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Bayesian Averaging vs. Dynamic Factor Models for Forecasting Economic Aggregates with Tendency Survey Data

Piotr Bialowolski, Tomasz Kuszewski, and Bartosz Witkowski

Abstract
The main goal of the article is to investigate forecasting quality of two approaches to modelling main macroeconomic variables without a priori assumptions concerning causality and generate forecasts without additional assumptions regarding regressors. With application of tendency survey data the authors develop methodology for application of the Bayesian averaging of classical estimates (BACE) but also construct dynamic factor models (DFM). Within the BACE framework they apply two diversified methods of regressors' selection: frequentist (FMA) and averaging (BMA). Because their models yield multiple forecasts for each period, subsequently the authors employ diversified approaches to combine forecasts. The assessment of the results is performed with in-sample and out-of-sample prediction errors. Although the results do not significantly differ, the best performance is observed in Bayesian models with frequentist approach. Their analysis conducted for Polish economy also shows that the unemployment rate turns out to be forecasted with highest precision, followed by the rate of GDP growth and the CPI. It can be concluded from their analyses that although their methods are atheoretical they provide reasonable forecast accuracy not inferior to that of structural models. Additional advantage of their approach is that the forecasting procedure can be mostly automated and the influence of subjective decisions made in the forecasting process can be significantly reduced.

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Keywords Bayesian averaging of classical estimates; dynamic factor models; tendency survey data; forecasting

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Introduction

In the history of macroeconomic forecasting two major trends can be observed and as a result they led to two diversified approaches to modeling and forecasting economic processes. One group of models is based on inclusion of stylized facts from macroeconomic theory and thus causal effects are incorporated in modelling, while the other group of methods is atheoretical and based only on the observed properties of time series from an economy. Although inclusion of structural relations seems well justified, there are studies showing that accuracy of predictions obtained from such models is low (Kolasa, Rubaszek, & Skrzypczyński, 2012; Rubaszek & Skrzypczyński, 2008). Due to this, we decided to pursue the second path, namely atheoretical modelling.

The idea of using economic models without referring to any economic theory in the process of forecasting is by no means new. The origins for our approach can be traced back to a brief comparison between seven structural models of the US economy and simple ARIMA forecasts (Cooper, 1972). The fundamental finding of the analyses conducted then was that the forecasts obtained from the time series models were more accurate than those produced by large scale structural models. Additionally, the effort associated with construction and testing of such a model was substantially lower than in the case of a structural one. Examples of such approaches are either ARIMA or VAR models. Novelty of our approach is that by application of large data sets from tendency surveys which describe behaviour of economic systems, we make an attempt to introduce data mining techniques to macroeconomic forecasting. In order to benefit out of the information carried by tendency survey data we propose approaches based on the Bayesian averaging and dynamic factor analyses.

The aim of the article[1] is first, to is to develop an effective system for forecasting macroeconomic variables in Poland with atheoretical framework by introducing a series of atheoretical models constructed in order to produce quarterly forecasts of the three main macroeconomic indicators: the GDP growth, the rate of unemployment and the CPI. In this context the term „atheoretical" means that no macroeconomic theory is used in the process of

[1] The paper is a modified and completed version of the authors' working paper „Prognozowanie podstawowych wskaźników makroekonomicznych z wykorzystaniem uśredniania bayesowskiego oraz modeli czynnikowych w oparciu o dane z testów koniunktury" financed within the frame of the National Bank of Poland grant competition for 2014. The authors gratefully acknowledge the financing from that institution.

model construction. As a rule it is proposed to use as explanatory variables lagged values of the above mentioned indicators as well as current and lagged balances of responses to the questions from different tendency surveys and composite indicators based on such balances. Second, we want to evaluate competing models with respect to their in-sample and out-of sample forecasting performance. Although arguments for the use of forecasting models with tendency survey data and application of Bayesian averaging of classical estimates were already stated (Białowolski, Kuszewski, & Witkowski, 2012, 2014a) we introduce an important novelty by conducting a twofold analysis with the use of both the approach known as „frequentist" (applied in the previous papers), which is based on the use of Bayesian averaging for the purpose of *selection* of the variables for the model and the approach known as "averaging", whose idea is not to select the independent variables but to average the results obtained in different model structures with all the possible regressors. Additionally, for the first time we use such a large set of Poland's tendency survey data in the dynamic factor framework for forecasting of the main macroeconomic variables. Thus we end up with three forecasting scenarios, which are subsequently evaluated.

Following the objectives the paper is structured as follows. The following section (Part 2) focuses on a brief overview of the methodology. In part 3 we present the data used for estimating the econometric models and on the statistical properties of the time series used. Part 4 describes the modeling results and in part 5 we evaluate the forecasts.

Forecasting models

Bayesian averaging models. Throughout the study it has been assumed that the main research interest is focused on explaining the GDP growth (GDP), the rate of inflation (CPI) and the rate of unemployment (UNE). Selection of variables was motivated by their importance in the assessment of economic situation but also accessibility of items in tendency surveys. The natural solution is a three equation model, in which it is assumed that all three time series GDP_t, UNE_t and CPI_t are interrelated, as well as each of these variables is strongly autocorrelated. Thus one possible approach would be to construct a three equation model which symbolically could be denoted as

$$
\begin{aligned}
&GDP_t = f_1(GDP_{t-1}, UNE_t, CPI_t, \boldsymbol{V}_1, \varepsilon_{1t}) \\
&UNE_t = f_2(UNE_{t-1}, GDP_t, CPI_t, \boldsymbol{V}_2, \varepsilon_{2t}) \qquad (1) \\
&CPI_t = f_3(CPI_{t-1}, GDP_t, UNE_t, \boldsymbol{V}_3, \varepsilon_{3t}), t = 1, \dots, T; k \in \{0,1,2,3,4\},
\end{aligned}
$$

where the $\boldsymbol{V}_1, \boldsymbol{V}_2$ and $\boldsymbol{V}_3$ stand for "any other specified explanatory variables". These might mean: the first or any further lags of GDP, UNE and CPI respectively, as well as any exogenous variables, such as economics situation indicators. Such a model can be viewed as a VAR and estimated as such. However, we adopt two different approaches in this paper (dynamic factor and Bayesian averaging) due to the following reasons. First, our main target is to provide a model which would be capable of providing short term forecasts of GDP, UNE and CPI. Thus the $\boldsymbol{V}_1, \boldsymbol{V}_2$ and $\boldsymbol{V}_3$ might only contain the lags of endogenous variables and such variables whose values are known for the near future. We believe that they are economic situation indicators among the series in our dataset which might serve as reasonable determinants of GDP, UNE and CPI and whose values are indeed known slightly in advance: they are available at the beginning of the quarter, which makes it possible to use them for forecasting purposes for the period of almost three months ahead. Furthermore, in the process of construction of leading indicators at the RIED (Research Institute for Economic Development at the Warsaw School of Economics), entrepreneurs and households are asked about their expectations regarding the economic situation in the near future. This makes it reasonable to use k-th lags of the business tendency indicators rather than their current values, which makes it possible to extend the horizon of forecast further by additional k periods (quarters). That unfortunately comes at a cost. The series of business tendency and consumer sentiment indicators described in the next section begins in 1996, and thus only 68 quarterly observations are available till the end of 2012, which makes adoption of VAR approach not feasible. Yet another problem is the issue of selection of "adequate" economic situation indicators for the model. Firstly, the number of available indicators is high, even if we just limit our attention to those provided by RIED. Not only would that mean very low (or even negative if additional lags of endogenous variables were also considered) number of degrees of freedom of the specified model, but also multicollinearity of them would be an issue. Naturally one could preselect just a few indicators for the $\boldsymbol{V}_1, \boldsymbol{V}_2$ and $\boldsymbol{V}_3$ sets, however it would certainly be difficult to give prior rationale for choosing a given subset of all the available economic situation indicators.

