Tail Dependence of Financial Stocks and CDS Markets – Evidence Using Copula Methods and Simulation-Based Inference

Paulo Pereira da Silva, Paulo Tomaz Rebelo, and Cristina Afonso

Abstract

Using copula methods and simulation-based inference the authors address the association between the performance of the stocks of European banks and the CDS markets. Their analysis has three purposes: (i) analysing the dependence structure of the markets when extreme events occur; (ii) checking the validity of the conclusion of Merton (*On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, 1974) and other similar structural models concerning the intensification of the relationship between stock prices and credit spreads during financial distress periods; (iii) analysing the auto-covariance of the dependence structure. First, the results show symmetric dependence and tail dependency equality between the two markets. This means that, surprisingly, the association between stock prices and spreads of the banking sector does not seem to surge in financial distress periods, contradicting the conclusions of Merton (1974) and other structural models, which could be related with a "too-big-to-fail" effect. Second, the authors do not detect structural breaks in the dependence structure in a period marked by the U.S. financial crisis (2008) and the European sovereign debt crisis (2010), which posed concerns on the European financial system health. Finally, they find evidence that the dependence between the markets is autoregressive and possibly time variant. The authors suggest that the inexistence of a higher negative tail dependence between the filtered returns may reside in the "too-big-to fail" effect, that is, credit holders receive a subsidy from governments protecting them from bankruptcy costs, contrary to what happens with equity holders whose capital is wiped-out if the bank fails.

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1. Introduction

The market for credit derivatives, and in particular the market for Credit Default Swaps (henceforth CDS), has experienced a remarkable development over the last two decades. The turnover of CDS markets has increased over the last years, mostly through transactions executed over-the-counter, posing some concerns about the transparency of these operations. In effect, some authors argue that these transactions contributed to a surge of the systemic risk in the financial sector during the financial crisis of 2008. More recently, these derivative instruments have received a special attention from regulators, practitioners and academics due to a surge of sovereign CDS spreads of some European countries.

This type of derivatives may be used to hedge risk or for speculation and allow investors to transact separately the credit risk of the reference entity and to split funding from default risk.[1] Financial institutions are one of the major participants in the CDS markets, since it allows them to hedge and diversify their exposure to illiquid bonds and/or loans/receivables. In fact, one argument in favour of these instruments is that they provide additional liquidity to the bond market, promote risk sharing between market participants and allow creating synthetic portfolios of bonds. The rapid growth of this market along with the severe financial crisis experienced in Europe induced a relevant discussion in the literature on the impact of credit risk derivatives on financial stability. In fact, this discussion had started in the years prior to the before mentioned crisis with some authors defending that CDS can stimulate financial stability through their ability of improving credit risk allocation, as a consequence of a more liquid and diversified market for credit risk transfers. For instance, Alan Greenspan argued that these new financial instruments allowed the sophisticated financial institutions to reduce their credit risk, transferring it to less leveraged market participants. However, if in one hand these financial instruments can reduce banks' exposure to default risk (for example, through mortgage backed securities); on the other hand they may leverage investors' exposure to new risks.

CDS spreads reflect the default risk of the underlying debt instrument. The final payoff of these over-the-counter contracts depends on a credit event and the spreads indicate the credit quality of the reference entity. In effect, these financial instruments provide us a way to assess the interaction between financial stocks performance and credit risk. The linkage between credit spreads and stock prices is sustained by credit risk structural models, such as the Merton (1974) model. The author values equity and debt as contingent claims over the firm's assets. According to Merton, the default probability of a company is a non-linear function of the equity price, the asset price volatility and the debt-equity ratio. Consequently, the returns of bonds and stocks should be correlated, particularly when default risk surges. In other words, the correlation between the returns of bonds and stocks should increase in financial distress situations.

Duffie (1999) shows that subject to some assumptions, a long position in a par priced floating rate note and the purchase of a CDS contract with the same face value of protection, creates a combined position with no credit risk in the event of default. Hence, the CDS spread should be equal to the credit spread of the par priced floating rate note. In that sense, one should expect a similar association between bond credit spreads and stock prices and between CDS spreads and stock prices, because bond credit spreads and CDS spreads are close substitutes. In theory, when the equity and debt

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[1] CDS is a bilateral financial contract in which one counterparty (the protection buyer or buyer) pays a periodic fee, typically expressed in basis points per annum on the notional amount, in return for a contingent payment by the other counterparty (the protection seller or seller) after a credit event of the reference entity. The contingent payment is designed to mirror the loss incurred by creditors of the reference entity in the event of a default. The settlement mechanism depends on the liquidity and availability of reference obligations.

rewards are not proper, arbitrage based in the firm capital structure is possible. Thus, if a company CDS spread is higher (lower) than it should be (given the stock price as well), an arbitrageur may obtain riskless profit from selling (buying) CDS contracts and buying (selling) shares. This way, arbitrage forces the equilibrium between the two markets.

Our research focuses on the banking sector. In effect, banks played an essential part in the trigger of the recent financial crisis, as well as being among the worst-hit players. Moreover, they still perform an important task in the economy, namely providing liquidity transformation and monitoring services. After the financial crisis started in 2007, the importance of credit risk in the banking sector has increased and CDS spreads are seen as an indicator of a bank's weakness. CDS spreads may be used to extract market perceptions about the financial health of banking institutions in particular of systemic banking firms. Thus, understanding the relationships between CDS spreads of the financial sector and stock markets is important to evaluate financial stability, and more precisely is of crucial importance in terms of supervision, regulation, market discipline and also for practitioners and academics. Moreover, it is important to evaluate the "too-big-to-fail" effect on the association between financial stock performance and credit risk, in particular for systemically large banks. On the other hand, CDS markets may frighten financial stability due to spillovers to other markets, namely the equity market and the bond market. Heyde and Neyer (2010) show that macroeconomic surrounding influences the impact of CDS markets on the stability of the banking sector. During recessions CDS markets impact the stability of the banking sector, regardless of the shock type (idiosyncratic or systematic), increasing the risk of a systemic crisis. However, in periods of moderate economic growth and during booms idiosyncratic shocks will increase the systemic risk only if there are other channels of contagion as well.

