Information Stickiness in General Equilibrium and Endogenous Cycles

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Abstract Traditionally, observed fluctuations in aggregate economic time series have been mainly modelled as being the result of exogenous disturbances. A better understanding of macroeconomic phenomena, however, surely requires looking directly at the relations between variables that may trigger endogenous nonlinearities. Several attempts to justify endogenous business cycles have appeared in the literature in the last few years, involving many types of different settings. This paper intends to contribute to such literature by investigating how we can modify the well-known information stickiness macro model, through the introduction of a couple of reasonable new assumptions, in order to trigger the emergence of endogenous fluctuations.

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'Chaos represents a radical change of perspective on business cycles. Business cycles receive an endogenous explanation and are traced back to the strong nonlinear deterministic structure that can pervade the economic system. This is different from the (currently dominant) exogenous approach to economic fluctuations, based on the assumption that economic equilibria are determinate and intrinsically stable, so that in the absence of continuing exogenous shocks the economy tends towards a steady state, but because of stochastic shocks a stationary pattern of fluctuations is observed.'

Barnett, Medio and Serletis (1997), pages 36-37.

1   Introduction

The benchmark macroeconomic paradigm is one in which the relations between relevant variables are essentially linear. Linear dynamic models allow to obtain one of two long-term outcomes: instability (divergence away from a fixed-point) or stability (convergence towards a fixed-point). This becomes a simplistic view of the economic system, since all sources of fluctuations in the long-run will be exogenous. A way to circumvent this excessively simplified view of the world is to look with further detail into the type of relations that explain the interaction among economic agents. This increased detail might allow to encounter nonlinearities that open the dynamic analysis to a wide range of possible long-term outcomes. Cycles of any periodicity or complete a-periodicity may be found, allowing for an intuitive endogenous explanation for business fluctuations. Periodic, a-periodic and even chaotic outcomes are forms of bounded instability that are compatible with the observed evolution of macro time series.

In macroeconomics, there have been many attempts to provide explanations for business cycles based on the notion of endogenous fluctuations [see Gomes (2006) for a survey]. In recent years, this field of study has remained active, with relevant contributions being published. Table 1 presents some meaningful studies published since 2007.

Author (year)	Type of model	Source of fluctuations
Fanti and Manfredi (2007)	Neoclassical labor market model	Consumption and leisure are modeled as weak substitutes
Jaimovich (2007)	Dynamic general equilibrium model	Interaction between firms' entry-and-exit decisions and changes in competition
Yoshida and Asada (2007)	Keynes-Goodwin model of the growth cycle	Lags in the implementation of stabilization policies
Chen, Li and Lin (2008)	Overlapping generations model with capital accumulation	Myopic and adaptive expectations
Fujio (2008)	Two-sector optimal growth model with a Leontief technology	The shape of the production function
Hallegatte, Ghil, Dumas and Hourcade (2008)	Non-equilibrium dynamic model that introduces investment dynamics into a Solow growth model	Investment-profit instability
Yokoo and Ishida (2008)	Economy with a continuum of firms that engage in innovation activities	Imperfect information
Dieci and Westerhoff (2009)	A model that integrates the stock markets of two countries via the foreign exchange market	Heterogeneous agents: technical traders and fundamentalists
Kikuchi and Stachurski (2009)	Two-country growth model	Interaction between unequal countries through credit markets
Stockmam (2009)	Two-sector growth model	Sector-specific externalities
Gomes (2010)	Sticky-information partial equilibrium macroeconomic model	Formation of expectations under a learning rule
Lines and Westerhoff (2010)	Macro model composed by Okun's law, expectations-augmented Phillips curve and an aggregate demand relation	Heterogeneous expectations (trend-following and rational expectations)
Sushko, Gardini and Puu (2010)	Hicksian trade-cycle model	Capital stock as a capacity limit (ceiling) for production

Table 1 – Recent literature on endogenous fluctuations.

As we observe in the table, there are many ways to justify the emergence of endogenous business cycles in relatively different contexts. If we

want to systematize this information, we might say that most of the mentioned studies are inspired in two or three successful approaches to the issue of endogenous volatility; we highlight the following: (i) the heterogeneous agents framework first developed by Brock and Hommes (1997, 1998), where fundamentalist agents work as a stabilizing force and technical traders as the force triggering temporary departures from stability; (ii) optimal growth models with non-conventional production functions and externalities in production, in the tradition of Nishimura and Yano (1995) and Christiano and Harrison (1999); and (iii) environments where bounded rationality in the formation of expectations have an important role, as in the case of Bullard (1994) and Schonhofer (1999).

In this paper, endogenous cycles are explored in a popular macroeconomic framework – the sticky-information general equilibrium (SIGE) model, developed by Mankiw and Reis (2006, 2007) and Reis (2009). The original goal of this model was to explain the gradual response or the inertia of aggregate variables to exogenous shocks. It allows for a steady-state analysis, where policy shocks may temporarily deviate the economy from its fixed-point long-run locus. This setup involves a dynamic result of stability, i.e., of convergence of any initial state towards a steady-state point, for the relevant macro variables. In the absence of exogenous disturbances, once the steady-state is accomplished, it will never be abandoned again.

How can endogenous cycles eventually emerge within this setup? The answer is given in this paper through the relaxation of two benchmark assumptions of the model. In the original framework, (i) perfect foresight or rational expectations hold independently of the distance in time between the moment in which expectations are formed and the moment they respect to; (ii) the pace of information updating is considered constant. Alternatively, we will consider that: (i) perfect foresight is not universal; (ii) information updating is counter-cyclical.

The two new assumptions are reasonable and introduce a larger degree of realism into the analysis: on one hand, economic agents will have difficulties in predicting future values with accuracy, when the future is distant in time. On the other hand, the degree of attentiveness to news about the state of the economy changes in time; in particular, it makes sense to recognize that periods of lower economic growth are necessarily periods of stronger exposure to news and, therefore, these will be periods of a more frequent in-

formation updating. Our conclusion will be that the introduction of further realistic details into the macro model allows to explain, at least partially, the observed volatility in the time series of aggregate variables. We will emphasize that the two new assumptions are, individually, necessary but not sufficient conditions for a long-term nonlinear outcome; only when we consider both simultaneously, we will be able to identify the presence of endogenous fluctuations.

