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Incentives in Supply Function Equilibrium

Henrik Vetter

Abstract

The author analyses delegation in homogenous duopoly under the assumption that the firm-managers compete in supply functions. In supply function equilibrium, managers' decisions are strategic complements. This reverses earlier findings in that the author finds that owners give managers incentives to act in an accommodating way. As a result, optimal delegation reduces per-firm output and increases profits to above-Cournot profits. Moreover, in supply function equilibrium the mode of competition is endogenous. This means that the author avoids results that are sensitive with respect to assuming either Cournot or Bertrand competition.

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Keywords Delegation; incentives; supply function equilibrium

Authors

Henrik Vetter, ✉ [Statsbiblioteket](mailto:hv@statsbiblioteket.dk), 8000 Aarhus C, Denmark, hv@statsbiblioteket.dk

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1. Introduction

In homogenous Cournot oligopoly, owners' strategic use of managerial compensation seems to encourage, in Reitman's (1993) terminology, overly aggressive behaviour. Fershtman and Judd (1987) and Sklivas (1987) analyse performance pay that is based on a measure that combines sales-revenue and profit. They show that delegation ends up in lower profits and outputs that are higher-than-Cournot outputs. The result, that owners' ability to tie the hands of their managers in fact promotes aggressive behaviour, depends on assumptions about the information structure in two ways. First, the occurrence of contracts giving owners less-than-Cournot profits presuppose that owners are ill informed at the time they design the contract. Without noise, contracts indexed on quantity sustain the Cournot profits. Second, the uncertainty is fully resolved before the manager maximises her pay-off. This implies that there are Stackelberg leadership gains and, in turn, it explains why owners give managers incentives to act aggressively. To see this, notice that, in linear homogenous oligopoly, when managers choose quantities their actions are strategic substitutes (Bulow et al., 1985). Therefore, if one manager acts more aggressively, her rival manager responds with more accommodating behaviour. Noticeably, this argument presupposes that each manager makes decisions knowing cost conditions, the other manager's motivation and a deterministic residual demand curve.

In this paper we assume that managers make their decisions before uncertainty is fully resolved. When demand is stochastic the manager cannot predict, in a precise way, the price that corresponds to some choice of quantity. In our context, the way managers adapt to uncertainty is relevant for the study of delegation's effects. In fact, under uncertainty the manager can adapt to changing market conditions by choosing a supply scheme rather than making commitment to some fixed quantity (Klemperer and Meyer, 1989). It is safe to conclude that in real life, firm-managers cannot, in a costless way, resolve uncertainty before competition occurs. Equally evident, uncertainty is important in determining economic decisions.

For this reason, it is relevant to consider incentives when managers compete in supply schemes. On the other hand, when owners delegate the daily running of their firm, the manager collects information that owners do not care about (Fershtman and Judd, 1987). In this way, the manager actually has more information than owners have. Nevertheless, that the firm's manager has superior information vis-à-vis firm-owners does not imply that the manager makes decisions in complete absence of uncertainty. It is closer to real life to assume that the manager also acts under some uncertainty rather than run the firm under complete knowledge of all relevant market conditions. To examine as simply as possible how incentives in delegation works when managers are ill-informed when they compete, we assume that managers and owners are equally ill-informed.

We consider an environment that equals the one examined in Fershtman and Judd (1987) and Sklivas (1987) except for our assumptions about managers' information. That is, in a first stage, profit-maximising owners set up incentive contracts for their managers. Managers make their decisions in the subsequent market stage. One part of the incentive contract is the specification of a performance measure which managers aim at maximising. Decisions in both stages are characterised by the presence of uncertainty. Of course, once the market stage takes place, the managers learn about the stochastic variables. Nevertheless, if managers in Cournot competition commit to a quantity before the uncertainty is resolved, they end up with a quantity-price combination that is different from the unconstrained *ex post* optimum. Put another way, managers would like to change their decisions upon learning the exact market conditions. For this reason, they do not want to stick to simple strategies such as fixing a quantity. Rather, as explained by Klemperer and Meyer (1989), in the presence of uncertainty managers are better off committing to a supply function rather than committing to a quantity (or price).¹

¹ To find a supply function equilibrium, demand is given by $D(p, \varepsilon)$ where it is assumed that the noise element is additive, that is, $D(p, \varepsilon) = D(p) + \varepsilon$; see for example, Klemperer and Meyer (1989), Anderson and Hu (2008), and Anderson (2013).

