Economics
The Open-Access, Open-Assessment E-Journal

Discussion Paper
No. 2012-33 | July 12, 2012 | http://www.economics-ejournal.org/economics/discussionpapers/2012-33

Fixed-income Portfolio Management in Crisis Period: Expected Tail Loss (ETL) Approach

Mehdi Mili

University of Sfax, Tunisia & CEREGE EA 1722, University of Poitiers, France

Abstract The purpose of this study is to develop an efficient strategy for managing fixed-income portfolios in crisis periods. We use the volatility ratio model of Briere and Szafarz (2008) and the Expected Tail Loss (ETL) approach of Litzenberger and Modest (2008). Our methodology is applied to U.S. and European markets of fixed-income products using interest rates at different maturities over the period 2002 through 2010. U.S. portfolio exhibits his optimum with small amounts of interest rates belonging to the short-term strategy and the European portfolio exhibits his optimum with small amounts belonging to the long-term strategy. The results show that the ETL is a better measure of the downside risk than the Value-at-Risk (VaR). For instance, the U.S. (European) portfolio has a VaR of -3.6% (-0.7%) against an ETL of -6% (-0.8%). Moreover, we find that, for these two geographical areas, the short-term interest rates make little contribution to the overall ETL of the American fixed-income portfolio and vice versa for the European portfolio.

JEL G11, G15, N20

Keywords Fixed-income portfolio; financial crisis; flight-to-quality; contagion; expected tail loss

Correspondence Mili Mehdi, University of Sfax, ISG Sousse, Rue 7 Novembre, Monastir, Bodheur, 5063, TUNISIA; e-mail: Mehdi.Mili@isgs.rnu.tn

1. Introduction

Financial crises are generally known by higher volatility and a negative return between most asset classes fixed-income. On the other hand, if there is contagion of crisis, the situation becomes difficult for investors because the correlations between assets returns increase and diversification will be more beneficial for the case of quiet period. The traditional management of fixed-income assets increases the risk of portfolios. To ensure a good hedge against this risk, the first part of our model reduces the effects of the crisis that exposed investors (Chow et al., 1999). Subsequently, the minimization of the volatility ratio, allows us to construct optimal[(1)] fixed income portfolios resistant to the effects of the financial crisis.

On the stock exchange, optimal fixed-income portfolio support managers of fixed-income assets to meet an estimated level of risk by investors. However, the levels of risk posed by our model show that the impact of the crisis varies with the portfolio composition and risk. Generally, the concept of an optimal fixed-income portfolio is justified when it is resistant to turbulent markets. Briere and Szafarz (2008) show that a portfolio of some safe assets is more resistant to the effects of the crisis that the optimal portfolio and this thanks to effective diversification. Our empirical framework used in this case to assess the benefits of risky fixed-income assets considered optimal to resist in times of financial crisis. To develop the empirical part of this research, we considered a study period situated between 2002 and 2010. Our database consists of daily returns of interest rates in several terms for European Union and United States.

The most popular measure which serves to measure the risk is the Value-at-Risk (VaR) which gives generally the maximal loss that can realize a financial institution, during the normal situations of the market, over a fixed period and for a certain probability α. In practice, the VaR calculation is summarized in the variation of the probability α and the determination of minimum capital at beginning of period that will later to face the maximum loss in the end. So, the advantage of the VaR calculation is easy and simple to determine. Moreover, the VaR calculation is based on two main approaches: a parametric approach and non-parametric approach. According to Litzenberger and Modest (2008), the first empirical evidence does not located on the straight thick tails and the second is based on the current portfolio and consider the gains and losses that are expressed by the portfolio during a past period. Generally, because of historical commonly used, non-parametric approach is still unable to detect losses located on the left tail thick.

Considering these limits, stress tests[(2)] and scenario analysis often complement this approach. Here, the expected tail loss (ETL) is the most effective measure of the downside risk than the VaR because it takes into account the distribution losses in the lower tail[(3)]. The ETL approach then allows a better measure of loss located on the tails of the distribution than

[(1)] That is to say the portfolios present low volatility in times of crisis. These optimal portfolios allow investors to reduce the effects of the volatility caused by the financial crisis. Moreover, such portfolios are not very demanding and therefore safer for investors.
[(2)] Stress tests are effective methods to assess the situation of a portfolio over a period of crisis. This test can also be used to assess the strength of financial institutions.
[(3)] This result is justified by Modest and Litzenberger (2008) in their paper "Crisis and Non-Crisis Risk in Financial Markets: A Unified Approach to Risk Management."

the VaR. In our context, this measure is considered a practical measure of risk because it ignores the non-normality of returns and is particularly advantageous in case heavy left tails arises. The second part of our model develops a framework for measuring the risk of loss that considers the markets for fixed-income products are characterized by quiet periods in most cases, varied by rare moments of crisis. Thus, our model is characterized by the following points: (i) it captures the movement of stress during the crisis period, (ii) it is compatible with empirical observations that are characterized by thick left tails, (iii) it allows determine the contribution of each asset in the fixed-income portfolio's overall ETL as well as its volatility. Once this approach is applied, we find that the extent and causes of losses differ according to the specificities markets for fixed-income products.

However, our empirical framework consists of two main sections. The first present the models and the second contains the results determined and shows the different interpretations that allow us to make a comparison between the two geographical areas: the United States of America and the European Union.

2. The model

2.1. Crisis risk and determination of the state of nature

The markets of fixed-income are often characterized by quiet periods (excluding crisis), sometimes interrupted by periods of crisis. Such that this type of markets is unpredictable, the fixed-income portfolio appears as the one who minimizes the volatility ratio between these two types of periods.

However, based on the following model, two regimes exist: a quiet regime *q* and a crisis regime *c* with a probability equal to half since we treat only one type of crisis considered as an event absolutely unpredictable. Assuming that there is *C* types of financial crises. For C>2, the following equation allows to determine the number of states of nature, *S*, for a large number of crises:

$$S = C + 2 + \sum_{K=2}^{C-1} \frac{C!}{K!(C-K)!}\,. \tag{1}$$

For example, for *C*=1, *S*=2: a state of crisis and a state of no crisis. For *C*=2, *S*=4: a state of no crisis, a two-state crisis and two states with a single crisis. Each state of nature has a probability π_s for a given period.

2.2. Optimal fixed-income portfolio resistant to the effects of crisis

The market for fixed-income is composed of *n* risky assets. Let $R = (R_1, \dots, R_n)$ a vector of stochastic return, chosen randomly for each of the multivariate distributions: with a covariance matrix Σ_q for the quiet period and Σ_c for the crisis period. Crises arise exogenously, but investors are always aware of the presence of crises and the effects which they can have (kole et al., 2006).

The composition of fixed-income portfolio P composed of *n* risky fixed-income assets is as follows: $\Phi = (\phi_1, \dots, \phi_n)$, $\sum_{i=1}^{n} \phi_i = 1$ with ϕ_i the proportion of asset i in P.