The baseline version of the model that we will take is the one in Gomes (2012), which is similar to the Mankiw-Reis framework, with only a few changes that help in treating the model from an analytical point of view. Nevertheless, these changes are innocuous in terms of the results one will obtain. The changes will appear later with the characterization of the model and they are essentially two:

1) the degree of information stickiness will be the same across the different types of economic agents (namely, price-setting firms, households who formulate consumption plans and wage-setting workers);

2) the monetary policy rule will ignore real stabilization, and it will focus solely on price stability (this allows to better highlight the condition under which monetary policy is active or aggressive).

Besides these remarks, we should stress that any kind of stochastic disturbance (e.g., technological innovations) will be overlooked, in order to emphasize the possible presence of endogenous fluctuations.

The remainder of the paper is organized as follows. Section 2 presents the model, through the characterization of profit maximization by firms, utility maximization by households and wage optimization by labor suppliers. In section 3, the two new assumptions, concerning the formation of expectations and the updating of information, are introduced. Section 4 confirms the stability result under perfect foresight. In sections 5 and 6 the model with the new assumptions is analyzed, respectively, under local and global perspectives. The study of global dynamics allows to detect endogenous fluctuations for reasonable values of parameters. Finally, section 7 concludes.

2 The Information-Stickiness General Equilibrium Model

Consider a general equilibrium setting in which firms and households behave optimally. Firms act with the goal of maximizing profits, while households have a two-fold concern: to optimize consumption plans and to select an efficient level of labor supply. In this environment, a source of rigidity exists, namely there is stickiness in the dissemination of information.

We start by addressing the problem faced by firms. There is an unspecified number of firms, in the unit interval, indexed by j. For each firm j, a production function is assumed, with labor as the unique input (capital is ignored and the technology level is implicitly normalized to 1). The production function takes the form $Y_{t,j} = N_{t,j}^{\beta}$, with $Y_{t,j}$ the output or income generated by firm j at time t and $N_{t,j}$ the amount of labor employed in production by the same firm at the same time period. Parameter $\beta \in (0,1)$ represents the output-labor elasticity and indicates that the production is subject to decreasing marginal returns.

Each firm produces a unique variety of the single assumed good, and does it by resorting to a unique variety of labor hired from households. The aggregate labor supply and the aggregate level of output may be presented under the form of Dixit-Stiglitz indexes:

$$N_t = \left(\int_0^1 N_{t,j}^{\frac{\gamma}{\gamma-1}} dj \right)^{(\gamma-1)/\gamma}$$

$$Y_t = \left(\int_0^1 Y_{t,j}^{\frac{v}{v-1}} dj \right)^{(v-1)/v}$$

with $\gamma > 0$ the elasticity of substitution between different varieties of labor and $v > 0$ the elasticity of substitution between different varieties of goods. The aggregate production function takes the form $Y_t = N_t^{\beta}$.

The model will be analyzed under a log-linear presentation of variables, and thus we define $n_t := \ln N_t$ and $y_t := \ln Y_t$. With these variables, $y_t = \beta n_t$.[1]

By solving the profit maximization problem of firms, we arrive to the

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[1] We will skip most of the derivation of the model and just present the main intuition and the main results. Details on the development of the optimization problems of the several agents can be found in the already cited references on the Mankiw-Reis framework.

following desired price: $p_t^* = p_t + mc_t$, with p_t the logarithm of the price level and mc_t a variable that represents real marginal costs, which are given by

$$mc_t = \frac{\beta}{\beta + \upsilon(1 - \beta)}(w_t - p_t) + \frac{1 - \beta}{\beta + \upsilon(1 - \beta)}y_t \tag{1}$$

Variable w_t is the logarithm of the nominal wage rate. According to (1), marginal costs increase whenever positive changes are observed in the real wage rate and in the level of output.

The desired price, p_t^*, is the price that all firms would like to set at time t (since firms are identical, except for the variety of labor they hire and the variety of the good they produce). The desired price rises above the aggregate price level whenever the measure of marginal costs mc_t is positive; the opposite occurs for $mc_t < 0$. Larger marginal costs lead to a desire for setting higher prices.

Now, we introduce into the analysis the assumption of sticky information. Firms will want to set price p_t^* but they are sluggish in the way they update information (firms face costs when acquiring, absorbing and processing information). This signifies that the information that is necessary to choose the mentioned price has been collected, by different firms, at different time periods in the past.

The infrequent information updating implies that a firm that last updated its information set j periods ago will generate the following expectation, $p_{t,j} = E_{t-j}(p_t^*)$. Note that the index j represents simultaneously different varieties of goods and the number of periods a firm remains inattentive; the implicit assumption is that a firm producing variety j is a firm that has formed expectations about prices j periods in the past.

We define $\lambda \in (0,1)$ as the share of firms that, at each time moment, recompute the optimal price by updating the corresponding information set. Looking from another angle, λ will also represent the probability of a firm updating its information set at the current time period. The consideration of this share allows presenting the aggregate price level under the form of a weighted average of past expectations about the current price level,

$$p_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j p_{t,j} \tag{2}$$

6

Let $\pi_t := p_t - p_{t-1}$ be the inflation rate and consider, as well, Δmc_t as being the change on the real marginal costs from $t-1$ to t. By applying first-differences to expression (2), we can present a central equation of the information stickiness analysis: the sticky-information Phillips curve.

$$\pi_t = \frac{\lambda}{1-\lambda} mc_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}(\pi_t + \Delta mc_t) \tag{3}$$

The Phillips curve in (3) involves a contemporaneous positive relation between marginal costs and inflation; inflation is also dependent on past expectations about the current state of the economy.

