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(In)Determinacy, Bargaining, and R&D Policies in an Economy with Endogenous Technological Change

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Abstract
In this paper, the author shows how the introduction of a bargaining game structure into a standard R&D endogenous growth model can be a potential source of local indeterminacy. He also shows that on a high-growth path, the government, by directly engaging in R&D activities and using R&D subsidies, may not enhance economic growth. On a low-growth path, the government, by directly engaging in R&D activities and using R&D subsidies, may enhance economic growth.

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Keywords Government R&D; innovation; endogenous growth; bargaining; indeterminacy

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1. Introduction

In endogenous growth models, especially the AK growth model, indeterminacy can take the form of multiple balanced growth paths along which the economy can persistently grow in the long run, see Benhabib and Farmer (1999), Benhabib and Rustichini (1994) and Benhabib and Gali (1995) survey the literature on indeterminacy in macroeconomics. Lucas (1993) indicates why two different countries would, such as South Korea and the Philippines, whose initial conditions were so close, differ so much in their later growth performance? Indeterminacy may explain why fundamentally similar economies can exhibit the same per capita income but grow at different rates. It is well know in literature that imperfect competition (Gali and Zilibotti, 1995; etc.), externalities (Chamley, 1993; Benhabib and Perli, 1994; and Boldrin and Rustichini, 1994; etc.), investment adjustment costs (Lai and Chin, 2010; Guo et al., 2009; etc.) and other government policies (Raurich, 2001; and Park and Philippopoulos, 2004; etc.) are sources of indeterminacy. The present model is highly related to a number of existing R&D-based growth models that exhibit indeterminacy. See, for example, Haruyama and Itaya (2006) and Arnold and Kornprobst (2008), show that indeterminacy may arise in R&D-based growth models when the elasticity of intertemporal substitution is greater than one. See also Chen and Chu (2010) relates to the literature on patent policy and economic growth. However, they do not include vertically connected imperfect competitive market structure. The present model is either imperfect competition in intermediate goods market or imperfect competition in final goods market. In a vertically connected imperfect competitive market, for example, expanding variety model of R&D in upstream industry and monopolistic competition in downstream industry, there is no literature to discuss the interaction between upstream and downstream industries influence the economic fundamental and then presents the indeterminate equilibrium. Therefore, this paper investigates the possibility of multiplicity of BGPs, in conjunction with indeterminacy of transitional dynamics, when it proceeds to bargaining between final goods and intermediate goods firms with endogenous technological change that leads to long-run growth.

Negotiating with each other, upstream and downstream firms, for their own interests is feasible. Bester (1993) mentions that “in many markets prices are the outcome of bilateral negotiations, so that both the seller and the buyer take an active part in setting the price. Examples include not only the bazaar of a less developed nation, but also the market for used cars, real estate, antiques, and inputs for manufacturing firms.”. Some literatures investigate contract bargaining in a vertically connected market structure, such as Villas-Boas (2007) used data on yogurt sold in a large urban area of the US. His results imply that wholesale prices are close to

marginal cost and retailers have great pricing power in this vertical chain. This is consistent with the non-linear pricing scheme of manufacturers or retailers with high bargaining power. Lee (2005) applies a bargaining model to estimate a settlement rate in the international telephone industry. He uses annual traffic data of forty bilateral markets from 1988 to 1995. Tirole (1998) stated the traditional franchise contract that upstream firm offers the contract and the downstream firm accepts the contract. In real life, however, it is often the case that the bargaining contract itself is not completely exogenous. People may have to choose one protocol in the process of reaching an agreement. Unlike Rubinstein (1982) which is an important noncooperative bargaining model, this paper introduces an equally popular and important bargaining which is the Nash (1950) bargaining model to analyze the franchise contract bargaining between the intermediate goods firms and the final goods firm. Since final goods firms prefer negotiating to lower the price of intermediate goods for reducing cost. On the other hand, intermediate goods firms prefer extracting more rent from the downstream industry. They firms all have incentives to progress contract bargaining. The present R&D-based growth model, only one literature (Wang et al., 2010), investigates contract bargaining between final and intermediate goods producers by extending the Grossman and Helpman (1991). But they lack dynamic analysis and the role of government for R&D activities in a complete macro model. This paper follows the bargaining structure of Wang et al. (2010) in successively imperfect competition market, and to investigate the bargaining between final and intermediate goods firms may cause local dynamic indeterminacy.

The role of the government cannot be ignored in the endogenous growth model, and accordingly in the 1990s there was an explosion of research on the growth effects of several government activities. In the R&D-driven endogenous growth models, Romer (1990), Aghion and Howitt (1992), and Barro and Sala-i-Martin (2004) all find that R&D subsidies encourage firms to devote more resources to R&D activities and as a result there is an increasing rate of economic growth in the long run. Jones and Williams (1998, 2000) point out that the decentralized economy typically under-invests in R&D when compared to what is socially optimal when using data for the US economy. Because of monopoly pricing and knowledge spillovers may result in too little private R&D, the present model has focused on subsidies for R&D, for example, Segerstrom (2000), and Zeng and Zhang (2007). They discuss the role of government policy in the field of the R&D-based endogenous growth model. Besides subsidy to R&D, the government also engages in R&D activities. For example, the economic development in Taiwan the government has played a leading role in investing in science and technology R&D such as establishing public research

organizations (i.e., Academia Sinica, and Industrial Technology Research Institute). The institutes research the blueprint, technology, or new production process in many fields, and transfer them to private industries for producing the new products. In addition, Glomm and Ravikumar (1994) present a model in which the economy grows thanks to public research. Pelloni (1997) allows the government to invest in public research so as to improve the growth performance of the economy. Park (1998) indicates that the share of research performed by the government varies across countries is generally higher among smaller R&D nations, and introduces public research in the model of expanding variety of products in Romer (1990) to analyze the impact of government research on long run growth. Morales (2004) finds the basic research performed at public institutions have unambiguously positive effects on growth, and performing applied research at public institutions could have negative growth effects. For this reason, we introduce not only R&D subsidy policy but also the government's R&D activities for too few private R&D activities in a decentralized economy to show the role of government in economic development in the country.