This paper pursues three research questions. First, the analysis explores the dependence structure of the markets when extreme events occur. Second, we aim to check the validity of the conclusion of Merton (1974) and other similar structural models concerning the intensification of the relationship between stock prices and credit spreads during financial distress periods. In that sense, we assess the "too-big-to-fail" effect on the association between financial stock performance and credit risk. Finally, we analyse the auto-covariance of the dependence structure of the performance of both markets.

This study extends the thriving academic literature on the interaction of CDS markets and stock markets. In doing so, we use the theory of copulas. Copula-based models provide a great deal of flexibility in modelling multivariate distributions, permitting the researcher to specify the models for the marginal distributions separately from the dependence structure (copula) that defines the joint distribution. In addition to flexibility, this method also facilitates the estimation of the model in phases, reducing the computational burden. We add to that analysis simulation-based inference with the aim of selecting the type of dependence structure that best fits the empirical data and to ascertain the robustness of the results.

This paper is structured as follows: section two contain a literature review on this subject; sector three conveys the theoretical framework and the sample; section four presents the empirical results; and finally section five displays the conclusions and a brief discussion of the implication of the results.

2. Literature Review

The empirical literature about the relationship between stock and debt markets performance is quite extensive. In the 90's, some empirical studies showed an empirical relation between stock returns and

bond yield changes. For instance, Blume *et al.* (1991), Cornell and Green (1991) and Fama and French (1993) report a contemporary and slightly positive but statistically significant association between stocks and bond returns. Kwan (1996) concludes that changes of bond yields are positively influenced by changes of Treasury bond yields and negatively affected by contemporaneous and lagged stock returns. More recently, Alexander and Ferri (2000) show a positive association between the raw daily returns of stocks and bonds of financially distressed firms in the period 1994-1997. However, when stocks abnormal returns are used instead of raw returns, the statistical association between the variables turns non-statistically significant. Hotchkiss and Ronen (2002) do not find evidence that stock markets led bond markets, although there is a modest and positive contemporaneous association between them.

Longstaff *et al.* (2003) examine Granger causality between (weekly) changes of CDS spreads, changes of bond credit spreads and stock returns. Their analysis focuses on the US markets and the results indicate that stock markets and CDS markets led corporate bond markets. Campbell and Taksler (2002) document an empirical relation between the volatility of stock returns and bond yields. Zhang (2007) shows that CDS spreads anticipate credit quality deterioration before stock markets. Norden and Weber (2009) study the relationships between stock markets, bond markets and CDS markets during the period 2000-2002 for a set of 58 firms [USA (35), Europe (20) and Asia (3)]. They find that (i) the CDS market reacts to the stock market movements, and that the magnitude of that reaction is affected by the credit quality of the firm and by the liquidity of the bond market; (ii) stock returns lead credit spreads and CDS spreads.

Bystrom (2005) analyses the association between the performance of a CDS iTraxx index and stock market returns during the period 2004-2005 and concludes that stock market returns Granger cause CDS spread changes, but the reverse does not occur. Fung *et al.* (2008) report a negative correlation between CDS indexes and stock indexes performance. That correlation is higher amid financial distressed firms and in the overall the correlation surged after July 2007. This outcome is consistent with Merton (1974) model: the decline of stock prices results in an increase of leverage, contributing to a rise of default risk and CDS spreads. Results also suggest that the stock market leads the CDS market, regardless of the firm's financial situation. However, the volatility spillovers from the CDS markets to the stock markets are higher than the reverse.

Avramov *et al.* (2009) show that the effects of rating downgrades on stock prices and CDS spreads are higher amid financially distressed firms. Forte and Peña (2009) show that the stock market leads the CDS market and the bond market in price discovery. Forte and Lovreta (2009) show that price discovery process changes with the financial situation of firms. The contribution of stock markets to price discovery is positively influenced by the turnover ratio of the stock market, the credit quality of the firm and by the reduced presence of negative adverse shocks. Stock markets appear to lead CDS markets, but that leadership has been decreasing over time.

The correlation between the two markets also appears to be asymmetric. For instance, Dupuis *et al.* (2009) conduct an empirical analysis on the influence of credit risk on the performance of stocks from of automobile industry using the theory of copulas. They show that stocks returns and the CDS spreads are negatively correlated, being that correlation higher in the tails of the probability density functions (henceforth, p.d.f.). Gatfaoui (2007) also presents evidence of an asymmetric relation between the CDS market and the stock market.

3. Data description

We perform our analysis using daily data for the period comprised between 03/12/2007 and 14/11/2012. We study the interaction between two well-known European indexes for the financial sector: EuroStoxx Banks 600 (Bloomberg ticker: SX7E) and iTraxx Europe Senior Financials 5Y TR from Markit (Bloomberg ticker: SNRFIN CDSI GENERIC 5Y Corp). Prices and spread from these two indexes are extracted from Bloomberg.

EuroStoxx Banks index is a capitalization-weighted basket and includes stocks from the banking sector (mostly large and systemically important banks) traded in countries that integrate the European Monetary Union. iTraxx indexes are often used as proxies for default risk. These baskets cover firms and sovereign entities from different sectors and regions of the world, and usually display high liquidity and low bid-ask spreads. The iTraxx Europe Senior Financials 5Y TR is a basket of CDS contracts having debt of European financial institutions' as underlying. It is an equally weighted index of 25 European financial institutions reference entities (also large and systemically important).

Figure 1 displays the performance of iTraxx Europe Senior Financials 5Y TR and EuroStoxx Banks 600. In the period before 2008, CDS spreads were small, denoting a reduced probability of default of the major European financial institutions. As of 2007, the default risk of financial institutions has surged sharply, in particular after the Bear Stearns' failure, with investors perceiving a higher probability of failure of financial companies. On the other hand, stock prices experienced a major decline between 2007 and mid-2009, and after 2010. Indeed, the figure suggests a negative co-movement between CDS spreads and stock prices. It is clear that bad news about a stock have a negative impact on prices. Simultaneously, CDS spreads should increase given a higher likelihood of a credit event. That impact should be even higher if the firm is near bankrupt.