Commitment to a supply function rather than commitment to a fixed quantity allows managers to adapt in a flexible way to the uncertainty in the market. By having a plan that specifies how much to supply for a given price, managers make decisions that are *ex ante* as well as *ex post* optimal. Hence, when managers learn about the stochastic parameters by observing the market equilibrium, they do not want to change their original decisions. That is, the flexibility of competing in supply functions makes sure that when acting in the market stage, managers reach their unconstrained optima. In this way, supply function equilibria are appealing when it is realistic to assume that managers make their decisions under some uncertainty.² With respect to the question of aggressive versus accommodating managerial behaviour, it is interesting that managers' decisions are strategic complements when they maximise performance by choosing a supply function. To see this, notice that each manager maximises performance given the residual demand function. If her rival acts more aggressively, residual demand goes down. Lower residual demand means that the negative effects of charging a higher price goes down (which it does, because sales goes as residual demand goes down). Therefore, the firm-manager acts more aggressively when the manager in the rival firm acts more aggressively showing that managers' decisions are strategic complements. Therefore, once managers compete in supply functions, competition takes place in strategic complements in spite of the goods being substitutes. The result of this is that there can be no Stackelberg leadership gains. In turn, owners are not tempted to have their manager act in an aggressive way. This paper hones in on this issue and explores the consequences for optimum managerial incentives.

To the best of our knowledge, the analysis of incentives in delegation that draws on the notion of supply function equilibria is novel. In addition, the distinction between strategic substitutes and complements

² In spite of this appealing characteristic, supply function equilibrium is used fairly little. Maybe this is because it is difficult to compute supply function equilibrium as the structure of a set of differential equations, rather than algebraic equations, as in Cournot or Bertrand equilibrium.

appears not to have been introduced into the literature on strategic delegation. Thus, there is no current literature that relates, in an obvious way, to our discussion.

2. Supply Functions and Managerial Incentives

The model used here follows the one analysed by Fershtman and Judd (1987) and Sklivas (1987). That is, we consider a linear homogenous duopoly with demand given by $p = \omega + \beta(q_1 + q_2)$ where $q_i, i = 1, 2$ is the output of firm i , $\omega > 0$ and $\beta < 0$. Market conditions are stochastic, in that the intercept of the demand curve is stochastic. The restrictions on the distribution of the stochastic parameter are that each firm's output is positive for all realisations. Firms use identical production technology and production costs given by $c_i(q_i) = \frac{1}{2}bq_i^2$. Each firm has a group of profit-maximising owners. Owners delegate the administration of the firm to a manager. Owners as well as managers are risk neutral. Part of delegation is that owners use incentive contracts, and they can ask managers to maximise the profit of the firm. Alternatively, owners use the total payment scheme strategically and set up other managerial objectives rather than set purely financial objectives. Both sides know the distribution of ω at the time when owners contract with a manager. More precisely, let π_i and s_i be profits and revenues, respectively, in firm i . Owners manipulate managers' behaviour by using performance pay that is co-determined by $\Omega_i = \alpha_i\pi_i + (1 - \alpha_i)s_i$. Following Fershtman and Judd (1987), Sklivas (1987) and Reitman (1993), the incentive contracts become public knowledge as soon as owners have made their decisions. That is, when the manager in firm i maximises Ω_i in a duopoly game with firm j , she knows α_j in addition to knowing about her own incentives. Symmetrically, the manager in firm j knows the value of α_i .

With respect to total managerial pay, owners pick from a large pool of potential identical candidates. A manager accepts to work for owners when she earns no less than her reservation wage. On the other hand,

when there are many potential managers available, owners see no point in offering a contract that yields higher pay than the reservation wage. Assuming that total compensation, in addition to performance payment, includes a flat salary, it is possible to adjust the fixed-pay component to secure equality between managers' expected pay and their reservation wage. Therefore, owners expect that payment to managers will equal the reservation wage irrespective of the actual level of demand. Hence, owners' concern is how managerial incentives affect managers' behaviour, and how behaviour in combination with the stochastic innovation determines profit. For this reason, as in Fershtman and Judd (1987), Sklivas (1987) and Reitman (1993), we ignore delegation costs when we look at the owners' decision, and, parallel to this, delegation does not result in any savings.

We consider a two-stage game. Owners decide on the incentive pay scheme in the first stage and managers subsequently maximise pay given the incentive scheme in the subsequent market stage. Consider the information structure. If owners have precise information about the value of the stochastic parameters, or if they obtain this information at some stage, they can increase profits by the use of contracts indexed on price or quantity. As in existing literature, we exclude this possibility and assume that owners have knowledge about the distribution of demand but are deliberately ignorant about the exact realisations of market conditions. By the end of the second period, owners observe profits and sales, and collect the residual between actual profits and managerial pay. When it comes to managers' information, they are ill-informed about the exact realisations of demand in the contract stage, and also ill-informed when they make decisions about how to act in the market. This assumption diverges from existing literature that assumes that managers somehow learn about the stochastic innovations at the beginning of the second stage.