In Bayesian approach in order to overcome these problems we propose the following approach. Firstly, we replace the model (1) with the following structure:

$$GDP_t = f_1(GDP_{t-1}, \boldsymbol{X}_{1,t-k}, \varepsilon_{1t}) \tag{2a}$$

$$UNE_t = f_2(\widehat{GDP}_t, UNE_{t-1}, \boldsymbol{X}_{2,t-k}, \varepsilon_{2t}) \tag{2b}$$

$$CPI_t = f_3(\widehat{GDP}_t, \widehat{UNE}_t, CPI_{t-1}, \boldsymbol{X}_{3,t-k}, \varepsilon_{3t}),\ t = 1, \dots, T; k \in \{0,1,2,3,4\}, \tag{2c}$$

where $\boldsymbol{X}_{i,t-k}, i = 1,2,3,$ stands for the set of economic situation indicators from period *t-k* influencing the GDP growth, the rate of unemployment and the rate of inflation respectively; $\varepsilon_{it}, i = 1,2,3,$ represents the error terms for subsequent equations, $f_i, i = 1,2,3,$ is a certain linear function, $\widehat{GDP}_t$ is the theoretical rate of GDP growth obtained from the equation (2a) and $\widehat{UNE}_t$ is the theoretical rate of unemployment obtained from the equation (2b). Estimating (1) on the equation-by-equation basis would not be adequate due to endogeneity of particular variables. In order to overcome the problem of endogeneity we use the 2SLS-type logic by replacing given variables with their theoretical values making the (2) feasible for recursive estimation with the use of a simple least squares estimator. The order of equations in (2) is based on our previous research: naturally one could order the dependent variables in (2a)-(2c) in six different ways, yielding six different sets of recursive equations. However, as shown in Białowolski et al. (2010), this way of ordering provided the set of equations that allowed for obtaining the most accurate forecasts in the past. We also adopt all the classical assumptions that make it possible to estimate the subsequent equations with the use of OLS: in particular we treat the error term as spherical.

The next issue is the problem of selecting the "best" $\boldsymbol{X}_{i,t-k}, i = 1,2,3$ for a given k. Firstly, it is not clear which lags of the economic situation indicators should be used so as to maximize the quality of the forecast, except that it seems obvious that those should not be lagged by too far. For that reason we estimate separately the set of (2a)-(2c) for different k between 0 (current values of economic situation indicators) up to their 4th lags, without mixing different lags in one equation. It would be tempting to use more lags of the same indicator in the same equations (say, 1st and 2nd lags of them in one model), this is however problematic due to very strong autocorrelation in the series of most indicators and high multicollinearity as its result. Next issue is: which of the indicators select for particular $\boldsymbol{X}_{i,t-k}, i = 1,2,3$ – clearly the set of indicators that would serve as best determinants of unemployment need not be the same as those used for the CPI or rate of GDP growth, thus each of the $\boldsymbol{X}$'s should be selected separately. Since the economic rationale is highly unclear and subjective in this case, we adopt the Bayesian model averaging approach for this purpose, which in the case where OLS is used for estimation purposes, degenerates to so called Bayesian averaging of classical estimates.

The technical details of the Bayesian model averaging can be found in numerous papers, such as the milestone article of Sala-i-Martin, Doppelhoffer and Miller (2004) or Próchniak and Witkowski (2013) and shall not be discussed here.

Further steps depend on the adopted approach. There are two types of Bayesian-averaging, which can be found in the literature: the "frequentist" and the "averaging" procedure (Moral-Benito, 2013). In this study, with regards to Bayesian averaging, three types of approaches were analysed: the averaging approach (BA), the frequentist approach (BF) and the frequentist approach with the control of collinearity (BFC). In the last one, after selecting the set of variables on the basis of their posterior probabilities, the variance inflation factors were checked and the regressors with highest VIFs were eliminated recursively until all VIFs were acceptable (the usual VIF<10 rule was adopted for this purpose).

Considering the fact, that 5 different sets of lags of $\boldsymbol{X}_{i,t-k}, i = 1,2,3$ were considered ($k = 0,1,2,3,4$) and three above described approaches (averaging, frequentist, frequentist with collinearity correction) were tested, a total of 15 model structures were found. For every k and approach, firstly the equation (2a) was estimated and the theoretical values of GDP_t were found. In the case of frequentist approach, those were the theoretical values of GDP from a single equation with "Bayesian-selected" economic situation indicators and the lagged GDP (having additionally eliminated the statistically collinear indicators in the collinearity corrected frequentist approach). In the case of the averaging approach, averaged parameter estimates for each regressor were found from all the estimated cases and those were used as is a single equation had been estimated with all the considered regressors to attain the theoretical GDP. Then the process was repeated for the equation (2b), except that the theoretical GDP from (2a) was used as one of the regressors (for each of the three considered approaches, theoretical GDP obtained with the same approach applied to equation 2a was used). Finally, the same process was applied to equation (2c), while theoretical GDP from (2a) and theoretical unemployment rate from (2b) were additionally used as independent variables. In all the Bayesian averaging models we decided to use only the prognostic variables from the tendency survey time series. Due to computational complexity of those methods but also research question oriented on forecasting, we decided to omit the indicators which were describing the current state of economic affairs or merely assessing the current climate. With such an approach we were able to significantly reduce the amount of computations required to obtain the results. The detailed description of the results achieved with the use of Bayesian approach can be found in (Białowolski, Kuszewski, & Witkowski, 2014b).

Due to considerable amount of estimates generated during the Bayesian averaging procedure, we decided to present only the set of regressors from the sets X in equations (2a)-(2c). In the BA method, following the philosophy of this method, in each the three equations and for each lag k, the set of regressors from the tendency surveys was the same and comprised the following indicators (please refer to the Appendix for definitions):

Ifo_be	gus2	gus4	gus7	gus11	ips_wo	biec_wwk	biec_wpi
biec_wrp	biec_wd	ind_q1f	ind_q2f	ind_q3f	ind_q4f	ind_q5f	ind_q6f
ind_q8f	hhs_q1	hhs_q2	hhs_q4	hhs_q6	hhs_q7	hhs_q9	hhs_q11

In the frequentist approach (BF, BFC) the set of regressors differed in models with collinearity correction and without it (Appendix, table A3).

Analysis of patterns of explanatory variables in the equations for macroeconomic variables enables to formulate the following conclusions:

- The cases with exactly the same the set of indicators for models with and without collinearity correction imply that the collinearity was not observed.
- The set of regressors depends on the lag (k). In the equations for GDP and CPI similarities are observed with in the sets: {k=0}, {k=1, k=2}, {k=3, k=4}, in the equations for UNE the sets are: {k=0, k=1}, {k=2, k=3, k=4}.
- A significant role is played by the regressors from consumer tendency surveys (CSO and RIED).
- The most frequently occurring indicators (except for the equation on GDP) are those of the Bureau of Investment and Economic Cycles - biec_xxx.