Consider now the behavior of households relating utility maximization. As firms, households are also indexed by j in the unit interval (each variety j of the assumed good is produced by a variety j of labor and consumed by a variety j of household). Consumer j possesses preferences given by the following utility function:

$$U(C_{t,j}; L_{t,j}) = \frac{C_{t,j}^{1-1/\theta} - 1}{1 - 1/\theta} - \frac{\varkappa L_{t,j}^{1+1/\psi}}{1 + 1/\psi}$$

The utility function has two arguments: consumption, $C_{t,j}$, and an index respecting to labor supply, $L_{t,j}$. Obviously, $\frac{\partial U}{\partial C_{t,j}} > 0$ and $\frac{\partial U}{\partial L_{t,j}} < 0$, i.e., utility increases with a larger level of consumption and additional hours of leisure.

Parameters $\theta > 0$ and $\psi > 0$ represent the intertemporal elasticity of substitution for consumption and the elasticity of labor supply, respectively. The value of $\chi > 0$ translates the relative weight attributed to leisure in the utility function. Taking a discount factor $\xi \in (0,1)$, the optimization problem faced by each household is

$$Max \sum_{t=0}^{\infty} \xi^t U(C_{t,j}; L_{t,j})$$

The above problem is subject to a conventional budget constraint, where the households' wealth increases with labor income and financial returns and decreases with consumption. By solving the optimal control problem, we encounter an Euler equation of the type:

$$c_{t,j} = -\theta E_{t-j}(R_t) \tag{4}$$

where $c_{t,j} := \ln C_{t,j}$. Variable $R_t = E_t \left(\sum_{i=0}^{\infty} r_{t+i} \right)$ represents the long real interest rate and r_t the real interest rate. In equation (4), we are already implicitly considering that households also update information infrequently and, thus, individual levels of consumption are obtained by taking into account past expectations on the expected value of the real interest rate. To simplify, we consider that the information stickiness parameter is for households the same we have already taken for firms, λ, and thus aggregate consumption under sticky-information will correspond to

$$c_t = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j c_{t,j} \tag{5}$$

Equation (5) might be transformed into an IS equation, after assuming that there is market clearing in the goods market, i.e., $c_t = y_t$. The expression of the sticky-information IS curve will be:

$$y_t = -\theta \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j}(R_t) \tag{6}$$

As for any other IS curve, the relation between the interest rate and the output is of opposite sign (higher expected real interest rates will encourage savings and, thus, will lower spending). Through the application of first-differences to equation (6), the economy's growth rate can be expressed by

$$g_t = -\theta \lambda R_t - \theta \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-1-j}[(1 - \lambda)R_t - R_{t-1}] \tag{7}$$

where $g_t := y_t - y_{t-1}$ is the growth rate of real output.

A third equation of motion will concern labor supply. The labor market is a monopolistically competitive market in the sense that workers have different varieties of skills. The optimal nominal wage rate is obtained also from the households' utility maximization problem and by taking into account the market clearing condition in the labor market, $L_t = N_t$. Sticky information is also present in this market, with the degree of information stickiness being the same one has already considered in the analysis of price setting behavior and of the choice of consumption plans, i.e., the measure of information updating or degree of attentiveness is again λ.

The aggregate wage index is defined by the sum of the individual wages,

weighted by parameter λ,

$$w_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j w_{t,j} \tag{8}$$

with $w_{t,j}$ the nominal wage rate that an agent who has updated her information set for the last time at period $t-j$ will desire, given the optimization problem she has solved. A worker who has last updated her information j periods in the past will have the following expectation for the desired nominal wage rate:

$$w_{t,j} = E_{t-j}\left[p_t + \frac{\gamma}{\gamma+\psi}(w_t - p_t) + \frac{1}{\beta(\gamma+\psi)}y_t - \frac{\psi}{\gamma+\psi}R_t\right] \tag{9}$$

According to (9), workers will demand a larger nominal wage whenever the values of the price level, the real wage rate and the real output are higher and when the real interest rate is expected to be lower.

The SIGE model is composed by the three derived relations, namely:

1) The sticky-information Phillips curve;

2) The sticky-information IS curve;

3) The sticky-information wage curve.

To close the model and present it under a tractable form, we need to make a couple of additional remarks. First, the real interest rate is given by the Fisher equation, $r_t = i_t - E_t(\pi_{t+1})$, with i_t the nominal interest rate. Second, we must define a monetary policy rule; the assumption is that the monetary authority is concerned exclusively with price stability and, hence, the Taylor rule takes the form:

$$i_t = \phi[E_t(\pi_{t+1}) - \overline{\pi}] \tag{10}$$

The value $\overline{\pi}$ is the target inflation rate that the central bank selects and ϕ is a policy parameter. As it is common in monetary policy analysis, we restrict our study to the case of an active monetary policy, i.e., a policy such that a one point change on the expected inflation rate will be fought by the central bank through a larger than one point change on the nominal interest rate. Active rules guarantee that the model's equilibrium is determinate and, in the simple case of rule (10) where real stabilization concerns are absent, the required condition is simply $\phi > 1$.

Basically, in an overall perspective, our framework involves three main original endogenous variables in a setting with three dynamic equations. These three original variables are p_t, y_t and w_t. For these, we define the steady-state as the point (p^*, y^*, w^*) such that

$$
\begin{aligned}
p^* &: = p_t = E_{t-j}(p_t) \\
y^* &: = y_t = E_{t-j}(y_t) \\
w^* &: = w_t = E_{t-j}(w_t), \forall t, j = 0, 1, 2, ...
\end{aligned}
$$

Applying the above definition to the set of relations one has derived, it is straightforward to arrive to the following outcome:

$$
p^* = w^*
$$
$$
y^* = 0
$$
$$
R^* = r^* = 0
$$
$$
\pi^* = i^* = \frac{\phi}{\phi - 1}\overline{\pi}
$$

In the long-run, prices and nominal wages will be identical and, therefore, the real wage will be equal to zero (recall that our variables are defined in logarithmic form). The level of output and the real interest rate are also zero. Prices and nominal wages will grow at a rate identical to the nominal interest rate. This rate depends on the inflation target, but it is larger than the value of $\overline{\pi}$; this is not a surprising result, since the adopted monetary policy rule is not an optimal rule. Note, in particular, that the more active or the more aggressive monetary policy is (larger ϕ), the more π^* approaches $\overline{\pi}$.

