International Transmission of Shocks, Money Illusion and the Velocity of Money

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Abstract Money illusion is "frequently invoked and frequently resisted" by economists. *Resisted* as it contradicts the maximizing paradigm of microeconomic theory and *invoked* since a tendency to think in nominal rather than real terms becomes evident in the behavior of agents. This paper rationalizes money illusion in an stylized open economy model considering that private agents learn nominal aggregate demand at a level different from the one imposed by rationality. We find that the welfare effects of a productivity shock are increasing in the degree of money illusion and decreasing in the degree of openness of the economy. Furthermore we introduce a velocity of money shock revisiting the Quantity Theory of Money within the open economy micro-founded framework. An incomplete information game between Home and Foreign policymakers with monetary policy rules is developed, where sudden unstable financial conditions arise in one country, to find that allowing for velocity shocks reinforces the need for optimal monetary policy rules and to open the economies in order to avoid welfare costs.

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1 Introduction

In a seminal paper - "Welfare and Macroeconomic Interdependence" - Corsetti and Pesenti [2001] present a highly stylized and intuitive utility maximizing framework to address international macroeconomic transmission of shocks in interdependent economies. This open economy framework, also referred to as the CP-OR model[1], is invaluable to restate open economy traditional theory in modern welfare micro-founded models and is considered a launching pad for the New Open Economy Macroeconomics (NOEM) literature on macroeconomic interdependence. In this first paper they introduce an elasticity of substitution between differentiated inputs higher than the elasticity of intratemporal substitution between the Home and the Foreign goods, equal to one, which enables analytical tractability with a closed form solution. The model has been further developed and specified in subsequent contributions, with special relevance to Corsetti and Pesenti [2005a].

Whereas in Corsetti and Pesenti [2001] the transmission of money and fiscal shocks are analyzed, in Corsetti and Pesenti [2005a] productivity shocks indicate the need for monetary and fiscal stabilization policies. One of the major findings is that, when producer currency pricing (PCP) is assumed, the Nash equilibrium solution for two interdependent economies is a first best for the world and thus there is no scope for international policy cooperation. This result is also due to the terms of trade externality, negative to the expansionary country but positive abroad.

Throwing doubts on the benefits of floating exchange rates Devereux and Engel [2003] investigate the expenditure-switching role played by flexible rates in open economies allowing for two different specifications of price setting: the traditional PCP assumption and LCP (local currency pricing), whereby prices are pre-set, and sticky, in the currency of the consumer. The main finding is that there is no case for flexible exchange rates under LCP since movements in exchange rates do not affect consumer prices. In this case optimal monetary policy will keep exchange rates fixed while the need for flexible exchange rates to adjust relative prices, as Friedman [1953] prescribes, exists only under the Friedman type of currency pricing.

Their strong result is revisited in Duarte and Obstfeld [2007], where it is shown that under the presence of nontraded goods even the complete absence of expenditure-switching effects need not nullify the case for flexible exchange rates under LCP. Nonetheless both papers suggest that low levels of exchange rate pass-through to consumer prices affect the optimal degree of exchange rate volatility, reducing it.

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[1]Model by Corsetti and Pesenti [2001, 2005a,b] and Obstfeld and Rogoff [2000].

Corsetti [2006] also restores the case for flexible exchange rates allowing for home bias in consumption preferences. This extension of his own model links LCP with the degree of openness in the economy and stresses that there is no allocation compatible with a fixed exchange rate regime. In the limiting case of a closed economy (strong home bias), nominal exchange rates fluctuate proportionally to productivity shocks even under LCP. Nevertheless, as the import content of consumption grows reflecting more identical preferences across border and approaching the no home bias case, optimal monetary policy do, in fact, generate limited exchange rate fluctuations since it stabilizes a weighted average of domestic and foreign marginal costs and thus the optimal monetary response to shocks in each country is similarly weighted.

Policy trade-offs in open economies with a specific international dimension are analyzed in Corsetti and Pesenti [2005b] considering intermediate degrees of pass-through of exchange rates into export prices. In between the two polar cases - PCP and LCP -, they argue, there is strategic interdependence among policymakers and thus optimal monetary policy is not completely inward-looking. In order to optimally reduce the volatility of uncertain profits, exposed to currency fluctuations, policymakers react to a productivity shock inducing the adjustment of the nominal exchange rate. Another key result of this paper is that the need for a specific open economy optimal policy design suggest the existence of welfare gains from cooperation.