Dynamic factor models. Application of dynamic factor models to forecasting of macroeconomic time series has been already extensively developed in the literature (Baranowski, Leszczyńska, & Szafrański, 2010; Boivin & Ng, 2006; Reijer, 2012; Stock & Watson, 2002, among others). Nevertheless, with minor exceptions it has been rarely focused on defining the dynamic factors with tendency survey data (Frale, Marcellino, Mazzi, & Proietti, 2010; Hansson, Jansson, & Löf, 2005; Kaufmann & Scheufele, 2013). However, it should be underlined that dynamic factor models have significant advantages over other approaches to modelling. Breitung and Eickmeier (2006) enumerate advantages of dynamic factor approach which can be summarized in following points: (1) Factor models can cope with many variables without running into low number of degrees of freedom, which can be often the case when we want to employ a lot of variables in a regression based modelling[2]; (2) In factor models idiosyncratic movements of specific variables, which possibly include measurement error and local shocks, can be eliminated; (3) with application of dynamic factor models it is possible for modellers to remain agnostic about the structure of the economy and do not rely on different assumptions, which is often the case in structural models.

With regards to forecasting, an especially important advantage of using dynamic factor models is elimination of noise from the data. Hansson et al. (2005) claim that idiosyncratic processes that are present in different sectors are probably rather not relevant to general economic processes in the economy. Eliminating them with factor approach might be of crucial importance, when the focus of analysis is on macroeconomic aggregates, which is the case in our analyses. We find dynamic factor models especially useful, as (see point 3 above) their structure and implied modelling strategy matches our initial assumptions regarding modelling with very limited influence of modellers on the forecasting process.

It needs to be taken into account that the dynamic factor models have also certain drawbacks. A disadvantage of common factor models is that factors are hardly (or even completely not) interpretable. Due to that, Stock and Watson (2002) suggest that they should be interpreted as diffusion indexes oriented on assessment of average economic activity. Naturally, there are also caveats associated with the number of indicators. Larger number of indicators is not always the most desirable case even in the dynamic factor specification. Boivin and Ng (2006) show that

2 Time series models usually contain no more than 10 time series (Boivin & Ng, 2006; Stock & Watson, 2002). Even our approach based on Bayesian Averaging was constructed in such a way that the optimal number of time series in an equation should be around 6.

adding a series that is highly correlated with other series might reduce rather than improve efficiency of the factor estimates. On the other hand, adding a 'noisy' time series, that share little common variance with other series also reduces the efficiency of factor estimates, because the average common component becomes smaller. So, our goal in establishing the common factors was to pick diversified data from tendency surveys in our data set but at the same time eliminate series providing noise in the final factor solutions.

Regardless of the character of time series data used, the structure of dynamic factor model is similar. Starting point for the analysis is approximate factor model with K factors, which takes the form:

$$\mathbf{X}_t = \mathbf{\Lambda F}_t + \boldsymbol{\varepsilon}_t, \text{ where}$$

$\mathbf{X}_t$ represents N x 1 vector of consumer and business tendency survey indicators (also composite indicators used in the analysis) measured at a given time point t, $\mathbf{\Lambda}$ is a matrix of factor loadings of dimension N x K, $\mathbf{F}_t$ is the K x 1 vector of period specific factor loadings, ε_t is a N x 1 vector of measurement errors in a given period.

Following the Stock and Watson (2002) approach we assume propose that the number of factors is determined based on the simple principal component approach.[3] Additionally, we assume that the number of factors is determined based on the standard Cattell criterion. In order to eliminate from certain factors those variables which have very low factor loadings, assumption from other factor models was adopted that the loadings need to be salient, which was assumed to be over 0.5. Brown (2006) suggests range between 0.4 and 0.6 for factor models based on individual data, however we assume the mid of the interval as an appropriate for dynamic factors. A drawback of dealing with static factors only, is that the dynamic structure, which is likely to exist between the factors, might not be accounted for. In order to account for this possible dynamics, based on the obtained static factors, dynamic component was introduced. The dynamic factor model is an extended form of the static, where the factors are assumed to follow dynamic, autoregressive process:

$$\mathbf{F}_t = \Phi(L)\mathbf{F}_{t-1} + \mu_t, \text{ where}$$

3 Naturally, for extraction of the common factors, a different factor analytical approach can be used, like exploratory factor analysis. Nevertheless, differences in the results (factor loadings) between various factor analytical approaches are usually very small and thus this issue was not subject to profound analysis.

$\Phi(L)$ is a lag polynomial describing the autoregressive structure of the data generating process of factors and μ_t describes the error. In our empirical approach, we assessed models with lag polynomial of the form: 1, L, L^2, L^3 and $1+L^3$, so we were interested in lags equal to 1,2,3,4 and 1 and 4 simultaneously. Selection of the appropriate lag is based on the Schwarz Information Criterion. Final step of the analysis oriented on forecasting with dynamic factor models, is inclusion of dynamic factors into the forecasting process of economic variables of interest. Standard specification of a model with dynamic factors used as forecasting tools can be presented by the following system of equations (Baranowski et al., 2010, among others; see Stock & Watson, 2002)]

$$\mathbf{y}_t = \boldsymbol{\alpha} + \sum_{m=1}^{L} \boldsymbol{\beta}_m \mathbf{y}_{t-m} + \sum_{n=0}^{L} \boldsymbol{\gamma}_n \mathbf{F}_{t-n} + \boldsymbol{\varepsilon}_t, \text{ where}$$

$\mathbf{y}_t$ represents vector of macroeconomic variables of interest, $\boldsymbol{\alpha}$ stands for a vector of constants, L is the number of lags included in the analysis, $\boldsymbol{\beta}_m$ is a vector of autoregressive coefficients standing by variables of interest lagged by m periods and $\boldsymbol{\gamma}_n$ is a vector of coefficients standing by dynamic factors lagged by *n* periods.

In our case due to the fact that we wanted to include interrelations between the current level of indicators, we followed a slightly modified approach. In our previous studies the established order in which macroeconomic variables should be related to each other is defined by equations (2a-2c). Inclusion of these interrelations between the macroeconomic variables results in a slightly modified framework with dynamic factors used for the forecasting purposes. Having $y_t = [GDP_t \; UNE_t \; CPI_t]^T$ but also additional assumptions that only one lag of the variable of interest is included in the equation for this variable and that dynamic factor estimates are taken only for a single quarter depending on the chosen lag (five possibilities of lags were checked k = 0,1,2,3,4), our final model can be presented by the following system:

$$GDP_t = \alpha_{GDP} + \beta_{GDP} GDP_{t-1} + \gamma_{GDP,k} F_{1,t-k} + \varepsilon_{t,GDP,k}$$
$$UNE_t = \alpha_{UNE} + \kappa_{UNE} \widehat{GDP}_t + \beta_{UNE} UNE_{t-1} + \gamma_{UNE,k} F_{2,t-k} + \varepsilon_{t,UNE,k}$$
$$CPI_t = \alpha_{CPI} + \kappa_{CPI} \widehat{GDP}_t + \lambda_{CPI} \widehat{UNE}_t + \beta_{CPI} CPI_{t-1} + \gamma_{CPI,k} F_{3,t-k} + \varepsilon_{t,CPI,k}$$

In the final specification, in the second equation (for UNE) estimated value of GDP for period *t* is included as exogenous variable, while in the third equation (for CPI) both estimates of GDP and UNE are included as exogenous variables. In addition to this, all dynamic factors are present in all equations (table 1).