The volatility of P depends on the following process:

$$\sigma^2(\mathrm{P}) = \begin{cases} \sigma_q^2 = \Phi^{'}\Sigma_{q^{\Phi}} \text{ during the quiet period,} \\ \sigma_c^2 = \Phi^{'}\Sigma_{c^{\Phi}} \text{ during the crisis period.} \end{cases} \quad (2)$$

However, the optimal fixed income portfolioP^*, is defined as a portfolio that minimizes the variance ratio between the two regimes mentioned above:

$$\frac{\sigma_c^2(\mathrm{P}^*)}{\sigma_q^2(\mathrm{P}^*)} = \min_{\mathrm{P}\in\left\{\substack{\text{realisable} \\ \text{portfolios}}\right\}} \frac{\sigma_c^2(\mathrm{P})}{\sigma_q^2(\mathrm{P})}. \quad (3)$$

The optimal fixed-income portfolio has several advantages that differ depending on the style of the manager of fixed-income assets. Among its advantages, we mention that transaction costs are reduced. On the other hand, the investor does not really need to rebalance the distribution of its fixed-income assets. In the case of active investors who react to small signals, the optimality of a fixed-income portfolio does not make sense because they adapt to any change in the market for fixed-income products. Unlike active investors, passive investors with a fixed-income portfolio well diversified always try a path that minimizes the impact of the crisis on their assets.

The composition of P^*, given by $\Phi^* = (\phi_1^*, \dots, \phi_n^*)$, where $\sum_{i=1}^{n} \phi_i^* = 1$, is such that:

$$\Phi^* = \operatorname{argmin} \frac{\Phi^{'}\Sigma_{c^{\Phi}}}{\Phi^{'}\Sigma_{s^{\Phi}}}. \quad (4)$$

Since we deal with fixed-income assets, our database consists precisely in interest rates in several maturities. However, this optimization that presents both equations (3) and (4) is applicable to any type of financial assets, provided that the structure (2) implies the existence of two regimes: quiet regime and crisis regime is verified.

To determine the two optimal fixed-income portfolios, we assume that each market consists of two fixed-income assets (i=1,2) with $\sigma_{iq} = 1,2$: the volatility of quiet period and $\sigma_{ic} = 1,2$: the volatility of crisis period. The variance given by equation (2), having both periods of fixed-income portfolio P, contains two proportions: a proportion $\phi \in [0,1]$ of a first fixed-income asset and a proportion $1 - \phi \in [0,1]$ of a second fixed-income asset. This variance is expressed as follows:

$$\sigma_q^2(\mathrm{P}) = \phi^2\sigma_{1q}^2 + (1-\phi)^2\sigma_{2q}^2 + 2\phi(1-\phi)\sigma_{1q}\sigma_{2q}\rho_q$$

$$(5)$$

$$\sigma_c^2(\mathrm{P}) = \phi^2\sigma_{1c}^2 + (1-\phi)^2\sigma_{2c}^2 + 2\phi(1-\phi)\sigma_{1c}\sigma_{2c}\rho_c \,,$$

with:

ρ_q : The correlation coefficient between the returns of two fixed-income assets during the quiet period.

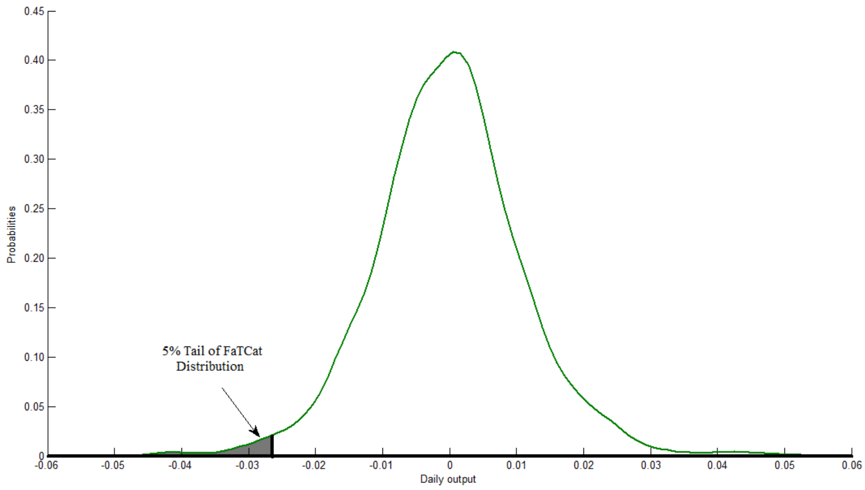
ρ_c : The correlation coefficient between the returns of two fixed-income assets during the crisis period.

In our case, this principle has been applied to distribute the six fixed-income assets by changing the composition of these two formulas in (5). Obviously, digital software was adopted to distribute such assets in each portfolio while minimizing the variance ratio presented in equation (3) to find the optimal allocation of each fixed-income asset. Once the optimal allocations of interest rates have been found, then we will develop the second part of our model which appears as a natural and effective measure of the risk of crisis.

2.3. Risk due to the financial crisis and Expected Tail Loss (ETL)

Litzenberger and Modest (2008) found significant results using the model discussed below to address hedge funds in case of a number of crises that are perfectly correlated. In this work, we adopted the same model to test the robustness of our two fixed-income portfolios. Unlike Litzenberger and Modest, we treat the case of one financial crisis. The expected tail loss (ETL) is calculated according to a percentile level $(1-\alpha)$ for a predetermined time horizon τ fixed in advance.

Figure 1: Expected tail loss of rates at 10 years E.U. (mixed distribution)



We present in this figure, the expected tail loss percentile for a level of 95%. After applying this model, we determine the ETL of each asset and we measure the contribution of these assets at the ETL of fixed-income portfolio. This same framework can be adopted in dealing with strategies in place assets. The non-normal returns with the global ETL of fixed-income portfolio which is expressed by weighting the probability of the states of nature, appears in the form of a distribution characterized by a thick tail.

This contrasts with the use of a mixture of non-normal distribution followed by independent probabilities between different fixed-income asset classes. Indeed, the ETL applied to fixed-income assets or on the portfolio P consisting of those assets is as follows:

$$\mathrm{ETL} = \sum_{s}^{S} \pi_s [\mu_{p,s}\, F\left(\frac{A-\mu_{p,s}}{\sigma_{p,s}}\right) - \sigma_{p,s} f\left(\frac{A-\mu_{p,s}}{\sigma_{p,s}}\right)]/\alpha\,, \tag{6}$$

with:

- S: number of states of nature.
- π_s: Probability of state *s* occurring.
- $\mu_{p,s}$: Mean of return on fixed-income portfolio for the state *s* suitable.
- $\sigma_{p,s}$: Standard deviation calculated from the yields of fixed-income portfolio for the state *s* suitable.
- A: Performance threshold determined from the choice of α percentile.

We can estimate A numerically using the following equation:

$$\alpha = \sum_s^S \pi_s F\left(A, \pi_{p,s}, \sigma_{p,s}\right). \tag{7}$$

In our framework, we assume a 5% probability for a given crisis applicable to each state of nature, α is equal to 0.05 to implement this model.

- f(.) : Probability density function for normalized returns of fixed-income portfolio.
- F(.) : Cumulative distribution function for standardized yields of fixed-income portfolio.

