

Stock Returns and Implied Volatility: A New VAR Approach

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Please cite the corresponding journal article:
<http://dx.doi.org/10.5018/economics-ejournal.ja.2013-3>

Abstract This study re-examines the return-volatility relationship and dynamics under a new VAR framework. By analyzing two model-free implied volatility indices—VIX (the U.S.) and VKOSPI (Korea)—and their corresponding stock market indices, we found an asymmetric volatility phenomenon in both developed and emerging markets. However, the VKOSPI, a recently published implied volatility index, shows impulse response dynamics that are clearly distinct from those for the VIX, an implied volatility index for the developed market.

JEL G10, G15

Keywords Asymmetric volatility; vector autoregression; VIX; VKOSPI

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1. INTRODUCTION

Financial economists have documented asymmetric return-volatility relationships in global stock markets. That is, changes in stock market returns and volatility are negatively related, and this relationship is more noticeable for negative returns than for positive returns. Within the dynamic framework, this indicates that negative return shocks have greater impacts on the change of volatility than positive ones do. This asymmetric volatility phenomenon has been explained through two famous hypotheses, the leverage hypothesis (Black, 1976; Christie, 1982) and the volatility feedback hypothesis (Campbell and Hentchel, 1992).

Although a number of studies have examined and reported the asymmetric volatility at the firm and market levels, it remains unclear whether previous studies' methods can fully explain the dynamic return-volatility relationship. This is because such methods rely on simple regression analyses or GARCH models.¹ In contrast to previous studies, this study re-examines the asymmetric relationship between return and volatility by taking a new vector autoregression (VAR) approach recently introduced in Lee's (2010) seminal paper. In addition, we examine the dynamic property of VKOSPI, a volatility index implied by KOSPI200 options, which represent the most liquid and remarkable options product in the world.² By analyzing and comparing the patterns of impulse responses of volatilities to positive and negative return shocks, we find that the asymmetric volatility phenomenon is detected in both developed (the U.S.) and emerging (Korea) markets, however, the dynamic relationship between stock market returns and implied volatility is unique to each market.

2. METHODOLOGY AND DATA

Lee (2010) employs his new VAR framework to examine the asymmetric effects of positive and negative inflation shocks that have the same size but opposite signs on stock market returns. In this section we briefly discuss Lee's framework, which can be used for investigating the asymmetric and dynamic relationship between any two economic variables.

$$\text{Bivariate Vector Autoregressive Representation (BVAR): } Y_t = A(L)Y_{t-1} + u_t \quad (1)$$

$$\text{Bivariate Moving Average Representation (BMAR): } Y_t = B(L)e_t \quad (2)$$

where $Y_t = [Y_{1t}, Y_{2t}]^T$, $u_t = [u_{1t}, u_{2t}]^T$, $e_t = [e_{1t}, e_{2t}]^T$, $\text{var}(u_t) = \Omega$, $\text{var}(e_t) = I$, $B^0 e_t = u_t$, and L is the lag operator. In addition, b_{ij}^0 , $B_{ij}(L)$, and $A_{ij}(L)$ are the elements of a 2-by-2 matrix of B^0 , $B(L)$, and $A(L)$, respectively ($i, j=1, 2$). The elements of $A(L)$ and Ω are obtained through the least squares estimation of Equation (1). By comparing Equations (1) and (2), we can obtain all elements of $B(L)$ if each

¹ Bekaert and Wu (2000) provide a good review of previous research.

² There is no published paper which thoroughly examines the VKOSPI.

element of B^0 is identified.³ We calculate B^0 by using the relationship $B^0(B^0)^T=\Omega$ and the additional restriction for the identification, $b_{11}^0+b_{11}^0=0$, which suggests that positive and negative shocks on the first variable Y_{1t} are the same size but opposite signs.⁴

We use this new VAR framework to examine the asymmetric return-volatility relationship. If we set the first variable as the stock market return and the second variable as the stock market volatility, then we can analyze the dynamic responses of the volatility to the same magnitude of positive and negative return innovations.

For the identification of the dynamic return-volatility relationship, implied volatility may be more appropriate than realized or historical volatility because implied volatility can gauge the expectation and sentiment of market participants. On the other hand, realized or historical volatility contains little information on investors' expectation of future states. Among implied volatility candidates, model-free implied volatility is known to have more explanatory power than those candidates dependent on option pricing models such as the Black-Scholes or Heston models.⁵ The most famous model-free implied volatility indicator is VIX, which represents the volatility index implied by the S&P500 option prices. Although a number of studies have examined the VIX, few have investigated the implied volatility index for the Korean financial market. Given that the KOSPI200 index options are top-tier options products in terms of high trading volume and investors' interest, there is an urgent need for research efforts using the model-free implied volatility of the emerging market. Since April 13, 2009, the Korea Exchange (KRX) has published the Korea's first implied volatility index, VKOSPI (Volatility Index of KOSPI200), which is calculated from the KOSPI200 option prices.