When the manager is unaware about the exact market conditions at the time of making decisions, commitment to a quantity (or a price) is only optimal *ex ante*. Clearly, depending on the exact realisation of the stochastic parameters, there is a range of performance-maximising outputs for each manager. *Ex post*, only the output corresponding to the actual realisation of demand is optimal. By committing to a supply function that is made up of all of the *ex-ante* optimal quantity-price combinations, the manager's decision is also *ex post* optimal (Klemperer and Meyer, 1989). Managers pick out a supply function in order to maximise $\Omega_i = \alpha_i \pi_i + (1 - \alpha_i) s_i$. Rewriting the performance measure as $s_i - \alpha_i c_i(q_i)$ it follows from Klemperer and Meyer (1989) and Anderson and Hu (2008) that the unique supply function equilibrium is given by $s_i(p) = \mu_i p$ where μ_i satisfies:

$$s_i(p) = \left(p - \alpha_i c_i'(s_i(p)) \right) \left(s_j'(p) - d'(p) \right), \quad (1)$$

where $d(p) = \beta^{-1}(p - \omega)$ is the inverse demand function.³ Equation (1) confirms the introduction's remarks on managers' decisions being strategic complements in spite of the fact that products are substitutes. To see this, consider the reaction of the manager in firm i if the manager in firm j acts more aggressively. The manager is more aggressive as the value of μ_j goes up. Firm i 's manager makes decisions under the restriction given by the residual demand function which is $d(p) - s_j(p)$. Moving along the residual demand curve, the sales loss following a price increase is $d'(p) - s_j'(p) = \beta^{-1} - \mu_j$. Lower sales affect the performance measure in the order of $p - \alpha_i c_i'(s_i(p))$. Therefore, the right-hand side of equation (1) shows the negative effect of increasing the price. Now as μ_j goes up, $\beta^{-1} - \mu_j$ becomes more negative. That is, from the point of view of the manager in firm i , the marginal loss of increasing the price increases as μ_j increases. The benefit of increasing the price is that revenue per unit sold goes up the higher μ_i goes (because of $q_i = \mu_i p$). In optimum, the marginal benefit of increasing the price is equal to the marginal cost of increasing the price. Hence, when the marginal cost increases as μ_j increases, the

³ Notice that the expression in equation (1) applies when $d_{p\omega}(p) = 0$. See Klemperer and Meyer (1989) for the symmetric case and Anderson and Hu (2008) and Anderson (2013) for the asymmetric case.

value of μ_i must increase in order to increase the marginal benefit. That is, managers' decisions are strategic complements.

3. Managers' behaviour

To see how the incentive contracts affect managers' choice of supply function, notice that the conditions in equation (1) reduce to:

$$\mu_1 = (1 - b\mu_1\alpha_1)(\mu_2 - \beta^{-1}), \quad (2)$$

$$\mu_2 = (1 - b\mu_2\alpha_2)(\mu_1 - \beta^{-1}). \quad (3)$$

Equations (2) and (3) define managers' response functions that are $\mu_1 = \mu_1(\alpha_1, \mu_2)$ and $\mu_2 = \mu_2(\alpha_2, \mu_1)$.

Owners' decisions enter into the best response function showing how owners, by their choice of incentives, manipulate the supply functions. The best response function of the manager in firm 1, which is $\mu_1 = \mu_1(\alpha_1, \mu_2)$, is shown in Figure 1. It is obvious that $\mu_2 = \beta^{-1} < 0$ implies $\mu_1 = 0$. Moreover, it is easy to see that the slope of $\mu_1 = \mu_1(\alpha_1, \mu_2)$ is positive and increasing for $\mu_1 < \alpha_1 b$ when α_1 is positive. The best response function of the manager in firm 2 is the mirror image. This response function is also shown in Figure 1. Evidently, we have a solution $(\mu_1(\alpha_1, \alpha_2), \mu_2(\alpha_1, \alpha_2))$ where $0 < \mu_1(\alpha_1, \alpha_2) < (\alpha_1 b)^{-1}$ and $0 < \mu_2(\alpha_1, \alpha_2) < (\alpha_2 b)^{-1}$.⁴

(Insert Figure 1 about here)

To see how the incentive contracts affect the market by affecting managers' behaviour, suppose that the owners of firm 1 increase the value of α_1 . If the manager in firm 2 were, in fact, not affected so that the value of μ_2 stays fixed, the only effect of an increase of α_1 is that μ_1 decreases. That is, the supply function

