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Endogenous Unrestricted Locations in Markets with Network Effects

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Abstract

The paper studies indirect network effects in a market composed by two incompatible intermediaries that choose price (short-term issue) in addition to location (long-term issue). The paper first shows that (i) when the network externality is sufficiently weak, only maximum differentiation prevails, (ii) the location equilibrium can be asymmetric for an intermediate level of the network externality, given that the first entrant locates at the city centre while the follower chooses an extreme (niche) positional location and (iii) tipping occurs favouring the leader in the location choice when the intensity of the network externality is sufficiently strong. Moreover, the paper concludes that the likelihood of an asymmetric location equilibrium is higher when there is no mismatch between the product space occupied by consumers and intermediaries. Finally, the author concludes that a penetration pricing strategy conducted by a third intermediary is more successful when the pre-entry condition is not the tipping equilibrium location.

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1 Introduction

The location of firms is a traditional topic covered by the economic literature since the "principle of minimum differentiation" of Hotelling (1929). As analyzed in d'Aspremont et al. (1979), the linear transportation cost is a critical assumption since it generates mathematical inconveniences to the purpose of reaching an equilibrium price. In this sense, the adoption of quadratic transportation costs relatively to the distance that each consumer travels to attend to a particular firm allowed the establishment of the "principle of maximum differentiation". The idea of sequential decisions starts with the canonical contribution of von Stackelberg (1952). Under a deterministic demand, the leader has a first mover advantage and the follower has an information advantage but, given the fact that accommodates the leader's decision relatively to a certain strategic variable, necessarily gets a lower equilibrium profit relatively to the case of a simultaneous choice.

There are several extensions conducted from the mentioned canonical models, one of them concerns with simple network effects (namely, in Serfes and Zacharias (2012)). Network effects are verifiable in many real-world situations (for instance, when a gambler plays poker in online platforms such as Pokerstars). In such cases, the intrinsic utility of a certain agent increases with the size of the attended network. Serfes and Zacharias (2012) assume that networks face location constraints, i.e., the authors restrict the product space occupied by the networks to be precisely an equivalent product space of the one occupied by agents. Note, however, that Lambertini (1997) and Tabuchi and Thisse (1995) extend d'Aspremont et al. (1979) by allowing firms to locate outside the city boundaries stimulated by the fact that firms have incentives to locate where there are no consumers.

The motivation of this manuscript is to extend the topic of product location in markets embracing simple network effects, allowing for an unrestricted product space occupied by networks relatively to the product space occupied by agents. The analysis of Serfes and Zacharias (2012) does not capture the analysis of real-world cases where networks and consumers do not necessarily share the same product space. In the context of markets embracing simple network effects, this issue becomes even more relevant. Let us detail our last sentence. Regarding the nightclub industry, many clubs are positioned outside the locations where consumers live;[1] moreover, there are industrial estates where firms are positioned but without residential use such that important aspects of land use planning lack for a better clarification.[2] In the console industry, players are attracted by other players, however, the great majority of the video games software releases have been commercial failures.[3] In this sense, the product space occupied by consoles is clearly larger than the product space occupied by players, that simply are not attracted to some consoles. Another example is the press industry. This industry depends in a crucial way on the possibility of financing an important fraction of its activities by advertising revenues. Therefore, as argued by Gabszewicz et al. (2002), this feature induces *"the editors of newspapers to moderate, in several cases, the political message they display to their readers, compared with the political opinions they would have expressed otherwise"*. However, this means that the *ex-ante* product space occupied by advertisers is clearly lower than the *ex-ante* product space occupied by newspapers. Summing up, the observation of a mismatch in the product space between networks and consumers should be also taken into consideration by policy-makers and researchers.

Then, the following research questions naturally arise. Firstly, what is the optimal location of the networks, given the mismatch between the product space occupied by networks and consumers? In such case, what is the role of the network effect? Secondly, following the seminal literature emerging after Farrell and Saloner (1986) allowing for a sequential location decision between networks, is it more likely the presence of an asymmetric equilibrium when networks and consumers share the same product space or, alternatively, when there exists a mismatch in product spaces? Finally, regarding entry, if a new entrant tries to obtain a fraction of market share by adopting a penetrating pricing strategy *à la* Gabszewicz and Wauthy (2012), in which type of equilibrium can the entrant more easily triumph? Is it when the pre-entry condition is maximum differentiation or,

[1] Note that in this industry there exists an inter-group externality since the agents that attend to a certain club (men and women) are mutually attracted.

[2] For a discussion of land use planning and the respective implications on consumers see, among others, Bárcena-Ruiz et al. (2014) and Matsumura and Matsushima (2012).

[3] The reader can check a list of commercial failures in video game industry in the beginning of the XXI century in the following Wikipedia link (accessed in 10/01/2015) http://en.wikipedia.org/wiki/List_of_commercial_failures_in_video_gaming

alternatively, the opposite situation of a tipping equilibrium favoring the first mover in the incumbents' market?

Regarding the first question, we find that when the network externality is sufficiently weak only maximum differentiation prevails. The location equilibrium can be asymmetric for an intermediate level of the network externality, given that the first mover locates at the city centre while the follower chooses an extreme (niche) positional location and tipping occurs favouring the leader in the location choice when the intensity of the network externality is sufficiently strong. Moreover, from our second research question, we conclude that the likelihood of an asymmetric location equilibrium is higher when there is no mismatch between the product space occupied by consumers and intermediaries. Finally, we observe that a penetration pricing strategy is more successful when the pre-entry condition is not the tipping equilibrium location.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 performs the analysis and main findings of the manuscript. Section 4 extends the benchmark to a triopoly by allowing entry of a third network with a penetrating pricing strategy and conclusions are drawn in section 5. Appendix with the proofs is attached in section 6.

2 Model

The market is composed by two horizontally differentiated networks, network A and network B. Network A is located at point a and network B is located at point b, with $\{a, b\} \in \mathbb{R}$. Without loss of generality, let $a \leq b$. We also consider that both intermediaries operate with a positive (and non-discriminatory) marginal cost c.

The intermediaries supply their customers. Each consumer buys exactly one unit of the good, which can be produced by either intermediary A or B. We consider a unit mass of pure *singlehoming* consumers[4], that are uniformly distributed across the linear

[4]Singlehoming means that each agent joins only one network and never attend to both. This holds particulary in markets with incompatible networks (see, among others, Caillaud and Jullien (2003)).

city, $x \in [0,1]$. Following Serfes and Zacharias (2012), we introduce the presence of (i) a *network effect*, $\alpha > 0$ and (ii) a *transportation cost* quadratic relatively to the distance d an agent has to 'travel' from his location to the location of the network, td^2, with $t > 0$. The first consideration means that each agent who joins a given intermediary cares about the number of agents that will join the same intermediary while the second consideration measures the degree of horizontal product differentiation between the two networks. Therefore, the utility of an agent attending to network i is given by:

$$u^i(x) \begin{cases} v + \alpha D^A - p^A - t(x-a)^2, & \text{if an agent attends to network A;} \\ v + \alpha D^B - p^B - t(b-x)^2, & \text{if an agent attends to network B.} \end{cases} \tag{1}$$

where $i = \{A, B\}$, v is a sufficiently high stand-alone benefit each agent receives independent of the number of participants such that the market is full covered, p^i is the price that network i charges, D^i is the number of agents that attend to network i and the parameters a and b represent the locations of platforms A and B, respectively. Solving the system of equations (1), the indifferent consumer is given by:

$$\widetilde{x} = \tfrac{1}{2t(b-a)} \left[\alpha(D^A - D^B) + p^B - p^A + t\left(b^2 - a^2\right)\right]. \tag{2}$$

Since the demand is normalized to 1 and assumed to be totally inelastic, the resulting interior market shares are given by[5]:

$$D^A\left(p^A, p^B\right) = \tfrac{p^B - p^A + t\left(b^2 - a^2\right) - \alpha}{2[t(b-a) - \alpha]}; \tag{3}$$

$$D^B\left(p^A, p^B\right) = 1 - D^A, \tag{4}$$

respectively. The profit of platform A is given by:

$$\pi^A\left(p^A, p^B\right) = \left(p^A - c\right) D^A\left(p^A, p^B\right), \tag{5}$$

and the profit of platform B is given by:

$$\pi^B\left(p^A, p^B\right) = \left(p^B - c\right) D^B\left(p^A, p^B\right). \tag{6}$$

We focus our analysis in the following game. In the first stage, networks choose the respective locations sequentially (assuming that network A is a leader and network B is a

[5] D^i is not only the quantity supplied by network i but also the market share of network i because the total inelastic demand is normalized to 1.

follower) and, then, in stage two networks engage in price competition. The agents, after observing network locations and prices, decide which network to join. The game is solved by the method of backward induction.

3 Analysis

Price competition. In the second stage of the game, given the locations of the intermediaries, the owners' face Bertrand competition. Deriving expression (5) relatively to price p^A and expression (6) relatively to price p^B, follows that:

$$p^A(a,b,\alpha) = c - \alpha + \tfrac{t(b-a)(2+b+a)}{3};\ p^B(a,b,\alpha) = c - \alpha + \tfrac{t(b-a)(4-b-a)}{3}. \tag{7}$$

Substituting expressions (7) in expressions (3) and (4), the market shares correspond to:

$$D^A(a,b,\alpha) = \tfrac{t(b-a)(2+b+a)-3\alpha}{6(t(b-a)-\alpha)};\ D^B(a,b,\alpha) = \tfrac{t(b-a)(4-b-a)-3\alpha}{6(t(b-a)-\alpha)}. \tag{8}$$

Therefore, replacing expressions (7) and (8) in expressions (5) and (6), the profits are given by:[6]

$$\pi^A(a,b,\alpha) = \tfrac{[t(b-a)(2+b+a)-3\alpha]^2}{18[t(b-a)-\alpha]};\ \pi^B(a,b,\alpha) = \tfrac{[t(b-a)(4-b-a)-3\alpha]^2}{18[t(b-a)-\alpha]}. \tag{9}$$

Location competition. Now, each network's owner decides the location of its corresponding platform. As in Serfes and Zacharias (2012), we assume that all agents join network A if both networks are equidistantly located from the city centre. When we fix the location of network B at a specific location b and examine the profits of network A when it moves from the left extreme $a = \widehat{a}_1$ to the right extreme $a = b$, we verify the presence of two determinant effects: (i) an intensification of price competition (lower prices) and (ii) demand creation on network A (since this network is closer to the attraction central point $\frac{1}{2}$). The first effect initially dominates but, due to the presence of a network externality, the latter effect overcomes the first effect.