This paper focuses on the financial resources that come in the form of subsidies out of government revenue. The type of government revenue that we consider is a specific tax which imposed on both final goods and intermediate goods to finance the subsidies and expenditure on R&D activities. This is because the ad valorem taxation (a tax proportional to the firm's revenue/profit) leads to the lower consumer price of a good even though firms would exit the market in a monopolistic competition case (Schröder, 2004). It is well known that the number of intermediate goods firms in monopolistic competition market is a key point in R&D-driven endogenous growth models because the more firms there are in the intermediate goods market, namely, the more variety there is, the more the economy grows. If the ad valorem taxation leads the firms to exit the market, it is harmful to the economic growth. On the other hand, Kitahara and Matsumura (2006) investigate how a specific tax and an ad valorem tax affect equilibrium location choice in a model of product differentiation which includes Hotelling and Vickrey-Salop spatial models. They find that the specific tax affects neither of the firms' equilibrium location, output quantity, nor profits. Therefore, a specific tax is a good tool for government to finance the revenue in an R&D growth economy. Hence, for the R&D-driven endogenous growth model we introduce the specific tax to analyze how the government's R&D policies (government engages in R&D activities and subsidizes the R&D cost of the firms) affect the rate of economic growth.

We present a four-stage model. In the first stage, the government levies specific taxes on final goods and intermediate goods to finance the government expenditure, to engage in R&D activities and to subsidize the R&D costs of the firms. In the second

stage, the final goods firms and the intermediate goods firms bargain over the franchise contract including over the franchise fee and the price of the intermediate goods according to Nash efficient bargaining. In other words, the upstream and downstream industries will vertically integrate to eliminate the double marginalization through the franchise contract. In the third stage, the final goods firms determine the prices of the final goods to maximize their profits. In the fourth stage, the consumers decide the expenditure plan to maximize their utility. We proceed by solving the model backward.

2. The model

The model is an extension of the endogenous growth model with the increasing variety model of Grossman and Helpman (1991, chapter 3) and the bargaining structure of Wang et al. (2010) in successively imperfect competition market. We consider the government not only implements a tax/subsidy policy but also engages in R&D activities and an imperfectly competitive final goods market. There are five agents in this model, R&D firms, the intermediate goods firms, the final goods producers, the government and the household. In this model, R&D investment creates new types of intermediate goods for final production. The price of intermediate goods is determined by the negotiation between the intermediate goods firms and final goods firms. The government levies a specific tax to finance the subsidy for too little R&D and engages in R&D activities. The household chooses a consumption/investment plan.

2.1 R&D

R&D technology is such that, to develop a new idea, a researcher needs a quantity of labor to develop ideas. The production function in the R&D sector is given by

$$\dot{n} = nL_A \tag{1}$$

where L_A is the amount of labor hired in the R&D sector which is from the R&D firms (L_R) and the government sector (L_G), both the government and private firms are engaged in R&D, $L_A = L_R + L_G = vL_A + (1-v)L_A$, v is the proportion of labor employed in the R&D sector between the R&D firms and the government, $\dot{n}$ is the number of new blueprints created for a given period of time, and n refers to the positive spillovers in the production of blueprints. The more workers the R&D sector employs or the more varieties of goods the intermediate goods market has, the more new blueprints are produced per unit of time.[1]

The research sector's after-subsidy profit flow is given by

$$\pi_A = p_A\dot{n} - (1-s)wvL_A \tag{2}$$

[1] To simplify our notation, the time arguments will all be dropped.

Where p_A is the after-subsidy cost or price of a new blueprint $\dot{n}$. s is a fraction of all research expenses paid by the government. w is the wage rate which is common to all sectors in the economy since labor is assumed to be perfectly mobile. Such a subsidy to R&D lowers the private cost.

2.2 Intermediate goods market

The typical intermediate firm produces its differentiated goods with a technology that requires one unit of labor per unit of intermediate goods ($x_i = l_i^x$). Each intermediate goods firm produces and sells a slightly unique variety of goods x_i to each final goods firm to maximize its profit since the good is protected by an infinitely-lived patent, taking the actions of all other producers in the intermediate goods sector as given

$$\pi_i = m(p_i^x x_i - w l_i^x) + m f_i \tag{3}$$

where l_i^x is the amount of labor used by firm i, p_i^x is the price of intermediate goods, m represents the number of final goods firms, and f_i is the franchise fee received from the final goods firm.[2]

2.3 Final goods

We consider a production economy with imperfectly competitive product markets. The consumption goods are produced by monopolistically competitive firms. Each consumption good is supposed to be produced by a single firm, that is, m also represents the number of firms which produce industry j goods. Therefore, a composite final good Y can be represented as

$$Y \equiv m\left(\frac{1}{m}\int_{j=0}^{m} y_j^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where σ is the constant elasticity of substitution. Each firm produces y_j by using a continuum of intermediate goods x_i. According to Dixit and Stiglitz (1977), the production function of firm j is

$$y_j \equiv \left(\int_0^n x_{ij}^{\frac{1}{\alpha}} di\right)^{\alpha}, \quad \alpha > 1 \tag{5}$$

where $i \in [0, n(t)]$ is the range of intermediate goods existing at time t. $-1/(1-\alpha)$

[2] To simplify the analysis, we assume that the fee paid to intermediate goods firms is identical to those in all contracts.

represents the elasticity of substitution between intermediate goods.

The producer j in the final goods sector chooses a price to maximize its profit

$$\Pi_j = q_j\left(\int_0^n x_{ij}^{\frac{1}{\alpha}} di\right)^{\alpha} - \int_0^n \hat{p}_{ij} x_{ij} di - \int_0^n f_{ij} di \tag{6}$$

Where q_j is the price of the final goods, $\hat{p}_{ij}$ is the after-tax price of the intermediate goods i, and f_{ij} represents the franchise fee that the final goods producer j has to pay to the intermediate goods firm in order to obtain the right and know-how to produce the final good by using these intermediated goods.

We assume that the government levies a specific tax on each final good and intermediate good, and that each tax is symmetric over time for analytical simplicity. The consumption goods price and the intermediate goods price become

$$p_j = q_j + \tau_y \tag{7}$$

$$\hat{p}_{ij} = p_{ij}^x + \tau^x \tag{8}$$

where p_j is the after-tax price of consumption good j. τ_y represents the specific tax imposed on the final goods and is the same for all j. τ^x represents the specific tax imposed on the intermediate goods i and is the same for all i.

2.4 Government

The government cannot borrow and thus satisfies the budget constraint

$$T_y + T^x = S + G \tag{9}$$

where $T_y = m\tau_y\left(\int_0^n x_{ij}^{\frac{1}{\alpha}} di\right)^{\alpha}$ and $T^x = m\tau^x \int_0^n x_i di$ are total tax revenues from final goods and intermediate goods markets, $S = vswL_A$ is the subsidy to defray the R&D cost of the firms, and $G = (1-v)wL_A$ is government expenditure to employ labor in the R&D sector. In considering the decomposition of government expenditures from the final and intermediate goods firms, T_y, and T^x, we assume

$$T_y = g(1 - v(1-s))wL_A \tag{10}$$

$$T^x = (1-g)(1 - v(1-s))wL_A \tag{11}$$

where the parameter $0 < g < 1$ is the share of government expenditure financed by

tax revenues from the final goods market and $1-g$ is the share of the government expenditure financed by tax revenues from the intermediate goods market. Since we like to analyze that once the government controls the R&D policies parameters, how the economy performance response. We consider that the parameters (g, s, v) are fixed and the vector of tax rates must adjust endogenously.[3] This will allow our results to easily show how the government's R&D subsidy policy and the government's R&D activities affect the dynamics of growth.