⁴ For $\mu_1 > (\alpha_1 b)^{-1} > 0$ we must have $\mu_2 - \beta^{-1} < 0$ or $\mu_2 < \beta^{-1} < 0$. Similarly, for $\mu_2 > (\alpha_2 b)^{-1} > 0$ we must have $\mu_1 < \beta^{-1} < 0$. This rules out symmetric solutions with $\mu_i > (\alpha_i b)^{-1} > 0$. In principle, incentive contracts can punish profits. Indeed, there is a symmetric solution with $\mu_1 = \mu_2 < \beta^{-1}$. Nevertheless, this solution would give negatively sloped supply curves passing through the origin and rule out the existence of equilibrium.

$p = \mu_1^{-1}q_1$ becomes steeper. Thus, the owners of firm 1 induce less aggressive behaviour by increasing the weight on profit. However, there is another effect of increasing α_1 . This works through μ_2 . If increasing α_1 makes the manager in firm 2 less aggressive (adjust the value of μ_2 down), then the accommodating effect of increasing α_1 is reinforced. This is because we have $d\mu_1/d\mu_2 > 0$, i.e., the supply function chosen by the manager becomes flatter when her opponent chooses a flatter supply function. To expand on this observation, notice that the equilibrium price follows from $p = \omega + \beta(\mu_1 + \mu_2)p$ and $q_1 = \mu_1p$ and $q_2 = \mu_2p$, or:

$$p = (1 - \beta(\mu_1 + \mu_2))^{-1}\omega. \quad (4)$$

Each manager maximises $\Omega_i = pq_i - \frac{1}{2}\alpha_i bq_i^2$. When the manager in firm i considers whether she should behave more aggressively or more accommodatingly, she looks at the marginal performance effect of changing strategy. This would be:

$$d\Omega_i/d\mu_i = p^2(1 - b\alpha_i\mu_i) + 2p^2(1 - \beta(\mu_1 + \mu_2))^{-1}(1 - \frac{1}{2}b\alpha_i\mu_i)\mu_i. \quad (5)$$

Clearly, because of the strategic interdependence between the duopolistic firms, the behaviour of the manager in firm j affects the considerations of the manager in firm i . In fact, we have:⁵

$$d^2\Omega_i/d\mu_j d\mu_i > 0. \quad (6)$$

⁵ The exact expression is:

$$d^2\Omega_i/d\mu_j d\mu_i = 2p(1 - b\alpha_i\mu_i) dp/d\mu_j + (1 - \beta(\mu_i + \mu_j))^{-1} (1 - \frac{1}{2}b\alpha_i\mu_i) \left(dp/d\mu_j 4p + 2p^2 (1 - \beta(\mu_i + \mu_j))^{-1} \mu_i \right).$$

Looking at expected values, since managers decide on the supply functions without knowing the exact market terms, equation (6) implies that $E(d^2\Omega_i/d\mu_j d\mu_i)$ is positive. This demonstrates that managers' strategies are complements.

4. Owners' decisions

When owners of firm i decide to increase α_i the immediate effect is that the firm's manager behaves more accommodatingly (μ_i goes down, meaning that the supply function becomes steeper). However, the manager in firm j will also change her behaviour. Because strategies are strategic complements, the manager in firm j will also act in a more accommodating way. In fact, assuming that the system defined by equations (2) and (3) is stable (meaning that it returns to equilibrium after a perturbation), the precise relationship follows from:

$$d\mu_i/d\alpha_i = -\Delta^{-1}b\mu_i(\mu_j - \beta^{-1})(1 + \alpha_j b(\mu_i - \beta^{-1})) < 0, \quad (7)$$

$$d\mu_j/d\alpha_i = -\Delta^{-1}b\mu_i(\mu_j - \beta^{-1})(1 - \alpha_j b\mu_j) < 0. \quad (8)$$

Stability implies that the determinant, Δ , is positive. The equations affirm that if the owners in one firm makes incentives more accommodating then both managers will behave more accommodating. Oppositely, in homogenous Cournot duopoly, when managers compete in quantities, more aggressive incentives in firm i , given firm j 's incentive contract, increase the output of firm i and reduce that of firm j . That is, $dq_i/d\alpha_i < 0$, and $dq_j/d\alpha_i > 0$ characterise the market stage (see, for example, equation (4b) in Fershtman and Judd (1987)). Using equations (7) and (8) we have:

$$dp/d\alpha_i = p(\beta^{-1} - \mu_i - \mu_j) \left(d\mu_i/d\alpha_i + d\mu_j/d\alpha_i \right) > 0. \quad (9)$$

Equation (9) reaffirms that the price increases when the owners in one of the firms, in the incentive contract, increase the relative weight on profits.