[6]Mandatory details regarding the price competition stage are relegated to Appendix 6.1.

Lemma 1 ***(Behavior of the profit of network*** **i** ***as a function of its location)*** *The profit function of network A (B), for any fixed location b (a) of network B (A), exhibits a $\cup$-shape.*

Proof. See Appendix 6.2. ■

The intuition regarding the shape of the payoff function of network A comes as follows. For a higher degree of horizontal product differentiation the market is shared and any movement closer to the attraction point $\frac{1}{2}$ intensifies the networks' price competition. However, after a certain threshold tipping occurs to the network that is closer to the city centre. The following Proposition summarizes the subgame perfect Nash equilibrium (SPNE).

Proposition 2 ***(Unrestricted location equilibria)*** *The SPNE where A is a leader and B is a follower in the location stage is characterized as it follows:*

Maximum differentiation equilibrium. *If $\alpha \in \left[0, \frac{t}{2}\right]$, then, $(a^*, b^*) = \left(-\frac{1}{4}, \frac{5}{4}\right)$ is a location equilibrium. The equilibrium prices, market shares and profits are given by:*

$$\left(p^{A*}_{(M)}, p^{B*}_{(M)}\right) = \left(c + \tfrac{3}{2}t - \alpha, c + \tfrac{3}{2}t - \alpha\right); \quad \left(D^{A*}_{(M)}; D^{B*}_{(M)}\right) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right);$$

$$\left(\pi^{A*}_{(M)}, \pi^{B*}_{(M)}\right) = \left(\tfrac{3}{4}t - \tfrac{\alpha}{2}, \tfrac{3}{4}t - \tfrac{\alpha}{2}\right).$$

Asymmetric location equilibrium. *If $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$, then, $(a^*, b^*) = \left(\frac{1}{2}, \frac{5}{4}\right)$ is a location equilibrium. Both platforms operate in the market. The equilibrium prices, market shares and profits are given by:*

$$\left(p^{A*}_{(A)}, p^{B*}_{(A)}\right) = \left(c + \tfrac{15}{16}t - \alpha, c + \tfrac{9}{16}t - \alpha\right); \quad \left(D^{A*}_{(A)}, D^{B*}_{(A)}\right) = \left(\tfrac{1}{2} + \tfrac{3t}{8(3t-4\alpha)}, \tfrac{1}{2} - \tfrac{3t}{8(3t-4\alpha)}\right);$$

$$\left(\pi^{A*}_{(A)}, \pi^{B*}_{(A)}\right) = \left(\tfrac{(15t-16\alpha)^2}{128(3t-4\alpha)}, \tfrac{(9t-16\alpha)^2}{128(3t-4\alpha)}\right).$$

Tipping location equilibrium. *If $\alpha \in \left[\frac{9t}{16}, \frac{3t}{2}\right)$, then, $(a^*, b^*) = \left(\frac{1}{2}, \frac{5}{4}\right)$ is a location equilibrium. Only the first-mover is active in the market. The equilibrium prices, market shares and profits are given by:*

$$\left(p^{A*}_{(T)}, p^{B*}_{(T)}\right) = \left(\alpha - \tfrac{3}{16}t, 0\right); \quad \left(D^{A*}_{(T)}, D^{B*}_{(T)}\right) = (1, 0); \quad \left(\pi^{A*}_{(T)}, \pi^{B*}_{(T)}\right) = \left(\alpha - \tfrac{3}{16}t - c, 0\right).$$

Proof. See Appendix 6.2. ■

When the network externality is weak, a network has no incentive to move closer to its rival in order to become dominant and the equilibrium locations are equivalent to Lambertini (1997). When the network externality is intermediate, the leader A always locates in the city centre. The follower network B can attract some agents and make strictly positive profits when it locates as most as possible to right of the city. Then, the leader has no incentive to move away from the middle point because, in this case, the follower B has a profitable deviation to locate closer to the middle point and leave A with a null market share. For a sufficiently strong network externality, the market tips in favor of the first mover. Note that the equilibrium locations differ from Serfes and Zacharias (2012), since in our model the product space occupied by networks mismatches from the product space occupied by consumers. In particular, the asymmetric location equilibrium and the tipping location equilibrium in Serfes and Zacharias (2012) is restricted to $(a^*, b^*) = \left(\frac{1}{2}, 1\right)$. Thus, with a mismatch in the product space between firms and consumers, the second mover is able to distance itself further from the city centre.

Therefore, the corresponding boundaries in each type of equilibrium also differ, which drives us into another motivating question. Farrell and Saloner (1986), among others thereafter, document a trade-off between 'standardization' and variety. Proposition 2 confirms that in equilibrium (i) when the externality is weak, neither standard is superior to the other, (ii) for an intermediate network intensity, there will be two 'standards' in equilibrium and, finally, (iii) when the externality is strong, there will be only one standard. Then, the following Lemma becomes straightforward.

Lemma 3 ***(Likelihood for each type of equilibrium)*** *When there exist a mismatch between the product space occupied by consumers relatively to the product space occupied by the networks follows that:* (i) *the likelihood of an asymmetric (tipping) equilibrium becomes lower (higher) and* (ii) *the likelihood of maximum differentiation is undeviating.*

Proof. See Appendix 6.2. ■

Lemma 3 shows that an equilibrium with two 'standards' is more likely to be verifiable in a restrictive location regime. Under an unrestricted location regime where the mismatch between product spaces stands, the leader network A anticipates that network B is, now, able to full exploit the role of horizontal product differentiation by locating further to the right of the city and behaves aggressively such that forces all agents to "get on board" (tipping) for a lower level of the network externality, thus, preventing network B to conquer any additional consumer. Therefore, the above Lemma highlights that a follower network prefers to fight for consumers in markets with location constraints. This finding justifies, for instance, the ascent of the 'Pensée Unique' in the press industry (see, among others thereafter, Gabszewicz et al. (2001)).

4 Entry deterrence with a penetration pricing strategy

Consider an extension of our benchmark to a triopoly. Let us allow the possibility of entry by a third network, and conduct a similar study as in Gabszewicz and Wauthy (2012).[7] Given that the incumbent networks and consumers do not share the same product space, we aim at understanding whether the network externality make it easier or harder for the incumbents to deter the entry of a third network, C. Specifically, we intend to find: (i) how does the network effect marginally influences the entry deterrence strategy conducted by the incumbents? (ii) And is it easier to the new entrant to fight for a market position under maximum differentiation or, alternatively, in the opposite situation where network A has previously tipped the market in its favor?

Suppose that the third network has the possibility of enter in the market, becoming positioned at a distance L from the city centre (i.e., from $x = \frac{1}{2}$), in a direction that is

[7] We follow the mechanism exposed in Gabszewicz and Wauthy (2012) when the authors consider entry by inferior-quality, i.e., we keep the pre-entry incumbent prices, market shares and profits for each type of equilibrium location and, assuming that the new entrant adopts a penetrating price equal to its marginal cost (assumed to be equal to the incumbents' marginal cost), we only focus on finding the maximum distance L of a new orthogonal segment relatively to the incumbent city $[0, 1]$, linked to the incumbent city at point $x = \frac{1}{2}$, above which entry is deterred.

orthogonal to the linear city. As in Gabszewicz and Wauthy (2012), suppose that networks A and B are charging the pre-entry equilibrium prices disposed in Proposition 2. Can network C attract any consumers by charging a penetration price equal to its marginal cost ($p^{C*} = c$), given the *ex-ante* situation that C has any consumers ($D^C = 0$)?[8]

Proposition 4 ***(Barriers to entry and network effects)****. The entry deterrence is easier attained* (*i*) *for a sufficiently strong level of the network externality,* $\alpha \in \left[\frac{9t}{16}, \frac{3t}{2}\right)$*;* (*ii*) *when the level of the network externality is lower,* $\alpha \in \left[0, \frac{9t}{16}\right)$*, with any marginally increment on the level of the network externality.*

Proof. See Appendix 6.2. ■

The network externality diminishes the maximum orthogonal distance between the new entrant and the incumbents above which entry is deterred, i.e., it helps the incumbents A and B to deter the entrance of the new network C, except when the network effect is too strong (tipping location equilibrium). Here, the critical distance is undeviating with any marginal change in the level of the network externality. However, the maximum threshold L (above which entry is blocked) is always lower than any distance L emerging from the other location equilibrium (maximum differentiation and asymmetric location) meaning, therefore, that market entry is more difficult. Thus, the intuition is that when the level of the network externality is too strong, the incumbent A displays a more aggressive behavior to protect its monopoly position relatively to the situation where the market is shared with the rival incumbent, network B.

Therefore, network C has a stronger incentive to enter in the market with a penetration pricing strategy when the network externality is not too strong. Indeed, in the case where

[8] We consider that the status quo is a coordination equilibrium assumed to be fully adverse to the entrant. Also note that the penetration pricing is a strategy where the initially price of a certain good is set at a lower level than the incumbents' market price. Such strategy is assiduously verifiable in many online markets, given the expectation that the new entrant has that consumers may switch. Our aim is, in this sense, to study penetration pricing as a marketing tool to get some "agents on board" in the new entrant, rather than allowing this new network to make strictly positive profit in the short term. Moreover, Kalish (1983) demonstrates that such penetration price can be changed in the future once the new entrant consolidates a significant amount of market share.

the maximum differentiation stands in the incumbent market, network C is able to enter in the market for a larger maximum distance L relatively to the maximum distance L under tipping (see Corollary 11 in Appendix). However, any positive marginal change in the intensity of the network externality is detrimental to the new entrant.

5 Conclusions

We study a duopoly that incorporates horizontal product differentiation and a network externality. In this framework, the product space occupied by networks and consumers is not forced to be equal. We conclude that there exists an interior equilibrium where networks locate symmetrically at a distance equal to $\frac{3}{4}$ of the city centre, when the level of the network effect is sufficiently weak. Then, given the sequentiality of location decision, there exists an intermediate level of network effect where both networks share the market and, finally, if the network intensity is too strong, only the leader network regarding the location decision is active. Secondly, when exists a mismatch between the space occupied by consumers relatively to the space occupied by the networks, we conclude that the likelihood of an asymmetric (tipping) equilibrium lower (higher). However, the likelihood for maximum differentiation equilibrium is undeviating whatever the restrictive (or unrestricted) regime of locations.