We know, from the above results, that the real interest rate converges to zero in the long-run. A convenient way to simplify the model consists in assuming that the expected rate of convergence of r_t from its current value towards the steady-state is constant; let this rate be $a \in (0, 1)$. The constant rate allows to present a simple relation between R_t and r_t:

$$
R_t = E_t\left(\sum_{i=0}^{\infty} r_{t+i}\right) = \sum_{i=0}^{\infty}(1-a)^i r_t = \frac{1}{a}r_t
$$

In order to close this section, we gather all the above information and

present the SIGE model under the form of a three-dimensional difference equations' system with three endogenous variables. The variables will be the inflation rate (π_t), the growth rate of the nominal wage $(\mu_t := w_t - w_{t-1})$ and the growth rate of real output (g_t),

$$
\begin{aligned}
\pi_{t+1} \;=\;& \frac{1}{1-\lambda}\pi_t + \frac{\lambda}{1-\lambda}\left(\Delta mc_{t+1} + \Delta mc_t\right) \\
& + \lambda\sum_{j=0}^{\infty}(1-\lambda)^j E_{t-j}\left[\pi_{t+1} + \Delta mc_{t+1} - \frac{1}{1-\lambda}\left(\pi_t + \Delta mc_t\right)\right]
\end{aligned}
$$

with $\Delta mc_t \;:\; = \dfrac{\beta}{\beta + \upsilon(1-\beta)}(\mu_t - \pi_t) + \dfrac{1-\beta}{\beta + \upsilon(1-\beta)}g_t$

$$
\begin{aligned}
\mu_{t+1} \;=\;& (1-\lambda)\mu_t + \lambda\left(\Delta z_{t+1} + \Delta z_t\right) \\
& + \lambda\sum_{j=0}^{\infty}(1-\lambda)^j E_{t-j}\left[(1-\lambda)\Delta z_{t+1} - \Delta z_t\right]
\end{aligned}
$$

with $\Delta z_t \;:\; = \pi_t + \dfrac{\gamma}{\gamma + \psi}(\mu_t - \pi_t) + \dfrac{1}{\beta(\gamma + \psi)}g_t - \dfrac{\psi}{\gamma + \psi}(R_t - R_{t-1})$

$$
\begin{aligned}
g_{t+1} \;=\;& -\theta\lambda\frac{\phi-1}{a}E_t(\pi_{t+1}) \\
& -\theta\lambda\sum_{j=0}^{\infty}(1-\lambda)^j E_{t-j}\left[(1-\lambda)\frac{\phi-1}{a}\pi_{t+2} - \frac{\phi-1}{a}\pi_{t+1}\right]
\end{aligned}
$$

3 Two New Assumptions

The SIGE model, as presented so far, corresponds, with minor changes, to the Mankiw-Reis framework, which serves the purpose of being a laboratory for the analysis of the behavior of variables resting in the steady-state when subject to some exogenous policy disturbances. As referred in the introduction, this is a model involving linear dynamics and a stability result under which relevant variables will converge from any initial state towards the steady-state that was characterized at the end of the previous section.

The stability result of the Mankiw-Reis setup is decisively linked to one of the underlying hypothesis of the analysis, namely rational expectations or, in the absence of exogenous shocks, plain perfect foresight. Perfect foresight implies that $E_{t-j}(\pi_t) = \pi_t, E_{t-j}(\mu_t) = \mu_t, E_{t-j}(g_t) = g_t, \forall j$. In the

discussed context, however, the perfect foresight assumption becomes somehow counter-intuitive, because it says that independently of how far in the past expectations are generated, agents will always maintain the capacity to exactly understand the evolution of the system that culminates in the current state.

In other words, the agents are equally capable of forecasting the value of a variable at time t, when the forecast is generated at $t-1$ or at, for instance, $t-100$. Producing an expectation within a 100 periods interval implies lacking a large quantity of information that most probably will make the forecast to deviate from the intended perfect foresight result. A more sensible assumption would be to consider that as we go back in time, agents lose the capacity to predict future values with accuracy and that they will more strongly interpret the current period as the long-run. In the long-run, in turn, variables should assume their steady-state values. The above reasoning might be analytically translated into the following:

$$
\begin{array}{rcl}
E_{t-j}(\pi_t) & = & \alpha^j \pi_t + (1-\alpha^j)\pi^*, \\
E_{t-j}(\mu_t) & = & \alpha^j \mu_t + (1-\alpha^j)\mu^*, \\
E_{t-j}(g_t) & = & \alpha^j g_t + (1-\alpha^j)g^*
\end{array}
$$

with $\alpha \in (0,1)$ the probability of formulating a perfect foresight expectation at $t-1$; $1-\alpha$ will be the probability of interpreting t as the steady-state, when formulating the expectation at $t-1$. Note that under the rules of formation of expectations presented above, if the expectation is formed at t concerning variables at t, perfect foresight holds. As we go back in time, the probability of generating perfect expectations will progressively fall in favor of interpreting the current period as the steady-state. In the limit case $j \to \infty$, the probability of generating perfect forecasts is zero and the present is fully understood as the long-term.

We may consider a value of α closer to 0 or closer to 1. The extreme cases are easy to interpret. When $\alpha = 0$, agents are unable to forecast the future and any expectation will interpret the current moment as the steady-state; if $\alpha = 1$, we are back at the full perfect foresight scenario where, no matter how far in the past, firms and households are able to predict with full accuracy contemporaneous observed values of macro variables. Thus, perfect foresight is a particular case of our expressions for α equal to 1, and therefore the

proposed assumption may be understood as a general platform that allows many possibilities relating the capacity of agents in forming expectations.