As noted, owners are ill informed about the exact market conditions. Therefore, owners do not know the exact realisation of the price, nor do they know the exact price-effect of a marginal increase in the weight that they assign to profits as given in (9). Nevertheless, $dp/d\alpha_i$ is positive for any realisation of stochastic demand. It follows that $E(dp/d\alpha_i)$ is positive. Of course, when owners design the incentive contract, the positive effect on price has to be balanced against the adverse effects on the firm's sales. Profit in firm i is $p q_i - \frac{1}{2} c q_i^2$, and the effect of incentives that make the manager behave in a more accommodating way is:

$$\frac{d\pi_i}{d\alpha_i} = p(1 - b\mu_i) \frac{dq_i}{d\alpha_i} + q_i \frac{dp}{d\alpha_i}, \quad (10)$$

where $q_i = \mu_i p$ has been used. From equation (9), the second term on the right-hand side is unambiguously positive. Noticing that $dq_i/d\alpha_i = p d\mu_i/d\alpha_i + \mu_i dp/d\alpha_i$, the output of firm i goes down as the firms' owners go for a contract that sustains less aggressive behaviour. We state this as Lemma 1 (proof in the Appendix).

Lemma 1. $dq_i/d\alpha_i$ is negative.

When owners maximise expected profit they balance the positive effect on price against the negative consequences for sales. Optimum incentives follow from $E(d\pi_i/d\alpha_i) = 0$. We show the next Lemma in the Appendix.

Lemma 2. In symmetric equilibrium, $E(d\pi_i/d\alpha_i) > 0$ when evaluated around $\alpha_i = \alpha_j = 1$.

The result in Lemma 2 is explained by two observations. First, Klemperer and Meyer (1989) show that a supply function equilibrium is somewhere in between the polar extremes of Cournot and Bertrand equilibrium. Hence, when owners ask managers to maximise profit (i.e., the incentive contracts set $\alpha_i = \alpha_j = 1$) they achieve less-than-Cournot profits. In this regard, there can be further gains if it is possible to sustain a more accommodating managerial behaviour. Second, when managers compete in supply functions, the strategic decisions or actions in the market stage are strategic complements. Therefore, when the owner-manager pair in firm i decides on an accommodating strategy they are “rewarded” by the other owner-manager pair since they also behave in an accommodating way. That is, it is in fact possible to sustain an accommodating behaviour. The implication of Lemma 2 is therefore that owners penalise sales irrespective of parameter values.

Theorem 1. In supply function equilibrium owners penalise sales.

This result is very different from the result of incentive contracts in symmetric homogenous Cournot oligopoly. Under Cournot competition in the market stage, the strategic decisions are characterised by $dq_i/dq_j < 0$. In turn, when owners in firm i , change the behaviour of their manager so that she acts less aggressively for instance, all they get is more aggressive behaviour by the rival owner-manager pair. This happens because the strategic variables are strategic substitutes. Indeed, Fershtman and Judd (1987) and Sklivas (1987) show that owners of the firms reward sales revenue and sometimes, for the appropriate parameter values, even penalise profits. In linear differentiated product duopoly with competition in prices, they show that owners penalise sales revenue. These results owe to the fact that prices are strategic complements in a linear differentiated product duopoly. That is, an accommodating behaviour in an owner-

manager pair increases the marginal gain of accommodating behaviour in the rival owner-manager pair explaining why sales are penalised in Nash equilibrium.⁶ Our results suggest that existing results on the difference between the optimum incentives in Cournot and Bertrand competitions owes to the specifics of the demand functions used rather than to assumptions about the nature of competition.

Theorem 1 shows that owners overcompensate managers for profit. The result does not show whether incentive contracts sustain lower outputs and therefore larger-than-Cournot profits. We show the proof of Theorem 2 in the Appendix.

Theorem 2. In supply function equilibrium, output is less than output in Cournot equilibrium but higher than the output that maximises joint profits.

Klemperer and Meyer (1989) report that the equilibrium adaptation to noise cannot be either a price or a quantity strategy. They also show that the supply function equilibrium is in between the polar extremes of a vertical supply curve as in Cournot competition, and a flat supply curve as in Bertrand competition.

Theorem 2 spells out the consequences of owners' strategic use of incentives in supply function equilibrium. Output is reduced to below-Cournot outputs. In turn, price and profits are increased beyond their Cournot values. Together, Theorems 1 and 2 show the opposite effect of incentive schemes in comparison to the effects reported in Fershtman and Judd (1987) and Sklivas (1987), who show that optimum incentives sustain overly aggressive behaviour in Cournot markets.