The ETL of fixed-income portfolio with the non-normality of returns ends in a distribution characterized by a thick tail. We infer then, that the period 2007-2010 is characterized by severe turbulence on the markets for fixed income. The sensitivity of expected tail loss of the portfolio due to changes in weight of fixed-income assets is presented according to the following equation:

$$\left.\frac{dETL}{dw_i}\right|_{w_{i=0}} = \frac{1}{\alpha}\sum_s^S \pi_s \left\{\left(\frac{d\mu_{p+,s}}{dw_i}\right) F(.) + \mu_{p+,s}\left(\frac{dF(.)}{dw_i}\right) - \left(\frac{d\sigma_{p+,s}}{dw_i}\right) f(.) - \sigma_{p+,s}\left(\frac{df(.)}{dw_i}\right)\right\}\Bigg|_{w_i=0}. \tag{8}$$

In this case, the derivatives applied to the weight of the fixed-income portfolio are valued at $w_i = 0$. Consider a fixed-income portfolio P^+ presenting an equal weight in w_i for the *nth* asset and $(1 - w_i)$ for the portfolio P.

The average rate of return linked to each state *s*: $\mu_{p+,s}$, and the standard deviation of the state *s*: $\sigma_{p+,s}$ of this new portfolio are given by the following equations:

$$\mu_{p+,s} = w_i\mu_{i,s} + (1 - w_i)\mu_{p,s}\ , \tag{9}$$

and

$$\sigma_{p+,s} = \left[w_i^2\sigma_{i,s}^2 + (1 - w_i)^2\sigma_{p,s}^2 + 2w_i(1 - w_i)\sigma_{ip,s}\right]^{\frac{1}{2}}\ . \tag{10}$$

The detailed procedures for calculating the sensitivity of the ETL appear in appendix.

To assess the marginal impact of each asset on the fixed-income portfolio's overall ETL, we differentiate equation (6) respecting each weight w_i presented by the equation (7) which is valued at $w_i = 0$. However, the fractional contribution of each asset ϕ_i^* to ETL overall optimal portfolio P^+ is represented by its weigh (w_{ip}). The ETL beta concerning the fixed-income portfolio P^+ is given by the following equation:

$$\beta_{i,p+}^{ETL} = 1 + \frac{\left.\frac{dETL}{dw_i}\right|_{w_i=0}}{ETL}. \tag{11}$$

We apply the same principle to determine the fractional contribution of each asset in the portfolio volatility. Indeed, the volatility beta is as follows:

$$\beta_{i,p+}^{\sigma} = 1 + \frac{\left.\frac{d\sigma_{p+}}{dw_i}\right|_{w_i=0}}{\sigma_{p+}}. \tag{12}$$

3. Results and interpretations

3.1. Study period, nature of the data and sample

The period of study extends from January 2002 until December 2010, to subdivide a quiet period from January 2002 until June 2007 and a period of crisis that stretches from July 2007 until December 31, 2010. We considered the date of July 2007 as a crisis because the subprime crisis erupted July 17, 2007 and key in practice most of the banking and financial system.

3.2. Descriptive tables

Table 1 presents descriptive statistics of all fixed-income assets that we treated during the entire period. For the U.S., interest rates at 3 months and 6 months have a higher average compared to other fixed-income assets with 1.124% and 0.005%, respectively. Rates at 3 months have a difference of 20.384 basis points compared to rates at 6 months of volatility. So, volatility reacts when there is a change in the yields of interest rates. The densities of the six fixed-income assets are sharp as they are all positive.

Regarding the European Union, we find that all fixed-income assets have negative averages, which explains their approximation in volatility. Unlike the United States, the highest volatilities are given by the assets belonging to the long-term strategy.

Table 1. Descriptive statistics of daily returns of fixed-income assets: January 1, 2002 - 31 December 2010 (mixed period)

	U.S.		E.U.
		rates at 3 months	
Mean (%)	1.124		-0.050
Volatility (%)	25.203		0.495
Skewness	19.175		-0.208
Kurtosis	453.191		19.728
		rates at 6 months	
Mean (%)	0.005		-0.040
Volatility (%)	4.819		0.547
Skewness	6.754		-0.139
Kurtosis	133.743		10.551
		rates at 1 year	
Mean (%)	-0.030		-0.031
Volatility (%)	3.368		0.814
Skewness	1.159		0.465
Kurtosis	17.242		8.393
		rates at 2 years	
Mean (%)	0.005		-0.021
Volatility (%)	3.860		2.652
Skewness	0.823		1.151
Kurtosis	12.262		18.623
		rates at 5 years	
Mean (%)	0.002		-0.018
Volatility (%)	2.640		1.633
Skewness	0.223		0.469
Kurtosis	8.989		7.265
		rates at 10 years	
Mean (%)	-0.003		-0.012
Volatility (%)	1.757		1.099
Skewness	-0.148		0.193
Kurtosis	8.972		4.507

Tables 2 and 3 provide descriptive statistics for the period of quiet and crisis, respectively. Logically, the crisis is known for its higher volatility. We note the increase by comparing the volatilities of the various fixed-income assets between the two periods. In fact, in United States during the quiet period, we find that there is little connection between the volatilities of the various fixed-income assets.

The rates most volatile are the ones in 1 year, 2 years and 5 years. In the European Union, we find that the rates most volatile are the ones in 2 years and 5 years with values equal to 1.672 % and 1.339 %, respectively. From another point of view, the United States yields on interest rates during the quiet period are all positive contrary to some fixed-income assets during the crisis period. So, these rates are suffering more than others in times of trouble.

Table 2. Descriptive statistics of daily returns of fixed-income assets: 01 January 2002 - June 30, 2007 (quiet period)

	U.S.	E.U.
	rates at 3 months	
Mean (%)	0.081	-0.005
Volatility (%)	1.439	0.392
Skewness	-0.075	0.365
Kurtosis	16.367	25.960
	rates at 6 months	
Mean (%)	0.080	0.002
Volatility (%)	1.453	0.585
Skewness	0.383	0.178
Kurtosis	13.921	9.732
	rates at 1 year	
Mean (%)	0.077	0.010
Volatility (%)	1.988	1.017
Skewness	0.704	0.323
Kurtosis	9.992	5.577
	rates at 2 years	
Mean (%)	0.062	0.011
Volatility (%)	2.470	1.672
Skewness	0.587	0.701
Kurtosis	7.675	7.494
	rates at 5 years	
Mean (%)	0.023	-0.002
Volatility (%)	1.747	1.339
Skewness	0.609	0.367
Kurtosis	7.010	4.533
	rates at 10 years	
Mean (%)	0.007	-0.014
Volatility (%)	1.265	0.980
Skewness	0.468	0.525
Kurtosis	5.487	4.499

In quiet period, it is only the rates at 3 months with a low skewness with a value of -0.075. Thus, the skewness of the other fixed-income assets means that returns during the quiet period are asymmetric to the right. The returns of the crisis period have high kurtosis compared to the quiet period. This increase means that these rates show a distinct peak near the average.