Finally, we study entry of a third network that adopts a penetrating pricing strategy. We conclude that the stronger the level of the network effect is, the more successful is the deterrence of an entrant. Even when the network effect is not too strong, any marginal change on the level of the network effect helps the incumbents to deter entry. In this sense, the framework highlights theoretical evidence that incumbent companies exhibit a more aggressive predatory behavior in the terminology of Ordover and Willig (1981) when the product space occupied by firms differs from the one occupied by consumers with the presence of a network externality. We let for future the introduction of strategic delegation as a relevant extension of this model.

6 Appendix

6.1 Appendix to support price competition stage

Profit function concavity

The profit functions are strictly concave in a platform's own prices if:

$$\alpha < t(b-a) \tag{10}$$

Alternatively, the above condition can be written as:

$$a < b - \frac{\alpha}{t} \ \cap \ b > a + \frac{\alpha}{t}$$

If (10) is satisfied, then the first order conditions are sufficient for profit maximization.

Positive market shares

The equilibrium market shares are given by (8). If (10) is satisfied, then the denominators of D^A and D^B in expression (8) are positive. To secure an interior equilibrium, the market shares need to be within $[0,1]$. This occurs if and only if:

$$\alpha < \min\left\{\tfrac{t(b-a)(2+b+a)}{3}, \tfrac{t(b-a)(4-b-a)}{3}\right\} \equiv \min\{\widehat{\alpha}, \widetilde{\alpha}\}.$$

Note that the ceiling threshold $\widehat{\alpha}$ prevents tipping to network B and the ceiling threshold $\widetilde{\alpha}$ prevents tipping to network A. The following claim is straightforward.

Claim 1 *In the unconstrained location regime:*

(i) for $D^A(a,b,\alpha)$ to be greater than zero (which implies that $D^B(a,b,\alpha)$ is less than one) we must have:

$$D^A > 0 \cap D^B < 1 \Leftrightarrow \alpha < \tfrac{t(b-a)(2+b+a)}{3} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} a < \widehat{a}_2 \equiv -1 + \frac{\sqrt{t[t(1+b)^2 - 3\alpha]}}{t}; \\ a > \widehat{a}_1 \equiv -1 - \frac{\sqrt{t[t(1+b)^2 - 3\alpha]}}{t}. \end{cases} \tag{11}$$

(ii) for $D^B(a,b,\alpha)$ to be greater than zero (which implies $D^A(a,b,\alpha)$ to be less than one) we must impose:

$$D^B > 0 \cap D^A < 1 \Leftrightarrow \alpha < \frac{t(b-a)(4-b-a)}{3} \Leftrightarrow$$
$$\Leftrightarrow \begin{cases} a < \widetilde{a}_2 \equiv 2 - \frac{\sqrt{t[t(2-b)^2+3\alpha]}}{t}; \\ a > \widetilde{a}_1 \equiv 2 + \frac{\sqrt{t[t(2-b)^2+3\alpha]}}{t}. \end{cases} \tag{12}$$

Proof. Fix the demand of both networks as a function of location a. Regarding point (i), let $f(a) = t(b-a)(2+b+a) - 3\alpha$. To prevent tipping in favor to network B must stand $f(a) > 0$. Since $\frac{\partial f(a)}{\partial a} = t(b-a) - t(b-a)(2+b+a)$, then $f(a)$ reaches to a local extreme in $a^* = -1$. Since $\frac{\partial^2 f(a)}{\partial a^2} = -2t < 0$, follows that a^* is a maximum and $f(a)$ is a concave function that must satisfy expression (11). Regarding point (ii), let $g(a) = t(b-a)(4-b-a) - 3\alpha$. To prevent tipping in favor to network A must stand $g(a) > 0$. Since $\frac{\partial g(a)}{\partial a} = -t(b-a) - t(b-a)(4-b-a)$, then $g(a)$ reaches to a local extreme in $a^* = 2$. Since $\frac{\partial^2 g(a)}{\partial a^2} = 2t > 0$, follows that a^* is a minimum and $g(a)$ is a convex function that must satisfy expression (12). ■

Combining simultaneously conditions (11) and (12), the existence of a sharing equilibrium is only attainable for:

$$a \in (\widehat{a}_1, \min\{\widehat{a}_2, \widetilde{a}_2\}). \tag{13}$$

The lower bound prevents tipping in favor to network B. The ceiling $\widehat{a}_2$ prevents tipping favouring platform B and the ceiling $\widetilde{a}_2$ prevents tipping in favor to platform A. The interpretation of the upper bounds in (13) is that for any given location b of platform B the market tips, either in favor of A or in favor of B, as network A is closer to network B since condition (10) does not guarantee *per se* the prevention of tipping. Note that condition (13) is different from Serfes and Zacharias (2012) where, $a \in (0, \min\{\widehat{a}_2, \widetilde{a}_2\})$, where consumers and networks share the same product space, restricted to $[0,1]$.

Definition and analysis of tipping

Tipping implies that when the market tips it is the platform that is closer to the attraction point $x = \frac{1}{2}$ that absorbs all the agents. However, an additional clarification regarding tipping is required: as in Zacharias and Serfes (2012), if both platforms are equidistantly located from the city centre, we consider that all agents will join platform A.

W.l.o.g., we consider in the main part of the paper $a \leq b$. Then, consider network A with location a and location b for network B with $\{a, b\} \in \mathbb{R}$. The thresholds (12) and (11) are mandatory to understand the dynamics of tipping. As in Serfes and Zacharias (2012), two relevant cases emerge: when $\widetilde{a}_2 < \widehat{a}_2$, then $b > \frac{1}{2} + \frac{\alpha}{2t} \equiv \overline{b}$ and, therefore, tipping is more likely to occur to network A. On the other hand, when $\widehat{a}_2 < \widetilde{a}_2$, then $b < \frac{1}{2} + \frac{\alpha}{2t} \equiv \overline{b}$ and, therefore, tipping is more likely to occur favouring network B. This means that, the more network B is closer (far away from) to its rival, the more this network is able to tip (not tip) the market. We split the analysis between those two cases:

Case 1. It turns out that the momentum that favors tipping in favor of the network A is such that $\widetilde{a}_2 < \widehat{a}_2$. This inequality yields for:

$$b \geq \tfrac{1}{2} + \tfrac{\alpha}{2t} \equiv \overline{b}.$$

When $\widetilde{a}_2 < \widehat{a}_2$ the lower binding threshold is $\widetilde{a}_2$ and the ceiling $\widehat{a}_2$ becomes irrelevant. Then, $\widetilde{a}_2$ corresponds to the maximum threshold above which network A tips the market.

Since $\widetilde{a}_2 < \widehat{a}_2 \Leftrightarrow b \geq \overline{b}$, as network A increases a it attracts all agents when $a > \widetilde{a}_2$ and the market remains tipped in favor of A until $a = b$. Note that under symmetric locations relatively to the city centre follows $a + b = 1 \Leftrightarrow a = 1 - b$. Then, when $b \geq \overline{b}$ implies $\widetilde{a}_2 \geq 1 - b$. Allowing a and b to be located outside $[0, 1]$, also holds $\widetilde{a}_2 \geq b - 1$.

Case 2. On the other hand, the momentum that favors tipping in favor of the network B is such that $\widehat{a}_2 \leq \widetilde{a}_2$. This inequality yields for:

$$b \leq \tfrac{1}{2} + \tfrac{\alpha}{2t} \equiv \overline{b}.$$

When $\widetilde{a}_2 \geq \widehat{a}_2$ the lower binding threshold is $\widehat{a}_2$ and the ceiling $\widetilde{a}_2$ becomes irrelevant. Then, $\widehat{a}_2$ corresponds to the maximum threshold above which network B tips the market.

Since $\widehat{a}_2 \leq \widetilde{a}_2 \Leftrightarrow b \leq \overline{b}$, network A's market share becomes zero when $a = \widehat{a}_2$. Above this point, the market will tip in favor of network A when $a = b-1$ (symmetric locations) and it will remain tipped in favor of network A until $a = b$. Note that since $a+b = 1 \Leftrightarrow a = 1-b$, then, $b \leq \bar{b}$ implies $\widehat{a}_2 \leq 1 - b$. Allowing a and b to be located outside $[0, 1]$, also holds $\widehat{a}_2 \leq b - 1$.

Finally, under this case, when $\widehat{a}_1 = \widehat{a}_2$ (so that only tipping to B occurs) implies $b = -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \leq \bar{b}$ meaning that, when network B moves closer to the city centre, the market share of network A is null. Note that this critical threshold is different from Serfes and Zacharias (2012), given the existence of a mismatch between the product space occupied by consumers relatively to the product space occupied by the networks. After helping the navigator reader with the above details, we now move towards the main proofs of the manuscript. □

6.2 Main Proofs

Proof of Lemma 1

The interior profit functions are given by expression (9). The second order condition (10) is always satisfied when the market has not tipped, i.e., under (13). W.l.o.g., we consider in the main part of the paper $a \leq b$. Let us analyze the payoff function of network A. We fix the location of network B at a specific point b and examine the payoff function of network A as $a \to b$.

The proof is divided in three steps. First, we compute the extremes of $\pi^A(a, b, \alpha)$ and study the corresponding roots. Then, we compute the zeros of $\pi^A(a, b, \alpha)$ and study the sign of $\frac{\partial \pi^A}{\partial a}$ in the domain points $a = -1$ and $a = 0$ such that it becomes unnecessary to study the second order conditions related with the global extremes. Finally, depending on case 1 ($\widetilde{a}_2 < \widehat{a}_2 \Leftrightarrow b \geq \bar{b}$) or case 2 ($\widehat{a}_2 \leq \widetilde{a}_2 \Leftrightarrow b \leq \bar{b}$,), we show the corresponding shape of the profit function of network A.