2.5 Households

The individuals inelastically supply labor service, L. Consumption loans in competitive labor and imperfectly competitive product markets. The representative household's preferences are defined over an infinite horizon

$$V = \int_0^{\infty} e^{-\rho t} U(C) dt \tag{12}$$

where

$$U(C) = \ln C \tag{13}$$

$$C \equiv m\left(\frac{1}{m}\int_{j=0}^{m} c_j^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 \tag{14}$$

Eqs. (12) and (13) indicate that utility is a unitary elasticity function and is discounted by a constant pure rate of time preference ρ. C is a composite consumption good which consists of a bundle of closely-related product varieties according to Eq. (14). This type of monopolistic competition CES functional form follows Dixit and Stiglitz (1977), m is the number of different varieties and is taken as exogenous for simply rescale utility, and c_j is a consumption good of variety j. $j \in [0, m]$ represents the varieties produced by different downstream firms.

The budget constraint, which describes how the household invests the new assets, is equal to the rate of return r earned on assets and total labor income plus the profit the household receives from the downstream firms minus total spending on consumption goods. It is therefore given by

$$\dot{a} = ra + wL + m\Pi - E \tag{15}$$

where

$$E = PC = \int_{j=0}^{m} p_j c_j dj \tag{16}$$

E is total spending on consumption goods, and P is the aggregate consumption

[3] Refer to Zeng and Zhang (2007, Journal of Economic Dynamics & Control), and Peretto (2007, Journal of Economic Theory).

price index. a is the household assets which is the value of the stock of the blueprints, $a = p_A n$ and $\dot{a} = p_A \dot{n} + n\dot{p}_A$.

Therefore, the budget constraint may be rewritten as

$$p_A \dot{n} + n\dot{p}_A = rp_A n + wL + m\Pi - PC \tag{17}$$

3. The market solution

The household chooses its consumption levels for each available product variety, c_j, in order to maximize the utility of Eq. (13), given the definition of composite consumption in Eq. (14) and the budget constraint in Eq. (16). The solutions for the consumption of variety j are obtained:

$$c_j = m^{-1}(\frac{p_j}{P})^{-\sigma} C \tag{18}$$

where

$$P = \left(m^{-1} \int_0^m p_j^{\,1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \tag{19}$$

Eq. (18) gives the downward-sloping demand curve for goods c_j which is faced by the final goods firms. Eq. (19) expresses the average price of the consumption goods.

Since the market will be symmetric in the various intermediates i, we have $x_{ij} = x_j$, $p_{ij}^x = p_j^x$, $\forall i$, in equilibrium associating with $y_j = c_j$. The final goods firms maximize the profit in Eq. (6) subject to the demand function in Eqs. (7), (8), (18), the typical final goods firm j will charge a monopolistic markup price to the consumers as follows

$$q_j = \frac{\sigma[n^{1-\alpha}(p_j^x + \tau^x)] + \tau_y}{\sigma - 1} \tag{20}$$

Nash bargaining solution: The firm j producing final goods and firm i producing intermediate goods bargain over the franchising contract (p^x , f) simultaneously.

The division of the rent between firm j producing final goods and firm i producing intermediate goods, using Eqs. (3), (6) and subject to Eqs. (5), (7), (8), (20), is obtained by maximizing the following Nash product

$$\max_{p^x, f} N = (\Pi_j - \Pi_0)^{\theta} (\pi_i - \pi_0)^{1-\theta} \tag{21}$$

Π_0 is the profit of firm j which is constant when the bargaining breaks down,

namely, the minimum profit of the final goods firm. π_0 is the profit of firm i which is constant when the bargaining breaks down, namely, the minimum profit of the intermediate goods firm. That is to say, if the bargaining breaks down, the final goods and intermediate goods firms will mark up their prices by marginal cost, respectively.[4] θ describes the bargaining power of firm j and lies in the interval $(0,1)$. With $\theta \to 0$, the model indicates that the intermediate goods firm i has full bargaining power to decide the intermediate goods price completely. To keep the analysis simple, we assume an identical bargaining power for all final goods firms with decentralized status. The same is true for all of the intermediate goods firms.

The decentralized bargaining means that all bargains take place simultaneously and the bargaining partners take all other intermediate goods prices and franchise fees as given.

According to the Nash bargaining solutions that are derived by maximizing Eq. (21), firm j and firm i select an optimal franchise fee and intermediate price as follows

$$p^x = w \tag{22}$$

$$f = \theta \frac{\pi_0}{m} + \frac{(1-\theta)}{n}\left[\frac{1}{\sigma-1}((w+\tau^x)+\tau_y n^{\alpha-1})\right] m^{-1} n^{1-\alpha} Y - (1-\theta)\frac{\Pi_0}{n} \tag{23}$$

Eqs. (22) and (23) describe the optimal bargaining contract in a vertically connected imperfectly competitive market structure. As the same as Wang et al. (2010), Eq. (22) is the pricing rule for intermediate goods, resulting from competition between the final goods firm and the intermediate goods firm, with both firms simultaneously engaging in optimization. The bargaining contract in our model is unlike the traditional franchise contract, in which the final goods firm does not have any bargaining power to determine the contract's content. The prices of the intermediate goods are equal to marginal cost which is unrelated to the bargaining power. This is a Nash efficient result. Because the aggregate rent/franchise fee is maximized by setting the prices of the intermediate goods equal to their marginal cost, this result is interpreted as stemming from the negotiations between the intermediate goods firm and the final goods firm or the competition between the upstream and downstream industries. They obtain the maximum aggregate rent at first and then extract the extra rent, respectively, according to their bargaining power through the franchise fee. This result, which characterizes the interaction of firms in this market structure, reflects the economic consequence that double marginalization does not occur. This is a vertical integration outcome through franchise contract bargaining. Unlike traditional models, in this paper the prices of intermediate goods are determined by negotiation, the intermediate goods firms charge a price based on marginal cost and not on markup to

[4] See Appendix A.

the final goods firms and then extract the profit through the franchise fee (Eq. (23)). Since vertical integration takes place, an inelasticity demand function of intermediate goods appears. Inside the square brackets on the right-hand side of Eq. (23) is the corporate income of firm j per unit of final good. The optimal franchise fee depends on the bargaining power θ. Firm i will extract all the rent if firm j has no bargaining power ($\theta \to 0$). Similarly, the rent will vanish if firm i has no bargaining power ($\theta \to 1$).