The expectations' hypothesis is the first assumption we introduce in order to address nonlinearities and endogenous fluctuations under the SIGE setup. The second assumption relates the way we interpret information updating. One of the main assumptions of the original model is that the degree of attentiveness by the various types of agents is constant through time and, most importantly, is constant independently of the faced economic conditions. Our argument at this level, which allows to change the mentioned assumption, will be based on the following observation:

'We (...) find evidence supporting that consumers update their expectations about the economy much more frequently during periods of high news coverage than in periods of low news coverage; high news coverage of the economy is concentrated during recessions and immediately after recessions, implying that 'stickiness' in expectations is countercyclical.'

Doms and Morin (2004), abstract.

This sentence presents a piece of evidence that we must take seriously. In fact, not only concerning the decisions of consumers, but also in what respects to the behavior of price-setting firms and wage-setting labor suppliers, it appears evident that there is a direct correlation between degree of attentiveness and news coverage of economic phenomena. The other argument is also undeniable, namely the idea that in periods of recession, the media attributes more attention to the behavior of the economy than in periods of expansion. Thus, we take as reasonable the intuition that information stickiness is counter-cyclical.

To model the counter-cyclicality of information updating, we let $\lambda_0 \in (0, 1)$ be the attentiveness rate for $g_t = 0$ and $\underline{\lambda} \in (0, \lambda_0)$ a benchmark minimal level of attention that asymptotically holds for very large growth rates. Attentiveness increases as the growth rate becomes smaller and full attentiveness, $\lambda = 1$, will be a virtual outcome for extremely negative growth rates. The function that captures the mentioned properties is:

$$\lambda(g_t) = \frac{1 + \underline{\lambda}}{2} - \frac{1 - \underline{\lambda}}{\pi} \arctan\left[g_t + \tan\left(\frac{\pi}{2} \frac{1 + \underline{\lambda} - 2\lambda_0}{1 - \underline{\lambda}}\right)\right] \qquad (11)$$

with $\pi = 3.14159...$

Figure 1 displays function (11) for $\lambda_0 = 0.25$ and $\underline{\lambda} = 0.1$: Note that in the vicinity of λ_0, $\lambda(g_t)$ is a decreasing and slightly convex function; this nonlinearity is a necessary ingredient for the result on fluctuations we will be able to obtain.
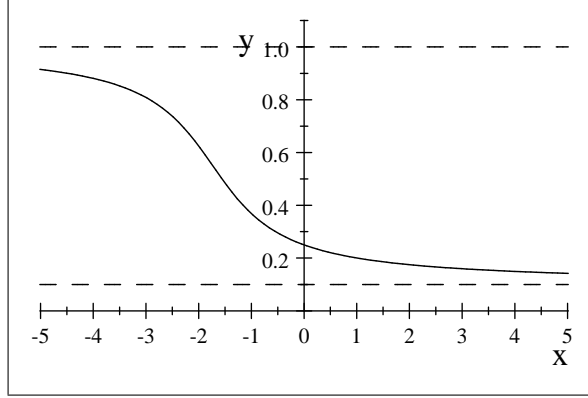

Fig. 1 - Information updating function

In this section, we have introduced two assumptions that allow the SIGE model to approach the observed reality: economic agents are certainly unable to predict with full accuracy no matter how far apart are the relevant time moments, and information updating tends to be countercyclical. With these new assumptions the system will be able to provide a rich set of possible long-term outcomes.

4 Perfect Foresight and Stability

Taking into account the new assumptions and defining the rate of change of the real wage by $\mu_t^R := \mu_t - \pi_t$, the dynamic SIGE system can be further rearranged and presented under the form of a pair of difference equations:

$$\begin{cases} g_{t+1} = f_{11}[\lambda(g_t)]g_t + f_{12}[\lambda(g_t)]\mu_t^R \\ \mu_{t+1}^R = f_{21}[\lambda(g_t)]g_t + f_{22}[\lambda(g_t)]\mu_t^R \end{cases} \quad (12)$$

where:

$f_{11}[\lambda(g_t)] = \alpha(1-\lambda) + \frac{\Omega_1}{(\phi-1)\Omega_2\Omega_6}$,

$f_{12}[\lambda(g_t)] = \frac{(1-\alpha)(1-\lambda)}{(1+\Omega_3)\Omega_6}$,

$f_{21}[\lambda(g_t)] = -\frac{\Omega_1}{(\phi-1)\Omega_2\Omega_6}\left(\Omega_6 + \frac{1-\beta}{\beta}\right)$,

$f_{22}[\lambda(g_t)] = \frac{1-\lambda}{1+\Omega_3}\left[1 + \alpha\Omega_3 - \frac{1-\alpha}{\Omega_6}\left(\Omega_6 + \frac{1-\beta}{\beta}\right)\right]$

and

$\Omega_1 := \frac{a}{\theta} \frac{1-\alpha(1-\lambda)}{\alpha^2 \lambda(1-\lambda)};$

$\Omega_2 := \frac{\lambda}{(1-\alpha)(1-\lambda)} \frac{\beta}{\beta + v(1-\beta)};$

$\Omega_3 := \frac{\lambda}{1-\alpha(1-\lambda)} \left[\frac{\beta}{\beta + v(1-\beta)} - \frac{\gamma}{\gamma + \psi} \right];$

$\Omega_4 := \frac{\lambda}{1-\alpha(1-\lambda)} \left[\frac{1-\beta}{\beta + v(1-\beta)} - \frac{1}{\beta(\gamma + \psi)} \right];$

$\Omega_5 := \frac{\lambda}{1-\alpha(1-\lambda)} \frac{\psi}{\gamma + \psi} \frac{\alpha}{a};$

$\Omega_6 := \frac{\Omega_4 - \Omega_1 \Omega_5}{1 + \Omega_3} - \frac{1-\beta}{\beta}$, with $\lambda = \lambda(g_t)$.