Table 1

Indicators of factors in the dynamic factor model

Factor 1	gus1 gus2 gus3 gus4 gus8 gus7 gus11 gus_wb gus_ww ips_wok ips_kg ips_sz ips_wb ips_wo biec_wrp biec_wd ind_q5f hhs_q1 hhs_q2 hhs_q3 hhs_q4 hhs_q7 hhs_q8 hhs_q9 hhs_q10 hhs_q11
Factor 2	pmi ifo_bc ifo_be ind_q1s ind_q1f ind_q2s ind_q2f ind_q3s ind_q3f ind_q6s ind_q6f ind_q7s ind_q7f ind_q8s ind_q8f constr
Factor 3	zew_ies ifo_bs gus1 gus2 biec_wwk biec_wpi biec_wrp ind_q1f ind_q2f ind_q3f ind_q4s ind_q4f ind_q5f hhs_q9 hhs_q12

Source: own estimates.

Thus, although the variable selection procedure is significantly different, the modelling strategy implemented in the dynamic factor framework shares with Bayesian approaches the final structure of forecasting models, which serve as a tool for generating the final forecasts.

Data – sources and preparation

In order to build forecasting models, quarterly data covering the years from 1996 to 2014 were collected. The data on the gross domestic product (GDP), the consumer price index (CPI) and the unemployment rate (UNE) come from a publication by Poland’s Central Statistical Office (CSO). The unemployment rate has been set on the basis of a Labour Force Survey. GDP, CPI and UNE serve in our models as endogenous variables. With respect to the previous research, the set of indicators was extended with time series on individual consumption, investment outlays, export and import but also value added in 16 sectors of the economy. Those additional variables were used as potential regressors.

In addition to the lagged endogenous variables and data from national accounts, tendency survey data are assumed to play the role of regressors in the designed econometric models either in their original form or as the variables explained by the presence of common factors. The tendency survey data is usually published in the form of monthly statistics. In line with the standard practice, business survey data for the first month of each quarter, i.e. January, April, July and October, are considered as a quarterly data. The database applied in the procedure comprises a time series from the Research Institute for Economic Development (RIED) at the Warsaw School of Economics (WSE), on sentiment in the manufacturing industry, trade and construction and among households. Data published by the Centre for European Economic Research (ZEW), the Leibniz Institute for Economic Research at the University of Munich (Ifo Institute), Bureau for Investments and Economic Cycles (BIEC), and the Purchasing Managers' Index (PMI) for Polish industry, were also collected and subsequently applied in the analysis. In addition to this, data on consumer confidence from the Central Statistical Office and IPSOS group were included in the analysis. The symbols adopted for the variables in the estimated models are presented in the Appendix.

Similarly to the most of empirical illustrations of economic processes, also in the conducted research, data generating processes were verified with respect to their stationarity. Most of the research provide verification of stationarity with respect to the mean, rarely stationarity with respect to variance is subject to verification. Lack of stationarity with respect to variance is usually accounted for by taking logarithm of the time series. However, such procedure appeared to be not necessary in the case of our series. The problem of stationarity with respect to the mean is usually accounted for by differencing the time series (difference order is usually described by letter d and stands for the order of integration). In our case, stationarity was checked with ADF and KPSS tests (Kwiatkowski, Phillips, Schmidt, & Schin, 1992) used in order to study an order of integration. No time series with an order of integration higher than 1 were identified in the database. Nevertheless, the analysis showed that it can be assumed that the time series for responses to business survey questions targeted at the industrial sector are stationary I(0) time series, while the time series for responses to business survey questions targeted at households are integrated I(1) time series. The remaining regressors time series appeared to be stationary ones. This explains why we decided against differentiating the values of the series I(1); instead we decided to study the statistical properties of the residual series of the estimated models. Stationarity of the time series of regressands has been investigated with KPSS test. Time series of GDP is stationary, but CPI and UNE are integrated of degree 1 (d=1).

Discussion regarding the seasonality of time series is constantly present in the literature (see, e.g.,Clements, Hendry, 2011). The voices of those in favour of deseasoning in economic modelling are more less equal to those having the opposite opinion. However, the seasonality treatment of the time series was omitted in our analysis because the results presented in Białowolski et al. (2014a) show its marginal influence in both deterministic and stochastic specification of seasonal factor. It follows a common econometric finding that with either version of the seasonality (deterministic or stochastic), due to the fact that different patterns of seasonality are present among regressors, it is hard to predict the influence of seasonality on parameter estimates and, more importantly, on the forecasts. Similar views are supported by Mycielski (2010).

In the literature one can find also arguments that deseasonised time-series are in fact obtained via estimation and due to this some of the information content of time series subject to deseasoning is lost (see e.g. Bloem, Dippelsman, & Maehle, 2001). It has been also pointed out that seasonality correction should be rather performed when the same months, quarters are compared to each other for different years in an analysis of a single times-series, while the seasonal correction is less justified when the time-series data serve for modelling of the economic processes (Manski, 2014). As an example, in the case of macroeconometric model for the Polish economy WK2009 (Welfe, 2013) based on quarterly data only not seasonally adjusted data were used.

The influence of deseasoning of a time-series on quality of estimates and testing of autoregressive models was assessed by Hecq (1998). He obtained a strong support for lack of seasonal treatment of time-series data. However, if time-series are to be used in different applications than econometric modelling, seasonal treatment might be more justified (Baranowski et al., 2010). Consequently, in all our models we decided to use raw time series.

Fitting forecasting models to the data

The fit of the forecasting models to the data is measured in two ways. The first one consists in the analysis of signs of the differences between the empirical and the theoretical regressands in subsequent periods. The comparison of the signs allows to judge the reaction of the models to the change of the direction of trends in the macroeconomic indicators. The second way to verify the quality of fit is to make use of one of the standard measures used in the forecast ex-post errors analysis. The appropriate measure is the mean absolute percentage error (MAPE). Its value provides information about the mean value of errors expressed in percent of the true value of the analysed variable. MAPE allow for comparisons of fit and forecast accuracy independently from the units used to code the regressand[4].

Table 2

Quality of the estimates of forecasting models measured with the use of the regressands' sign difference and MAPE for the 1996q1 – 2012q4 period

Regressand	DFM models					BA models				
	k = 0	k = 1	k = 2	k = 3	k = 4	k = 0	k = 1	k = 2	k = 3	k = 4
	The number of coinciding signs of the differences of empirical and theoretical values									
GDP	47	43	44	44	43	45	44	42	39	42
UNE	37	39	40	40	36	45	45	42	45	49
CPI	48	47	44	45	44	45	46	47	41	45
	MAPE									
GDP	0.31	0.31	0.31	0.34	0.34	0.62	0.39	0.37	0.50	0.38
UNE	0.05	0.05	0.05	0.05	0.06	0.11	0.12	0.11	0.12	0.06
CPI	0.15	0.17	0.20	0.21	0.21	0.87	0.25	0.26	0.40	0.40
Average	0.17	0.18	0.19	0.20	0.20	0.53	0.25	0.24	0.34	0.28
	BF models					BFC models				
	k = 0	k = 1	k = 2	k = 3	k = 4	k = 0	k = 1	k = 2	k = 3	k = 4
	The number of coinciding signs of the differences of empirical and theoretical values									
GDP	44	45	41	42	41	44	43	41	42	39
UNE	45	47	43	44	45	44	47	40	40	44
CPI	49	44	46	44	45	47	43	48	43	40
	MAPE									
GDP	0.24	0.24	0.28	0.25	0.28	0.24	0.24	0.28	0.27	0.27
UNE	0.04	0.04	0.04	0.04	0.03	0.04	0.04	0.04	0.04	0.04
CPI	0.16	0.15	0.17	0.17	0.17	0.15	0.15	0.18	0.18	0.19
Average	0.14	0.15	0.16	0.15	0.16	0.14	0.15	0.17	0.16	0.16

Source: own estimates. Remark: for every k which represents the delay of lag a different number of differences were compared; 66 for k = 0, 66 for k = 1, 65 for k = 2, 64 for k = 3 and 63 for k = 4.