Figure 1 – DJ Eurostoxx Banks 600 and iTraxx European Financial SNR prices
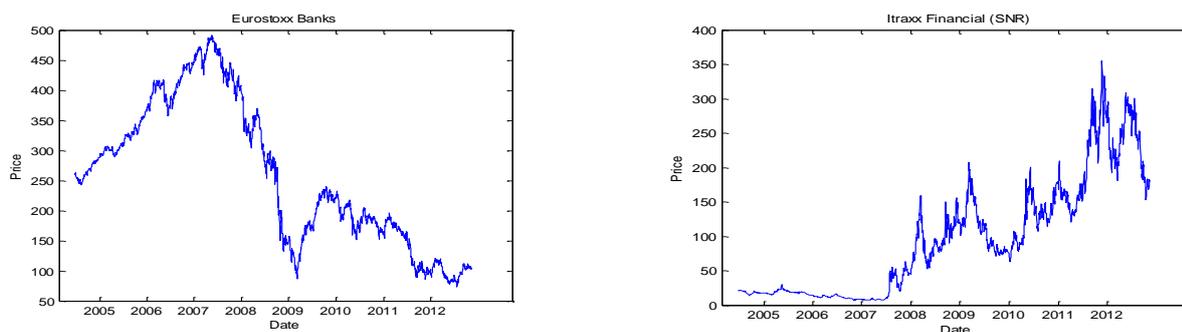
4. Methodology

The classical theory of portfolio management and risk management is based on the assumption that returns follow multivariate normal i.i.d. distributions. This assumption is very convenient because it allows practitioners to use correlations as a measure of dependence. However that assumption might not be a very realistic assumption about the behaviour of returns on financial markets. For instance, equity returns take joint negative extreme values more often than joint positive extremes, leading to

the statement that "stocks tend to crash together but not boom together." The opposite tends to take place in the CDS market, where the correlation is larger when higher positive extreme values occur.

Another way to assess the correlation structure of the series relies on the concept of copulas. Copula-based multivariate models permit modelling the marginal distributions separately from the dependence structure (copula) that links these distributions to form the joint distribution. This method increases the degree of flexibility in specifying the model, in comparison to other methods.

In some cases, such as in portfolio management, the concordance between extreme (tail) values of random variables is of interest. Very often the marginal distributions are asymmetric and/or the tail dependence is non-linear. This means that correlation makes no sense as a dependence metric, given that it requires an elliptical multivariate distribution. In our analysis, we address the interaction of the equity markets and CDS markets, and in particular we assess the tail dependence between the two markets. Tail dependence captures the behaviour of the random variables during extreme events. In our analysis, we are interested in the co-movement of CDS spreads and stock prices not only in normal conditions, but especially in extreme situations. This requires a dependence measure for the upper and the lower tails of the multivariate distribution of the series. Such a dependence measure is essentially related to the conditional probability that one index exceeds some value, given that another exceeds some value.

The copula of two variables is simply the function that maps the univariate marginal distributions to a joint distribution. The estimation by the copula method is performed in several stages. Firstly, the marginal distributions are estimated separately from the dependence structure, simplifying the study of high-dimension multivariate problems. Before modelling the dependence structure between the series, one must first model their conditional marginal distributions.

$$Y_{i,t} = \mu_i(Z_{t-1}) + \sigma_i(Z_{t-1}) \times \varepsilon_{i,t}$$

For i=1,2

$$Z_{t-1} \in \mathcal{F}_{t-1} \sim F_i(0,1)$$

Within this setup, it is assumed that each series will have potential time-varying conditional mean and variance and that the standardized residual $\varepsilon_{i,t}$ is a white noise, that is, has a constant conditional distribution (with zero mean and a variance of one). Thus, in a first pass, we model the conditional means and variances of the returns of the two indexes. In order to capture the conditional mean, we calculate and plot the ACF and PACF of the returns of the two time series of the returns (results not reported). Along with the Ljung-Box-Pierce test and the Breusch-Godfrey LM test (see Table A1 in the Appendix), the evidence indicates the presence of higher than one autocorrelation in the analysed time series.

To model the conditional mean, we use ARMA models: (i) we fit a ARMA(1,2) for the returns of the banking sector stock index; and a ARMA(1,2) for the returns of the iTraxx Europe Financials SNR. The autocorrelation of the original series is removed after applying the ARMA filters. To model the volatility of the returns, we use ARCH models. The results of some common used tests to detect autoregressive conditional variance, such as the Ljung-Box-Pierce test applied to the square residuals and the Breush-Pagan LM test are presented in the Appendix. Both tests indicate the presence of ARCH effects of order 4 in the series of the stock index returns. Thus, we filter that variable with a GARCH(4,1), removing the conditional variance from the residuals. As for the CDS index, a GARCH(1,1) process is used to remove the ARCH effects. Finally, with the aim of testing if the

filtered returns (residuals of the ARMA/GARCH model) are i.i.d., we perform the BDS test and the runs test. These tests do not reject the null hypothesis of i.i.d. returns in neither of the series.

The standardized residuals are calculated as:

$$\hat{\varepsilon}_{i,t} = \frac{Y_{i,t} - \mu_i(Z_{t-1}; \hat{\alpha})}{\sigma_i(Z_{t-1}; \hat{\alpha})}$$

where $\widehat{\alpha}$ is the vector of estimated parameters of the ARMA/GARCH model. Estimating the dependence structure between the series entails the transformation of the standardized residuals into a uniform distribution (F_i). The estimation of F_i may be performed using a parametric or a non-parametric model. Many choices are possible for the parametric model of F_i, including the Normal, the standardized Student's t and the skewed t of Hansen. However, due to its simplicity and flexibility, we choose a non-parametric estimate, the empirical distribution function (EDF):

$$\hat{F}_i(\varepsilon) \equiv \frac{1}{T+1} \sum_{t=1}^{T} 1 * \{\varepsilon_{i,t} < \varepsilon\}$$

Combining the use of the empirical distribution function (EDF) of the standardized residuals with parametric models for estimating the conditional means and variances turns our model semi-parametric. Inference on the estimated dependence statistics can be performed either using the asymptotic distribution of the parameters of the model or using a bootstrap approach (assuming that the true conditional copula is constant through time). As in Rémillard (2010), we are assuming that the estimated parameters of the ARMA/GARCH model do not affect the asymptotic distribution of the dependence statistics and thereby the conditional mean and variance may be estimated independently of the copula.