To analyze system (12) under perfect foresight, we just need to recall that this corresponds to the case where condition $\alpha = 1$ applies. In this case, dynamics are reduced to

$$\begin{cases} g_{t+1} = [1 - \lambda(g_t)] \, g_t \\ \mu_{t+1}^R = [1 - \lambda(g_t)] \, \mu_t^R \end{cases} \tag{13}$$

The dynamic behavior of system (13) is straightforward to characterize. The result is synthesized in proposition 1.

Proposition 1 *Under perfect foresight, there is stability in the SIGE model. This result holds for constant information updating and for counter-cyclical information updating.*

Proof. The linearization of system (13) in the vicinity of the steady-state point $[g^*, (\mu^R)^*] = (0,0)$ allows to write it under matricial form:

$$\begin{bmatrix} g_{t+1} \\ \mu_{t+1}^R \end{bmatrix} = \begin{bmatrix} 1 - \lambda_0 & 0 \\ 0 & 1 - \lambda_0 \end{bmatrix} \cdot \begin{bmatrix} g_t \\ \mu_t^R \end{bmatrix}.$$

Recall that λ_0 is the steady-state level of $\lambda(g_t)$ when information updating is taken as counter-cyclical. The system is precisely the same for a constant $\lambda = \lambda_0$. Because $\lambda_0 \in (0,1)$, both eigenvalues of the Jacobian matrix are inside the unit circle and, therefore, stability holds, i.e., we observe convergence towards $[g^*, (\mu^R)^*] = (0,0)$ independently of parameter values and initial state ∎

The result in proposition 1 indicates that the way we approach information updating or the degree of information stickiness is not relevant for the model's dynamics as long as we maintain that agents formulate expectations under perfect foresight.

In perfect foresight settings, parameter λ_0 just indicates the velocity of convergence towards the steady-state when taking an initial point (g_0, μ_0^R) in the vicinity of that state, but it cannot change the stable nature of the

system. The linearized system has the following solution,

$$g_t = (1 - \lambda_0)^t g_0$$
$$\mu_t = (1 - \lambda_0)^t \mu_0$$

The velocity of convergence is given precisely by parameter λ_0, which indicates that the more sluggish information updating is, the slower will be the process of convergence. However, since λ_0 is positive, the model remains stable. Under perfect foresight, any remark about the degree of information stickiness may be used to evaluate how fast the steady-state is reached, but the stability result cannot be questioned.

5   Partial Perfect Foresight: Local Analysis

In this section, we address the stability properties of system (12) for $\alpha < 1$, i.e., in the absence of full perfect foresight. A first result relates to the case of a constant attentiveness share λ.

Proposition 2 *Independently of the degree in which perfect foresight prevails in the formation of past expectations about current events, as long as the updating of information remains constant in time, nonlinearities will not exist.*

Proof. Just observe that for a constant value of the parameter λ, system (12) is linear. Thus, only two outcomes are conceivable: stability or instability (convergence or divergence relatively to the steady-state). The finding of a stable or of an unstable outcome will depend on the values of the parameters of the model ■

The analysis of local and global dynamics under constraint $\alpha < 1$ cannot be feasibly undertaken for the model on its generic form. We need to proceed with a numeric example and we adopt the same values of parameters as in Mankiw and Reis (2006): $\psi = 4$, $\beta = 2/3$, $\theta = 1$, $\gamma = 10$, $\upsilon = 20$. Besides these, we take as well the following: $\alpha = 0.75$, $a = 0.01$. Relatively to these two last values, changing them would have no significant impact on the qualitative results as long as they remain bounded below 1. The policy parameter ϕ will be our bifurcation parameter in the analysis. For now, we consider that information is updated every four periods, if the corresponding

parameter is constant or, in the case of counter-cyclical information updating, if the economy's growth rate is zero; hence, $\lambda = \lambda_0 = 0.25$. With the described data, the following result is obtained.

Proposition 3 *For the considered array of parameter values, the SIGE model with partial perfect foresight is stable for $\phi > 1.1808$.*

Proof. The linearized SIGE model with the assumed parameter values is:

$$\begin{bmatrix} g_{t+1} \\ \mu_{t+1}^R \end{bmatrix} = \begin{bmatrix} -0.2547/(\phi - 1) + 0.5625 & -0.2167 \\ -0.2149/(\phi - 1) & 0.6709 \end{bmatrix} \cdot \begin{bmatrix} g_t \\ \mu_t^R \end{bmatrix}$$

Local dynamics are identical for constant information updating and counter-cyclical inattentiveness as long as $\lambda = \lambda_0$, as it is the case. Stability conditions are:

(i) $1 - Det = 0.622\,6 + 0.2174/(\phi - 1) > 0$;

(ii) $1 - Tr + Det = 0.143\,6 + 0.0373/(\phi - 1) > 0$;

(iii) $1 + Tr + Det = 2.\,6107 - 0.4721/(\phi - 1) > 0$.

with Tr and Det representing, respectively, the trace and the determinant of the Jacobian matrix of the above system. The first two conditions are satisfied for any $\phi > 1$; the third stability condition requires $\phi > 1.1808$ ∎

The result in proposition 3 is graphically depicted in figure 2. This figure represents the relation between the trace and the determinant; the three lines that form the inverted triangle are the bifurcation lines and the area inside the triangle represents the region of stability. The bold line translates the dynamics of the system; while inside the stability area, this line implies a value of ϕ larger than 1.1808. When ϕ equals this value, the bifurcation line $1 + Tr + Det = 0$ is crossed (a flip bifurcation occurs), and the stability region is abandoned for values of ϕ below the referred threshold value.
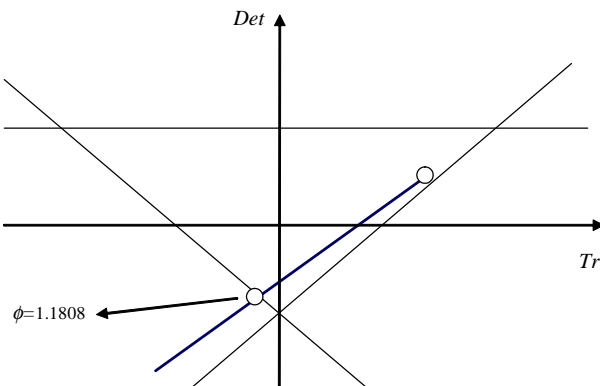
Fig. 2 – Trace-determinant diagram in the partial perfect foresight case

The obtained result is relevant and intuitive: it indicates that a departure from perfect foresight will require a more active policy by the monetary authorities, in order for stability to hold. Agents with a less than perfect capacity in forecasting future values will turn harder monetary policy implementation, because it will need to be more aggressive than in the benchmark case.