The conclusions from the comparison of the accuracy of the models' fit to the data in the 1996q1 to 2012q4 period are the following:

- Considering the number of coinciding signs of the differences of the empirical and the theoretical values of the regressands, none of the methods is clearly superior to others. This means that all of the models are correct in identifying the turning points in short term

[4] In the literature there has long been an ongoing discussion on the sensibility of the use of different measures of accuracy. Some believe that MAPE is not an appropriate measure. Hyndman and Koehler (2005) propose measures that stem from MAPE but differ from it in their construction. However, these give no additional interpretation possibilities in this paper.

trends in approximately the same percentage of cases (which roughly exceeds 2/3 of all the attempts).

- The number of coinciding signs of the differences for all the models and all the macroeconomic indicators decreases in line with the increase of the lag of the variables taken from tendency surveys.
- The analysis of the value of MAPE allows for ordering of the forecasting models, starting with the one that fits the data best and ending with the one which fits the data worst. They are the Bayesian frequentist approach that are characterized by the basically lower values of the MAPE than in the case of models identified with the use of the dynamic factor analysis, while models identified with the use of the classical bayesian model averaging are the worst in this respect.
- The values of MAPE increase in line with the size of lag of the variables from tendency surveys.
- Whichever algorithm was used, in each of the model it is the equation which explains the UNE variable which fits the data best, followed by the CPI equation, while the GDP equation proves to be the least accurately fit one. Considering the fact that the order of the equations in the model is the same in each case, it can be clearly seen that the errors made in estimation of the GDP are not accumulated with the errors related with the other two variables' estimation.

In the next step we consider the forecasting errors in the 2013q1 – 2014q2 period. However, the model identified with the use of the classical Bayesian model averaging is eliminated from further analysis as the one that fits the data in the worst way.

Forecasting

In the discussion on the construction of the forecasting model sit has been emphasized many times that due to different lengths of lags of the variables that come from tendency surveys and describe business climate it is possible to make more than one forecast of each macroeconomic indicator. Let us call the set of forecasts of each indicator that is obtained with the use of the same empirical observations a *portion of forecasts*. We shall make use of the fact that following the tradition, the empirical values of tendency surveys which correspond to the 1st quarter come from January, those for the 2nd quarter come from April, those for the 3rd quarter – from July while those for the 4th quarter – from October. This fact enables forecasting with the k=0 delay (zero lag). Furthermore, models used to forecast with lags k=1,2,3,4 have also been prepared. Thus an example of portions of forecasts obtained in the frequentist approach with the co-linearity correction (table 3) include 15 values for each regressand: there is a one sole value for k=0 and five values for k=4. Table 3 presents the forecasts for the case of the empirical data ending in the first quarter of 2014 and the series is prolonged with the use of the 2014q2 data afterwards.

Table 3

Forecasts from BFC model

k	Last period of data 2014q1. Forecasts for:					Last period of data 2014q2. Forecasts for:				
	2014q2	2014q3	2014q4	2015q1	2015q2	2014q3	2014q4	2015q1	2015q2	2015q3
	GDP									
0	3.8					3.2				
1	3.8	4.5				4.1	4.2			
2	3.8	4.1	4.4			3.6	3.9	4.2		
3	3.5	3.6	3.6	3.2		3.4	3.4	3.0	3.5	
4	3.1	2.8	2.4	2.2	2.1	3.0	2.5	2.4	2.2	2.2
	UNE									
0	9.6					9.1				
1	10.4	10.0				8.7	8.4			
2	10.8	10.6	10.9			9.1	9.3	8.4		
3	10.5	10.1	9.6	9.7		8.8	8.5	8.7	7.8	
4	9.8	8.8	7.8	7.5	5.8	7.9	6.8	6.3	4.5	4.3
	CPI									
0	0.6					0.6				
1	0.6	1.2				0.8	0.6			
2	0.9	1.2	1.7			0.4	0.7	0.5		
3	0.5	0.4	0.2	-0.1		0.1	-0.3	-0.6	-0.6	
4	0.6	0.4	0.2	-0.2	-0.3	0.1	-0.2	-0.5	-0.7	-0.7

Source: own estimates.

The analysis of ex post forecast accuracy based on the separate portions of forecasts answers the question about the stability of the forecasting process as the subsequent observations are gradually added to the time series of the regressors. Recall that the models were identified in the sense of their general functional form (that is the set of regressors) with the use of 1996q1-2012q4 time series of the independent variables. The estimates of structural parameters change (are re-estimated on the time-constant set of regressors) with each new observation added to the series. For example, let us compute the ex post forecasts accuracy based on the portion of forecasts obtained with the use of 2012q4-2014q1 time series (table 4).

Table 4

The MAPE-measured accuracy of the portion of forecasts based on the gradually extended time series

Regressand	Last period of data and number of forecasts in portion					
	2012q4	2013q1	2013q2	2013q3	2013q4	2014q1
	15	15	14	12	9	5
	DFM models					
GDP	0.75	0.25	0.30	0.16	0.18	0.13
UNE	0.14	0.27	0.10	0.07	0.07	0.17
CPI	0.76	0.56	1.00	0.88	1.18	1.22
Average	0.55	0.36	0.47	0.37	0.48	0.51
	BF models					
GDP	0.52	0.37	0.31	0.19	0.18	0.13
UNE	0.11	0.19	0.11	0.07	0.08	0.13
CPI	0.90	1.27	1.56	1.38	0.93	1.00
Average	0.51	0.61	0.66	0.55	0.40	0.42
	BFC models					
GDP	0.51	0.31	0.30	0.16	0.18	0.11
UNE	0.09	0.19	0.11	0.07	0.07	0.12
CPI	0.81	0.73	1.00	0.88	1.18	1.06
Average	0.47	0.41	0.47	0.37	0.48	0.43

Source: own estimates.

In result of extending the time series during a few quarters it is possible to compare the values of MAPE. It is worth noting that these obtained with the use of portions of forecasts of different sizes, ranging from 5 to 15 values. The unemployment rate turns out to be forecasted with greatest precision, followed by the rate of GDP growth and the CPI. Let us recall that basing on the comparison of the fit of particular equations they were the GDP growth equations that seemed to be the least accurate.

Combined forecasts

Basing on the structure of the forecasts it can be noticed that many forecasts of a single macroeconomic indicator are available to the researcher in any quarter. However, if a single, point forecast was preferred as a result, the task would be to “average” the attained forecasts. In the described situation the additional difficulty stems from the different number of forecasts depending on the number of the preceding quarters used in the forecasting process. Furthermore, it needs to be considered that the forecasts may lose accuracy as the lag which separates the forecasting period and the period of the last observation increases.