Substituting Eqs. (20) and (22) into the aggregate consumption price index, Eq. (19), is given respectively as

$$P = \frac{\sigma}{\sigma - 1}((w + \tau^x) + \tau_y n^{\alpha - 1})n^{1-\alpha} \tag{24}$$

Then, by substituting the results, Eqs. (18)-(20), (22)-(24) associate with (7)-(8), into Eqs. (6) and (3), the profits can be written as

$$\Pi = \left[\theta Y - (\theta - \frac{(1-\theta)\sigma}{\sigma - 1})Y_0\right]\frac{1}{m}\frac{1}{\sigma - 1}((w + \tau^x) + \tau_y n^{\alpha - 1})n^{1-\alpha} \tag{25}$$

$$\pi = \left[(1-\theta)Y + (\theta - \frac{(1-\theta)\sigma}{\sigma - 1})Y_0\right]\frac{1}{n}\frac{1}{\sigma - 1}((w + \tau^x) + \tau_y n^{\alpha - 1})n^{1-\alpha} \tag{26}$$

where the subscript 0 denotes the value of the bargaining breakdown.[5]

In R&D sector, by substituting the production function, Eq. (1), into Eq. (2) and due to the property of perfect competition in the R&D sector ($\pi_A = 0$), the blueprint cost or value is as follows

$$p_A = \frac{v(1-s)w}{n} \tag{27}$$

Eq. (27) indicates that the value of the blueprint is equal to its cost.

Anyone can have free entry into the business of being an inventor as long as the R&D cost secures the net present value of the profit in intermediate goods,[6] we obtain

$$r = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A} \tag{28}$$

Eq. (28) is a non-arbitrage condition which states that the rate of return on bonds, r, equals the rate of return to investing in R&D. The R&D rate of return equals the profit

[5] Since we would like to analyze the macro-economy, we assume that the numbers of firms in the intermediate goods and final goods markets are the same for the economics of the bargaining and the breakdown in negotiations, $m_0 = m$, $n_0 = n$. Furthermore, the R&D activities take place in the first period of the game structure, namely, the government's expenditure on R&D activities takes place first. We then assume that $\tau_0^x = \tau^x$, $\tau_{y0} = \tau_y$.

[6] Differentiating the free entry condition ($p_A = \int_t^\infty \pi(\omega)e^{-\int_0^\omega r(\upsilon)d\upsilon}d\omega$) with respect to time.

rate, π/p_A, plus the rate of capital gain or loss, $\dot{p}_A/p_A$.

The government's budget constraint rewritten as

$$m\tau_y n^{\alpha-1}nx + m\tau^x nx = (1-v(1-s))wL_R \tag{29}$$

where

$$m\tau_y n^{\alpha-1}nx = g(1-v(1-s))wL_A \tag{30}$$

$$m\tau^x nx = (1-g)(1-v(1-s))wL_A \tag{31}$$

The representative household chooses the optimal consumption and investment plan to maximize its discounted utility, Eq. (12), subject to the budget constraint, Eq. (17). The familiar Euler equation derived from the household's intertemporal optimization is

$$\frac{\dot{C}}{C} = r - \rho - \frac{\dot{P}}{P} \tag{32}$$

To determine the aggregate dynamics of this economy, we have to find the equilibrium on the labor market and the final goods market. The labor market equilibrium condition states that total labor demand is equal to total labor supply, i.e., the optimal allocation of the given supply of labor (L) to the three sectors, $L_x + L_G + L_R = L$, and that labor is perfectly mobile across the intermediate goods sector and the blueprint industry. Since the quantity of labor allocated to the intermediate goods sector is $L_x = mnl^x$ and that allocated to the R&D industry is $L_A = \dot{n}/n$, the labor market equilibrium condition will be rewritten as

$$L_x + \frac{\dot{n}}{n} = L \tag{33}$$

Next, by combining Eq. (14) with $c = y$ and $x = l^x$, and considering the clearing condition for the final goods market in the symmetric equilibrium, we have

$$C = Y = n^{\alpha-1}L_x \tag{34}$$

Eq. (34) is the resource constraint of the economy (see Appendix B).

3.1 Dynamics

Eqs. (24)-(34) fully define the dynamics of the economy. Since we like to analyze the government policies and activities with regard to R&D, we assume that a vector of tax rates (τ_y, τ^x) is endogenous. Using Eq. (34) and the labor market equilibrium condition ($L_A = L - L_x$), Eqs. (30) and (31) may be rewritten as

$$\tau_y = \frac{g(1-v(1-s))w(L-L_x)}{n^{\alpha-1}L_x} \tag{35}$$

$$\tau^x = \frac{(1-g)(1-v(1-s))w(L-L_x)}{L_x} \tag{36}$$

Substituting Eqs. (35) and (36) into Eq. (24), we obtain

$$P = \frac{\sigma}{\sigma-1}\frac{L_x+(1-v(1-s))(L-L_x)}{L_x}n^{1-\alpha}w \tag{37}$$

By multiplying Eq. (37) by $v(1-s)/n$, we obtain

$$p_A = \frac{v(1-s)}{n}\frac{\sigma-1}{\sigma}\frac{L_x n^{\alpha-1}}{L_x+(1-v(1-s))(L-L_x)}P \tag{38}$$

Differentiating Eq. (38) with respect to time

$$\frac{\dot{p}_A}{p_A} = (\alpha-2)\frac{\dot{n}}{n}+\frac{\dot{L}_x}{L_x}+\frac{\dot{P}}{P}-\frac{v(1-s)\dot{L}_x}{L_x+(1-v(1-s))(L-L_x)} \tag{39}$$

Substituting Eqs. (24), (34)-(36) into Eq. (26) and dividing by Eq. (27), we obtain

$$\frac{\pi}{p_A} = \frac{1}{(\sigma-1)v(1-s)}\left[(1-\theta)L_x+(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}\right]\frac{L_x+(1-v(1-s))(L-L_x)}{L_x} \tag{40}$$

where L_{x0} which is constant is the quantity of labor employed in the intermediate goods market in a successively imperfectly competitive economy.[7]