Now, let us consider other possible values for λ or λ_0. Table 2 shows how the stability condition changes when λ changes.

λ	Stability Condition	λ	Stability Condition
0.1	$\phi > 1.9787$	0.6	$\phi > 1.0311$
0.2	$\phi > 1.2720$	0.7	$\phi > 1.0200$
0.3	$\phi > 1.1293$	0.8	$\phi > 1.0118$
0.4	$\phi > 1.0750$	0.9	$\phi > 1.0053$
0.5	$\phi > 1.0475$	1	$\phi > 1$

Table 2 – Stability condition for various degrees of agents' attentiveness.

The interpretation of table 2 is straightforward, and we synthesize it in the following proposition,

Proposition 4 *For the chosen array of parameter values, in the case of partial perfect foresight, in order for stability to hold, the stronger the level of inattentiveness the more aggressive monetary policy is required to be.*

Proof. Table 2 furnishes the data that is necessary to confirm this result ∎

Figure 3 illustrates the result in proposition 4.
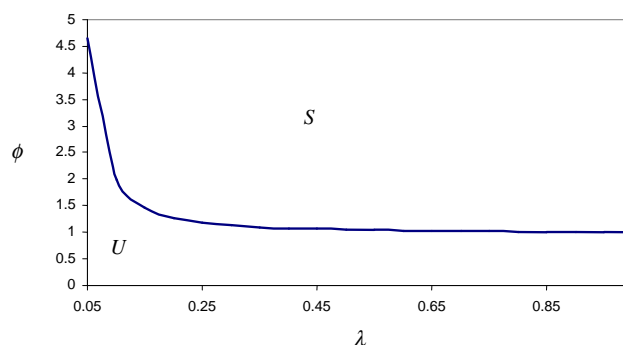
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Fig. 3 – Stability in the space of parameters

The above result is robust to changes in parameter values. Any other numerical experimentation, using admissible parameter values, will lead to a same kind of outcome. This is also an intuitive result: given some rule for the formation of expectations, the more inattentive agents are, the more the monetary authority needs to intervene (with a more aggressive policy), in order for stability to hold.

6 Partial Perfect Foresight: Global Analysis

Until now, the results of the constant attentiveness case and of the counter-cyclical attentiveness scenario have coincided: under perfect foresight, a result of stability holds in any of the cases. With partial perfect foresight, a same bifurcation condition separates, for both cases, regions of stability from regions of instability. This region of instability, however, will have different meanings under the two different assumptions about inattentiveness: constant information updating implies that the model is linear, local and global dynamics will coincide and there will be no exogenous fluctuations. On the opposite, counter-cyclical information updating triggers the formation of endogenous cycles in the region one has identified of being of local instability.

Recover the values $\lambda_0 = 0.25$ and $\underline{\lambda} = 0.1$ and remember that, in this case, stability holds for $\phi > 1.1808$. Figure 4 illustrates the long-term behavior of the model for the output variable g_t, and considering an interval of possible values of ϕ. The displayed bifurcation diagram allows to confirm where the region of stability is placed and to observe how the system

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behaves for values of ϕ between 1 and 1.1808. There is a period doubling bifurcation process that culminates in a small region of chaotic cycles, after which cycles of low periodicity return. In this way, we confirm the possibility of endogenous cycles of various periodicities and complete a-periodicity in the model of counter-cyclical attentiveness and partial perfect foresight: endogenous volatility is associated with a not sufficiently aggressive monetary policy. We can infer, from the analysis, that periods of larger volatility in the time paths of the main macroeconomic variables can be at least partially explained by a policy that is not active or aggressive enough given economic conditions relating agents' inattentiveness and agents' ability to accurately predict the future.
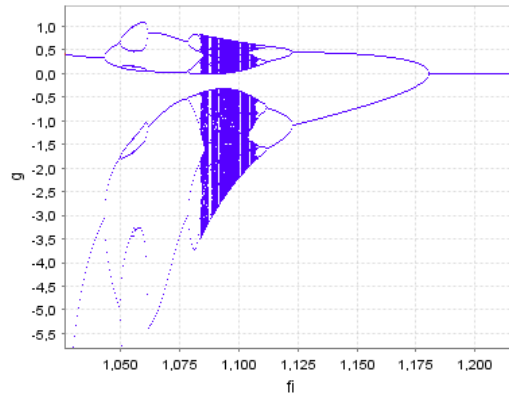

Fig. 4 – Bifurcation diagram

Figure 5 shows the type of strange attractor that emerges when a point of the system located at the chaotic zone is considered. The diagram shows all the possible points representing pairs of values (g_t, μ_t^R) that are obtainable in the long-run for a policy parameter value $\phi = 1.1$.
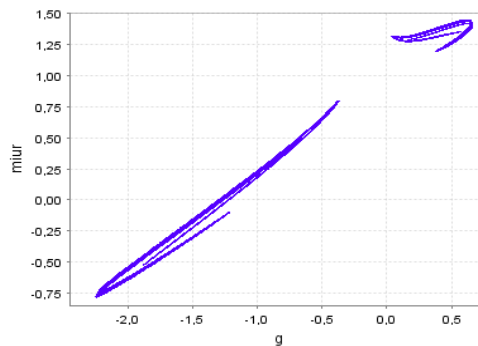

Fig. 5 – Attractor

7   Conclusion

The analysis has shown how a benchmark macroeconomic general equilibrium model with information stickiness can be adapted, by including two reasonable assumptions that allow to approach real life conditions, in order to display endogenous fluctuations on a setting that is, otherwise, inherently stable. The two assumptions, departures from perfect foresight and counter-cyclical information updating are, individually, necessary but not sufficient conditions for the generation of endogenous cycles. One needs to consider both in order to achieve the mentioned outcome. With the provided interpretation of macro relations, we have proven that the perspective put forward in the paper's initial sentence by Barnett, Medio and Serletis (1997) can be adapted to a macro environment involving information stickiness.