For a fair comparison, we use daily VIX/VKOSPI data for the period from April 13, 2009, the date of the VKOSPI announcement, to September 9, 2011. To analyze the stationary process, we use the first-order differentiated volatility and the corresponding stock market index (that is, S&P500 and KOSPI200) returns.

3. DISCUSSION

We estimate the VAR model by using U.S. market data (S&P500 index returns and VIX) and Korean market data (KOSPI200 index returns and VKOSPI) separately. The first variable (Y_{1t}) is set to the log return on the stock index and the second variable (Y_{2t}) is set to the differenced implied volatility index in each market. Thus, the first error e_{1t} indicates a positive return shock and the second error e_{2t} indicates a negative return shock. During the estimation procedure, we endogenously determined the lag-order of the VAR model by conducting a sequential likelihood ratio test, as in Rapach (2001).

³ Note that $B(L)=[I-A(L)L]^{-1}B^0$.

⁴ For a more detailed discussion, please refer to Section 3 in Lee (2010).

⁵ If we derive implied volatility by using option pricing models, then it contains some model bias, of which representative examples are volatility smiles or smirks of the Black-Scholes model.

Figures 1 and 2 plot the dynamic impulse responses of stock returns and implied volatility to positive and negative return shocks for the U.S. and Korean market data, respectively.⁶ The figures illustrate the asymmetric effects of positive and negative return innovations which structurally have the same magnitude. They present the response of stock return to positive and negative return shocks (*Panel A*) and the response of volatility to positive and negative return shocks (*Panel B*).

Figure 1 indicates that changes in VIX are negatively influenced by the initial S&P 500 return shocks. Positive stock returns induce a decrease in volatility, and negative stock returns induce an increase in volatility. However, the magnitudes of the effects of positive and negative shocks are quite different. Negative return shocks are much more likely to influence volatility changes than positive ones do.

The dynamic return-volatility relationship in the Korean market is slightly different from that in the U.S. market. As shown in *Panel B* of Figure 2, both positive and negative return shocks initially influence volatility in the same direction, inducing an increase in volatility. However, a positive return shock induces only a slight initial increase in volatility, whereas a negative return shock induces a sharp initial increase in volatility. That is, negative return-volatility relationships from negative return shocks dominate positive relationships from positive return shocks. As a result, we can also detect negative and asymmetric return-volatility relationships in the Korean market by using KOSPI200 index return and VKOSPI data.

Table 1 reports the forecast error variance decomposition of stock returns and that of volatility and shows the proportion of returns or volatility that can be explained by positive and negative return shocks. In the U.S. as well as in Korea, most of the volatility forecast error variance is explained by negative return shocks. More than 90% of the forecast error variance of volatility is explained by negative return shocks. For the three trading days immediately after the arrival of the return shock, the negative return shock explains more than 99% of the forecast error variance of volatility in the U.S. market. On the other hand, it explains relatively lower portion (about 96%) in the Korean market. This suggests that the asymmetric volatility phenomenon is clearer in the U.S. market. In terms of the Korean market, although the negative return shock still play a dominant role, the positive return shock maintains relatively significant influence on the dynamic stock-volatility relationship.

4. CONCLUSION

Considering the new VAR framework of Lee (2010), this study detects negative and asymmetric relationships between stock market returns and implied volatility in the U.S. and Korean markets. However, we also find slight differences in impulse response dynamics between the U.S. (a developed

⁶ All values of the impulse response functions are within their corresponding standard error bands, which are generated using the bootstrap method, as in Rapach (2001). For brevity, we show only the coefficients for impulse responses in each figure.

market) and Korea (an emerging market).

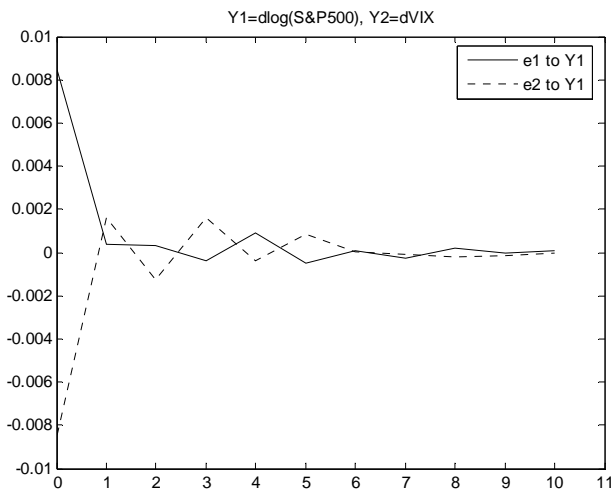
This paper demonstrates that the new VAR framework can be used to investigate the asymmetric and dynamic relationship between two economic variables. A number of studies have examined various properties of the VIX, including asymmetric volatility, but few have focused on the VKOSPI. In this regard, this paper contributes to the literature by being the first to analyze the asymmetric and dynamic responses of the VKOSPI under the new VAR framework. This study is expected to be a stepping-stone for further empirical research on the VKOSPI and other implied volatility indices of global financial markets.