Table 3. Descriptive statistics of daily returns of fixed-income assets: 01 July 2007 - December 31, 2010 (crisis period)

	U.S.		E.U.
		rates at 3 months	
Mean (%)	2.758		-0.094
Volatility (%)	40.300		0.577
Skewness	11.930		-0.252
Kurtosis	176.246		15.733
		rates at 6 months	
Mean (%)	-0.111		-0.083
Volatility (%)	7.504		0.503
Skewness	4.636		-0.724
Kurtosis	58.703		11.308
		rates at 1 year	
Mean (%)	-0.197		-0.072
Volatility (%)	4.786		0.538
Skewness	1.046		0.452
Kurtosis	10.541		16.219
		rates at 2 years	
Mean (%)	-0.086		-0.052
Volatility (%)	5.357		3.356
Skewness	0.741		1.069
Kurtosis	7.999		14.092
		rates at 5 years	
Mean (%)	-0.032		-0.035
Volatility (%)	3.623		1.882
Skewness	0.136		0.494
Kurtosis	5.928		7.092
		rates at 10 years	
Mean (%)	-0.019		-0.010
Volatility (%)	2.328		1.206
Skewness	-0.266		0.010
Kurtosis	6.721		4.252

Table 4.a) Statistical tests of equality of volatilities of daily returns for the U.S. interest rates (quiet period compared to the crisis period)

		Quiet period					
		rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
Crisis period	rt (3 m)	0					
	rt (6 m)		0				
	rt (1 y)			0			
	rt (2 y)				0		
	rt (5 y)					0	
	rt (10 y)						0

pour α=5%

Referring to statistical tests of equalities and based on the assumptions presented, we find that there are no equalities between the volatility of interest rates over the two periods for the two geographical areas. This confirms the results found in the descriptive tables and justify that the crisis period is characterized by an increase in volatility.

Table 4.b) Statistical tests of equality of volatilities of daily returns for the E.U. interest rates (quiet period compared to the crisis period)

		Quiet period					
		rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
Crisis period	rt (3 m)	0					
	rt (6 m)		0				
	rt (1 y)			0			
	rt (2 y)				0		
	rt (5 y)					0	
	rt (10 y)						0

pour α=5%

However, tests of equality of means of two periods, confirms our results found in some instances and contradicts them in others. Indeed, equality displayed by these tests is due to the strong correlation between interest rates both in the European and American market.

Table 5.a) Statistical tests of equality of means of daily returns for the U.S. interest rates (quiet period compared to the crisis period)

		Quiet period					
		rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
Crisis period	rt (3 m)	0.012					
	rt (6 m)		0.350				
	rt (1 y)			0.055			
	rt (2 y)				0.364		
	rt (5 y)					0.620	
	rt (10 y)						0.724

pour α=5%

Table 5.b) Statistical tests of equality of means of daily returns for the U.S. interest rates (quiet period compared to the crisis period)

		Quiet period					
		rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
Crisis period	rt (3 m)	0.000					
	rt (6 m)		0.000				
	rt (1 y)			0.016			
	rt (2 y)				0.567		
	rt (5 y)					0.637	
	rt (10 y)						0.937

pour α=5%

Statistical tests of Jarque-Bera were performed to verify the various fixed-income assets follow the normal distribution. We have applied this type of testing for all interest rates and over three different periods.

Table 6.a) Tests of normality Jarque-Bera U.S.

	Prob. Mixed period	Prob. Quiet period	Prob. Crisis period
rates at 3 months	0.001*	0.001*	0.001*
rates at 6 months	0.001*	0.001*	0.001*
rates at 1 year	0.001*	0.001*	0.001*
rates at 2 years	0.001*	0.001*	0.001*
rates at 5 years	0.001*	0.001*	0.001*
rates at 10 years	0.001*	0.001*	0.001*

(*) Non-normal series.

Table 6.b) Tests of normality Jarque-Bera E.U.

	Prob. Mixed period	Prob. Quiet period	Prob. Crisis period
rates at 3 months	0.001*	0.001*	0.001*
rates at 6 months	0.001*	0.001*	0.001*
rates at 1 year	0.001*	0.001*	0.001*
rates at 2 years	0.001*	0.001*	0.001*
rates at 5 years	0.001*	0.001*	0.001*
rates at 10 years	0.001*	0.001*	0.001*

(*) Non-normal series.

The results presented in these two tables, confirm the non-normality of the series of returns for the two regions.

3.3. Correlation matrix

Tables 7 to 10 show the correlations of daily returns of fixed-income assets for different periods and for both regions. In all cases, the different rates exhibit positive correlations for both blocks: U.S. and E.U., this means that both correlated rates move in the same direction. However, we find that the correlations between the fixed-income assets representing the long-term strategy are higher values that the correlations of the short-term strategy. These properties show the importance of the maturity of interest rates in reducing the risk posed by financial crises.

Table 7.a) Correlation Matrix: fixed-income assets U.S., mixed period
Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	25.360	21.773	14.342	10.500	7.226
rt (6 m)		-	58.971	42.233	35.696	31.230
rt (1y)			-	73.658	65.322	57.391
rt (2 y)				-	87.979	76.882
rt (5 y)					-	93.778
rt (10 y)						-

Table 7.b) Correlation Matrix: fixed-income assets E.U., mixed period
Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	82.005	57.797	9.987	8.662	4.256
rt (6 m)		-	88.752	19.205	20.146	14.432
rt (1 y)			-	25.328	28.333	21.938
rt (2 y)				-	70.486	54.173
rt (5 y)					-	74.808
rt (10 y)						-

The correlations between rates at 3 months and other rates have values situated between 31.536 % and 72.221% for the United States and between 11.715 and 76.955%% for the European Union during quiet period. For the most part, the correlations fall sharply during the crisis, with values between 7.847% and 91.109% for the United States and 0.230% and 90.991% for the European Union.

Table 8.a) Correlation Matrix: fixed-income assets U.S., quiet period
Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	72.221	48.780	39.053	35.174	31.536
rt (6 m)		-	83.112	67.068	60.389	54.458
rt (1 y)			-	87.410	78.104	71.055
rt (2 y)				-	91.216	82.913
rt (5 y)					-	95.590
rt (10 y)						-

Table 8.b) Correlation Matrix: fixed-income assets U.S., quiet period
Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	76.955	56.760	19.216	19.512	11.715
rt (6 m)		-	91.109	37.315	35.531	24.655
rt (1 y)			-	46.429	44.281	32.250
rt (2 y)				-	81.297	65.904
rt (5 y)					-	80.296
rt (10 y)						-

Table 9.a) Correlation Matrix: fixed-income assets U.S., crisis period
Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	25.481	23.717	15.707	11.408	7.847
rt (6 m)		-	57.893	40.765	34.051	29.874
rt (1 y)			-	69.616	61.499	53.120
rt (2 y)				-	86.874	74.807
rt (5 y)					-	93.229
rt (10 y)						-

Table 9.b) Correlation Matrix: fixed-income assets U.S., crisis period
Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	90.991	76.112	6.910	3.364	0.230
rt (6 m)		-	90.813	11.276	8.553	5.379
rt (1 y)			-	16.797	14.853	11.076
rt (2 y)				-	67.841	51.333
rt (5 y)					-	71.900
rt (10 y)						-

The effect FTQ (flight-to-quality) among the various fixed-income assets is clearly observed during the crisis period. This can be seen clearly in the tables (10.a) and (10.b) that display negative values determined from the difference between the correlations in a crisis and those in quiet period. The effect FTQ is a result of lower correlations in times of stress. For the United States, all correlations express FTQ, it means effectively that crises lead to low correlations. For example, on the American market during the period of disorder, the correlation between the rates at 6 months and 1 year falling more than 25%. The correlation that is resistant to most is that the crisis of the last two rates with a value of -2.361%.