We estimate nine different time-invariant copulas:

- Normal Copula - the normal copula is flexible in that it allows for equal degrees of positive and negative dependence and includes both Fréchet bounds in its permissible range.

- Clayton's Copula - the Clayton copula cannot account for positive dependence. It has been used to study correlated risks because it exhibits strong left tail dependence and relatively weak right tail dependence.

- Rotated Clayton Copula – similar to Clayton Copula, but can only account for negative dependence.

- Plackett copula – also enjoys the property of symmetry.

- Frank Copula - dependence in the tails of the Frank copula tends to be relatively weak compared to the Gaussian copula, and the strongest dependence is centred in the middle of the distribution, which suggests that the Frank copula is most appropriate for data that exhibit weak tail dependence.

- Gumbel Copula - Gumbel does not allow for negative dependence. It exhibits strong right tail dependence and relatively weak left tail dependence. If outcomes are known to be strongly correlated

at high values but less correlated at low values, then the Gumbel copula is an appropriate choice for modelling the concordance of the series.

- Rotated Gumbel Copula - similar to Gumbel Copula, but can only account for negative dependence.

- Student's t Copula – provides higher tail dependence than the Normal Copula.

- Symmetrised Joe-Clayton Copula.

Along with time-invariant copulas, we estimate three time-varying copulas: time-varying Normal copulas, time-varying Rotated Gumbel copulas and time-varying SJC copulas.

The next section presents the results and the conclusions derived from the copula estimation.

5. Results

As a first step, we estimate the quantile dependence of the two time series. The quantile dependence assesses the strength of the dependence between two variables in the joint lower, or joint upper, tails of their support. Quantile dependence provides a good description of the dependence structure of two series. Figure 2 shows the (estimated) quantile dependence plot along with a 90% confidence interval based on a bootstrap simulation. The dependency between the two series is concentrated in the median of the marginal distributions. The dependence between the two time series is weak in both tails, but the standard deviation (computation based on a bootstrap simulation) of the upper tail quantile dependence is higher. Therefore, confidence intervals are narrower in the lower tail and wider in the upper tail (values of q near 1). By estimating the strength of the dependence between the two variables as we move from the centre of the distribution to the tails, and by comparing the left tail with the right tail we are able to capture more exhaustive information about the dependence structure than it is provided by a scalar indicator such as the linear correlation or the rank correlation. In effect, some copulas such as the Normal, the Frank and the Student's t-copulas, assume a symmetric dependence between the variables, and as a consequence this information is useful to choose the right copula.

Figure 2 - Quantile dependence for the Eurostoxx Banks 600 filtered returns and the iTraxx Financial Europe SNR filtered returns
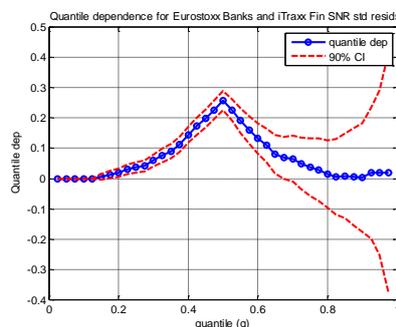

Figure 3 presents the difference between the upper and lower tails of the previous plot, along with a pointwise confidence interval for the differential. It seems that there is no difference between the upper and lower tail quantile dependence frequencies. Notice that this evidence is also supported by the tests of tail dependence equality and asymmetric dependence shown on Table 1, which we will analyse in more detail later.

Figure 3 – Difference in upper and lower quantile dependence
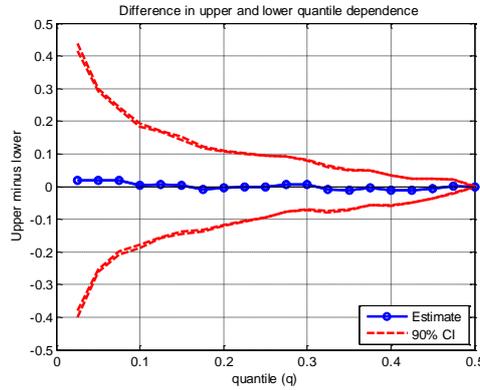
We use two tests to measure symmetric dependence and tail dependency equality among variables. Under symmetric dependence we have:

$$\lambda^q = \lambda^{1-q} \ \forall \in [0,1]$$

Where, λ is the dependence measure. Rémillard (2010) proposes a Chi-square test to test jointly asymmetric dependence for a set of different q's. Instead of testing each q separately we follow Rémillard (2010) and run a co-joint significance test over the dependence measure at different quantiles:

$$H_0 : R\lambda = 0$$

where $\lambda = [\lambda^{q1}, \lambda^{q2}, \lambda^{q3} \dots \lambda^{qk}]$ and $q \in \{025; 0:05; 0:10; 0:975; 0:95; 0:90\}$. The test for jointly asymmetry provides a necessary but not sufficient condition for symmetric dependence. Rémillard (2010) proposes a bootstrap estimate to implement the Chi-square test, which we also adopt in this analysis. See further details about this test on Rémillard (2010) or Patton (2012). The test fails to reject the null hypothesis of a symmetric dependence between the variable (Table 1 –Panel A).