Substituting Eqs. (28), (39) and (40) into Eq. (32), we obtain

$$\begin{aligned}\frac{\dot{C}}{C} &= \frac{1-\theta}{\sigma-1}L_x+\frac{(1-\theta)(1-v(1-s))}{(\sigma-1)v(1-s)}L+\frac{1}{\sigma-1}(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}\\ &+\frac{1-v(1-s)}{(\sigma-1)v(1-s)}(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}\frac{L}{L_x}+(\alpha-2)\frac{\dot{n}}{n}+\frac{\dot{L}_x}{L_x}\\ &-\frac{v(1-s)\dot{L}_x}{(1-v(1-s))L+v(1-s)L_x}-\rho\end{aligned} \tag{41}$$

Differentiating Eq. (34) with respect to time

$$\frac{\dot{C}}{C} = (\alpha-1)\frac{\dot{n}}{n}+\frac{\dot{L}_x}{L_x} \tag{42}$$

Proposition 1. *There is a necessary and sufficient condition that leads the economy to indeterminacy. A high equilibrium and a low equilibrium in the economy will take place. (i) if final goods firm has no bargaining power* $\theta \to 0$*, then there exists a unique but unstable equilibrium. (ii) if the final goods firm's bargaining power is greater than intermediate goods firm's multiply by markup* $\theta > (1-\theta)\sigma/(\sigma-1)$*, then there exist two equilibria, one is unstable and determinate and the other is stable and indeterminate.*

[7] See Appendix A.

Proof. From Eqs. (41), (42) and (33), we find the dynamic equation for L_x

$$\frac{(\sigma-1)L_x v(1-s)\dot{L}_x}{(1-v(1-s))L+v(1-s)L_x}$$
$$=(\sigma-\theta)L_x{}^2+\left[\frac{(1-\theta)-(\sigma-\theta)v(1-s)}{v(1-s)}L+(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}-(\sigma-1)\rho\right]L_x$$
$$+\frac{1-v(1-s)}{v(1-s)}(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}L \tag{43}$$

Assume that $\Omega=\left[\frac{(1-\theta)-(\sigma-\theta)v(1-s)}{v(1-s)}L+(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}-(\sigma-1)\rho\right]$, and $\Gamma=\frac{1-v(1-s)}{v(1-s)}(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}L$.

In the steady state $\dot{L}_x=0$, we obtain

$$\tilde{L}_x=\frac{-\Omega\pm\sqrt{\Omega^2-4(\sigma-\theta)\Gamma}}{2(\sigma-\theta)} \tag{44}$$

Eq. (44) indicates that the economy exhibits an indeterminate solution if $\Omega<0$, $\Gamma>0$. The necessary and sufficient conditions are as follows

$$\frac{(1-\theta)-(\sigma-\theta)v(1-s)}{v(1-s)}L+(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}-(\sigma-1)\rho<0 \tag{45}$$

$$\frac{1-v(1-s)}{v(1-s)}(\theta-\frac{(1-\theta)\sigma}{\sigma-1})L_{x0}L>0 \tag{46}$$

If final goods firm has no bargaining power $\theta\to 0$, Eq. (46) is always negative ($\Gamma<0$), thus eliminating the indeterminacy and there exists a unique but unstable equilibrium. If the final goods firm's bargaining power is greater than intermediate goods firm's multiply by markup $\theta>(1-\theta)\sigma/(\sigma-1)$, then there exist two equilibria, one is unstable and determinate and the other is stable and indeterminate. Besides, to satisfied Eq. (45) the more firm's R&D activities v has or the less R&D subsidy policy s has, the more possibility indeterminacy occur. In addition, when elasticity of substitution σ is large, Eqs. (45) and (46) are easy to satisfied, then dual equilibria emerge. For example, if final goods market is too perfect competition, then the economy represents indeterminacy.

According to Eq. (43), the first-order condition and second-order condition are as follows

$$\frac{\partial\dot{L}_x}{\partial L_x}=2(\sigma-\theta)L_x+\Omega\overset{>}{\underset{<}{}}0 \tag{47}$$

$$\frac{\partial^2 \dot{L}_x}{\partial L_x{}^2} = 2(\sigma - \theta) > 0 \tag{48}$$

Eqs. (47) and (48) imply that there are two equilibria for $\tilde{L}_x$ in the steady state. One is stable, namely, the low equilibrium ($\breve{L}_x$), and the other is unstable, namely, the high equilibrium ($\widehat{L}_x$).

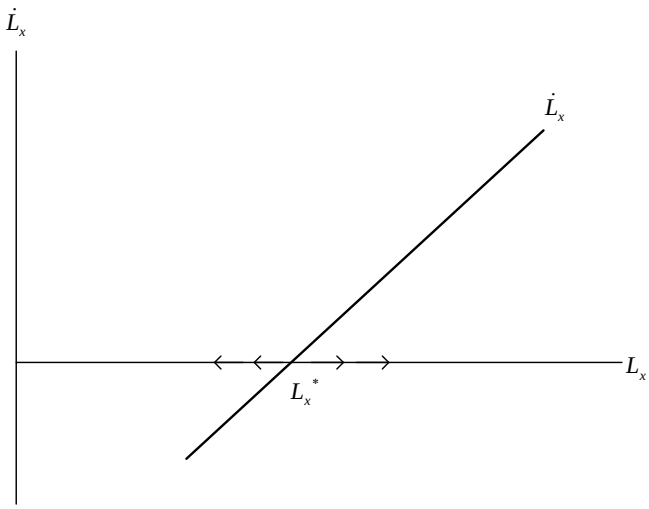

Figure 1. A unique and determinate equilibrium; $\theta \to 0$

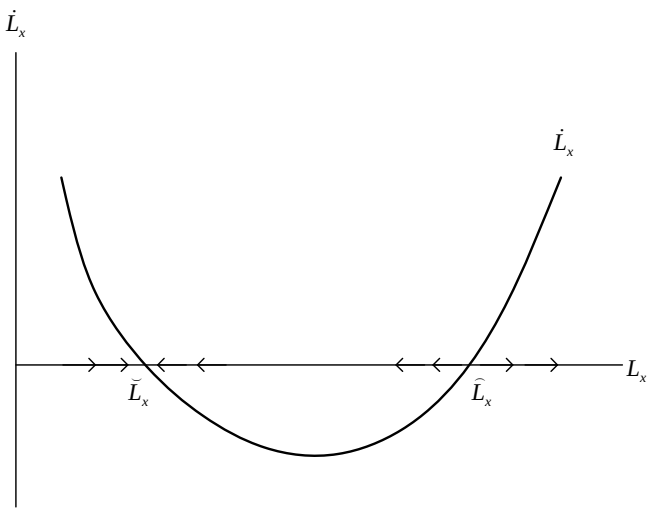

Figure 2. Two and indeterminate equilibria; $\theta > (1-\theta)\dfrac{\sigma}{\sigma - 1}$

Figures 1 and 2 provide a graphical illustration of Proposition 2. Figure 1 depicts case (i) where there is only one unstable and hence globally determinate steady-state equilibrium, while Figure 2 presents the case where there are two steady-state

equilibria. Point $\breve{L}_x$ represents a locally stable and indeterminate equilibrium. Point $\widehat{L}_x$, on the other hand, is a locally unstable and hence determinate equilibrium.