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Figure 1. Dynamic impulse response of stock market return and volatility to positive and negative return shocks (e_{1t} and e_{2t}) for the U.S. market (the S&P500 index and VIX)

Panel A. The impulse responses of the change of S&P500 return



Panel B. The impulse responses of the change of VIX

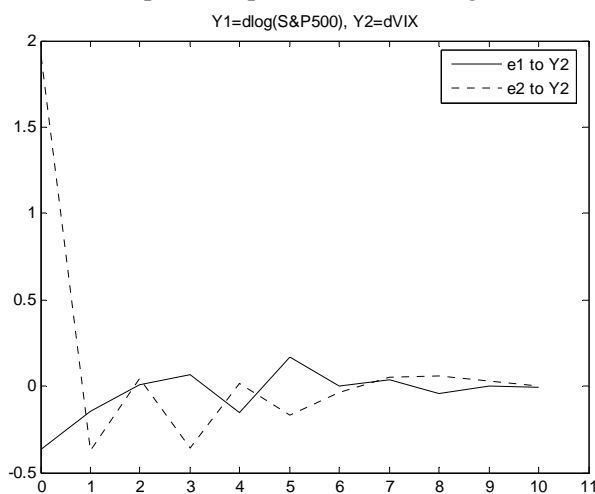
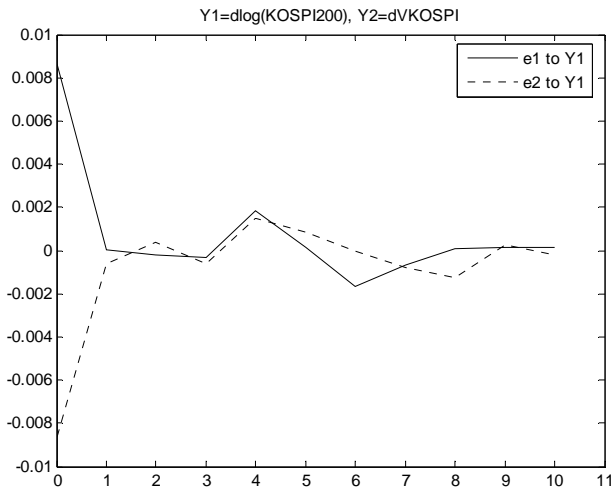


Figure 2. Dynamic impulse response of stock market return and volatility to positive and negative return shocks (e_{1t} and e_{2t}) for the Korean market (the KOSPI200 index and VKOSPI)

Panel A. The impulse responses of the change of KOSPI200 return



Panel B. The impulse responses of the change of VKOSPI

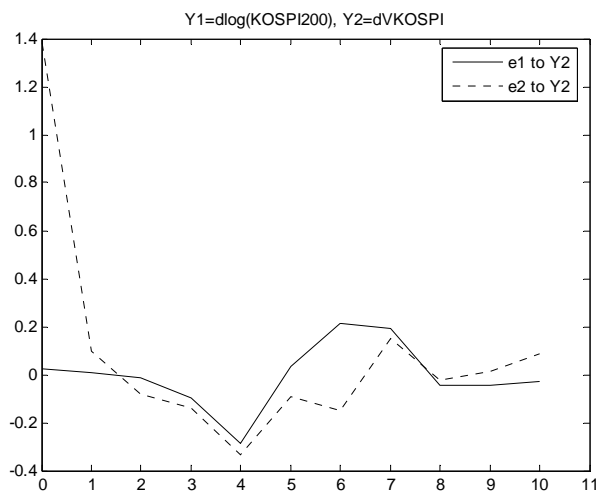


Table 1. Forecast error variance decomposition of stock returns and volatility

Panel A. United States (S&P500 return and VIX)

Days-ahead	0	1	2	3	4	5	6	7	8	9	10
Percent of return variance attributable to positive return shock	50.000	49.869	49.834	49.736	50.137	49.912	50.807	50.735	50.229	50.217	50.212
Percent of return variance attributable to negative return shock	50.000	50.131	50.166	50.264	49.863	50.088	49.193	49.265	49.771	49.783	49.788
Percent of IV variance attributable to positive return shock	0.032	0.035	0.042	0.529	4.373	4.403	6.348	7.783	7.866	7.944	7.948
Percent of IV variance attributable to negative return shock	99.968	99.965	99.958	99.471	95.627	95.597	93.652	92.217	92.134	92.056	92.052

Panel B. South Korea (KOSPI200 return and VKOSPI)

Days-ahead	0	1	2	3	4	5	6	7	8	9	10
Percent of return variance attributable to positive return shock	50.000	49.135	48.624	47.866	48.085	47.941	47.943	47.962	47.961	47.957	47.958
Percent of return variance attributable to negative return shock	50.000	50.865	51.376	52.134	51.915	52.059	52.057	52.038	52.039	52.043	52.042
Percent of IV variance attributable to positive return shock	3.496	3.889	3.890	3.878	4.426	5.077	5.076	5.109	5.150	5.149	5.150
Percent of IV variance attributable to negative return shock	96.504	96.111	96.110	96.122	95.574	94.923	94.924	94.891	94.850	94.851	94.850

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