The second test addresses tail dependency equality in the tails, namely whether the tail dependence coefficients (i.e., the limits of the quantile dependence functions) are equal. More precisely we test if:

$$\lambda^U = \lambda^L$$

The test is implemented using bootstrap inference methods (see Patton (2012) for more details). The t-stat associated to this test is 0.1225, that is, not statistically significant (Table 1 – Panel B).

**Table 1 – Testing for asymmetric dependence and tail dependence equality**

Panel A - Testing for asymmetric dependence

	Chi-stat	p-value
Testing for asymmetric dependence	0.0176	0.999

Panel B - Testing for tail dependence equality

	t-stat	p-value
Testing for tail dependence equality	0.1225	0.902

Figure 4 plots the 60 days rolling rank correlation for the standardized residuals series and respective bootstrap confidence interval. The rank correlation between the standardized residuals range between -0.3 and -0.9 in the time frame covered in the analysis. Notice that the correlation is higher in the last third of the sample, period marked by the sovereign debt crisis in Europe. The linear correlation and the rank correlation between the series of raw returns are -0.52 and -0.36, respectively.

Figure 4 - 60 day rolling rank correlation for Eurostoxx Banks and iTraxx Fin SNR filtered returns
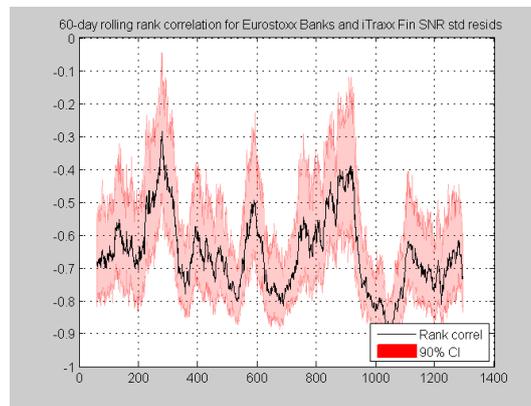
The variability of the rank correlation through time suggests the presence of time-varying dependence. Hence, testing the presence of time-varying dependence could be informative, for example, before specifying a functional form for a time-varying conditional copula model. The tests usually considered assume a constant conditional copula under the null hypothesis (see Rémillard, 2010). We focus on three tests that evaluate changes in rank correlation and are used to assess structural breaks or time-varying dependence. The first test evaluates a break in the rank correlation at some specified point in the sample. Under the null hypothesis, the dependence measure before and after the breakpoint is equal to:

$$H_0: \rho^1 = \rho^2$$

Where ρ^1 and ρ^2 denote the rank correlation before and after the breakpoint. The critical value for this test derives from a iid bootstrap simulation. By generating the bootstrap samples we obtain draws that impose the null hypothesis. Even though simple to implement, this entails a prior knowledge by the researcher about the dependence structure of the variables. The critical value for the difference between the rank correlations of both sides of the sample (before and after some specified point in the sample) is obtained using iid bootstrap. The p-values are obtained through 1000 bootstrap simulations. The three different break points considered lie in the 15, 50 and 85% points of our sample. In all three, the null hypothesis of no structural break is not rejected, as one can see in Table 2.

A second test for time-varying dependence checks the break in the rank correlation coefficient at some unknown date. We follow Andrews (1993) in the implementation of the test. A critical value for this test is obtained again by using an iid bootstrap. The null hypothesis of no structural break is again not rejected (see Table 2).

The final test concerning time-varying dependence is based in the "ARCH LM" test for conditional variance proposed by Engle (1982). Instead of testing for one discrete one-time breaks in the dependence structure, it addresses the autocorrelation of a measure of dependence (rank correlation), using an autoregressive-type test. The null hypothesis of no autocorrelation of the dependence structure of the variables is rejected by the test. Indeed, the evidence suggests a lag 5 autoregressive-type of time-varying dependence.

The table below summarises the results of the tests for time varying correlation between the standardized residuals of the index DJ Eurostoxx Banks 600 and the index iTraxx Fin SNR. All in all, we do not detect structural breaks in the rank correlation, although some evidence points towards order 5 autocorrelation of this measure, meaning that the best copula structure to characterize the data could be time varying.

Table 2 – Testing for time-varying dependence

		p-value
Break	0.15	0.39
	0.50	0.71
	0.85	0.46
	Anywhere	0.59
AR (p)	1	0.39
	5	0.04
	10	0.44

Next, we estimate several copulas in order to find the one that better fits to the data. Copulas are written in terms of random variables U1 and U2 with standard uniform marginal distributions. So, along with the estimation of F_i as described in section 3, a K-S test is performed for each of the standard uniform variables. The results of the tests are presented in Table 3. K-S tests do not reject the null hypothesis that the transformed standardized residuals are uniformly distributed.

Table 3 – Kolgomorov-Smirnov test (uniform cdf)

	K-S statistic	p-value
u	0	1
v	0	1

Because copulas separate the marginal distributions from the dependence structures, the appropriate copula for a particular application is the one that best captures the dependence features of the filtered returns. One way to choose the copula that best fits the data consists in evaluating AIC and BIC measures. Since the variables display negative dependence, the estimation of Clayton and Gumble's copulas is useless because these copulas can only account for positive dependence. Rotated Clayton and Gumble's copulas should be used instead. Table 4 shows the log likelihood, the lower and the upper tail derived from the estimated parameters.

Table 4 – Summary results from the copula estimation

	LL	Lower Tail	Upper Tail
Normal Copula	-406.1	0.0	0.0
Clayton's copula	0.1	0.0	0.0
Rotated Clayton copula	0.1	0.0	0.0
Plackett copula	-395.5	0.0	0.0
Frank copula	0.0	0.0	0.0
Gumbel copula	115.8	0.0	0.12
Rotated Gumbel copula	115.1	0.12	0
Student's t copula	-412.4	0.000006	0.000006
Symmetrised Joe-Clayton copula	32.1	0.000002	0.000002

In order to select the copula that best suits the data, we calculate the AIC and BIC measures for each estimated copula. As one can observe by the results in Table 5, the time-varying normal copula and the Student's t copula are the copulas that better adjust to the data.