⁶ However, when the demand function is of the constant elasticity type, Cournot competition can involve competition in strategic complements, and price competition might imply that decisions are strategic complements (Klemperer and Meyer, 1989).

5. Conclusion

We have examined optimum managerial incentives in homogenous symmetric duopoly. As is standard, profit-maximising owners choose incentive contracts in a first period and managers act in the market in a second period. Our assumptions about uncertainty are that managers, just like owners, know only the distribution of demand at the time they decide to behave in an accommodating or an aggressive way in the market. When managers make decisions before uncertainty is resolved, it is reasonable to model equilibrium in the market stage as supply function equilibrium (Klemperer and Meyer, 1989).

In supply function equilibrium, managers commit to a combination of price and quantities. This makes certain that decisions are *ex post* as well as *ex ante* optimal. In contrast is the *ex-ante* optimality of committing to either price or quantity. In supply function equilibrium, managers' decisions in the market stage are strategic complements and this gives owners an incentive to reward an accommodating behaviour. In turn, the strategic use of incentives lowers output and sustains higher-than-Cournot profits, in spite of the fact that products are substitutes.

Earlier results on incentives in delegation relationships show that optimal incentives are very sensitive to assumptions about the mode of competition. When there is Cournot competition, owners choose incentives that sustain aggressive managerial behaviour. On the other hand, owners give managers incentives to behave in an accommodating way when managers compete in prices in symmetrically differentiated oligopoly. One way to understand the supply function equilibrium model is as a way to make the price endogenous against quantity competition. When the manager commits to a set of price-quantity combinations made up of conditionally optimal combinations—the condition being the exact realisation of demand—there is no question of price versus quantity. In this perspective, our paper suggests a more

clear-cut effect of delegation. In other words, delegation is beneficial because it sustains an accommodating behaviour and, in this way, brings about higher profits to owners.

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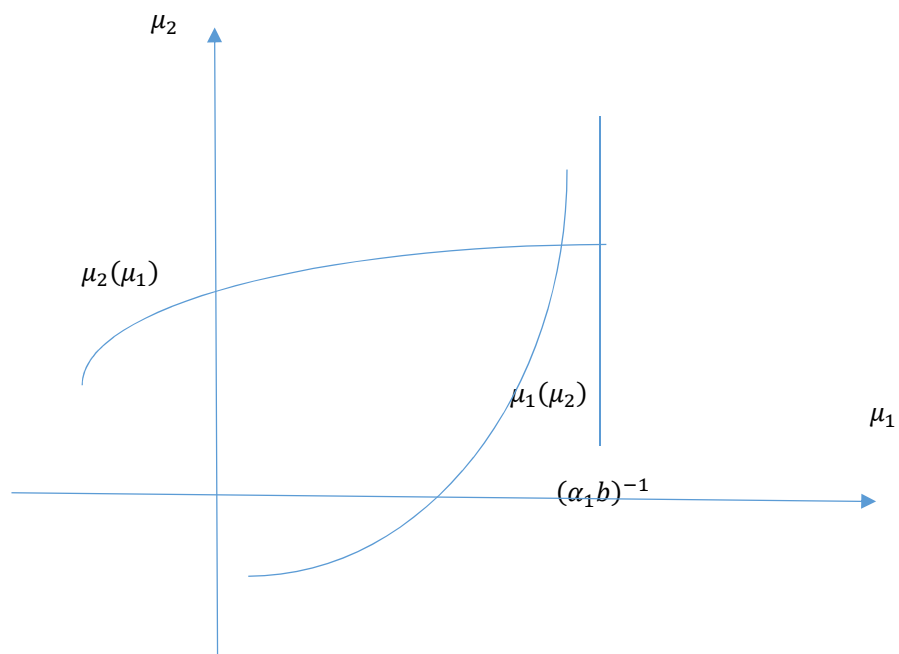


Figure 1. Depiction of equations (2) and (3).

Appendix

Proof of equations (4) and (5).