Table 10.a) Correlation spread matrices: U.S. fixed-income assets

Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	-46.740	-25.063	-23.346	-23.766	-23.689
rt (6 m)		-	-25.219	-26.303	-26.338	-24.584
rt (1 y)			-	-17.794	-16.604	-17.935
rt (2 y)				-	-4.342	-8.106
rt (5 y)					-	-2.361
rt (10 y)						-

(*) Differences between the correlations in crisis and correlations in quiet periods. The cells in grey correspond to the presence of FTQ (lower correlation).

Table 10.b) Correlation spread matrices: E.U. fixed-income assets

Daily returns (%)

	rt (3 m)	rt (6 m)	rt (1 y)	rt (2 y)	rt (5 y)	rt (10 y)
rt (3 m)	-	14.036	19.352	-12.307	-16.148	-11.485
rt (6 m)		-	-0.296	-26.039	-26.978	-19.276
rt (1 y)			-	-29.632	-29.427	-21.174
rt (2 y)				-	-13.456	-14.571
rt (5 y)					-	-8.397
rt (10 y)						-

(*) Differences between the correlations in crisis and correlations in quiet periods. The cells in grey correspond to the presence of FTQ (lower correlation).

For the European Union, the effect FTQ observed in thirteen cases out of fifteen, which probably detects the presence of a sort of contagion between these types of fixed-income assets. Thus, these positive differences mean that, during crises, holders of fixed-income assets Europeans consider the first two short-term rates as risky. Correlation spreads and volatility ratios are key elements to determine optimal fixed-income portfolios resistant to crisis. In addition, the information disclosed in the tables from 10 to 11, help us to identify the effects of the crisis and the benefits of diversification.

3.4. Volatility ratios

The volatility ratio is normally a measure of the randomness of a series of returns. In our context, the volatility ratio is calculated by dividing the volatility of returns in crisis period by the volatility of returns in quiet period for the fixed-income assets concerned.

Table 11. a) Volatility ratios(*), daily returns of fixed-income assets U.S.

rt (3 m)	rt (6 m)	rt (1 y)	tx (2 ans)	tx (5 ans)	tx (10 ans)
28.008	5.163	2.407	2.168	2.074	1.840

(*) volatility during crisis period / volatility during quiet period.

By comparing the volatility ratios of the two markets, we find that the ratios of short-term strategies are higher than long-term strategies for the United States and the opposite case for the European Union, which shows the effects of the crisis on the rates.

Table 11. a) Volatility ratios(*), daily returns of fixed-income assets U.S.

rt (3 m)	rt (6 m)	rt (1 y)	tx (2 ans)	tx (5 ans)	tx (10 ans)
1.472	0.860	0.529	2.008	1.406	1.231

(*) volatility during crisis period / volatility during quiet period.

As our model allows the minimization of the volatility ratio for all fixed-income assets to be included in an optimal portfolio, we can always discuss that the rates with minimum volatility ratios are considered as strong rates.

3.5. Distribution of interest rates in optimal fixed-income portfolios

For each geographical area, we determine an optimal fixed-income portfolio consists of six interest rates at various maturities. In both cases, only one parameter is considered to indicate the composition of each portfolio, this is the ratio of volatility minimized for all rates. The four figures below plot the conditional volatility of daily returns of interest rates based on an estimate GARCH (1,1). We selected two samples of each strategy to that representation. Each sample covers the entire period of study.

In terms of short-term strategy, we note that the rates at 6 months U.S. strongly growing between mid 2007 and early 2009 and then decrease abruptly in the form of shock for the rest of the period. The volatility of European rate at 1 year is constant during the period 2002 to 2008 and increased sharply from 2008 and then down in the form of shocks between 2009 and 2010. This justifies the increasing impact of the financial crisis on these fixed-income assets and the decrease is a result of the collapse of the markets for fixed-income products, which makes these types of the most risky rates.

Figure 2: Conditional volatility of rates at 6 months U.S. (period: 2002-2010)

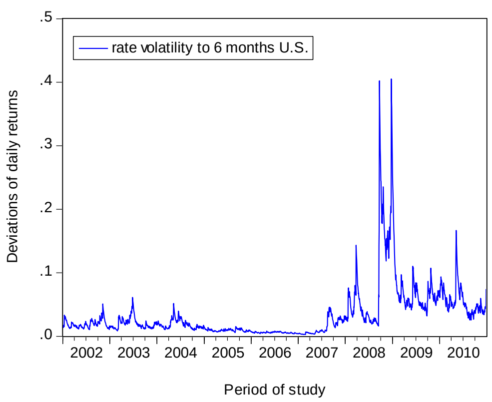
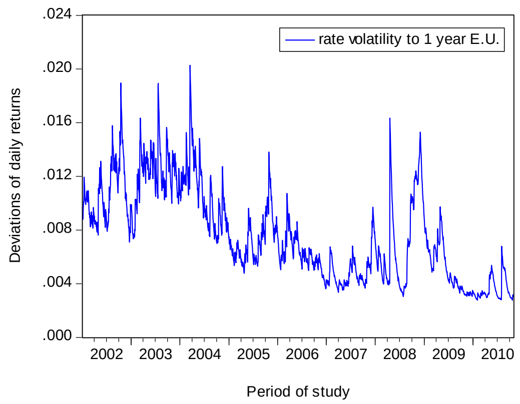


Figure 3: Conditional volatility of rates at 1 year E.U. (period: 2002-2010)



In terms of long-term strategy, we find that the rates at 5 years American have an overall trend upward in 2009. These rates show an increase in volatility from mid 2007, which shows the effect of the crisis and confirm the exact date of its outbreak. The European rates at 10 years have two peaks on the rise.

Figure 4: Conditional volatility of rates at 5 years U.S. (period: 2002-2010)

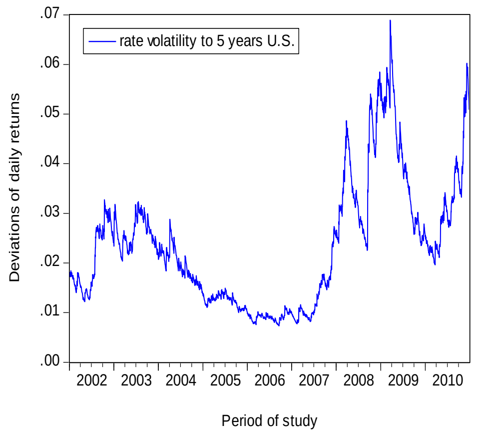
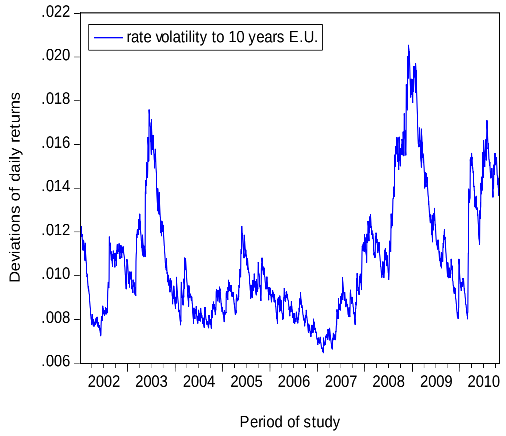


Figure 5: Conditional volatility of rates at 10 years E.U. (period: 2002-2010)



The optimal allocation is determined from the optimal weights based on empirical distributions of daily returns (each period separately) by using a programming software.

