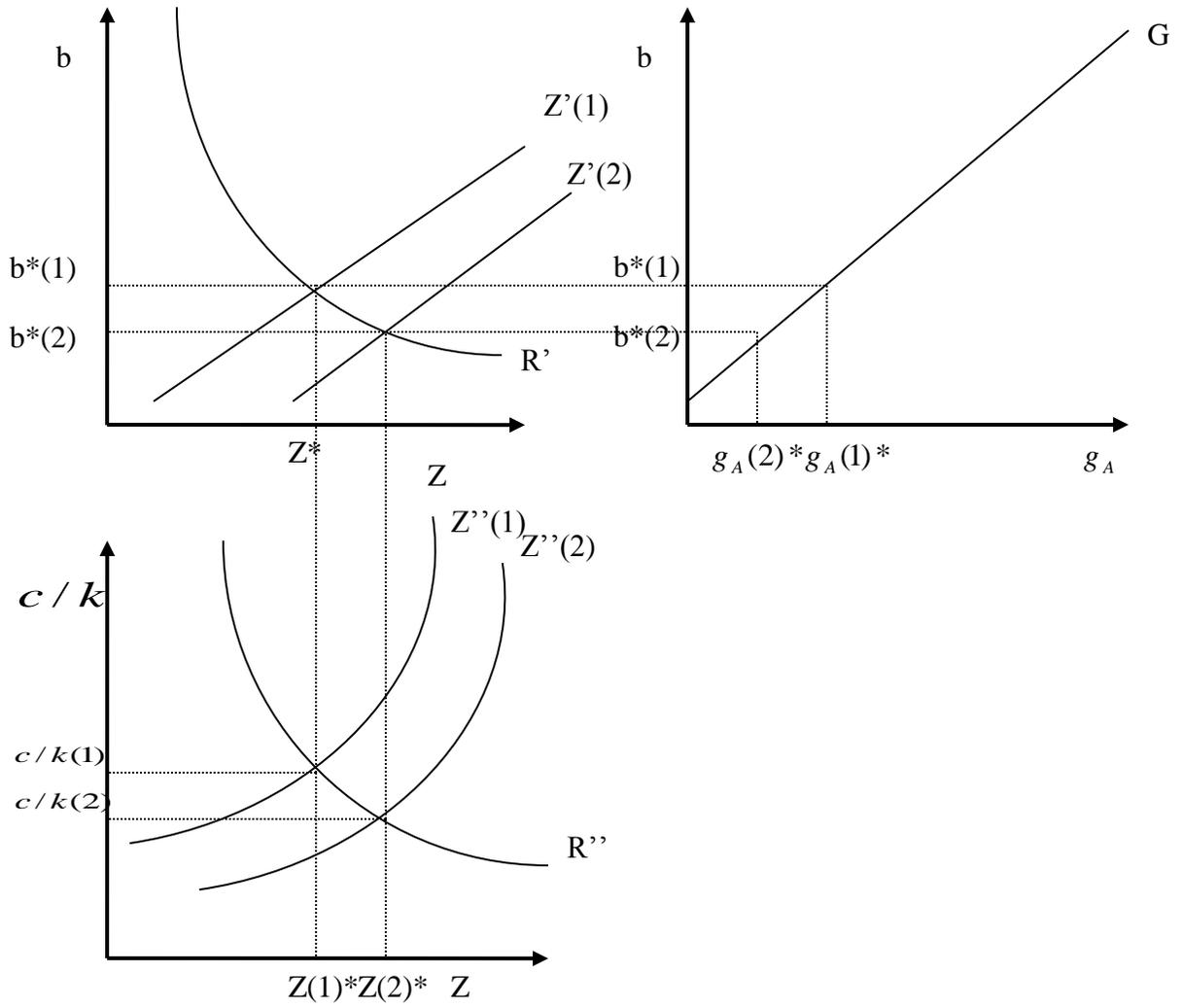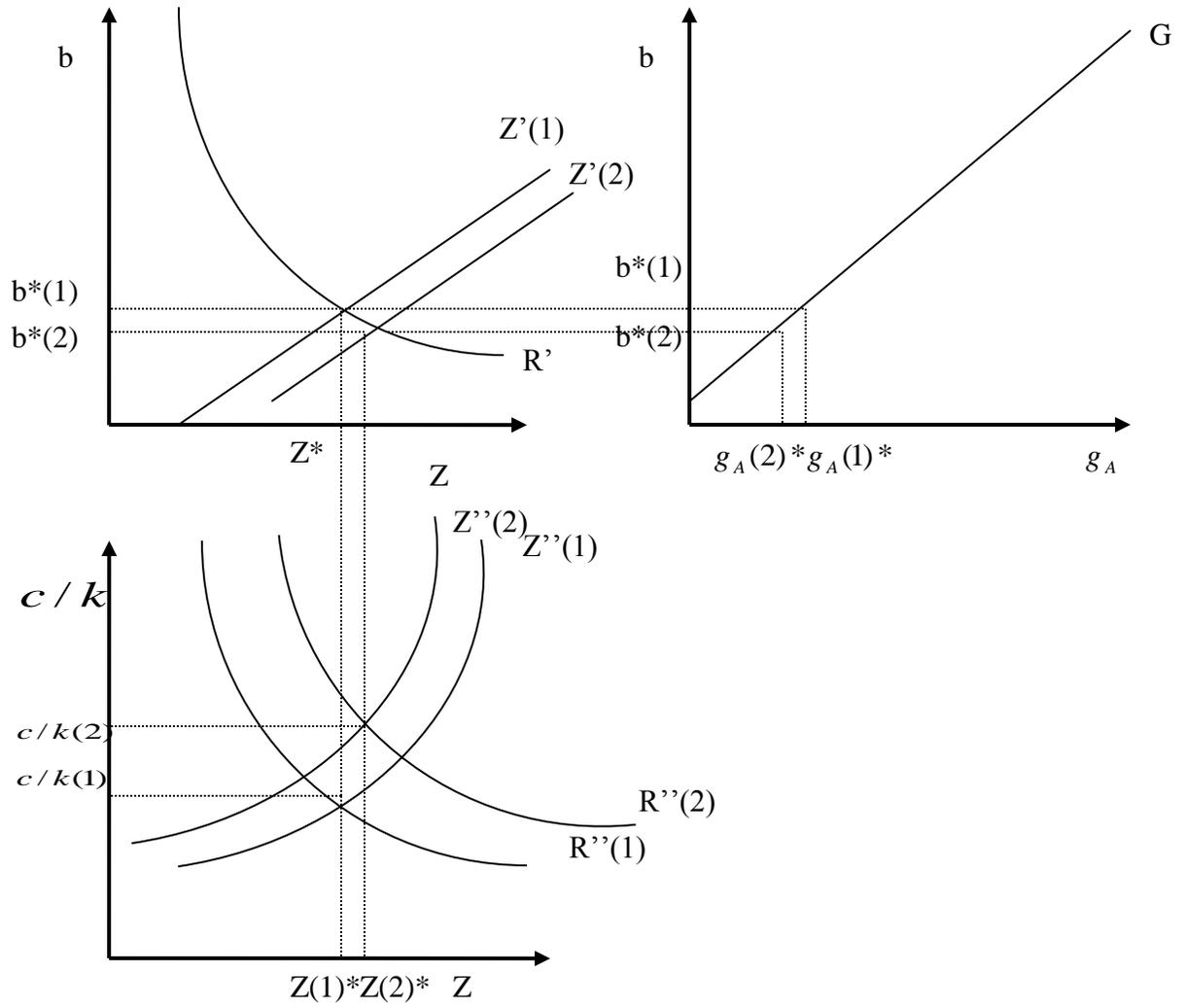
Figure 3. An Asset Tax and the Steady State

Figure 4. The Cost of Child Rearing and the Steady State

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