Roots

The derivative of network's A payoff function (9) with respect to a is given by:

$$\frac{\partial \pi^A(a,b,\alpha)}{\partial a} = \frac{t[t(b-a)(2+b+a)-3\alpha][t(a-b)(2-b+3a)+\alpha(1+4a)]}{18(t(b-a)-\alpha)^2}. \tag{14}$$

The extremes of $\pi^A(a,b,\alpha)$ and, thus, the zeros of (14) are given by:

$$a = a_1 = \widehat{a}_1 \equiv -1 - \frac{\sqrt{t[t(1+b)^2-3\alpha]}}{t}; \tag{15}$$

$$a = a_2 = \widehat{a}_2 \equiv -1 + \frac{\sqrt{t[t(1+b)^2-3\alpha]}}{t}; \tag{16}$$

$$a = a_3 \equiv \frac{2(tb-\alpha)-t}{3t} - \frac{\sqrt{t^2(1+b)^2+\alpha[4\alpha+t(1-8b)]}}{3t}; \tag{17}$$

$$a = a_4 \equiv \frac{2(tb-\alpha)-t}{3t} + \frac{\sqrt{t^2(1+b)^2+\alpha[4\alpha+t(1-8b)]}}{3t}. \tag{18}$$

The above roots are not excluded due to the imposition of a restriction on the location of the networks because $\{a,b\} \in \mathbb{R}$. However, recall the definition of tipping: when the market tips it is the platform that is closer to the attraction point $\frac{1}{2}$ that absorbs all the agents and if both platforms are equidistantly located from the city centre, we consider that all agents will join platform A.

Root a_1 is clearly negative since is lower than -1. Note that $a_1|_{\alpha=0} = -2-b \gtreqless 0$ if $b \lesseqgtr -2$ and $\frac{\partial a_1}{\partial b} = -\frac{t(1+b)}{\sqrt{t[t(1+b)^2-3\alpha]}} < 0$. Therefore, if B locates to the left then a_1 goes to the right but network A would react locating in the city centre to attract all the agents.

Root a_2 is higher than -1 and $a_2|_{\alpha=0} = b \gtreqless 0$ if $b \gtreqless 0$ and $\frac{\partial a_2}{\partial b} = \frac{t(1+b)}{\sqrt{t[t(1+b)^2-3\alpha]}} > 0$. Therefore, if B locates closer to the city centre, network A must react locating also in the city centre to attract all the agents.

Regarding root a_3, note that it is negative because $a_3|_{\alpha=0} = -\frac{2-b}{3} \gtreqless 0$ if $b \gtreqless 2$ and $\frac{\partial a_3}{\partial \alpha} = \frac{1}{t}\left(-\frac{t-8b(t-\alpha)}{6\sqrt{t[t(1+b)^2-3\alpha]}} - \frac{2}{3}\right) < 0$ if $b \in \left(-\frac{1}{4}, \frac{5}{4}\right)$. Therefore, if B locates to the right so that a_3 turns positive, network A would react locating in the city centre and would attract all the agents.

Relatively to root a_4, follows that $a_4|_{\alpha=0} = b \gtreqless 0$ if $b \gtreqless 0$ and $\frac{\partial a_4}{\partial \alpha} = \frac{t-8b(t-\alpha)}{6t\sqrt{t[t(1+b)^2-3\alpha]}} > 0$ if $b \in \left(-\frac{1}{4}, \frac{5}{4}\right)$. Therefore, if B locates closer to the city centre, network A must react locating also in the city centre to attract all the agents. After this brief explanation, the following two Corollaries are required.

Corollary 5 *Let us define $\mu = \widehat{a}_2 - \widehat{a}_1 > 0$. To secure a sharing equilibrium ($\mu = \widehat{a}_2 - \widehat{a}_1 > 0$), it must be the case that $b > b_2 \equiv -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}}$.*

Proof. When $\widehat{a}_2 < \widetilde{a}_2$ (case 2 of the above analysis of tipping), to secure that there exists a sharing equilibrium it must be the case that $\widehat{a}_2 - \widehat{a}_1 > 0$. Let us define $\mu = \widehat{a}_2 - \widehat{a}_1 > 0$. Using (15) and (16), follows:

$$\mu = \frac{2\sqrt{t[t(1+b)^2 - 3\alpha]}}{t}.$$

Computing $\mu > 0$ implies to study the function:

$$b^2 + 2b + \left(1 - \frac{3\alpha}{t}\right) > 0. \tag{19}$$

The polynomial (19) holds two zeros at:

$$b_1 = -1 - \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \cap b_2 = -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}}.$$

Therefore, in a sharing equilibrium, any $b \in [b_1, b_2]$ is impossible because $\mu \leq 0$ is incompatible with a sharing equilibrium. Since the threshold b_1 becomes irrelevant given that for any $b \leq b_2$ tips in favor to network B, then, our Claim is straightforward. ■

Given Corollary 5, the following is also mandatory to the purpose of our proof regarding Lemma 1. The goal is to prove that whatever the location b of network B where a sharing equilibrium yields, the root a_3 constitutes a location domain point of that is always higher than the location domain point $a = -1$.

Corollary 6 $\forall\ b > b_2 \equiv -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \Rightarrow a_3 > -1.$

Proof. Let us define $\Delta = a_3 + 1$. If $\Delta > 0$, then, $a_3 > -1$. Using (17) follows:

$$\Delta = \frac{2t(1+b) - 2\alpha - \sqrt{t^2(1+b)^2 + t\alpha(1-8b) + 4\alpha^2}}{3t}.$$

Computing $\Delta > 0$ implies to study the function:

$$2t(1+b) - 2\alpha - \sqrt{t^2(1+b)^2 + t\alpha(1-8b) + 4\alpha^2} > 0, \tag{20}$$

Rearranging (20) implies that is an equivalent expression to:

$$b^2 + 2b + \left(1 - \frac{3\alpha}{t}\right) > 0,$$

which is precisely expression (19). The polynomial (19) holds two zeros at:

$$b_1 = -1 - \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \cap b_2 = -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}}.$$

Therefore, for any $b > b_2$ follows that $\Delta > 0$ and, therefore, the location domain point a_3 is at the right of the location domain point $a = -1$. ■

Zeros of the profit function π^A and the signs of $\left.\frac{\partial \pi^A}{\partial a}\right|_{a=-1}$ and $\left.\frac{\partial \pi^A}{\partial a}\right|_{a=0}$

The zeros of π^A are given by:

$$a = a_1 = \widehat{a}_1 \equiv -1 - \frac{\sqrt{t[t(1+b)^2 - 3\alpha]}}{t} \cap a = a_2 = \widehat{a}_2 \equiv -1 + \frac{\sqrt{t[t(1+b)^2 - 3\alpha]}}{t}.$$

Therefore, two of the extremes are also zeros of the profit function of network A.

To avoid study the extremes, we only evaluate the derivative of the profit function relatively to a in the points $a = -1$ and $a = 0$. If $\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=-1} > 0$ and $\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=0} < 0$, then, the profit function $\pi^A(a,b,\alpha)$ yields a minimum at a_1, a maximum at a_3 (because $\pi^A(a,b,\alpha)$ is an increasing function at $a = -1$ and according to Corollary 6 follows $a_1 < -1 < a_3$) and a minimum at a_4 or at a_2 (depending on whether we rely on case 1 or on case 2, respectively).

Regarding $\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=0}$, the proof that $\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=0} < 0$ is given by Serfes and Zacharias (2012) and, thus, omitted.[9]

Corollary 7 $\forall\ b > b_2 \equiv -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \Rightarrow \left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=-1} > 0$.

Proof. Relatively to $\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=-1}$, we have:

$$\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=-1} = \frac{\left[t(1+b)^2 - 3\alpha\right]^2}{9[t(1+b) - \alpha]}. \tag{21}$$

Solving (21) equal to zero results the polynomial (19), and, therefore, two roots with respect to b:

$$b_1 = -1 - \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \cup b_2 = -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}}.$$

As shown above, let $\mu \equiv \widehat{a}_2 - \widehat{a}_1 = \frac{2\sqrt{t[t(1+b)^2 - 3\alpha]}}{t}$. As shown above, follows that $\widehat{a}_2$ becomes $\widehat{a}_1$ so that $\mu \leq 0$ when $b \in (b_1, b_2)$. Therefore, the feasible root b_1, becomes

[9] The proof is available in the working paper versions of Serfes and Zacharias (2009) and Serfes and Zacharias (2010) and in the final version Serfes and Zacharias (2012).

irrelevant since when $b \leq b_2$ the market tips in favour to network B. Then, it is immediate that $\left.\frac{\partial \pi^A(a,b,\alpha)}{\partial a}\right|_{a=-1} > 0$ for any $b > b_2 = -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}}$. ■

Shape of the profit function

Case 1 ($\widetilde{a}_2 < \widehat{a}_2 \Leftrightarrow b \geq \overline{b}$). We have that $\widetilde{a}_2 < a_2 = \widehat{a}_2$ such that only the root a_4 and the threshold $\widetilde{a}_2$ are relevant. Two situation can occur: $a_4 < \widetilde{a}_2 \cup \widetilde{a}_2 < a_4$. Whether the first situation holds or not depends the overall impact of the network externality in equilibrium prices and market shares. Given that only a_4 and $\widetilde{a}_2$ are relevant, the fact that (i) the profit function of network A is strictly increasing at $a = -1$ and (ii) the profit function of network A is strictly decreasing at $a = 0$ (corresponding to interior points where the sharing equilibrium exists), the profit function attains: (i) a local minimum at $a = a_1$ that remains at the left of $a_1 = \widehat{a}_1$ (because for $a_1 \leq \widehat{a}_1$ tipping in favor to network B occurs), (ii) a local maximum at $a = a_3$ and (iii) a local minimum at $a = a_4$, if $a_4 < \widetilde{a}_2$ or a local minimum at $a = \widetilde{a}_2$, if $\widetilde{a}_2 < a_4$. As explained above, in this case, the two networks share the agents when $a \in (\widehat{a}_1, \widetilde{a}_2)$. As network A moves closer to its rival, the price competition is intensified (the price of network A reduces according to (7)) but the market share of network A increases (demand creation according to (8)), in particular, after $a = b - 1$ where network A is closer to the city centre than network B. That is why a minimum may be attained at $a = a_4$ and after this point the payoff function increases. This will not occur if $\widetilde{a}_2 < a_4$, in which the profit function of network A is decreasing until $a = \widetilde{a}_2$. Network B is losing market share and at $a = \widetilde{a}_2$ tipping occurs in favor of network A. Profits increase for A as it moves closer to B because its distance to the marginal consumer, located at the right extreme $x = 1$, is reduced. Figure 1 below depicts the behavior of π^A in both cases, $a_4 < \widetilde{a}_2$ and $\widetilde{a}_2 < a_4$, respectively.