Table 12. Composition of optimal fixed-income portfolios

Strategy by block	Fixed-income assets affected in the portfolio	Allocation (%)	Minimum volatility ratio
Fixed-income portfolio U.S. (*mixed strategy*)	rates at 3 months	0.149	1.560
	rates at 6 months	4.926	
	rates at 1 year	12.200	
	ratse at 2 years	9.702	
	rates at 5 years	21.317	
	rates at 10 years	51.706	
Fixed-income portfolio E.U. (*mixed strategy*)	rates at 3 months	26.234	0.763
	rates at 6 months	34.412	
	rates at 1 year	30.121	
	rates at 2 years	0.773	
	rates at 5 years	2.462	
	rates at 10 years	5.998	

In considering Table 12, we find that the U.S. finds its optimum portfolio with small allocations for interest rates belonging to the short-term strategy. The lowest proportion is the rates at 3 months with 0.149% followed by the rates at 6 months and rates at 2 years with 4.926% and 9.702%, respectively. The European Portfolio present small proportions for the last three rates, unless there is a risk of contagion between the fixed-income assets, which is not the case for the United States. For an optimal fixed-income portfolio perfectly balanced, the optimal ratio most advantageous for Europeans is equal to 0.763 and the best ratio for Americans is equal to 1.560. However, the difference between the portfolio compositions is due to the anomalies caused by the crisis.

3.6. Approach ETL (Expected Tail Loss)

The two figures below plot the distribution of sample rates for each region. For the United States, we plot the distribution of daily returns for the rates at 5 years. For the European Union, we plot the distribution of rate at 10 years. The blue distribution represents the non-crisis period; the red distribution is subject to a crisis.

Figure 6: Graphical illustration of two regime stress loss framework (rates at 5 years U.S.)

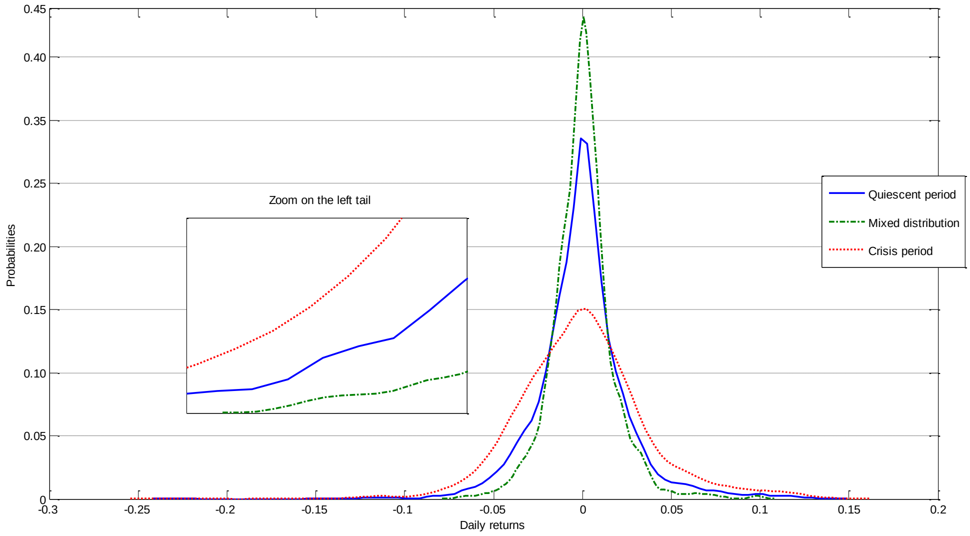
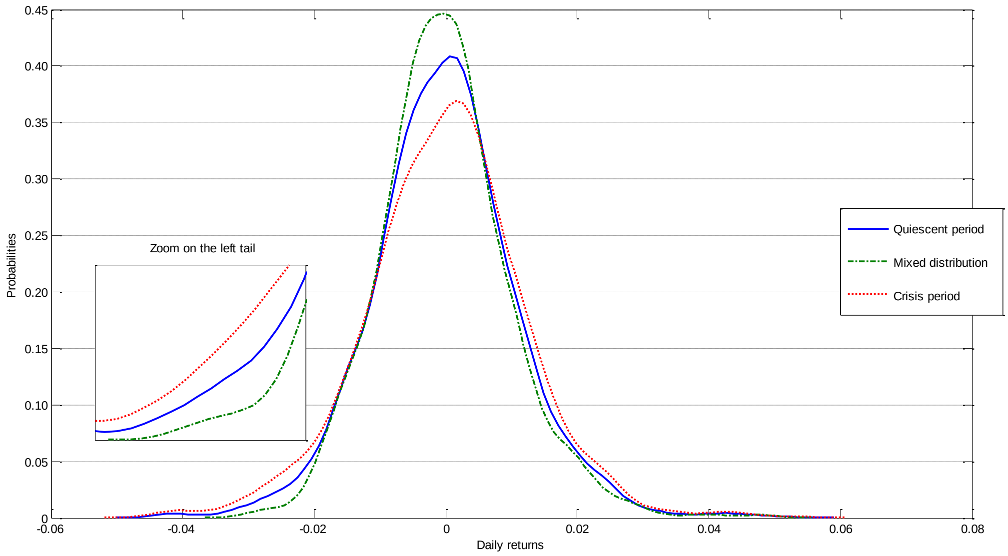


Figure 7: Graphical illustration of two regime stress loss framework (rates at 10 years E.U.)



The conditional distribution in times of crisis is marked by an average decrease related to the decrease of correlations in periods of stress, as well as an increase of the volatility with regard to quiet period. As shown in these figures, the left part shows the negative returns (losses) and the right side contains the earnings of each fixed-income asset. Distribution in times of crisis shows a thicker left tail than the other distributions. The zoom on the left side of the tail shows the extent of losses during crisis period.

Table 13. Analysis of fixed-income asset allocation strategies

Strategy by block	Fixed-income assets distributed	Allocation (%)	Volatility of returns (%)	VaR (%)	ETL (%)	β^{ETL}	Contribution to the ETL Portfolio (%)	β^{σ}	Contribution to the volatility of the portfolio (%)	Excess of expected return (%)
mixed strategy U.S.	rates at 3 months	0.149	25.203	-40.331	-76.665	1.044	0.104	1.873	0.187	1.127
	rates at 6 months	4.926	4.819	-7.920	-16.294	1.036	5.077	1.144	5.604	0.008
	rates at 1 year	12.200	3.368	-5.570	-10.471	1.146	13.983	1.220	14.881	-0.027
	rates at 2 years	9.702	3.860	-6.345	-11.005	1.382	13.401	1.604	15.557	0.008
	rates at 5 years	21.317	2.640	-4.341	-7.261	1.159	24.694	1.167	24.852	0.005
	rates at 10 years	51.706	1.757	-2.893	-4.544	0.632	32.651	0.753	38.919	0.000
mixed strategy E.U.	rates at 3 months	26.234	0.495	-0.864	-1.324	0.758	19.861	0.693	18.164	-0.012
	rates at 6 months	34.412	0.547	-0.941	-1.133	0.939	32.290	0.943	32.426	-0.003
	rates at 1 year	30.121	0.814	-1.370	-1.810	1.220	36.734	1.362	40.982	0.007
	rates at 2 years	0.773	2.652	-4.384	-6.787	1.158	0.926	1.685	1.348	0.017
	rates at 5 years	2.462	1.633	-2.705	-3.521	1.072	2.679	1.179	2.946	0.019
	rates at 10 years	5.998	1.099	-1.819	-2.279	0.802	4.810	0.689	4.134	0.026