An extension of the model with a nontradable good sector and allowing for home bias in the consumption of tradables has been applied to investigate the costs of a single welfare-optimizing monetary policy within a currency union when shocks are asymmetric in Corsetti [2006a]. The paper highlights that insufficient stabilization by a central bank in a monetary union translates into higher preset product prices and lower output and consumption relative to the flexible-price benchmark.

In this paper we revisit core issues on traditional monetary theory as money illusion and the velocity of money under the edge of the new microfounded open economy approach. We carry out welfare analysis considering, beyond the benchmark stylized choice-theoretic model, that private agents have money illusion and learn nominal aggregate demand at a level different from the one imposed by rationality. Furthermore, to call into question the traditional Quantity Theory of Money within an open economy microfounded model, the approach pursued is to develop an incomplete information game between Home and Foreign policymakers with monetary policy rules, whereby velocity of money shocks may happen in one country under sudden unstable financial conditions or else technological or regulatory changes while the other country does not observe the realization of that shock.

This paper is organized as follows. Section 2 lays out the CP-OR model for open economies setting the stage for welfare analysis. Section 3 introduces money illusion and reconsiders its role for the new set up of international monetary models. Section 4 further introduces uncertainty through an incomplete information game with a velocity of money shock. Section 5 concludes.

2 The CP-OR model for open economies

In this Section we outline the main features of the CP-OR model that lead to the aggregate demand and supply functions developed in Corsetti and Pesenti [2001, 2005a,b]. The world consists of two countries, each specialized in the production of a traded good produced in a number of differentiated varieties defined over a continuum of unit mass.

Households and prices: There is a continuum of households in each country, immobile across borders, with population size normalized to 1. Households are indexed by j in the Home country and by j^* in the Foreign country. Household $j \in [0, 1]$ has a lifetime expected utility given by,

$$U_t(j) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\ln C_\tau(j) + \chi \ln \frac{M_\tau(j)}{P_\tau} - \kappa l_\tau(j) \right] \qquad \beta, k, \chi > 0 \quad (1)$$

where $\beta < 1$ is the discount rate, real balances $M(j)/P$ provide liquidity services, P is the price of one unit of consumption (Consumer Price Index, or CPI), χ is a parameter measuring the utility from real balances, l is labor supplied by the household (hours worked) which, along with κ that measures the discomfort associated with work, captures the disutility of labor effort. Consumption is represented by $C(j)$, a constant-elasticity-of-substitution index (CES) aggregating across consumption of the Home good $C_H(j)$ and the Foreign good $C_F(j)$ with a unitary elasticity[2], which implies a Cobb-Douglas basket:

$$C_t(j) = C_{H,t}(j)^{1-\gamma} C_{F,t}(j)^\gamma \qquad 0 < \gamma < 1 \qquad (2)$$

where γ is the share of the Foreign good in preferences and thus measures openness. In the benchmark CP-OR model $\gamma = 1/2$ and thus consumption

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[2]A unitary elasticity of intratemporal substitution between the Home and Foreign good implies that a 1% decrease on the relative price of the Home good induces a 1% increase on consumption of the Home good relative to the Foreign good. Thus, the expenditure share of each good is constant.

of each good is equally weighted in the basket. Each good is represented by a CES basket of differentiated varieties,

$$C_{H,t}(j) = \left(\int_0^1 C_t(h,j)^{\frac{\theta-1}{\theta}} dh\right)^{\frac{\theta}{\theta-1}} \qquad C_{F,t}(j) = \left(\int_0^1 C_t(f,j)^{\frac{\theta-1}{\theta}} df\right)^{\frac{\theta}{\theta-1}} \quad (3)$$

where $1 < \theta < \infty$ is the elasticity of substitution across varieties, which are imperfect substitutes of each other[3]. In accordance to this specification households substitute more among varieties of the same good than between the two goods. Preferences and the consumption indexes of the Foreign country, $C_F^*(j^*)$ and $C_H^*(j^*)$, are analogously defined (Foreign variables are indexed by asterisks; when applied to prices an asterisk means price in Foreign currency).