Table 5 –Summary results from the copula estimation II

	LL	AIC	BIC
Normal Copula	-406.10	-812.20	-812.20
Clayton's copula	0.06	0.12	0.13
Rotated Clayton copula	0.06	0.12	0.13
Plackett copula	-395.52	-791.04	-791.04
Frank copula	0.01	0.03	0.03
Gumbel copula	115.78	231.56	231.56
Rotated Gumbel copula	115.14	230.28	230.28
Student's t copula	-412.36	-824.72	-824.72
Symmetrised Joe-Clayton copula	32.12	64.24	64.24
Time-varying normal Copula	-418.78	-837.55	-837.53
Time-varying rotated Gumbel copula	0.16	0.33	0.35
Time-varying SJC copula	62.05	124.12	124.14

In order to construct confidence intervals for the estimates, a bootstrap simulation is again performed. As Chen *et al.* (2006) and Rémillard (2010) we use the estimated standardized residuals as the true standard errors. Under the assumption that the copula is constant over time, we perform a iid bootstrap to calculate standard errors and confidence intervals: (i) we randomly draw with replacement from the matrix of standardized residuals; (ii) and estimate the dependence measures from the bootstrapped sample; (iii) the before mentioned procedure is repeated 1000 times t; (iv) extract the $\frac{\alpha}{2}$ and $(1-\frac{\alpha}{2})$ confidence interval for the parameters.

When the conditional copula is time-varying, this procedure is not valid. In effect, the parameter estimation error from the models for the conditional mean and variance should not be ignored (Rémillard, 2010).

If one disregards time-varying copulas, the Student's t copula seems to be the one that best fits the data, followed by the Normal Copula. As seen before the results point towards the rejection of the presence of asymmetric heavy tail dependence on the series. That result is now confirmed by the outcome of the copulas estimation. Asymmetric heavy tail copulas are "rejected" by the data: even Frank and Plackett Copulas yield better results than the Rotated Gumble and the Rotated Clayton copulas that assume an asymmetric dependence setup. In spite of the results one may conclude that the dependency structure is not concentrated in one of the tails of the distribution, but rather in the centre, notwithstanding the dependence in the tails being higher than predicted by the Gaussian copula.

Table 6 shows the bootstrap standard errors and confidence intervals for the parameters of each estimated copula. Notice that for the Student's t copula, the confidence interval for the Pearson correlation is very narrow and the standard deviation is small. Nonetheless, v (the degrees of freedom of the copula distribution) presents high standard errors and wider confidence intervals.

Table 6 – Bootstrap standard errors and 95 % confidence intervals

	Estimated Parameters		Bootstrap Standard Errors		95% Lower Confidence Interval		95% Upper Confidence Interval	
	Parameter 1	Parameter 2	Parameter 1	Parameter 2	Parameter 1	Parameter 2	Parameter 1	Parameter 2
Normal Copula	-0.5417		0.0177		-0.5732		-0.5052	
Clayton's copula	0.0001		0.0000		0.0001		0.0001	
Rotated Clayton	0.0001		0.0000		0.0001		0.0001	
Plackett copula	0.1784		0.0108		0.1589		0.2008	
Frank copula	0.0001		0.0000		0.0001		0.0001	
Gumbel copula	1.1000		0.0000		1.1000		1.1000	
Rotated Gumbel	1.1000		0.0000		1.1000		1.1000	
Student's t copula	-0.5463	7.2195	0.0157	1.6544	-0.5757	5.4164	-0.5127	11.8269

Concurrently, we perform a Monte Carlo Simulation also with the aim of extracting standard errors and confidence intervals for the parameters of the Student's t copula. To do so, we first generate random numbers for the Student's t copula (using the parameters estimated previously) and marginal normal distributions. Using a Copula random number generator we simulate values for the disturbance term of the two dependent variables. Subsequently, we use the simulated shocks and the estimated conditional mean and variance equation to simulate new values for the raw returns, using the mean of the raw returns and its long term unconditional variance as starting values.

In a second stage, and using the simulated sample for the raw returns, the Student's t copula is re-estimated and the obtained estimated parameters are saved. This process is repeated 1,000 times. Using the estimated parameters provided by the simulations, the standard errors and the confidence intervals for the Student's t copula are calculated. This analysis shows that the parameters estimated from the original time series lie inside the parametric 95% confidence interval (Table 7), which might mean that t-copulas fit the data and the model is correctly specified.

Table 7 – Parametric standard errors and the 95% confidence intervals for the Student's t copula

	Par. 1	Par. 2
Standard Error	0.022	43.087
95% Lower Confidence Interval	-0.588	4.996
95% Upper Confidence Interval	-0.501	100.000

Concomitantly, we estimate three time-varying copulas: time-varying normal Copula, time-varying rotated Gumbel copula and time-varying SJC copula. AIC and BIC criterions suggest that the time-varying Normal copula is the one that best fits to our data. In fact, according to these two criterions the time-varying Normal copula outperforms the constant Student's t copula in terms of suitability (see Table 5). Notice that previously, we have already concluded that the filtered returns rank correlation displayed autocorrelation. That is also consistent with a time varying copula.
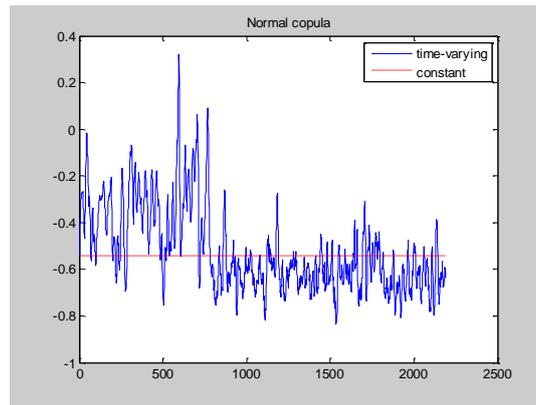
Figure 5 – Time-varying copula

Figure 5 shows a dissimilar pattern of the correlation before and after the European sovereign debt crisis. This pattern is captured by the time-varying normal copula, but not by the constant copula. In this sense, the time-varying normal copula is more appropriated than the constant normal copula and the t-student copula.