This table consists of two fixed-income portfolios following a mixed strategy. Each of our portfolios represents a geographical area and consists of the entire interest rates in our sample. The U.S. fixed-income portfolio has a volatility of returns equal to 2.180% and the European portfolio exhibits a volatility of its returns equal to 0.557%. After calculating the VaR for each of fixed-income assets, we note that the ETL provides more significant results in all cases. This is verified by comparing the same VaR and overall ETL for each portfolio. Indeed, for a confidence level α = 5%, the U.S. fixed-income portfolio has a VaR equal to -3.588% against an ETL equal to -5.967%. The fixed-income portfolio representing the European Union displays a VaR equal to -0.753% against an ETL equal to -0.820%.

The highest ETL is the one of rates at 3 months with -76.665% and a VaR equal to -40.331%, followed by the ETL of rates at 6 months with -16.294% and VaR equal to -7.920%. So, in this regard, more allocation weakens, more volatility is big and the ETL increases. The VaR of rates at 1 year, 2 years and 5 years shows damage with respective values that are between -4.341% and -6.345%. This can be seen by referring to ETL of these fixed-income assets with values between -7.261% and -11.005%. The lowest ETL appropriates to the rates at 10 years with -4.544%, this is true in terms of its low volatility with 1.757%. Although rates at 3 months and 6 months have relatively high volatility, they tend to grow better during periods of stress when the prices of fixed-income assets move in a sustained manner. In the Euro zone, we find a small difference from the United States. However, the results displayed in columns 4, 5 and 6 of the mixed strategy of the European Union are lowered in a descending order. These properties indicate that more maturity rate increases, more the risk from the asset increases. The first three rates of the European Union are considered as the safest fixed-income assets.

The two variables: ETL beta and beta volatility are two ways to represent the contributions of each strategy to the ETL and the volatility of fixed-income portfolio. Thus, as regards to the United States, we find that the rates with short-term strategy contribute less to the ETL fixed-income portfolio than other rates representing the long-term strategy. About Europe and as already mentioned, the fixed-income assets behave in a manner quite different. For the United States, the two rates with the lowest betas are the rates at 10 years with 0.632 and the rates at 6 months with 1.036. In the case of the Euro zone, we find that the two rates with the lowest ETL beta rates are the rates at 3 months followed by rates at 10 years with respectively, 0.758 and 0.802. The major contributors to the volatility of U.S. fixed-income portfolio are driven by rates at 3 months, rates at 6 months, rates at 1 year and rates at 2 years. Rates at 5 years with an important allocation of 21.317% present relatively high volatility during periods of non-crisis. Regarding the European Union, the major contributors to the volatility of fixed-income portfolio are the rates at 2 years with 1.685 and the rates at 1 year with 1.362. Rates at 5 years Europeans provide high volatility during quiet periods. At this point, we find that for two blocks, the two types of contributions are the consequences of fixed-income asset allocation.

The results presented in the last column of Table 13 show the excess of expected return for each rate for each fixed-income portfolio. These excess of expected return are compatible with the weight of optimal fixed-income portfolios. The overall expected return is 0.187% for the U.S. fixed-income portfolio and 0.009% for the European fixed-income portfolio. In addition, the side of the United States, we observe that only rates at 1 year contribute negatively to the overall portfolio yield. For its part, the six fixed-income assets composing its portfolio, Europe has only two rates that contribute negatively to its fixed-income portfolio. However, the last four rates show positive returns ranging between 0.007% and 0.026%.

4. Conclusion

The circumstances recorded by the markets for fixed- income push investors to take the necessary precautions when a crisis occurs. Every crisis has its own characteristics: some start slowly, others quickly, some are short, and others are long-term. However, until the onset of a crisis to rebalance a fixed-income portfolio can be a huge risk. To confront these harmful events, our setting allows building an optimal fixed-income portfolio with a minimum volatility ratio during a period situated between stability and stress. Our work is to show that the introduction of risky fixed-income assets in a portfolio can resist to the increase in volatility due to the crisis. Thus, the presence of some risky fixed- income assets is associated with the onset of the effect of flight-to-quality (FTQ). The same situation occurs when the contagion is observed instead FTQ. Of course, the contagion is an undesirable phenomenon for the managers of fixed-income portfolios through its unpleasant consequences. For its part, FTQ contributes to the protection of interest rates by reducing the effect of the financial crisis.

Throughout this research, we developed two-regimes on the risk of loss of a fixed-income portfolio. This context includes the idea that markets for fixed-income products are characterized by quiet periods mostly interrupted by occasional periods of crisis. Our structure allows associating a random number of crises with transactions to help the investor to make the right decisions in a given period. Since the returns are characterized by the non-normality, the conditional distribution of returns has heavy tails left. Our risk structure can capture the volatility of the quiet period and the extreme losses during the crisis period.

This framework intensely opposed to other risk measures and especially the VaR for measuring potential losses during normal periods, then to complete them with adequate loss scenarios. The expected tail loss (ETL) is considered as a relevant measure in our framework. We determine the excess of expected return for each rate composing his fixed-income portfolio; this return must be compatible with all weights of the optimal portfolios. In practice, the manager of fixed-income assets cares generally about the volatility of its portfolio. In this case, the framework allows the investor to make decisions based on the excess of expected return, weighted average of the ETL and the volatility during quiet period.

Appendix

This appendix is dedicated to providing the steps that we followed to determine the Expected Tail Loss (ETL) and the derivatives necessary to allow the calculation of β^{ETL} and β^{σ} for each fixed- income asset i with respect to portfolio P^{+}.

In a formal way, the ETL applied on a fixed-income portfolio P can be expressed as follows:

$$\mathrm{ETL} \equiv \mathrm{E}\left[\tilde{\mathrm{R}}_{\mathrm{p}} \middle| \tilde{\mathrm{R}}_{\mathrm{p}} \leq \mathrm{A}\right] = \sum_{s}^{S} \pi_{s} \mathrm{E}\left[\tilde{\mathrm{R}}_{\mathrm{p}} \middle| \tilde{\mathrm{R}}_{\mathrm{p}} \leq \mathrm{A}, \mathrm{s}\right] \qquad (A.1)$$

with:

- S: number of states of nature.
- π_s: Probability of state *s* occurring.
- R_p: portfolio rate of return in excess of the risk-free rate.
- $\mu_{p,s}$: Mean of return on fixed-income portfolio for the state *s* suitable.
- $\sigma_{p,s}$: Standard deviation calculated from the yields of fixed-income portfolio for the state *s* suitable.
- A: Performance threshold determined from the choice of α percentile.