3.2 Steady states

In the generalized case, we showed that there can be two long-run equilibria, and both of them can be locally determinate or indeterminate. Thus, we cannot exclude any of them on the grounds of stability. That is to say, the conditions, Eqs. (45) and (46), are satisfied. In the steady state $\dot{L}_x = 0$, and by totally differentiating Eq. (43), the results of the comparative static state are as follows

$$\frac{\partial \tilde{L}_x}{\partial \theta} = \frac{1}{2(\sigma-\theta)\tilde{L}_x + \Omega} \times (\tilde{L}_x + \frac{1-v(1-s)}{v(1-s)} L) \times \left[\tilde{L}_x - (1+\frac{\sigma}{\sigma-1}) L_{x0} \right] \begin{matrix} > \\ < \end{matrix} 0 \tag{49}$$

$$\frac{\partial \tilde{L}_x}{\partial s} = \frac{-1}{2(\sigma-\theta)\tilde{L}_x + \Omega} \times \frac{(1-\theta)L}{v(1-s)^2} \times$$

$$\left[\tilde{L}_x - (\frac{\theta}{1-\theta} - \frac{\sigma}{\sigma-1}) \frac{1}{\sigma} \tilde{L}_x - (\frac{\theta}{1-\theta} - \frac{\sigma}{\sigma-1}) \left(\frac{1-v(1-s)}{v(1-s)\sigma} L - L_{x0} \right) \right] \begin{matrix} > \\ < \end{matrix} 0 \tag{50}$$

$$\frac{\partial \tilde{L}_x}{\partial v} = \frac{1}{2(\sigma-\theta)\tilde{L}_x + \Omega} \times \frac{(1-\theta)L}{v^2(1-s)} \times$$

$$\left[\tilde{L}_x - (\frac{\theta}{1-\theta} - \frac{\sigma}{\sigma-1}) \frac{1}{\sigma} \tilde{L}_x - (\frac{\theta}{1-\theta} - \frac{\sigma}{\sigma-1}) \left(\frac{1-v(1-s)}{v(1-s)\sigma} L - L_{x0} \right) \right] \begin{matrix} > \\ < \end{matrix} 0 \tag{51}$$

The signs of Eqs. (49)-(51) depend on the sign of $2(\sigma-\theta)\tilde{L}_x + \Omega$.[8]

From Eq. (33), in the steady state the growth rates of innovation depend on the state of $\tilde{L}_x$ such that

$$\breve{\gamma}_n = L - \widehat{L}_x \tag{52}$$

otherwise

$$\widehat{\gamma}_n = L - \breve{L}_x \tag{53}$$

[8] In Eq. (49), the third term on the right hand side is positive since the equilibrium exists a subgame perfection. And in Eqs. (50) and (51), we assume that the third term on the right hand side is positive for analysis the R&D policies' effects.

where $\gamma_n = \dot{n}/n$. Eq. (52) denotes the low balanced equilibrium growth rate of innovation and Eq. (53) the high balanced equilibrium growth rate of innovation. Hence, the effects of the government's R&D policies and bargaining power on the balanced equilibrium growth rate depend on Eqs. (49)-(51), that is the sign of $2(\sigma-\theta)\tilde{L}_x + \Omega$.

Proposition 2. *The effects of the government direct expenditure on R&D activities and subsidies to defray the R&D costs of firms on the economy have entirely different results depend on whether there is a high balanced growth path or a low balanced growth path. In addition, the bargaining power between the firms producing intermediate goods and final goods also has reverse effects on the dual economy.*

Proof. From Eq. (44), we have

$$\tilde{L}_x = \begin{cases} \breve{L}_x, & \text{if } \ 2(\sigma-\theta)\tilde{L}_x + \Omega < 0 \\ \hat{L}_x, & \text{if } \ 2(\sigma-\theta)\tilde{L}_x + \Omega > 0 \end{cases} \tag{54}$$

When the economy is in a low equilibrium where labor is employed by intermediate goods firms, $\breve{L}_x$, the denominator of the first fraction on the right-hand side of Eqs. (49)-(51), is negative, i.e., $2(\sigma-\theta)\tilde{L}_x + \Omega < 0$. On the other hand, when the economy is in a high equilibrium where labor is employed by intermediate goods firms, $\hat{L}_x$, the denominator of the first fraction on the right-hand side of Eqs. (49-(51) is positive, i.e., $2(\sigma-\theta)\tilde{L}_x + \Omega > 0$. Hence, the effects of the exogenous parameters are entirely reversed between the high balanced growth path economy and low balanced growth path economy.

Differentiating Eq. (52) with respect to θ, s, and v, and associate with Eqs. (49)-(51), we obtain

$$\text{sign}(\frac{\partial \breve{\gamma}_n}{\partial \theta}) = -\text{sign}(\frac{\partial \hat{L}_x}{\partial \theta}) = \text{sing}(2(\sigma-\theta)\tilde{L}_x + \Omega) \tag{55}$$

$$\text{sign}(\frac{\partial \breve{\gamma}_n}{\partial s}) = -\text{sign}(\frac{\partial \hat{L}_x}{\partial s}) = -\text{sing}(2(\sigma-\theta)\tilde{L}_x + \Omega) \tag{56}$$

$$\text{sign}(\frac{\partial \breve{\gamma}_n}{\partial v}) = -\text{sign}(\frac{\partial \hat{L}_x}{\partial v}) = \text{sing}(2(\sigma-\theta)\tilde{L}_x + \Omega) \tag{57}$$

Differentiating Eq. (53) with respect to θ, s, and v, and associate with Eqs.