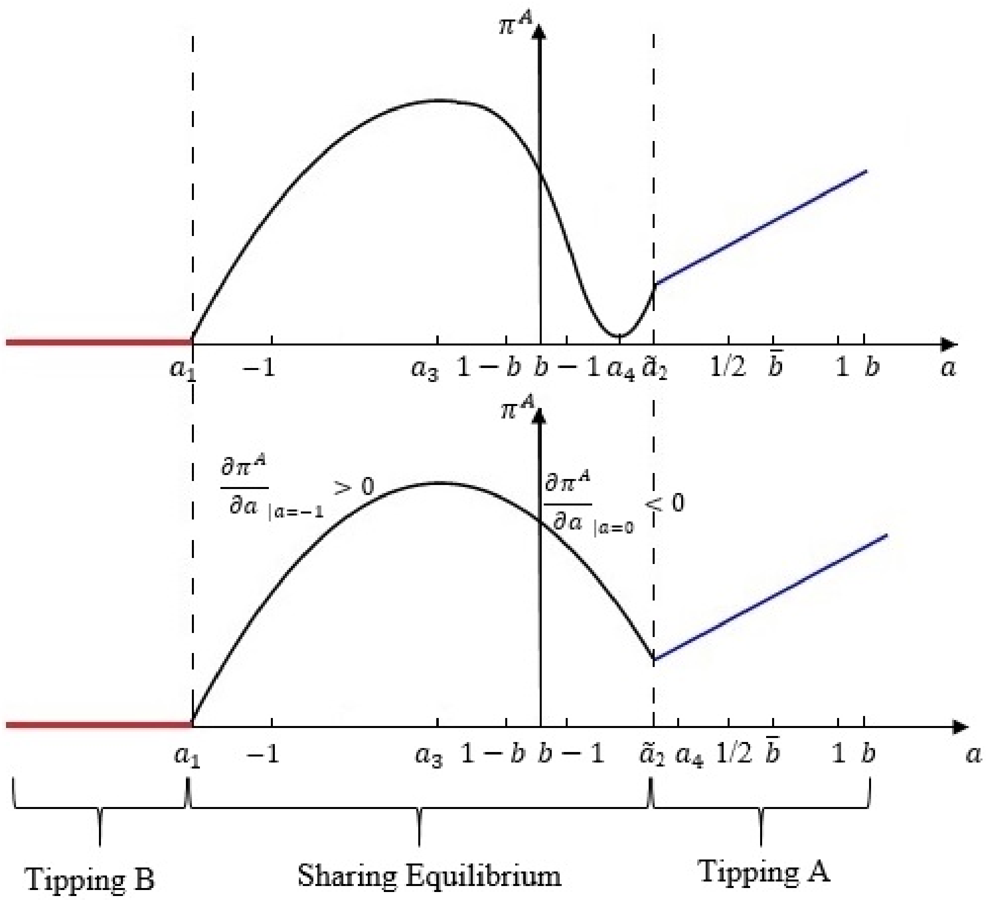

Figure 1

Case 2 ($\widehat{a}_2 < \widetilde{a}_2 \Leftrightarrow b \leq \bar{b}$). We have that $a_2 = \widehat{a}_2 < \widetilde{a}_2$ such that only the root $a_2 = \widehat{a}_2$ is relevant. The fact that (i) the profit function of network A is strictly increasing at $a = -1$ and (ii) the profit function of network A is strictly decreasing at $a = 0$ (corresponding to interior points where the sharing equilibrium exists), the profit function attains: (i) a local minimum at $a = a_1$ that remains at the left of $a_1 = \widehat{a}_1$ (because for $a_1 \leq \widehat{a}_1$ tipping in favor to network B occurs), (ii) a local maximum at $a = a_3$ and (iii) a local minimum at $a = a_2 = \widehat{a}_2$ (since, now, a_4 is not a relevant root because at $a = a_2 = \widehat{a}_2$ the market tips in favor of network B and the slope of A's profit function becomes zero. Therefore, this implies that a_4 cannot be less than $a_2 = \widehat{a}_2$, since if that was the case there should be one more root in that range). As explained above, in this case, the two networks share the agents when $a \in (\widehat{a}_1, \widehat{a}_2)$, unless $b \leq -1 + \frac{\sqrt{3}\sqrt{\alpha}}{\sqrt{t}} \leq \overline{b}$, in which case network A's market share is zero at $a = a_1 = \widehat{a}_1 = \widehat{a}_2$. As network A moves closer to network B both prices and market shares fall (because now B is closer to the city centre than in case 1) and at $a = a_2 = \widehat{a}_2$ the market tips in favor of B. Then, at $a = b - 1$ the market tips in favor of A. Therefore, in both cases, the profit function of network A is U-shaped up to $a = b$. Finally, just a remark to highlight that $\overline{b}$ is higher than 1 for $\alpha \in (t, t(b-a)]$, which does not violate condition (10) since we impose $a \leq b$. Figure 2 below depicts this case.

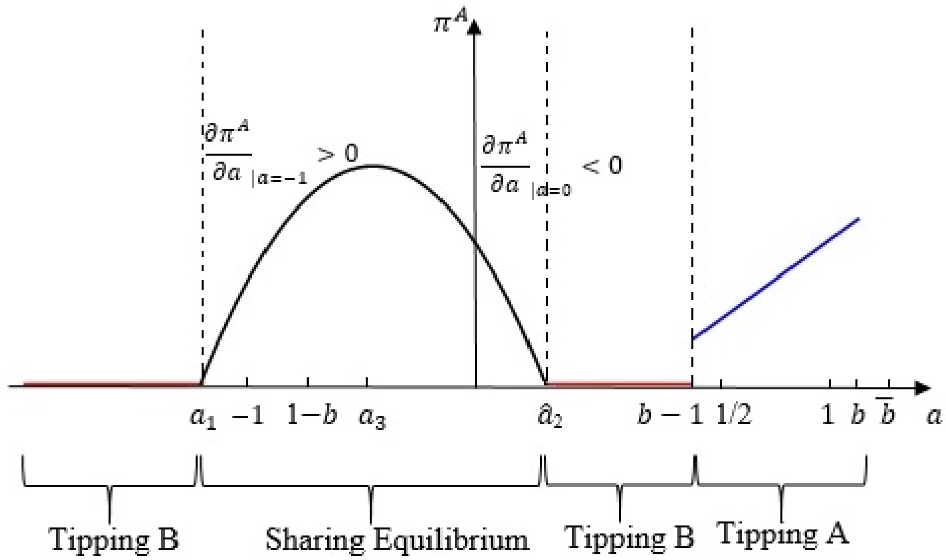

Figure 2

With a simple relabeling the same applies, *mutatis mutandis*, to network B with respect to b for any $b \geq a$. $\square$

Proof of Proposition 2

The proof is divided in three steps. First, we study maximum differentiation, then we study the asymmetric location equilibrium and, finally, the tipping equilibrium.

Preliminary details

Consumers are within the city limits, $x \in [0, 1]$, such that tipping in favor of network B

corresponds to a situation where B gets the whole market on both sides while network A has no market share and charges a zero price. In such a situation:

$$D^A = 0 \cap D^B = 1 \cap p^A = 0.$$

If, for some reason, B locates arbitrarily close to 0, to attract the whole market, the maximum price that network B can charge is such that the customers located at $x = 0$ are indifferent between the two platforms:

$$u^A(0) = u^B(0) \Leftrightarrow -t(0-a)^2 = \alpha - p^B - t(b-0)^2 \Leftrightarrow p^B = \alpha + ta^2 - tb^2.$$

Therefore, the resulting profit of network B is:

$$\pi^{B(T)} = \alpha + ta^2 - tb^2,$$

while the profit of platform A is obviously null. Platform B can never obtain a profit that is greater than α. This is due to the fact that:

Platform B action: $\max_b \pi^B$	Platform A action: $\min_a \pi^B$
$\frac{\partial \pi^{B(T)}}{\partial b} = 0 \Leftrightarrow b^* = 0$	$\frac{\partial \pi^{B(T)}}{\partial a} = 0 \Leftrightarrow a^* = 0$
$\frac{\partial^2 \pi^{B(T)}}{\partial b^2} = -2t < 0$	$\frac{\partial^2 \pi^{B(T)}}{\partial a^2} = 2t > 0$

Thus, tipping in favor of platform B would imply minimum differentiation in the point $(a, b) = (0, 0)$ and the maximum profit that platform B obtains is given by:

$$\pi^{B(T)*}(0,0) = \alpha. \tag{22}$$

A similar argument stands for network A but in the point $(1, 1)$ such that:

$$\pi^{A(T)*}(1,1) = \alpha. \tag{23}$$

Note that network B (A) does not have the incentive to move to the left (right) of point 0 (1), since this action would increase the rival's tipping profit. Therefore, we conclude that the maximum profit of each network if the market tips is equal to α. This result is only relevant to evaluate the range where maximum differentiation prevails.However, it is very important (specially in the case of asymmetric location) to understand that the points $(0,0)$ or $(1,1)$ are not attraction points. We showed in Lemma 1 that network A's profit function is U-shaped with respect to a for any $a \leq b$. Since network B (the follower) will either locate next to network A and tip the market in B's favor or will locate at the farthest point to the right of the city, then, given network B's reaction, it is network A's dominated strategy to locate at $a \in \left(\widehat{a}_1, \frac{1}{2}\right)$. Therefore, if network B finds it profitable to locate at $b = \frac{5}{4}$, network A's profits are maximized at $a = -\frac{1}{4}$. This is because network B can tip the market in its favor (unless $a = \frac{1}{2}$). However, choosing to locate at $b = \frac{5}{4}$, it must mean that the externality is not so strong. In this case, A is better off locating at $a = -\frac{1}{4}$ first. If, on the other hand, network B finds it profitable to locate right next to A (and closer to the center $\frac{1}{2}$), then A is better off locating first at $\frac{1}{2}$. So, in any subgame perfect equilibrium we have either $a = -\frac{1}{4} \cup a = \frac{1}{2}$. Also, note that network A can always secure strictly positive profits because it can always locate at the city centre and tip the market in its favor (since even if B locates at the centre, the tipping assumption implies that all agents join network A).

<u>Maximum differentiation</u>

Our purpose is to investigate under which condition the interior location equilibrium $(a,b) = (-\frac{1}{4}, \frac{5}{4})$ is verified. We will assume that network B locates at $\frac{5}{4}$ and seek to find whether locating at $-\frac{1}{4}$ is a best response for network A. If so, the equilibrium candidate is a location equilibrium. Consider network A and w.l.o.g $a \leq \frac{1}{2}$..