By replacing the operator of the expected value, the ETL can be rewritten as:

$$\mathrm{ETL} = \sum_{s}^{S} \pi_{s} \int_{-\infty}^{A} x f\left(x, \mu_{p,s}, \sigma_{p,s}\right) \mathrm{d}x / x\ , \qquad (A.2)$$

with

- f(.) : Probability density function for normalized returns of fixed-income portfolio.

As indicated in section (2.3), the threshold of normalized return (a_s) corresponding to the state *s* is as follows:

$$a_s = \frac{A - \mu_{p+,s}}{\sigma_{p+,s}}\ , \qquad (A.3)$$

Analytically by integrating the right side of equation (A.2), we obtain the solution over the ETL. Also, consider a fixed-income portfolio P^{+} with a weight w_i in *nth* fixed-income asset and $(1 - w_i)$ in the fixed-income portfolio P. The conditional solution on A with α the performance of portfolio for a non-normal distribution is such that:

$$\mathrm{ETL} = \sum_{s}^{S} \pi_{s} [\mu_{p+,s} \mathrm{F}(a_s) - \sigma_{p+,s} f(a_{s)}] / \alpha\ , \qquad (A.4)$$

where

- $F(.)$: Cumulative distribution function for standardized yields of fixed-income portfolio.

However, equation (A.4) is subject to the following constraint:

$$\alpha = \sum_{s}^{S} \pi_s \, F(a_s) \ . \tag{A.5}$$

The ETL in each state of nature *s* depends on the average rate of return on fixed-income portfolio tied to each state *s*: $\mu_{p+,s}$, and the standard deviation calculated from the yields of fixed-income portfolio: $\sigma_{p+,s}$. The latter two are shown by the following equations:

$$\mu_{p+,s} = w_i \mu_{i,s} + (1 - w_i)\mu_{p,s} \ , \tag{A.6}$$

and

$$\sigma_{p+,s} = [w_i^2 \sigma_{i,s}^2 + (1 - w_i)^2 \sigma_{p,s}^2 + 2w_i(1 - w_i)\sigma_{ip,s}]^{\frac{1}{2}} \ , \tag{A.7}$$

With $\sigma_{ip,s}$ which means the covariance of the state *s* between the yields of fixed-income assets i and yields of fixed-income portfolio P. Taking into account the weight w_i, the derivative of the ETL is:

$$\left.\frac{dPAQ}{dw_i}\right|_{w_{i=0}} = \frac{1}{\alpha} \sum_{s}^{S} \pi_s \left\{ \left(\frac{d\mu_{p+,s}}{d_{w_i}}\right) F(a_s) + \mu_{p+,s} \left(\frac{dF(a_s)}{dw_i}\right) - \left(\frac{d\sigma_{p+,s}}{dw_i}\right) f(a_s) - \sigma_{p+,s} \left(\frac{df(a_s)}{dw_i}\right) \right\} \right|_{w_i=0} \ . \tag{A.8}$$

To solve this derivative, we assume that the weight of the fixed-income portfolio is valued at$w_i = 0$. On the other hand, equation (A.8) is subject to the following constraint:

$$\sum_{s}^{S} \pi_s \left.\frac{dF(a_s)}{dw_i}\right|_{w_i=0} = 0 \ . \tag{A.9}$$

The derivative presented by equation (A.8) is an essential element to calculate β^{PAQ} of the fixed-income asset relative to the fixed-income portfolio P^+. We present now the analytical derivatives that make up equation (A.8). The derivatives of the mean of return on fixed-income portfolio and the standard deviation of the state *s* suitable by considering the weight w_i are:

$$\left.\frac{d\mu_{p+,s}}{dw_i}\right|_{w_i=0} = \mu_{i,s} - \mu_{p,s} \ , \tag{A.10}$$

$$\left.\frac{d\sigma_{p+,s}}{dw_i}\right|_{w_i=0} = \rho_{i,p,s}\sigma_{i,s} - \sigma_{p,s} \ , \tag{A.11}$$

$\rho_{i,p,s}$ represents the correlation between the returns of the nth fixed-income asset and fixed-income portfolio P in state *s*. Referring to the chain rule, we can present the derivatives of the cumulative distribution and the derivatives of the probability density for the standardized returns of fixed-income portfolio in accordance with its weight as:

$$\left.\frac{dF(a_s)}{dw_i}\right|_{w_i=0} = \left.\left(\frac{da_s}{dw_i}\right)\left(\frac{dF(a_s)}{da_s}\right)\right|_{w_i=0} \ , \tag{A.12}$$

$$\left.\frac{df(a_s)}{dw_i}\right|_{w_i=0} = \left.\left(\frac{da_s}{dw_i}\right)\left(\frac{df(a_s)}{da_s}\right)\right|_{w_i=0} . \tag{A.13}$$

The derivatives of the components (A.10) and (A.11) are:

$$\left.\frac{da_s}{dw_i}\right|_{w_i=0} = \frac{1}{\sigma_{p+,s}}\left\{\left.\frac{dA}{dw_i}\right|_{w_i=0} - (\mu_{i,s} - \mu_{p,s}) - a_s(\rho_{i,p,s}\sigma_{i,s} - \sigma_{p,s})\right\}, \tag{A.14}$$

$$\frac{dF(a_s)}{da_s} = f(a_s), \tag{A.15}$$

$$\frac{df(a_s)}{da_s} = -a_s f(a_s) \ . \tag{A.16}$$

By replacing the right side of equation (A.14) for $\left.\frac{da_s}{dw_i}\right|_{w_i=0}$ in each of the relations (A.12) and (A.13) and using (A.15) and (A.16), we obtain the following derived:

$$\left.\frac{dF(a_s)}{dw_i}\right|_{w_i=0} = \frac{f(a_s)}{\sigma_{p+,s}}\left\{\left.\frac{dA}{dw_i}\right|_{w_i=0} - \left(\mu_{i,s} - \mu_{p,s}\right) - a_s(\rho_{i,p,s}\sigma_{i,s} - \sigma_{p,s})\right\}, \tag{A.17}$$

$$\left.\frac{df(a_s)}{dw_i}\right|_{w_i=0} = -a_s \left.\frac{dF(a_s)}{dw_i}\right|_{w_i=0} . \tag{A.18}$$

Substituting the right side of equation (A.17) in (A.9) and solving for α % of derived upper tail while considering the weight of the fixed income-portfolio (that is to say $\left.\frac{dA}{dw_i}\right|_{w_{i=0}}$), we obtain:

$$\left.\frac{dA}{dw_i}\right|_{w_i=0} = \sum_s^S \pi_s \frac{f(a_s)}{\sigma_{p,s}}\left[\left(\mu_{i,s} - \mu_{p,s}\right) + a_s\left(\rho_{i,p,s}\sigma_{i,s} - \sigma_{p,s}\right)\right] \div \left(\sum_s^S \pi_s \frac{f(a_s)}{\sigma_{p,s}}\right) . \tag{A.19}$$

For this resonator, we show that A is the percentile (1-α) of the Value-at-Risk (VaR) for the state of nature *s*. Therefore, we interpret this last one as derived of the sensitivity of the

VaR associated with the change in weight of fixed-income assets in the portfolio P. Consequently, this analysis complements the derived analytical $\left.\frac{\mathrm{dPAQ}}{\mathrm{dw_i}}\right|_{\mathrm{w_{i=0}}}$.

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