(49)-(51), we obtain

$$\text{sign}(\frac{\partial \widehat{\gamma}_n}{\partial \theta}) = -\text{sign}(\frac{\partial \breve{L}_x}{\partial \theta}) = -\text{sing}(2(\sigma - \theta)\tilde{L}_x + \Omega) \tag{58}$$

$$\text{sign}(\frac{\partial \widehat{\gamma}_n}{\partial s}) = -\text{sign}(\frac{\partial \breve{L}_x}{\partial s}) = \text{sing}(2(\sigma - \theta)\tilde{L}_x + \Omega) \tag{59}$$

$$\text{sign}(\frac{\partial \widehat{\gamma}_n}{\partial v}) = -\text{sign}(\frac{\partial \breve{L}_x}{\partial v}) = -\text{sing}(2(\sigma - \theta)\tilde{L}_x + \Omega) \tag{60}$$

4. The effects of R&D policies on economic growth

4.1 High balanced growth path economy

When the economy is in a low equilibrium where labor is employed by intermediate goods firms, $\breve{L}_x$, the denominator of the first fraction on the right-hand side of Eqs. (49)-(51) is negative, i.e., $2(\sigma - \theta)\tilde{L}_x + \Omega < 0$. At this time, the economy has a high balanced growth path ($\widehat{\gamma}_n$). Therefore, the effects of the parameters on the economic growth are as follows

$$\frac{\partial \widehat{\gamma}_n}{\partial \theta} = -\frac{\partial \breve{L}_x}{\partial \theta} > 0 \tag{61}$$

$$\frac{\partial \widehat{\gamma}_n}{\partial s} = -\frac{\partial \breve{L}_x}{\partial s} < 0 \tag{62}$$

$$\frac{\partial \widehat{\gamma}_n}{\partial v} = -\frac{\partial \breve{L}_x}{\partial v} > 0 \tag{63}$$

Eq. (61) illustrates that increasing the bargaining power of final goods firms will increase the high balanced growth rate of innovation. Increased bargaining power of a final firm will acquire more profits. This will decrease profits of the intermediate firm, and the intermediate firm will thus employ less labor. Then, labor is transferred to the R&D sector, which stimulates economic growth. That is to say, the final goods firm plays an important role in a high balanced growth path economy in boosting the rate of economic growth. In addition, Eq. (62) indicates that the government's policy of subsidizing the R&D cost of the firms has a negative effect on the high balanced growth rate. In other words, the government raises the rate of the subsidy which will cause the growth rate to slow down. This is because subsidy policy will increase the profit of intermediate goods firm, hence the intermediate goods firm hire more labor to produce, thus decrease the labor to do R&D. The less R&D activities has, the less economy grows. Moreover, if the government directly increases the expenditure on the R&D activities, it will decrease the growth rate, too (Eq. (63)). Therefore, the

government's expenditure on R&D will crowd out the private R&D activities. This result supports real data that OECD (1989) indicates the share of government research has fallen since the mid-1970s.

4.2 Low balanced growth path economy

When the economy is in a high equilibrium where labor is employed by intermediate goods firms, $\widehat{L}_x$, the denominator of the first fraction on the right-hand side of Eqs. (49)-(51) is positive, $2(\sigma-\theta)\widetilde{L}_x+\Omega>0$. At this time, the economy is on a low balanced growth path ($\breve{\gamma}_n$). Therefore, the effects of the parameters on the economic growth are as follows

$$\frac{\partial\breve{\gamma}_n}{\partial\theta}=-\frac{\partial\widehat{L}_x}{\partial\theta}<0 \tag{64}$$

$$\frac{\partial\breve{\gamma}_n}{\partial s}=-\frac{\partial\widehat{L}_x}{\partial s}>0 \tag{65}$$

$$\frac{\partial\breve{\gamma}_n}{\partial v}=-\frac{\partial\widehat{L}_x}{\partial v}<0 \tag{66}$$

Eq. (64) illustrates that an increase in the bargaining power of intermediate goods firms will increase the low balanced growth rate of innovation. That is to say, the intermediate goods firms play an important role in a low balanced growth path economy to enhance the rate of economic growth. When the intermediate goods firms extract more franchise fee from final goods market, there are the more profit provided to R&D activities, thus the more labor to do R&D the more economy grows. In addition, Eq. (65) indicates that the effect of a government's subsidy policy on the R&D cost of the firms in a low balanced growth path economy is positive. In other words, a government that raises the ratio of the subsidy to the R&D cost of the firms will cause the growth rate to speed up. Since subsidy policy will encourage labor to do R&D and decrease the labor hired in intermediate goods market, thus economy grows. Moreover, if the government directly increases its expenditure on R&D activities, it will increase the growth rate, too (Eq. (66)). This result as the same as OECD (1989) illustrate the share of research performed by the government varies across countries is generally higher among smaller R&D nations. The intuition for this result is that the more labor government hired to do R&D, which will transfer some labor in intermediate goods market to R&D sector. Hence, the government plays an important role in enhancing the rate of economic growth, and the policies on R&D activities are helpful in a low balanced growth path economy.

5. Conclusion

By analyzing the implications of R&D policies in an R&D endogenous growth model with a bargaining game structure in the vertically connected imperfect competitive market, we have shown that the bargaining power may give rise to multiple growth paths with global indeterminacy. With regard to R&D policies implications, the government not only subsidizes the R&D cost of the firms but also engages in R&D activities, we have demonstrated how R&D policies can serves a potential tool which can improve growth performance in dual economies.

We find that the economy is characterized by two balanced equilibrium growth rates which comprise a high balanced growth equilibrium and a low balanced growth equilibrium. In a high growth rate economy the government's subsidy policy and the R&D activities will crowd out the private R&D activities, and hence the R&D policies are of no help to the economic growth. This result supports real data that OECD (1989) indicates the share of government research has fallen since the mid-1970s. In other words, the final goods firms play an important role in driving the economic growth, and the stronger the bargaining power of the final goods firms is, the more the economy grows. On the contrary, in a low growth path economy the government that directly engages in R&D activities plays an important role in economic growth. The R&D policies of the government have a positive effect on the economic growth. This result as the same as OECD (1989) illustrates the share of research performed by the government varies across countries is generally higher among smaller R&D nations. In addition, the intermediate goods firms play an important role in driving the economic growth, and the stronger the bargaining power of the intermediate goods firms is, the more economy grows.

This paper finds entirely different effects on a high growth rate economy and a low growth rate economy. In different economies, the government and the firms that manufacture intermediate goods and final goods play different roles in the process of economic growth.

Appendix A

A successively imperfect competitive economy consists of two types of firms – intermediate goods firm and final goods firm. They achieve their maximizing profits respectively by a traditional method described as follows.

Firm j in the final goods market optimizes its production plan. The maximizing profit problem is

$$\max_{x_{ij0}} \Pi_{j0} = q_{j0} y_{j0} - \int_0^{n_{t0}} (p_{ij0}^x + \tau_0^x) x_{ij0} di \tag{A1}$$

subject to Eqs. (4), (7), (18) and $y_{j0} = c_{j0}$. (Note: Subscript 0 denotes the case of

traditional pricing, corresponding to negotiation breakdown. That is, the intermediate goods firm and the final goods firm do not integrate.)