We start by naively considering the system of equations of demands, that only applies in an interior equilibrium, and set $b = \frac{5}{4}$. From (3), this implies that:

$$D^A\left(p^A, p^A, a, \tfrac{5}{4}\right) = \frac{p^B - p^A - \alpha + t\left[\left(\frac{5}{4}\right)^2 - a^2\right]}{2t\left(\frac{5}{4} - a\right) - \alpha} \tag{24}$$

By expression (10), it should be clear that the profit function is strictly concave in network A's own price if and only if $\alpha < t\left(\frac{5}{4} - a\right)$. If this condition is not verified, then there can only exist tipping equilibria.

Lemma 8 *If $\alpha < t\left(\frac{5}{4} - a\right)$, there exists an unique equilibrium that is interior, with prices* *and market shares given by:*

$$\begin{aligned} p^A\left(a,\tfrac{5}{4}\right) &= c+\tfrac{t(13+4a)(5-4a)}{48}-\alpha;\ p^B\left(a,\tfrac{5}{4}\right)=c+\tfrac{t(11-4a)(5-4a)}{48}-\alpha; \\ D^A\left(a,\tfrac{5}{4}\right) &= \tfrac{t(13+4a)(5-4a)-48\alpha}{24[t(5-4a)-\alpha]};\ D^B\left(a,\tfrac{5}{4}\right)=1-D^A\left(a,\tfrac{5}{4}\right). \end{aligned}$$

Proof. Using (24) and multiplying by p^A-c follows that:

$$\pi^A\left(p^A,p^B,a,\tfrac{5}{4}\right)=\left(p^A-c\right)\left(\frac{p^B-p^A-\alpha+t\left[\left(\frac{5}{4}\right)^2-a^2\right]}{2t\left(\frac{5}{4}-a\right)-\alpha}\right)$$

and for B multiplying by p^B-c follows that:

$$\pi^B\left(p^A,p^B,a,\tfrac{5}{4}\right)=\left(p^B-c\right)\left(1-\frac{p^B-p^A-\alpha+t\left[\left(\frac{5}{4}\right)^2-a^2\right]}{2t\left(\frac{5}{4}-a\right)-\alpha}\right).$$

Solving the FOC with respect to p^A and p^B the results are, now, straightforward. ■

By locating at the left position ($a=-\frac{1}{4}$) with $b=\frac{5}{4}$, platform A obtains a profit that is equal to:

$$\pi^{A*}\left(-\tfrac{1}{4},\tfrac{5}{4}\right)=\tfrac{3}{4}t-\tfrac{\alpha}{2}. \tag{25}$$

From expression (23), follows that the maximum tipping profit that network A can obtain is equal to:

$$\pi^{A(T)*}(1,1)=\alpha.$$

Combining expression (25) and expression (22), $(a,b)=(-\frac{1}{4},\frac{5}{4})$ is a location equilibrium if and only if:

$$\pi^{A*}\left(-\tfrac{1}{4},\tfrac{5}{4}\right)\geq\pi^{A(T)*}(1,1)\Leftrightarrow\alpha\leq\tfrac{t}{2}. \tag{26}$$

which completes the proof regarding maximum differentiation.

Asymmetric location

Substituting $(a^*,b^*)=\left(-\frac{1}{4},\frac{5}{4}\right)$ in expression (10), the profit functions are strictly concave in platform's own price as long as:

$$\alpha<\tfrac{3}{2}t, \tag{27}$$

Therefore, given (26) and (27), the level of the network intensity for which asymmetric and tipping equilibrium location exist relies on $\alpha \in \left(\frac{t}{2}, \frac{3}{2}t\right)$. Below a certain threshold $\widetilde{\alpha} \in \left(\frac{t}{2}, \frac{3}{2}t\right)$, both networks will participate in the market but above the threshold $\widetilde{\alpha} \in \left(\frac{t}{2}, \frac{3}{2}t\right)$, only the leader network A operates in the market and tipping occurs. Note that location stage is sequential and that the only location points that are considered attraction points are $a = -\frac{1}{4} \cup a = \frac{1}{2}$ (in the case of network A) and $b = \frac{1}{2} \cup b = \frac{5}{4}$ (in the case of network B).

Since A is the leader and B is the follower, if $\alpha > \frac{t}{2}$, network A will locate at $\frac{1}{2}$ and B replies by locating at 1 (the right extreme where the indifferent agent is located such that network B is able to conquer some market share and profit). Given $a = \frac{1}{2}$, network B has no profitable deviation from this point. Moreover, we find the critical level $\widetilde{\alpha}$ below which the market share of network B (the follower) is not null.

Lemma 9 *If $\alpha < \frac{9}{16}t$, considering network A the leader and network B the follower, there exists an unique equilibrium that is interior, with prices, market shares and profits given by:*

$$\begin{aligned}
p^A\left(\tfrac{1}{2}, \tfrac{5}{4}\right) &= c + \tfrac{15t}{16} - \alpha;\ p^B\left(\tfrac{1}{2}, \tfrac{5}{4}\right) = c + \tfrac{9t}{16} - \alpha; \\
D^A\left(\tfrac{1}{2}, \tfrac{5}{4}\right) &= \tfrac{1}{2} + \tfrac{3t}{8(3t-4\alpha)};\ D^B\left(\tfrac{1}{2}, \tfrac{5}{4}\right) = \tfrac{1}{2} - \tfrac{3t}{8(3t-4\alpha)}; \\
\pi^{A*}\left(\tfrac{1}{2}, \tfrac{5}{4}\right) &= \tfrac{(15t-16\alpha)^2}{128(3t-4\alpha)};\ \pi^{B*}\left(\tfrac{1}{2}, \tfrac{5}{4}\right) = \tfrac{(9t-16\alpha)^2}{128(3t-4\alpha)}.
\end{aligned}$$

Proof. Network A is a first mover by locating in the city centre. Thus, replace $a = \frac{1}{2}$ in expression (3). Therefore, we obtain:

$$D^A\left(p^A, p^B, \tfrac{1}{2}, b\right) = \frac{p^B - p^A + t\left(b^2 - \frac{1}{4}\right) - \alpha}{2\left[t\left(b - \frac{1}{2}\right) - \alpha\right]};\ D^B\left(p^A, p^B, \tfrac{1}{2}, b\right) = 1 - D^A\left(p^A, p^B, \tfrac{1}{2}, b\right) \tag{28}$$

We intend to verify what is the optimal location b of network B, given the location a of network A. The game is solved backwards. Multiplying D^A by $p^A - c$ follows that:

$$\pi^A\left(p^A, p^B, \tfrac{1}{2}, b\right) = \left(p^A - c\right)\left(\frac{p^B - p^A + t\left(b^2 - \frac{1}{4}\right) - \alpha}{2\left[t\left(b - \frac{1}{2}\right) - \alpha\right]}\right) \tag{29}$$

and multiplying D^B by $p^B - c$ follows that:

$$\pi^B\left(p^A, p^B, \tfrac{1}{2}, b\right) = \left(p^B - c\right)\left(1 - \frac{p^B - p^A + t\left(b^2 - \frac{1}{4}\right) - \alpha}{2\left[t\left(b - \frac{1}{2}\right) - \alpha\right]}\right). \tag{30}$$

In the last stage, networks compete for prices. Solving the FOC with respect to p^A and p^B follows that the equilibrium prices as a function of location b is:

$$p^A(b) = c - \tfrac{5-4b(2+b)}{12} - \alpha; \tag{31}$$

$$p^B(b) = c - \tfrac{7-4b(4-b)}{12} - \alpha. \tag{32}$$

Turning into the stage where B decides its location b and replacing (31) and (32) in (30), the profit of the follower network B is given by:

$$\pi^B\left(p^A, p^B, \tfrac{1}{2}, b\right) = \tfrac{\{t[7-4b(4-b)]+12\alpha\}^2}{144[t(2b-1)-2\alpha]}. \tag{33}$$

Deriving (33) relatively to b and making it equal to zero, it is straightforward that the profit of platform B is maximized at $b^* = \frac{5}{4}$. Substituting $b^* = \frac{5}{4}$ in (28), (31), (32), (29) and (33) our Lemma is, now, straightforward. As expected, the equilibrium prices and market shares of the leader A are higher relatively to the follower B and also the equilibrium prices, market shares and profits of both networks are strictly positive for any:

$$\alpha < \tfrac{9}{16}t, \tag{34}$$

and the threshold $\widetilde{\alpha} = \frac{9}{16}t$ is, now, found. ■

Tipping location

Given (27) and (34), for any $\alpha \in \left[\frac{9}{16}t, \frac{3}{2}t\right)$, network B's market share is zero. We assume that network B stays at $b = \frac{5}{4}$ (its profit is zero regardless of where it locates, such that, in this sense the equilibrium is not unique) because network A is located at $a = \frac{1}{2}$ and, hence, network A conquers all the agents and it's price (and profit when the marginal cost is zero) is $\alpha - \frac{3}{16}t$.

Lemma 10 *If $\alpha \in \left[\frac{9}{16}t, \frac{3}{2}t\right)$, considering network A the leader and network B the follower, there exists a tipping equilibrium, with prices, market shares and profits given by:*

$$\begin{aligned}
p^A\left(\tfrac{1}{2}, \tfrac{5}{4}\right) &= \alpha - \tfrac{3}{16}t;\ p^B\left(\tfrac{1}{2}, \tfrac{5}{4}\right) = 0; \\
D^A\left(\tfrac{1}{2}, \tfrac{5}{4}\right) &= 1;\ D^B\left(\tfrac{1}{2}, \tfrac{5}{4}\right) = 0; \\
\pi^A\left(\tfrac{1}{2}, \tfrac{5}{4}\right) &= \alpha - \tfrac{3}{16}t - c;\ \pi^B\left(\tfrac{1}{2}, \tfrac{5}{4}\right) = 0.
\end{aligned}$$

Proof. Using (1) and for $D^A\left(\frac{1}{2},\frac{5}{4}\right)=1\cap D^B\left(\frac{1}{2},\frac{5}{4}\right)=0$ follows:

$u^A(1)\big|_{\left(a=\frac{1}{2},b=\frac{5}{4}\right)}=u^B(1)\big|_{\left(a=\frac{1}{2},b=\frac{5}{4}\right)}\Leftrightarrow -\alpha-p^A-t\left(1-\frac{1}{2}\right)^2=-t\left(\frac{5}{4}-1\right)^2\Leftrightarrow p^A=\alpha-\frac{3}{16}t.$