The structure of accessible forecasts as well as the relation between the length of lag and the number of forecasts can be illustrated on the basis of contents of table 4. Regarding GDP forecast for the 1st quarter 2014. For the first time it has been forecasted in the model with last observation of data in the 4th quarter 2014, when the lag order was assumed to be k=4. In the following step, when information regarding the 1st quarter 2013 was already at hand, two forecasts were accessible (for k=3 and k=4). Finally, when the data up to the 4th quarter 2013 were gathered, forecast for the 1st quarter 2014 was performed with k=0,1,2,3,4. Consequently, having the information gathered up to the 4th quarter 2013, we were able to obtain 15 forecasts obtained in five different quarters. The dispersion of the values and the number of the generated forecasts for the 2014q1 period is illustrated in figure A1-A3 (Appendix). It can be seen that the shorter the lag length of the regressors from tendency surveys is, the closer to reality the forecasts of a given macroindicator are.

In the process of aggregation of the forecasts obtained in different periods weights are applied. They should be non-negative real numbers with sum equal to one. It is also assumed that the forecast made in period t for a given quarter is more important than forecast made at period t–1. Finally, it is assumed that the second derivative of a weight with respect to t is nonnegative. The last condition is driven by the assumption that the difference in importance between the information from time point t and information from point t–1 is at least as high as the difference in importance between the information present at t–1 and that present at t–2. A family of weight functions fulfilling this condition can be shown (Czerwiński & Guzik, 1980). The most popular are harmonic, linear and exponential weights (table 5). The weights are usually described by a sequence of m observations ordered with respect to t (t=1,2,…,m) given the following formulas:

- harmonic weights $\boldsymbol{w_t^m = w_{t-1}^m + \frac{1}{m(m-t+1)}, t = 1, 2, \ldots, m;\ w_0^m = 0;}$

- linear weights $w_t^m = \frac{2t}{m(m+1)}, \mathrm{t} = 1,2, \ldots, m;$

- exponential weights $w_t^m = \frac{(1-q)q^{m-1}}{1-q^m}, \mathrm{t} = 1,2, \ldots, m;\ 0 < \mathrm{q} < 1.$

Growth of harmonic weight are proportional to the difference between m and t. Differences in the linear specification of weights are constant. Differences of exponential weights grow with the growth of t. Exponential weights have an additional important feature. By taking an adequate value of q, the decline of importance of observations from older periods can be managed.

Table 5

The weights for different forecast lags

Weights	Weights for the forecasts made a given number of quarters back				
	5 quarters	4 quarters	3 quarters	2 quarters	1 quarter
Harmonic	0.04	0.09	0.16	0.26	0.45
Linear	0.07	0.13	0.20	0.27	0.33
Exponential	0.00	0.01	0.03	0.16	0.80

Source: own estimates.

The procedure used to obtain the combined forecast consists of two parts. Firstly, the arithmetic mean of all the forecasts available for the given period is found. Thus for example using the information about the value of regressors from 2013q4 all the 5 forecasts for the 2014q1 were computed (using k=0,1,2,3,4 lags) and their mean was found. The distance between the forecast period and the period of the last empirical value observed was one quarter (m=1). With the use of the information about the value of regressors in 2013q3, 4 forecasts for the 2014q1 (with the use of k=0,1,2,3 lags) were computed and averaged. The distance between the forecast period and the period of the last empirical value observed was two quarters (m=2). Then in the second part of the averaging process three types of weights for different lags of forecasts (m=1,2,3,4,5) were used and the so called combined forecasts were attained (table 6).

Table 6

Combined forecasts for quarters 2014q1 and 2014q2

Last period of data		2012q4	2013q1	2013q2	2013q3	2013q4	2014q1
Number of forecasts for 2014q1		1	2	3	4	5	
Average from forecasts	GDP	1.9	2.3	2.0	2.9	3.4	
	UNE	11.5	11.8	11.5	10.5	10.1	
	CPI	0.9	0.4	-0.3	0.6	0.7	
Number of forecasts for 2014q2			1	2	3	4	5
Average from forecasts	GDP		3.3	3.0	3.5	4.2	3.6

UNE		12.0	10.4	10.6	10.1	10.2
CPI		0.8	0.4	0.9	1.0	0.6
Combined forecasts for	**2014q1**			**2014q2**		
Weights	Harmonic	Linear	Exp	Harmonic	Linear	Exp
GDP	2.9	2.7	3.3	3.7	3.7	3.7
UNE	10.7	10.8	10.3	10.3	10.4	10.2
CPI	0.5	0.5	0.7	0.7	0.8	0.7
Real GDP	3.4			3.5		
Real UNE	10.6			9.1		
Real CPI	0.6			0.3		

Source: own estimates.

The reason for providing the 2014q1 and 2014q2 forecasts is that the full set of 15 forecast values were available in these quarters. It would be difficult to recommend any particular method of averaging on the basis of just this information since the number of forecasts made this way seems to be too low to enable us to draw any definite conclusions. The above described computation should rather be viewed as an illustration if the forecasting practice is continued. Białowolski et al. (2014b) compare quarterly forecasts for the 2013q1 – 2014q1 period and compare those with the forecasts made by the Economic Institute of the National Bank of Poland as well as the forecasts prepared by the Institute of Research on the Market Economy.

Concluding remarks

In this study, we construct a prognostic model for three key macroeconomic indicators: GDP growth, the unemployment rate and the consumer price index. We use three approaches. Two of them comprise a variation of Bayesian averaging methods (“averaging” and “frequentiest” approach) and the third one is the result of dynamic factor approach. In all models we use the set of indicators from tendency surveys. The way in which the business and consumer sentiment indicators are collected but also approach in which lagged values of tendency survey data are used as regressors enables to generate forecasts without any additional assumptions regarding their values. Such an approach eliminates from the estimation process all subjective assumptions made by forecasters regarding economic processes in the economy. It might be stated that forecaster’s intuition is replaced by aggregated intuition present in the business and consumer tendency survey data.

We confront the forecasts from the Bayesian approaches with those obtained from dynamic factor model. The results show the best performance of the "frequentist", which is characterized by the lowest in sample and out of sample mean absolute percentage errors. The differences in forecasting error between the Bayesian approach and the dynamic factor models is very small, which suggests similar forecasting efficiency of both approaches. It is especially confirmed by very narrow differences in aggregated forecasts for the 1st and the 2nd quarter 2014.

It is worth underlining that parameters of all prognostic models were estimated based on observations of time series up to the 4th quarter 2012. Over the next six quarters the models have not been re-estimated and kept the forecasting ability comparable to other forecasting approaches.

An important feature of our approach is that the forecasting procedure can be mostly automated and the influence of subjective decisions made in the forecasting process can be significantly reduced. It seems that the proposed forecasting methods combine methodology of statistics and econometrics with data mining approach.