6. Conclusions

Merton (1974) provides the setup for the analysis of the relationship between CDS markets and equity market performance. According to the model, a high debt-equity ratio would imply a high correlation between stock and bond prices, because stock and credit spreads are influenced by the market value of assets. The value of liabilities would reflect the difference between the value of a riskless bond and a put option over the firms' assets, having as strike price the face value of liabilities. A similar relationship might be established between stock prices and CDS spreads. In that sense, the relationship between stock prices and CDS spreads should increase with financial distress. This implies a non-linear association, where the co-movement intensifies during distressed periods.

We focus our analysis on the banking sector. Understanding the relationships between CDS spreads of the financial sector and stock markets is important to evaluate financial stability, and more precisely is of crucial importance in terms of supervision, regulation and market discipline. Moreover, it is important to evaluate the "too-big-to-fail" effect on the association between financial stock performance and credit risk, in particular for systemically large banks.

Using a copula-based approach we address the association between stocks of European financial institutions and CDS markets. We aim to accomplish three purposes: (i) analysing the dependence structure of the markets when extreme events occur, having into account that sometimes banks are too big to fail; (ii) checking the validity of the conclusion of Merton (1974) and other similar structural models regarding the intensification of the relationship between stock prices and CDS spreads during financial distress periods; (iii) analysing the auto-covariance of the dependence structure. First, our results show a symmetric dependence and tail dependency equality in the two markets, which means that, surprisingly, the association between stock prices and CDS spreads does not seem to change in financial distress periods, contrary to the conclusions of Merton (1974). Second, we do not detect structural breaks in the dependence structure in a period marked by the U.S. financial crisis (2008) and the European sovereign debt crisis (2010), which posed concerns on the financial system health. Notwithstanding, we find evidence that the dependence between the markets is autoregressive.

A copula-based approach also rejects the hypothesis that the dependence structure is more intense in the tails. On the contrary, heavy tail copulas such as the Rotated Gumbel and Rotated Clayton copulas

present a poor performance fitting the data when compared to the Gaussian and the student t copula. Student t copula and the Gaussian time-variant copula are the copulas that better fit the data, meaning that the dependence structure is more heavy-tailed than is assumed by the constant Gaussian copula and is possibly time variant. One possible reason for the inexistence of a higher negative tail dependence between the filtered returns may reside in the too big to fail effect, that is, credit holders receive a subsidy from governments protecting them from bankruptcy costs, contrary to what happens with equity holders whose capital is wiped-out if the bank fails.

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Appendix

Table A1– Autocorrelation diagnosis I:

The *Ljung-Box-Pierce test and the Breush-Godfrey test confirm the presence of autocorrelation.*

	Ljung-Box-Pierce test		Breush-Godfrey test	
	Stocks	CDS	Stocks	CDS
1	6.44**	24.35***	4.4**	17.44***
2	7.26**	25.43***	5.65*	18.93***
3	13.56***	34.46***	8.73**	21.62***
4	13.82***	49.01***	9.38*	22.56***
5	18.06***	51***	12.54**	22.95***
6	18.23***	52.25***	12.58*	23.25***
7	18.25**	52.78***	12.67*	23.09***
8	18.56**	52.8***	12.7	22.97***
9	18.57**	56.19***	12.7	24.42***
10	18.57**	56.28***	12.75	24.72***
11	18.66*	56.95***	13.1	24.69**
12	19.52*	58.79***	14.07	28.62***

(***), (**), (*) denotes statistically significance at a 1%,5%, 10% level, respectively.

Figure A1 – ACF and PACF of the square of the ARMA(1,2) errors (Index DJ Eurostoxx Banks 600)

The graphical representation of the ACF and PACF of the square of the ARMA(1,2) errors suggests the presence of ARCH effects of order 4. As one can observe, after order 4 the PACF drops drastically to zero.
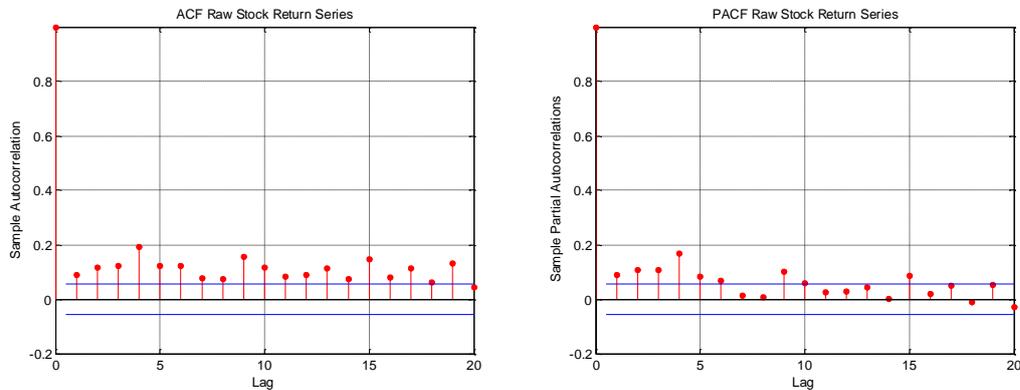
Figure A2 – ACF and PACF of the square of the ARMA(1,2) errors (iTrax European Financial SNR)