Since $\pi^{A(T)*}=\left(p^{A(T)*}-c\right)D^{A(T)*}$ follows that:

$$\pi^A=\alpha-\frac{3}{16}t-c,$$

while the price (and profit) of network B is, obviously, null. ■

By other words, given that the marginal consumer is at $x=1$ and given the location of the network B (A) at $b=\frac{5}{4}$ $\left(a=\frac{1}{2}\right)$, this suggests that the difference in transportation cost between the two networks - in favor of B - is $\frac{3}{16}t$ and the difference in the network benefit - in favor of A - is α. The proof of Proposition 2 is, now, completed. □

Proof of Lemma 3

Let us define the likelihood or the probability for each one of the equilibrium location of type i, $i=\{M,A,T\}$ as follows:

$$p_i=\frac{lengh\ of\ location\ type\ i}{total\ lengh\ of\ the\ profit\ concavity}=\frac{\overline{\alpha}_i-\underline{\alpha}_i}{\overline{\overline{\alpha}}-\underline{\underline{\alpha}}}.$$

The numerator is defined by the upper boundary $\alpha=f(t)$ and by the lower boundary $\alpha=f(t)$ for each location equilibrium type (maximum differentiation, asymmetric location and tipping) and the denominator defines the whole range $\alpha=f(t)$ that simultaneously sustains the three location equilibrium types. Therefore, according to Proposition 2, the denominator is equal to $\frac{3}{2}t$ (since $\overline{\overline{\alpha}}=\frac{3}{2}t\cap\underline{\underline{\alpha}}=0$ since profit concavity requires $\alpha<\frac{3}{2}t$). For $i=\{M\}$, the numerator equals to $\frac{1}{2}t-0$, for $i=\{A\}$, the numerator equals to $\frac{9}{16}t-\frac{1}{2}t=\frac{1}{16}t$ and for $i=\{T\}$, the numerator equals to $\frac{3}{2}t-\frac{9}{16}t=\frac{15}{16}t$. Now, conduct the same procedure to the case of restrictive locations analyzed in Serfes and Zacharias (2012). Relying on Proposition 2 of Serfes and Zacharias (2012), the denominator equals t (since $\overline{\overline{\alpha}}=t\cap\underline{\underline{\alpha}}=0$ since profit concavity requires $\alpha<t$). The following Table summarizes the

results.

	Location type i		
	M	A	T
Restrictive locations			
Numerator of p_i	$\frac{1}{2}t$	$\frac{5}{12}t$	$\frac{1}{4}t$
Likelihood of p_i	$\frac{1}{3} \approx 33,(3)\%$	$\frac{5}{12} \approx 41,(6)\%$	$\frac{1}{4} \approx 25\%$
Unrestricted locations			
Numerator of p_i	$\frac{1}{2}t$	$\frac{1}{16}t$	$\frac{15}{16}t$
Likelihood of p_i	$\frac{1}{3} \approx 33,(3)\%$	$\frac{1}{24} \approx 4,1(6)\%$	$\frac{5}{8} \approx 62,5\%$

Table 1

From Table 1, we can conclude that: (i) the likelihood for maximum differentiation is the same under restrictive or unrestricted locations; (ii) the likelihood for an asymmetric equilibrium is higher in the case of restrictive location and (iii) the likelihood of tipping is higher in the case of an unrestricted location. $\square$

Proof of Proposition 4

The proof is divided in three steps: the analysis is conducted for each type of equilibrium location.

<u>Maximum differentiation</u>

Maximum differentiation holds for $\alpha \in \left[0, \frac{t}{2}\right]$. Given the pre-entry conditions: $a = -\frac{1}{4}$, $b = \frac{5}{4}$, $x = \frac{1}{2}$, $p^{A*} = p^{B*} = c + \frac{3}{2}t - \alpha$, $p^C = c$, $D^{A*} = D^{B*} = \frac{1}{2}$ and $D^C = 0$, network C attracts the indifferent consumer located at $x = \frac{1}{2}$ if and only if:

$$u^A\left(\tfrac{1}{2}\right) \leq u^C\left(\tfrac{1}{2}\right) \Leftrightarrow v - \tfrac{\alpha}{2} - \left(c + \tfrac{3}{2}t - \alpha\right) - t\left[\tfrac{1}{2} - \left(-\tfrac{1}{4}\right)\right]^2 \leq v - c - t\left(L^M - \tfrac{1}{2}\right)^2 \Leftrightarrow$$

$$\left(L^M\right)^2 - L^M - \left(\frac{5}{16} + \frac{3(1-\alpha)}{2t}\right) \leq 0. \tag{35}$$

From inequality (35) two roots hold:

$$\underline{L_1^M} = \frac{1}{2} - \frac{1}{4}\sqrt{9 + \frac{24(1-\alpha)}{t}} \cap \overline{L_2^M} = \frac{1}{2} + \frac{1}{4}\sqrt{9 + \frac{24(1-\alpha)}{t}}. \tag{36}$$

Note that the network C enters with a penetration pricing strategy for any threshold $L \in \left(\underline{L_1^M}, \overline{L_2^M}\right)$ whilst the incumbents deter entry above $\overline{L_2^M}$ or below $\underline{L_1^M}$ (if positive).

Corollary 11 ***(Entry deterrence and maximum differentiation as the pre-entry condition)***

(i) Let $t \leq \frac{24}{7}$. Then, $\forall \alpha \in \left[0, \frac{t}{2}\right]$ follows $\underline{L_1^M} < 0$ and, therefore, the maximum distance above which entry is deterred is given by $\overline{L_2^M}$ and $\frac{\partial \overline{L_2^M}}{\partial \alpha} = -\frac{3}{\sqrt{t}\sqrt{9t+24(1-\alpha)}} < 0$.

(ii) Let $t > \frac{24}{7}$. Then, $\forall \alpha \in \left[0, \frac{t}{2}\right]$ follows $\widetilde{\alpha} < \frac{t}{2}$, with $\widetilde{\alpha} \equiv 1 + \frac{5t}{24}$ representing the critical threshold where $\underline{L_1^M} = 0$. Therefore:

a) $\forall \alpha \in [0, \widetilde{\alpha}]$ follows $\underline{L_1^M} < 0$ and, thus, the maximum distance above which entry is deterred is given by $\overline{L_2^M}$ and yields $\frac{\partial \overline{L_2^M}}{\partial \alpha} = -\frac{3}{\sqrt{t}\sqrt{9t+24(1-\alpha)}} < 0$;

b) $\forall \alpha \in \left(\widetilde{\alpha}, \frac{t}{2}\right]$ follows $\underline{L_1^M} > 0$. The region where entry occurs relies in any $L \in \left[\underline{L_1^M}, \overline{L_2^M}\right]$. Let $\Phi \equiv \overline{L_2^M} - \underline{L_1^M} = \frac{\sqrt{9t+24(1-\alpha)}}{2\sqrt{t}}$. *Then: $\frac{\partial \Phi}{\partial \alpha} = -\frac{6}{\sqrt{t}\sqrt{9t+24(1-\alpha)}} < 0$. Entry never occurs for any $L \leq \underline{L_1^M}$ and $\frac{\partial \underline{L_1^M}}{\partial \alpha} = \frac{3}{\sqrt{t}\sqrt{9t+24(1-\alpha)}} > 0$.*

Proof. First, $\overline{L_2^M} \geq 0$, $\forall \alpha \in [0,1] \cap t > 0$. Secondly, we compute for which critical value $\underline{L_1^M}$ becomes positive. $\underline{L_1^M} \gtreqless 0 \Leftrightarrow \alpha \gtreqless 1 + \frac{5t}{24} \equiv \widetilde{\alpha}$. Now, we compute for which threshold, the critical value $\widetilde{\alpha}$ is above or below the upper bound where maximum differentiation prevails. $\widetilde{\alpha} \gtrless \frac{t}{2} \Leftrightarrow t \lessgtr \frac{24}{7}$. Therefore, when t is sufficiently low, $\widetilde{\alpha}$ overcomes the upper bound $\frac{t}{2}$ meaning that $\underline{L_1^M}$ is strictly negative for any $\alpha \in \left[0, \frac{t}{2}\right]$. Thus, the (positive) distance where the incumbents do not prevent entry is attained for $L \in \left[0, \overline{L_2^M}\right]$. According to (36), follows that $\frac{\partial \overline{L_2^M}}{\partial \alpha} < 0$, meaning that an increment on the intensity of the network externality diminishes the maximum orthogonal distance between the new entrant and the incumbents above which entry is deterred, i.e., it helps the incumbents A and B to deter the entrance of the new network C. On the other hand, when t is above the threshold $\frac{24}{7}$ (the degree of horizontal product differentiation is sufficiently high) means that $\widetilde{\alpha}$ is bellow the upper bound $\frac{t}{2}$ meaning that $\underline{L_1^M}$ is not strictly negative for any $\alpha \in \left[0, \frac{t}{2}\right]$. Indeed, $\underline{L_1^M}$ is strictly non-positive for any $\alpha \in [0, \widetilde{\alpha}]$ and $\underline{L_1^M}$ is a positive root for any

$\alpha \in \left(\widetilde{\alpha}, \frac{t}{2}\right]$. In the first situation, the (positive) distance where the incumbents do not prevent entry is attained for $L \in \left[0, \overline{L_2^M}\right]$ and according to (36), follows that $\frac{\partial \overline{L_2^M}}{\partial \alpha} < 0$. In the second situation, $\underline{L_1^M}$ is a positive root and, therefore, the (positive) distance where the incumbents do not prevent entry is attained for any $L \in \left[\underline{L_1^M}, \overline{L_2^M}\right]$. Given that $\Phi \equiv \overline{L_2^M} - \underline{L_1^M} = \frac{\sqrt{9t+24(1-\alpha)}}{2\sqrt{t}}$, follows that $\frac{\partial \Phi}{\partial \alpha} < 0$, meaning that an increment on the intensity of the network externality diminishes the maximum orthogonal distance between the new entrant and the incumbents above which entry is deterred, i.e., it helps the incumbents A and B to deter the entrance of the new network C. Note that, under these circumstances, for any $L < \underline{L_1^M}$ entry never occurs. Note that $\frac{\partial \underline{L_1^M}}{\partial \alpha} = \frac{3}{\sqrt{t}\sqrt{9t+24(1-\alpha)}} > 0$, meaning that an increment on the intensity of the network externality increases the maximum orthogonal distance between the new entrant and the incumbents bellow which entry never occurs, i.e., increases the distance $\underline{L_1^M} - 0$ and, thus, it acts as a detrimental effect in order to let the new network C to get access to the market. ■

<u>Asymmetric equilibrium location</u>

We start the analysis of asymmetric location equilibrium, $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$, by establishing the following claim.