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Appendix. Description of variables used in the analysis

spingd – households final consumption expenditure index,
nakinw – investment outlays index,
eksptiu – exports of goods and services index,
imptiu – imports of goods and services index,
wdb_xxx – gross value added in xxx sector index (xxx = industry, construction, trade, transport and storage, accommodation and catering, information and communication, financial and insurance activities, real estate activities, professional an scientific activities, administrative and support service activities, public administration and defence, education, human health and social work activities, arts and entertainment, other service activities),
gus_xxx – balance of responses to question 'xxx' from a consumer sentiment survey CSO (Table A2),
gus_wb – current consumer confidence indicator (CSO),
gus_ww – leading consumer confidence indicator (CSO)
ips_wok – consumer sentiment indicator (IPSOS),
ips_kg – economic climate indicator (IPSOS),

ips_sz – advantage to make purchases indicator (IPSOS),
ips_wk – current consumer confidence indicator (IPSOS),
ips_wo – leading consumer confidence indicator (IPSOS),
zew_ies – ZEW indicator of economic sentiment,
ifo_bs – Ifo business situation indicator,
ifo_be – Ifo business expectations indicator,
biec_wwk – BIEC leading index,
biec_wpi – BIEC future inflation index,
biec_wrp – BIEC future unemployment rate index,
biec_wd – BIEC well-being index,
pmi - Purchasing Managers' index (PMI) for Polish industry,
ind_xxx - balance of responses to question 'xxx' from a business sentiment survey in industry RIED (Table A1),
hhs_xxx - balance of responses to question 'xxx' from a consumer sentiment survey RIED (Table A2),
trade - business sentiment indicator RIED in trade,
agri - business sentiment indicator RIED in agriculture,
cons - business sentiment indicator RIED in construction.

Table A1

Questions from the business sentiment survey in industry

Symbol	Question (ind_xxs – current state, ind_xxf – projection)
ind_q1	Production
ind_q2	total orders
ind_q3	export orders
ind_q4	stock of finished products
ind_q5	prices of goods produced by enterprise
ind_q6	Employment
ind_q7	financial standing
ind_q8	Poland's macroeconomic performance
Business sentiment survey in industry, Research Institute for Economic Development, Warsaw School of Economics	

Table A2
Questions from the consumer sentiment survey CSO & RIED

Symbol	Question
hhs_q1, gus1	Assessment of household financial status, compared with the situation 12 months earlier
hhs_q2, gus2	Projected household financial status in the next 12 months
hhs_q3, gus3	Performance of the Polish economy in the last 12 months
hhs_q4, gus4	Projected performance of the Polish economy in the next 12 months
hhs_q5	Comparison of maintenance costs now and 12 months earlier
hhs_q6	Projection for the inflation rate in the next 12 months
hhs_q7, gus7	Projection for the unemployment rate in the next 12 months
hhs_q8, gus8	An advantage to make major purchases at the present time
hhs_q9	Projected spending on durable consumer goods over the next 12 months in relation to the level reported in the last 12 months
hhs_q10	Assessment of savings and the climate for saving in the context of the country's macroeconomic performance
hhs_q11,gus11	Projected household's saving in the next 12 months
hhs_q12	Financial position of the household
Survey of households, Central Statistical Office, Research Institute for Economic Development, Warsaw School of Economics	

Table A3
Variables in the frequentist approach models

Regressor	Frequentist approach without collinearity correction															Frequentist approach with collinearity correction														
	k=0			k=1			k=2			k=3			k=4			k=0			k=1			k=2			k=3			k=4		
	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI	GDP	UNE	CPI
ifo_be			1				1			1	1		1		1			1				1			1	1		1		1
gus2			1	1				1	1	1																				
gus4			1	1		1		1		1								1	1		1		1							
gus7								1	1	1	1				1															
gus11		1		1	1	1	1	1	1	1	1		1	1			1		1	1	1	1	1	1	1	1		1	1	
gus_ww																														
ips_wo										1	1		1	1	1										1	1		1	1	1
biec_wwk				1			1	1		1	1	1	1		1				1			1			1			1		
biec_wpi		1	1	1		1	1		1	1	1		1	1	1		1		1		1	1			1	1		1	1	1
biec_wrp		1	1	1	1	1					1			1	1		1		1	1	1									
biec_wd								1															1							
ind_q1f					1			1	1		1									1			1	1		1				
ind_q2f		1				1		1		1	1	1	1	1	1		1				1				1		1	1	1	1
ind_q3f	1					1		1	1	1	1		1	1	1	1					1					1		1		
ind_q4f		1									1		1	1			1									1		1	1	
ind_q5f		1				1		1	1	1			1	1							1		1	1	1			1	1	
ind_q6f						1		1	1	1		1	1	1	1						1		1	1	1		1		1	1
ind_q7f		1			1									1						1										
ind_q8f	1		1	1			1				1			1	1	1		1	1			1		1		1			1	1
hhs_q1			1								1				1			1								1				
hhs_q2	1	1		1		1		1						1	1	1	1		1		1									
hhs_q4				1		1		1			1	1		1							1						1		1	
hhs_q6						1				1			1								1			1	1			1		
hhs_q7					1	1			1		1		1		1					1	1			1		1		1		
hhs_q9	1					1	1	1				1		1	1	1					1	1	1				1		1	1
hhs_q11								1						1	1								1						1	1

Source: own estimates. “1” means that the given variable was included in the equation that describes the variability of the given indicator.