Using equations (2) and (3):

$$\begin{pmatrix} 1 + \alpha_1 b(\mu_2 - \beta^{-1}) & -(1 - b\mu_1 \alpha_1) \\ -(1 - b\mu_2 \alpha_2) & 1 + \alpha_2 b(\mu_1 - \beta^{-1}) \end{pmatrix} \begin{pmatrix} d\mu_1 \\ d\mu_2 \end{pmatrix} = \begin{pmatrix} -b\mu_1(\mu_2 - \beta^{-1})d\alpha_1 \\ -b\mu_2(\mu_1 - \beta^{-1})d\alpha_2 \end{pmatrix}.$$

The determinant of the system is:

$$\Delta = (1 + \alpha_1 b(\mu_2 - \beta^{-1}))(1 + \alpha_2 b(\mu_1 - \beta^{-1})) - (1 - b\mu_2 \alpha_2)(1 - b\mu_1 \alpha_1)$$

and $\Delta < 0$ follows from stability. We have:

$$d\mu_1/d\alpha_1 = \Delta^{-1} \begin{vmatrix} -b\mu_1(\mu_2 - \beta^{-1}) & -(1 - b\mu_1 \alpha_1) \\ 0 & 1 + \alpha_2 b(\mu_1 - \beta^{-1}) \end{vmatrix},$$

and

$$d\mu_2/d\alpha_1 = \Delta^{-1} \begin{vmatrix} 1 + \alpha_1 b(\mu_2 - \beta^{-1}) & -b\mu_1(\mu_2 - \beta^{-1}) \\ -(1 - b\mu_2 \alpha_2) & 0 \end{vmatrix},$$

and so on, which gives the expressions in the text. ■

Proof of Lemma 1. We know that $dp/d\alpha_i$ is positive. Therefore $dq_i/d\alpha_i + dq_j/d\alpha_i$ must be negative.

Next, notice that $q_j = \mu_j/\mu_i \cdot q_i$. Using Figure 1, consider now an increase of α_i . A change of α_i moves the $\mu_1(\mu_2)$ -curve upwards so that the equilibrium moves along the $\mu_2(\mu_1)$ -curve in the direction of the origin.

This shows that μ_j/μ_i goes up. Thus it follows from $q_j = \mu_j/\mu_i \cdot q_i$ that q_j goes up if q_i goes up. But this would increase total output which is impossible. Thus q_i falls as α_i goes up. ■

Proof of Lemma 2. Write $d\pi_i/d\alpha_i$ as:

$$d\pi_i/d\alpha_i = p \left\{ (1 - b\mu_i) \left(dp/d\alpha_i \right)^{-1} dq_i/d\alpha_i + \mu_i \right\} dp/d\alpha_i.$$

We know that $dp/d\alpha_i$ is positive. Therefore, we have that $d\pi_i/d\alpha_i$ is positive when the term in $\{ \cdot \}$ is positive. Now

$$\begin{aligned} & \mu_i + (1 - b\mu_i) \left(dp/d\alpha_i \right)^{-1} dq_i/d\alpha_i = \\ & \mu_i + (1 - b\mu_i) \left(dp/d\alpha_i \right)^{-1} \left(\mu_i dp/d\alpha_i + p d\mu_i/d\alpha_i \right) \end{aligned}$$

From $q_i = \mu_i p$. That is:

$$\begin{aligned} & \mu_i + (1 - b\mu_i) \left(dp/d\alpha_i \right)^{-1} dq_i/d\alpha_i = \\ & \mu_i + (1 - b\mu_i) \left(\mu_i + p \left(dp/d\alpha_i \right)^{-1} d\mu_i/d\alpha_i \right) \end{aligned}$$

Now:

$$dp/d\alpha_i = p(\beta^{-1} - \mu_i - \mu_j) \left(d\mu_i/d\alpha_i + d\mu_j/d\alpha_i \right)$$

so that:

$$\begin{aligned} & \mu_i + (1 - b\mu_i) \left(dp/d\alpha_i \right)^{-1} dq_i/d\alpha_i = \\ & \mu_i + (1 - b\mu_i) \left(\mu_i + \left((\beta^{-1} - \mu_i - \mu_j) \left(d\mu_i/d\alpha_i + d\mu_j/d\alpha_i \right) \right)^{-1} d\mu_i/d\alpha_i \right) \end{aligned}$$

Now, because $\beta^{-1} - \mu_i - \mu_j < 0$ and $\left(d\mu_i/d\alpha_i + d\mu_j/d\alpha_i\right)^{-1} d\mu_i/d\alpha_i < 1$ we have:

$$\beta^{-1} - \mu_i - \mu_j < \left((\beta^{-1} - \mu_i - \mu_j) \left(d\mu_i/d\alpha_i + d\mu_j/d\alpha_i \right) \right)^{-1} d\mu_i/d\alpha_i.$$

Therefore:

$$\begin{aligned} \mu_i + (1 - b\mu_i) \left(\mu_i + \left((\beta^{-1} - \mu_i - \mu_j) \left(d\mu_i/d\alpha_i + d\mu_j/d\alpha_i \right) \right)^{-1} d\mu_i/d\alpha_i \right) > \\ \mu_i + (1 - b\mu_i)(\mu_i + \beta^{-1} - \mu_i - \mu_j) = \mu_i + (1 - b\mu_i)(\mu_i\beta^{-1} - \mu_j) = 0 \end{aligned}$$