The graphical representation of the ACF and the PACF of the square of the ARMA(1,2) residuals suggests the presence of ARCH effects.
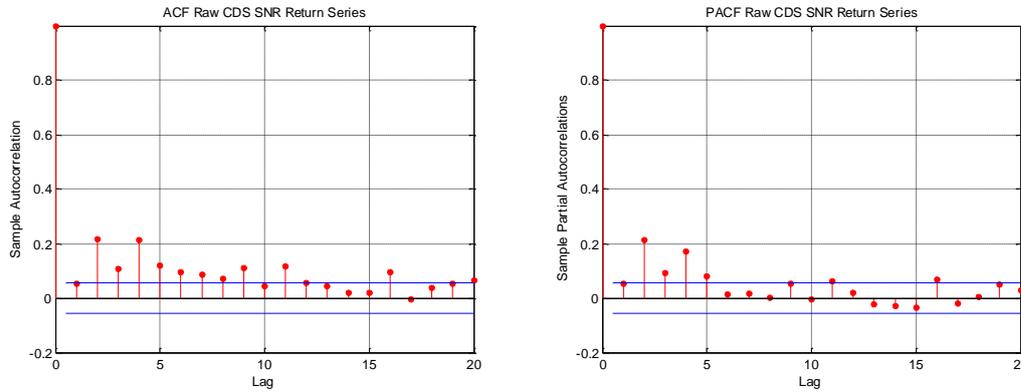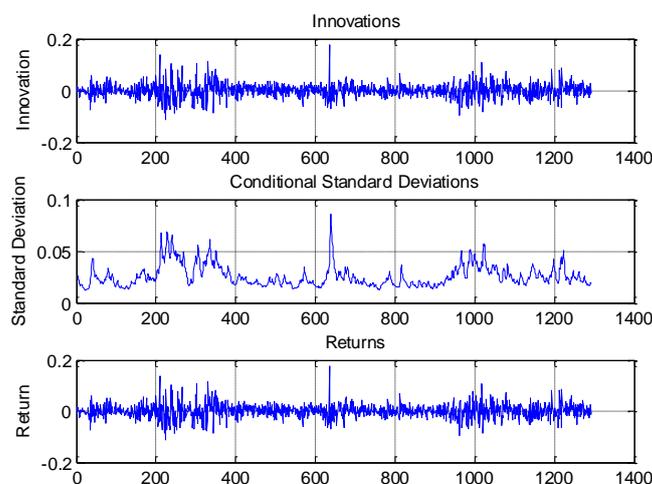

Table A2 – ARCH diagnosis (returns of the Index DJ Eurostoxx Banks 600 filtered by the ARMA(1,2) process)

In order to examine the presence of autoregressive conditional variance on the returns of the Index DJ Eurostoxx Banks 600 filtered by the ARMA(1,2) process, the ARCH test and the Ljung-Box-Pierce Test of the square residual are performed. Both tests allow detecting the presence of ARCH effects in the time series. We filter the square residual by a GARCH(4,1) process, removing completely the ARCH effects.

	ARMA (1,2) model		ARMA(1,2)/GARCH(4,1)	
	Ljung-Box_Pierce Test	ARCH test	Ljung-Box_Pierce Test	ARCH test
1	10.44***	10.41***	0.02	0.02
2	27.9***	25.56***	0.02	0.02
3	47.75***	39.93***	0.04	0.04
4	95.63***	74.94***	1.57	1.56
5	115.63***	83.15***	2.17	2.15
6	134.8***	88.85***	3.31	3.27
7	142.31***	88.93***	5.23	5.13
8	149.67***	88.92***	6.85	6.79
9	181.25***	100.95***	7.65	7.74
10	198.62***	104.77***	8.03	8.14
11	207.48***	105.32***	8.04	8.19
12	217.63***	106.14***	9.06	9.27

(***), (**), (*) denotes statistically significance at a 1%,5% and 10% level, respectively.

Figure A3 - Innovations and Conditional Variance of the returns of the Index DJ Eurostoxx Banks 600 filtered by the ARMA (1,2) process


Table A3– ARCH diagnosis (returns of the iTrax European Financial SNR filtered by the ARMA(1,2) process)

With the aim of examining the presence of autoregressive conditional variance in the iTrax European Financial SNR filtered returns, the ARCH test and the Ljung-Box-Pierce Test of the square residual is performed. Both tests point toward the presence of ARCH effects in the time series. We filter the square residual by a GARCH(1,1) process, removing completely the ARCH effects.

	ARMA (1,2) model		ARMA(1,2)/GARCH(1,1)	
	Ljung-Box_Pierce Test	ARCH test	Ljung-Box_Pierce Test	ARCH test
1	3.47*	3.46*	2.15	2.14
2	64.02***	62.42***	2.54	2.46
3	79.08***	72.83***	2.54	2.45
4	139.07***	108.85***	4.87	4.75
5	158.09***	116.28***	8.48	8.8
6	170.01***	116.41***	8.5	8.79
7	179.95***	116.68***	9.11	9.52
8	186.82***	116.62***	9.14	9.51
9	202.93***	119.89***	9.49	9.71
10	205.36***	119.82***	10.63	10.93
11	222.8***	124.36***	12.72	12.94
12	227.02***	124.71***	12.82	13.2

(***), (**), (*) denotes statistically significance at a 1%,5% and 10% level, respectively.

Figure A4 – Innovations and Conditional Variance of the returns of the iTraxx Financial SNR filtered by the ARMA (1,2) process
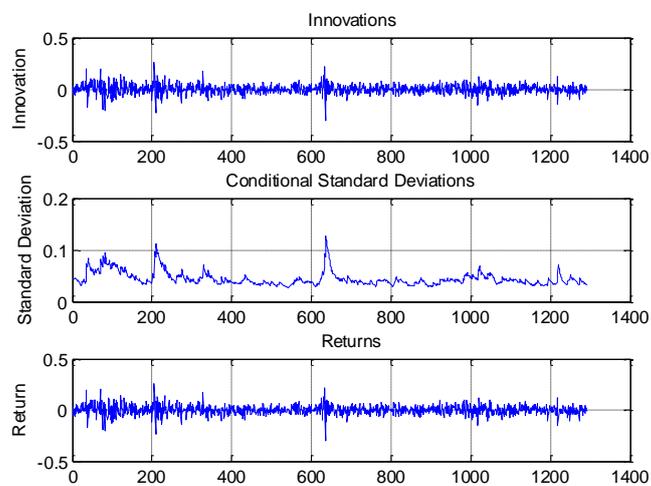
Table A4–BDS test and Run's test on filtered returns

	BDS p-value	RUN test p-value
Stocks	0.869	0.151
CDS SNR	0.979	0.139