Figure A1

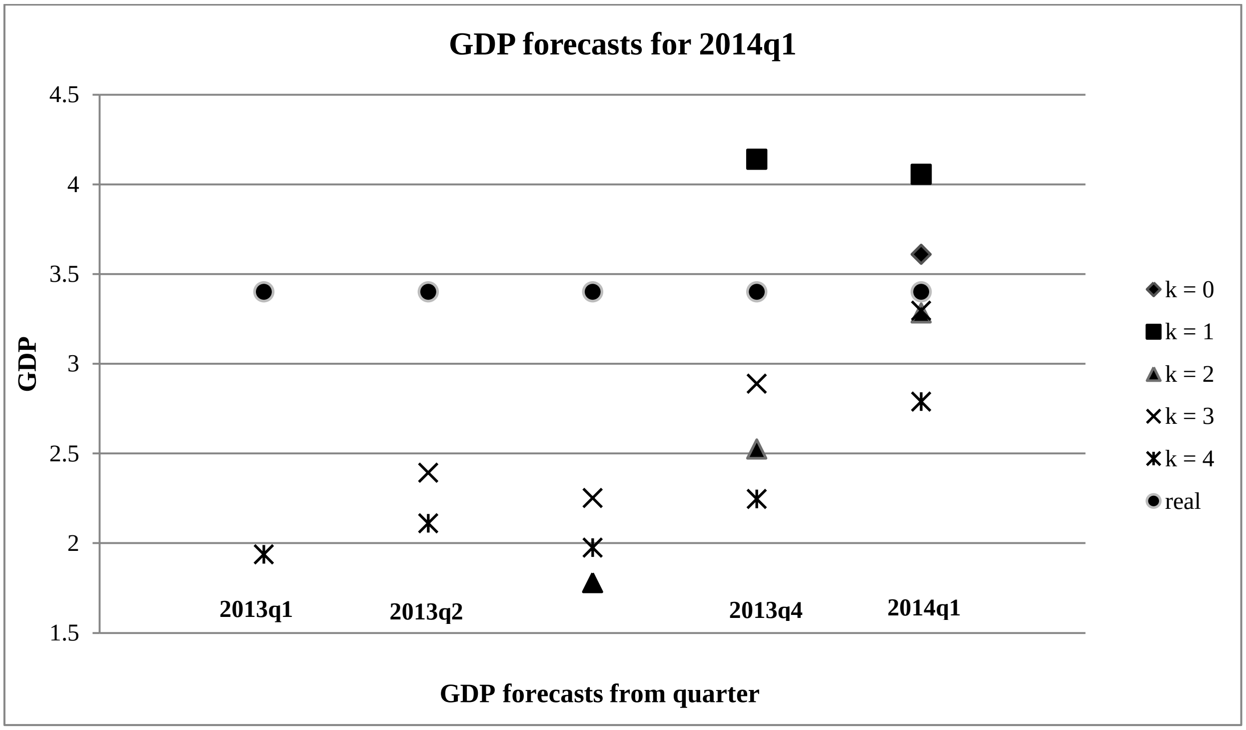


Figure A2

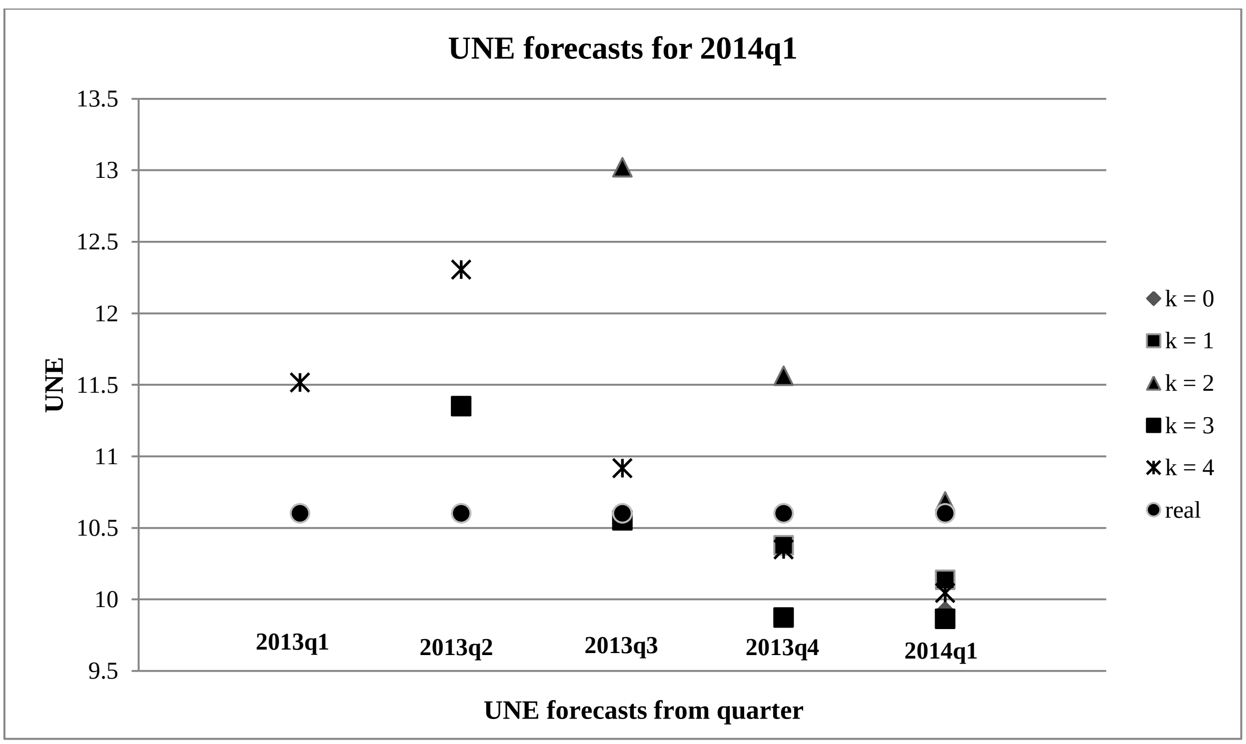
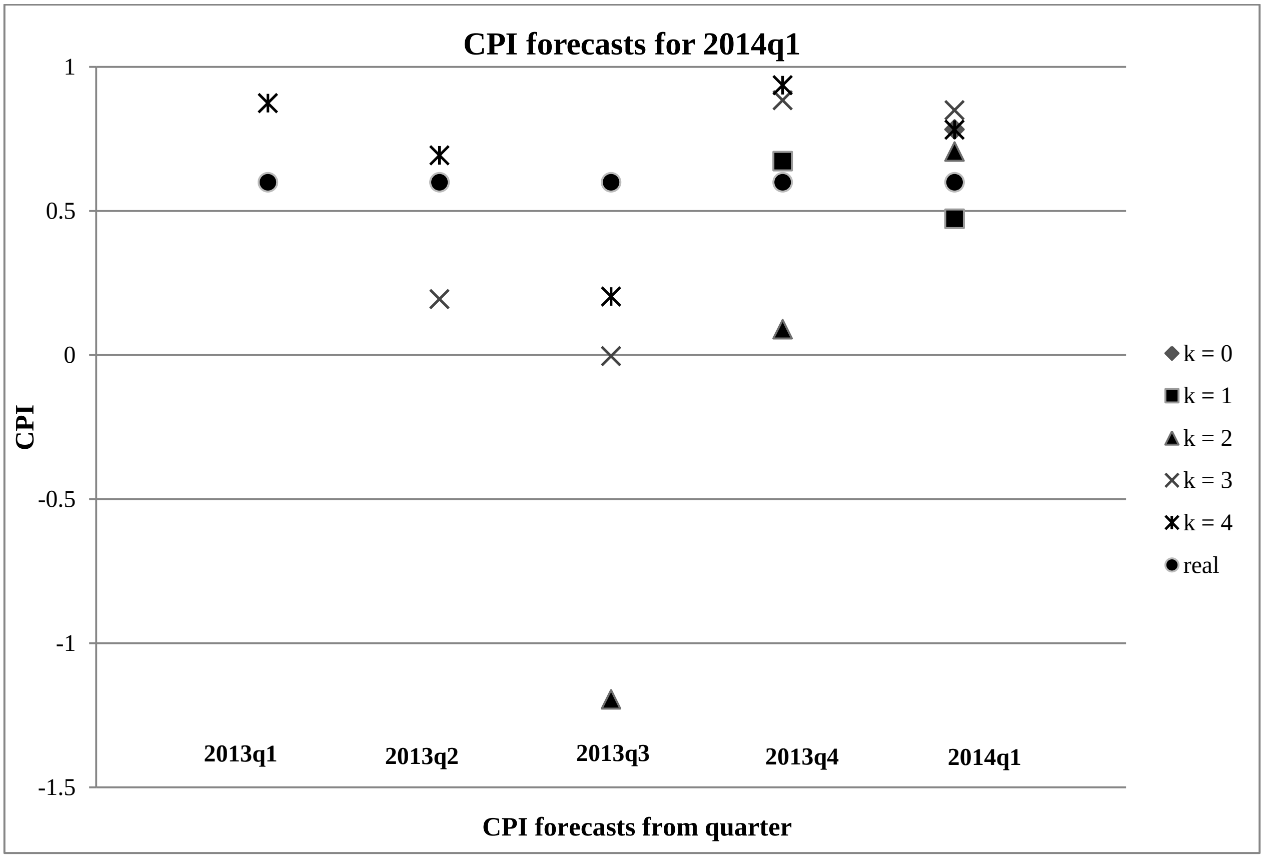


Figure A3



CPI forecasts for 2014q1
1
0.5
0
-0.5
-1
-1.5
CPI
2013q1
2013q2
2013q3
2013q4
2014q1
CPI forecasts from quarter
k = 0
k = 1
k = 2
k = 3
k = 4
real