The study offers some intuitive results: it says that endogenous volatility arises through the combination of, on one hand, two anomalies relatively to what can be interpreted as an efficient behavior of economic agents – inattentiveness and non pervasive perfect foresight – with, on the other hand, an eventual difficulty of the central bank in understanding how aggressive its behavior must be, given the departures from 'perfect behavior' by the private agents. Thus, a sound monetary policy must be oriented towards two achievments: (i) to allow households and firms to have access to information and to help in equipping them with the ability of correctly forecasting the future and (ii) to recognize that private agents will not be able to always re-compute their decisions in an optimal way, what implies that monetary policy should be ready to perceive how active it should be in order to avoid excessive volatility.

References

[1] Barnett, W.A.; A. Medio and A. Serletis (1997). *Nonlinear and complex dynamics in economics*, EconWPA working paper number 9709001.

[2] Brock, W. A. and C. H. Hommes (1997). "A rational route to randomness." *Econometrica*, vol. 65, pp.1059-1095.

[3] Brock, W. A. and C. H. Hommes (1998). "Heterogeneous beliefs and routes to chaos in a simple asset pricing model." *Journal of Economic Dynamics and Control*, vol. 22, pp. 1235-1274.

[4] Bullard, J. B. (1994). "Learning equilibria." *Journal of Economic Theory*, vol. 64, pp. 468-485.

[5] Chen, H-J.; M-C. Li and Y-J. Lin (2008). "Chaotic dynamics in an overlapping generations model with myopic and adaptive expectations." *Journal of Economic Behavior and Organization*, vol. 67, pp. 48-56.

[6] Christiano, L. and S. Harrison (1999). "Chaos, sunspots and automatic stabilizers." *Journal of Monetary Economics*, vol. 44, pp. 3-31.

[7] Dieci, R. and F. Westerhoff (2009). "Heterogeneous speculators, endogenous fluctuations and interacting markets: a model of stock prices and exchange rates." *Journal of Economic Dynamics and Control*, vol. 34, pp. 743-764.

[8] Doms, M. and N. Morin (2004). *Consumer Sentiment, the Economy, and the News Media*, FRBSF working paper 2004-09.

[9] Fanti, L. and P. Manfredi (2007). "Neoclassical labour market dynamics, chaos and the real wage Phillips curve." *Journal of Economic Behavior and Organization*, vol. 62, pp. 470-483.

[10] Fujio, M. (2008). "Undiscounted optimal growth in a Leontief two-sector model with circulating capital: the case of a capital intensive consumption good." *Journal of Economic Behavior and Organization*, vol. 66, pp. 420-436.

[11] Gomes, O. (2006). "Routes to chaos in macroeconomic theory." *Journal of Economic Studies*, vol. 33, pp. 437-468.

[12] Gomes, O. (2010). "The sticky-information macro model: beyond perfect foresight." *Studies in Nonlinear Dynamics and Econometrics*, vol. 14, issue 1, article 1, pp. 1-35.

[13] Gomes, O. (2012). "Transitional dynamics in sticky-information general equilibrium models." *Computational Economics*, vol. 39, issue 4, pp. 387-407.

[14] Hallegatte, S.; M. Ghil; P. Dumas and J. C. Hourcade (2008). "Business cycles, bifurcations and chaos in a neo-classical model with investment dynamics." *Journal of Economic Behavior and Organization*, vol. 67, pp. 57-77.

[15] Jaimovich, N. (2007). "Firm dynamic and markup variations: implications for sunspot equilibria and endogenous economic fluctuations." *Journal of Economic Theory*, vol. 137, pp. 300-325.

[16] Kikuchi, T. and J. Stachurski (2009). "Endogenous inequality and fluctuations in a two-country model." *Journal of Economic Theory*, vol. 144, pp. 1560-1571.

[17] Lines, M. and F. Westerhoff (2010). "Inflation expectations and macroeconomic dynamics: the case of rational vs extrapolative expectations." *Journal of Economic Dynamics and Control*, vol. 34, pp. 246-257.

[18] Mankiw, N. G. and Reis, R. (2006). "Pervasive stickiness." *American Economic Review*, vol. 96, pp. 164-169.

[19] Mankiw, N. G. and Reis, R. (2007). "Sticky information in general equilibrium." *Journal of the European Economic Association*, vol. 5, pp. 603-613.

[20] Nishimura, K. and M. Yano (1995). "Nonlinear dynamics and chaos in optimal growth: an example." *Econometrica*, vol. 63, pp. 981-1001.

[21] Reis, R. (2009). "A sticky-information general-equilibrium model for policy analysis." NBER working paper n° 14732.

[22] Schonhofer, M. (1999). "Chaotic Learning Equilibria.", *Journal of Economic Theory*, vol. 89, pp. 1-20.

[23] Stockman, D. R. (2009). "Chaos and sector-specific externalities." *Journal of Economic Dynamics and Control*, vol. 33, pp. 2030-2046.

[24] Sushko, I.; L. Gardini and T. Puu (2010). "Regular and chaotic growth in a Hicksian floor/ceiling model." *Journal of Economic Behavior and Organization*, vol. 75, pp. 77-94.

[25] Yokoo, M. and J. Ishida (2008). "Misperception-driven chaos: theory and policy implications." *Journal of Economic Dynamics and Control*, vol. 32, pp. 1732-1753.

[26] Yoshida, H. and T. Asada (2007). "Dynamic analysis of policy lag in a Keynes-Goodwin model: stability, instability, cycles and chaos." *Journal of Economic Behavior and Organization*, vol. 62, pp. 441-469.

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