Where the last equality follows from equations (2) and (3) setting $\alpha_i = \alpha_j = 1$. ■

Proof of Theorem 2. The supply functions are determined from equations (2) and (3):

$$\mu_1 = (1 - b\mu_1\alpha_1)(\mu_2 - \beta^{-1}) \quad (2)$$

$$\mu_2 = (1 - b\mu_2\alpha_2)(\mu_1 - \beta^{-1}) \quad (3)$$

This determines $\mu_1 = \mu_1(\alpha_1, \alpha_2)$ and $\mu_2 = \mu_2(\alpha_2, \alpha_1)$. Optimum incentives are determined by:

$$E_\omega \left(p \left\{ (1 - b\mu_1) \left(dp/d\alpha_1 \right)^{-1} dq_1/d\alpha_1 + \mu_1 \right\} dp/d\alpha_1 \right) = 0$$

$$E_\omega \left(p \left\{ (1 - b\mu_2) \left(dp/d\alpha_2 \right)^{-1} dq_2/d\alpha_2 + \mu_2 \right\} dp/d\alpha_2 \right) = 0$$

Due to symmetry the solution is $\alpha_1 = \alpha_2 = \alpha$, and $\mu_1(\alpha_1, \alpha_2) = \mu_2(\alpha_2, \alpha_1) = \mu$.

First part of proof. Now, suppose that $\alpha_1 = \alpha_2 = \alpha_c$ sustains the Cournot outcome. Thus the price in supply function equilibrium equals the price that would obtain when managers compete in quantities. This means:

$$(1 - 2\mu\beta)^{-1}\omega = (b - 3\beta)^{-1}(b - \beta)\omega,$$

which solves for $\mu = (b - \beta)^{-1}$. Notice that $\mu = (1 - b\mu\alpha)(\mu - \beta^{-1})$ is independent of ω meaning that owners can design the optimum incentive scheme without knowing ω .

By choosing α (from $\mu = (1 - b\mu\alpha)(\mu - \beta^{-1})$) so that $\mu = (b - \beta)^{-1}$ owners know that $p + \beta q_i - b q_i = 0$ since this is the first-order condition under Cournot competition. Now, in the contract stage owners evaluate the expected value of $\pi_i = p q_i - \frac{1}{2} b q_i^2$ so that:

$$d\pi_i/d\alpha_i = (p - b q_i) dq_i/d\alpha_i + q_i dp/d\alpha_i.$$

Since $p - b q_i = -\beta q_i$ at $\alpha_1 = \alpha_2 = \alpha_c$ we have

$$d\pi_i/d\alpha_i = -\beta q_i dq_i/d\alpha_i + q_i dp/d\alpha_i.$$

Now, we are on the demand function so that $dp/d\alpha_i = \beta(dq_i/d\alpha_i + dq_j/d\alpha_i)$ and we have:

$$d\pi_i/d\alpha_i = \beta q_i dq_j/d\alpha_i > 0.$$

Second part of proof. Now, suppose that $\alpha_1 = \alpha_2 = \alpha_m$ sustains the outcome under joint profit-maximising behaviour. Joint profit-maximising behaviour implies $Q = (\frac{1}{2}b - 2\beta)^{-1}\omega$. The corresponding price is $p = (\frac{1}{2}b - 2\beta)^{-1}(\frac{1}{2}b - \beta)\omega$ and therefore:

$$(1 - 2\mu\beta)^{-1}\omega = (\frac{1}{2}b - 2\beta)^{-1}(\frac{1}{2}b - \beta)\omega,$$

which solves for $\mu = \frac{1}{2}(\frac{1}{2}b - \beta)^{-1}$ and combining with $\mu = (1 - b\mu\alpha)(\mu - \beta^{-1})$ we find α_m . Following the derivation above, but around $\alpha_1 = \alpha_2 = \alpha_m$, we find:

$$d\pi_i/d\alpha_i = \frac{1}{2}\beta Q \left(-dq_i/d\alpha_i + dq_j/d\alpha_i \right),$$

showing that $d\pi_i/d\alpha_i$ is negative at $\alpha_1 = \alpha_2 = \alpha_m$ when $-dq_i/d\alpha_i + dq_j/d\alpha_i > 0$ which follows since $|d\mu_i/d\alpha_i| > |d\mu_j/d\alpha_i|$. The last inequality is immediately obtainable from equations (7) and (8) assuming symmetry. ■

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