In the symmetric equilibrium, $x_{ij0} = x_{j0}$, $p^x_{ij0} = p^x_{j0}$, $\forall i$, the first-order condition is given by

$$x_{j0} = [(p^x_{j0} + \tau^x_0)n_0^{1-\alpha} + \tau_{y0}]^{-\sigma}(\frac{\sigma}{\sigma-1})^{-\sigma} m_0^{-1} n_0^{-\alpha}(P_0)^{\sigma} C_0 \tag{A2}$$

Eq. (A2) states the demand of intermediate goods.

Firm i in the intermediate goods market chooses the price to maximize its profit

$$\max_{p^x_{i0}} \pi_{i0} = m_0 p^x_{i0} x_{i0} - m_0 w l^x_{i0} \tag{A3}$$

subject to its production function ($x_{i0} = l^x_{i0}$) and Eq. (A2).

The first-order condition is thus given by

$$p^x_0 = \frac{\sigma w + \tau_{y0} n_0^{\alpha-1} + \tau^x_0}{\sigma - 1} \tag{A4}$$

Eq. (A4) indicates that firm i of intermediate goods charges the markup price above the marginal cost to firm j of final goods. This is the solution in the traditional R&D growth model.

Substituting Eq. (A4) into (A2) and using Eq. (4) and (18), we obtain the equilibrium demand of intermediate goods. Then the price of the final goods is given by

$$p_{j0} = \frac{\sigma}{\sigma-1}\left[\frac{\sigma(w+\tau^x_0)n_0^{1-\alpha} + \tau_{y0}}{\sigma-1} + \tau_{y0}\right] \tag{A5}$$

Eqs. (A5) and (A4) state that double marginalization takes place due to successive markups. Namely, both the intermediate goods firm and the final goods firm set the markup price for their consumers.

Following Eq. (A5), we obtain the average price of the consumption goods (P_0), and profits (Π_0, π_0) of the intermediate goods firm and the final goods firm respectively in a successive monopolistic competitive economy. The results are shown as follows.

$$P_0 = \frac{\sigma}{\sigma-1}\left[\frac{\sigma(w+\tau^x_0)n_0^{1-\alpha} + \tau_{y0}}{\sigma-1} + \tau_{y0}\right] \tag{A6}$$

$$\Pi_0 = \frac{\sigma}{(\sigma-1)^2}\frac{1}{m_0}((w+\tau^x_0)n_0^{1-\alpha} + \tau_{y0})Y_0 \tag{A7}$$

$$\pi_0 = \frac{1}{\sigma - 1}\frac{1}{n_0}((w + \tau_0^x)n_0^{1-\alpha} + \tau_{y0})Y_0 \tag{A8}$$

The dynamics of the non-integrated economy are:

$$(\frac{\dot{C}}{C})_0 = \frac{(1 - v(1-s))L + v(1-s)L_{x0}}{v(1-s)(\sigma - 1)} + (\alpha - 2)(\frac{\dot{n}}{n})_0 + \frac{\dot{L}_{x0}}{L_{x0}}$$

$$- \frac{(1 - (vs + (1-v)))\dot{L}_{x0}}{L_{x0} + (vs + (1-v))(L - L_{x0})} - \rho \tag{A9}$$

$$(\frac{\dot{n}}{n})_0 = L - L_{x0} \tag{A10}$$

$$(\frac{\dot{C}}{C})_0 = (\alpha - 1)(\frac{\dot{n}}{n})_0 + \frac{\dot{L}_{x0}}{L_{x0}} \tag{A11}$$

Combining Equations (A9)-(A11), we find the dynamic equation for L_{x0}

$$\frac{(1 - (vs + (1-v)))\dot{L}_{x0}}{L_{x0} + (vs + (1-v))(L - L_{x0})} = \frac{1 - v(1-s)\sigma}{v(1-s)(\sigma - 1)}L + \frac{\sigma}{\sigma - 1}L_{x0} - \rho \tag{A12}$$

Since the coefficient $\sigma/(\sigma - 1)$ of L_{x0} is positive, we have $\dot{L}_{x0} = 0$. Its steady state value is

$$L_{x0} = \frac{v(1-s)\sigma - 1}{v(1-s)\sigma}L + \frac{\sigma - 1}{\sigma}\rho \tag{A13}$$

Appendix B

From household's budget constraint

$$p_A\dot{n} + n\dot{p}_A = rp_An + wL + m\Pi - PC \tag{B1}$$

Substituting the zero profit condition: $p_A\dot{n} = v(1-s)wL_A$, labor market equilibrium: $L_x + L_A = L$, and non-arbitrage condition: $r = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A}$ into Eq. (B1)

$$v(1-s)wL_A + n\dot{p}_A = (\frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A})p_An + w(L_x + L_A) + m\Pi - PC \tag{B2}$$

Rewriting Eq. (B2) to

$$v(1-s)wL_A = n\pi + w(L_x + L_A) + m\Pi - PC \tag{B3}$$

Substituting π and Π, namely, Eqs. (25) and (26), into Eq. (B3), we obtain

$$v(1-s)wL_A = Y\frac{1}{\sigma - 1}((w + \tau^x) + \tau_y n^{\alpha-1})n^{1-\alpha} + w(L_x + L_A) - PC \tag{B4}$$

Since the taxes are endogenous, substituting the government budget constraint $\tau_y = \frac{g(1 - v(1-s))wL_A}{n^{\alpha-1}L_x}$, and $\tau^x = \frac{(1-g)(1 - v(1-s))wL_A}{L_x}$ into Eq. (B4), we obtain

$$v(1-s)wL_A = Y\frac{1}{\sigma - 1}\frac{L_x + (1 - v(1-s))L_A}{L_x}wn^{1-\alpha} + w(L_x + L_A) - PC \tag{B5}$$

Owing to $Y = n^{\alpha-1}L_x$,

$$v(1-s)wL_A = \frac{1}{\sigma-1}(n^{1-\alpha}Y + (1-v(1-s))L_A)w + w(n^{1-\alpha}Y + L_A) - PC \tag{B6}$$

Rewriting to

$$v(1-s)wL_A = \frac{\sigma}{\sigma-1}n^{1-\alpha}wY + \frac{1}{\sigma-1}(1-v(1-s))wL_A + wL_A - PC \tag{B7}$$

Substituting $P = \frac{\sigma}{\sigma-1}\frac{L_x + (1-v(1-s))wL_A}{L_x}n^{1-\alpha}w$ into (B7), we obtain

$$0 = \frac{\sigma}{\sigma-1}n^{1-\alpha}wY - \frac{\sigma}{\sigma-1}n^{1-\alpha}wC \tag{B8}$$

Hence, we obtain the resource constraint as follows

$$Y = C \tag{B9}$$

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