Claim 2 *It is indifferent for the new entrant to fight for the niche market or for the larger market.*

Proof. (i) **Fighting for the larger market**. In this case, given the pre-entry conditions: $a = \frac{1}{2}$, $x = \frac{1}{2} + \frac{3t}{8(3t-4\alpha)}$, $p^{A*} = c + \frac{15}{16}t - \alpha$, $p^C = c$, $D^{A*} = \frac{1}{2} + \frac{3t}{8(3t-4\alpha)}$ and $D^C = 0$, network C attracts the indifferent consumer located at $x = \frac{1}{2} + \frac{3t}{8(3t-4\alpha)}$ that attends to A if and only if:

$$u^A\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) \leq u^C\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) \Leftrightarrow$$

$$v - \alpha\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) - \left(c + \frac{15}{16}t - \alpha\right) - t\left[\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) - \left(\frac{1}{2}\right)\right]^2 \leq$$
$$v - c - t\left(\frac{3t}{8(3t-4\alpha)} + L^A - \left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right)\right)^2 \Leftrightarrow$$

$$\left(L^A - \frac{1}{2}\right)^2 \leq \frac{9t^3 + 60t(3t-4\alpha)^2 - \left[96\alpha(3t-4\alpha)^2 + 24\alpha t(3t-4\alpha)\right]}{64t(3t-4\alpha)^2}. \tag{37}$$

From inequality (37) two roots hold:

$$\underline{L_1^A} = \frac{1}{2} - \frac{\sqrt{3}}{8t(3t-4\alpha)}\sqrt{t\gamma} \cap \overline{L_2^A} = \frac{1}{2} + \frac{\sqrt{3}}{8t(3t-4\alpha)}\sqrt{t\gamma}, \tag{38}$$

with $\gamma = 183t^3 - 792t^2\alpha + 1120t\alpha^2 - 512\alpha^3 > 0, \forall\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$.

(ii) **Fighting for the niche market**. In this case, given the pre-entry conditions: $b = \frac{5}{4}$, $x = \frac{1}{2} + \frac{3t}{8(3t-4\alpha)}$, $p^{B*} = c + \frac{9}{16}t - \alpha$, $p^C = c$, $D^{B*} = \frac{1}{2} - \frac{3t}{8(3t-4\alpha)}$ and $D^C = 0$, network C attracts the indifferent consumer located at $x = \frac{1}{2} + \frac{3t}{8(3t-4\alpha)}$ that attends to B if and only if:

$$u^B\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) \leq u^C\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) \Leftrightarrow$$

$$v - \alpha\left(\frac{1}{2} - \frac{3t}{8(3t-4\alpha)}\right) - \left(c + \frac{9}{16}t - \alpha\right) - t\left[\left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right) - \left(\frac{5}{4}\right)\right]^2 \leq$$
$$v - c - t\left(\frac{3t}{8(3t-4\alpha)} + L^A - \left(\frac{1}{2} + \frac{3t}{8(3t-4\alpha)}\right)\right)^2 \Leftrightarrow$$

From the above inequality also two roots hold:

$$\underline{L_1^A} = \frac{1}{2} - \frac{\sqrt{3}}{8t(3t-4\alpha)}\sqrt{t\gamma} \cap \overline{L_2^A} = \frac{1}{2} + \frac{\sqrt{3}}{8t(3t-4\alpha)}\sqrt{t\gamma},$$

which is precisely expression (38) and, therefore, the Claim is satisfied. ■

Given the above proof, it is straightforward that, as in the case of maximum differentiation, network C enters with a penetration pricing strategy for any threshold $L \in \left(\underline{L_1^A}, \overline{L_2^A}\right)$ whilst the incumbents deter entry above $\overline{L_2^A}$ or below $\underline{L_1^A}$ (if positive). Our focus is only to prove that an increment on the intensity of the network externality diminishes the maximum orthogonal distance between the new entrant and the incumbents above which entry is deterred, i.e., it helps the incumbents A and B to deter the entrance of the new network C.[10] Therefore, using the upper bound $\overline{L_2^A}$ and evaluating its sign computing $\frac{\partial \overline{L_2^A}}{\partial\alpha}$, follows:

$$\frac{\partial \overline{L_2^A}}{\partial\alpha} = \eta\frac{\sqrt{3}}{(3t-4\alpha)\sqrt{t\gamma}}, \tag{39}$$

[10] We do not provide a full description of entry deterrence under asymmetric location (as in the case of maximum differentiation) since the methodological approach is similar and adds nothing significant since the results and the intuition towards the results are qualitatively equivalent to the case of maximum differentiation equilibrium.

with $\eta = 128\alpha^3 - 57t^3 + 222t^2\alpha - 288t\alpha^2$. If $\eta < 0$, then, $\forall \alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$ follows that $\frac{\partial \overline{L_2^A}}{\partial \alpha} < 0$ and, therefore, the part (ii) of Proposition 4 is straightforward.

Corollary 12 ***(Impact of an incremental network externality in the asymmetric location equilibrium)*** *If $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$, then, follows that $\eta < 0$ and, therefore, $\frac{\partial \overline{L_2^A}}{\partial \alpha} < 0$.*

Proof. Let us compute the zeros of $\eta(\alpha, t) = 128\alpha^2\left(\alpha - \frac{9t}{4}\right) - 57t^2\left(t - \frac{74\alpha}{19}\right)$. It follows that we obtain a zero for:

$$\alpha_0 = \frac{6t}{8} - \frac{t^2}{8\left(-3t^3 + t^3\sqrt{10}\right)^{\frac{1}{3}}} + \frac{\left(-3t^3 + t^3\sqrt{10}\right)^{\frac{1}{3}}}{8}. \tag{40}$$

Rearranging (40), follows that:

$$\alpha_0 = \left[\frac{6 + \left(-3 + \sqrt{10}\right)^{\frac{1}{3}} - \left(3 - \sqrt{10}\right)^{\frac{1}{3}}}{8}\right] t \approx 0.589t.$$

Given $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$, consider a transformation of the function $\eta(\alpha, t)$ into a function $\eta(\alpha(t), t)$. Replacing α by the lower bound $\alpha(t) = \frac{t}{2}$ follows that $\eta(\alpha(t), t) = -2t^3 < 0, \forall t > 0$. Also replacing α by the upper bound $\alpha(t) = \frac{9t}{16}$ follows that $\eta(\alpha(t), t) = -\frac{15}{32}t^3 < 0, \forall t > 0$. Therefore, whatever the interval $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$, the function $\eta(\alpha(t), t)$ is strictly negative, although, as α increases (from the lower bound to the upper bound), the function $\eta(\alpha(t), t)$ becomes less negative (since $-2t^3 < -\frac{15}{32}t^3$).

Thus, for any threshold $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$ yields $\eta(\alpha(t), t) < 0$. Since the asymmetric location equilibrium stands in any $\alpha \in \left(\frac{t}{2}, \frac{9t}{16}\right)$ which yields below the critical threshold α_0, it is immediate that $\eta(\alpha, t) < 0$ and, thus, also $\frac{\partial \overline{L_2^A}}{\partial \alpha} < 0$. Note that $\eta(\alpha, t)$ is an increasing concave function in α since the first and second derivatives of $\eta(\alpha, t)$ relatively to α are given by:

$$\frac{\partial \eta(\alpha, t)}{\partial \alpha} = 222t^2 - 576t\alpha + 384\alpha^2 \tag{41}$$

and

$$\frac{\partial^2 \eta(\alpha, t)}{\partial \alpha^2} = -576t + 768\alpha,$$

respectively. Note that for any $\alpha_j \in \left(\frac{t}{2}, \frac{9t}{16}\right)$ we obtain $\left.\frac{\partial \eta(\alpha, t)}{\partial \alpha}\right|_{\alpha = \alpha_j} > 0$ and $\left.\frac{\partial^2 \eta(\alpha, t)}{\partial \alpha^2}\right|_{\alpha = \alpha_j} < 0, \forall t > 0$ (the same is verified in the limit boundaries since: (i) $\left.\frac{\partial \eta(\alpha, t)}{\partial \alpha}\right|_{\alpha = \frac{t}{2}} = 30t^2$ and $\left.\frac{\partial^2 \eta(\alpha, t)}{\partial \alpha^2}\right|_{\alpha = \frac{t}{2}} = -192t$ and (ii) $\left.\frac{\partial \eta(\alpha, t)}{\partial \alpha}\right|_{\alpha = \frac{9t}{16}} = \frac{39}{2}t^2$ and $\left.\frac{\partial^2 \eta(\alpha, t)}{\partial \alpha^2}\right|_{\alpha = \frac{9t}{16}} = -144t$). ■

Tipping location

Tipping holds favouring network A for $\alpha \in \left[\frac{9t}{16}, \frac{3t}{2}\right)$. Given the pre-entry conditions: $a = \frac{1}{2}$, $x = 1$, $p^{A*} = \alpha - \frac{3}{16}t$, $p^C = c$, $D^{A*} = 1$ and $D^C = 0$, network C attracts the indifferent consumer located at $x = 1$ if and only if:

$$u^A(1) \leq u^C(1) \Leftrightarrow v - \alpha - \left(\alpha - \frac{3}{16}t\right) - t\left(1 - \frac{1}{2}\right)^2 \leq v - t\left[\left(1 - \frac{1}{2} + L^T\right) - 1\right]^2 \Leftrightarrow$$

$$L^T \leq \frac{1}{4}.$$

Therefore, $\forall \alpha \in \left[\frac{9t}{16}, \frac{3t}{2}\right)$, if $L^T \leq \frac{1}{4}$ then the incumbent avoids the entrant C. Such distance is lower than the maximum distance under maximum differentiation and asymmetric location equilibrium (comparing with the maximum distance emerging from Corollary 11 since $\overline{L_2^M} = \frac{1}{2} + \frac{1}{4}\sqrt{9 + \frac{24(1-\alpha)}{t}} > \overline{L^T} = \frac{1}{4}$ and from Corollary 12 since $\overline{L_2^A} = \frac{1}{2} + \frac{\sqrt{3}}{8t(3t-4\alpha)}\sqrt{t\gamma} > \overline{L^T} = \frac{1}{4}$) and is undeviating with any change in the intensity of the network externality. Part (i) of Proposition 4 is now, straightforward. $\square$

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