The utility-based CPI that correspond to this preferences[4] is,

$$P_t = 2P_{H,t}^{1/2} P_{F,t}^{1/2} \qquad (4)$$

The producer price of the Home good in Home currency P_H and the producer price of the Foreign good in Home currency P_F are given, respectively, by the following utility-based price of a consumption bundle of Home varieties and utility-based price of a consumption bundle of Foreign varieties:

$$P_{H,t} = \left(\int_0^1 p_t(h)^{1-\theta} dh\right)^{\frac{1}{1-\theta}} \qquad P_{F,t} = \left(\int_0^1 p_t(f)^{1-\theta} df\right)^{\frac{1}{1-\theta}} \qquad (5)$$

where $p(h)$ is the price of variety h in Home currency and $p(f)$ is the price of variety f in Home currency. Household j demands variety h, $C(h,j)$, and variety f, $C(f,j)$, in accordance to relative prices and consumption of each good:

$$C_t(h,j) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}(j) \qquad C_t(f,j) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}(j) \quad (6)$$

His demand for each good - Home and Foreign - is a function of relative prices and total consumption,

$$C_{H,t}(j) = \frac{P_t}{P_{H,t}} \frac{C_t(j)}{2} \qquad\qquad C_{F,t}(j) = \frac{P_t}{P_{F,t}} \frac{C_t(j)}{2} \qquad (7)$$

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[3]$\theta > 1$ captures the idea that varieties of the same consumption good are substitutes. The closer to one θ, the less substitutes are the varieties and the higher the market power of each firm.

[4]The consumption-based price index P is the price of one unit of C that minimizes expenditure $P_{H,t}C_{H,t}(j) + P_{F,t}C_{F,t}(j)$.

Firms and price setting: Each variety h is produced by a single Home firm and sold domestically and abroad in conditions of monopolistic competition[5]. The production function is linear on labor,

$$Y_t(h) = Z_t l_t(h) \tag{8}$$

where Z_t is a country-specific productivity process[6] and $l(h)$ is firm's h labor demand. The resource constraint for variety h reflects, in an open economy, total Home consumption and total Foreign consumption for it (or Home exports),

$$Y_t(h) \geqslant \int_0^1 C_t(h,j)dj + \int_0^1 C_t^*(h,j^*)dj^* \tag{9}$$

Firms minimize costs subject to the production function from what follows the condition for nominal marginal costs (MC), identical across firms:

$$MC_t(h) = MC_t = \frac{W_t}{Z_t} \tag{10}$$

where W is the nominal wage. Supply is demand-determined, as Home firms set prices at the beginning of each period and stand ready to meet demand at this price during the period in which the price is sticky. Firms set prices to maximize profits $D(h)$ taking into account the downward-sloping demand for their variety. The following expressions define optimal prices in a flexible price and in a sticky price world, respectively:

$$p_t(h) = P_{H,t} = \varepsilon_t p_t^*(h) = \frac{\theta}{\theta - 1} MC_t \tag{11}$$

$$p_t(h) = P_{H,t} = \varepsilon_t p_t^*(h) = \frac{\theta}{\theta - 1} E_{t-1}(MC_t) \tag{12}$$

where $\frac{\theta}{\theta - 1}$ is a constant markup reflecting the market power of the firm and ε is the nominal exchange rate (Home currency per unit of Foreign currency). An increase of ε represents a depreciation of the exchange rate. Households in the world have identical preferences and there are no barriers to trade so that the law of one price holds and variety h sells at the same price - but in

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[5]In this model goods market operates in imperfectly competitive conditions. Imperfect competition may either exist in the goods market or in the labor market, being this distinction not essential for the results. If the goods market is imperfectly competitive there are a continuum of differentiated varieties of each country's good; if the labor market is imperfectly competitive the output of each country is produced with a continuum of differentiated labor inputs supplied by agents.

[6]A positive productivity shock happens when $Z_t > 1$, while $0 < Z_t < 1$ represents a negative shock and $Z_t = 1$ a no shock situation.

different currencies - in both countries and thus $p_t(h) = \varepsilon_t p_t^*(h)$. Moreover, variety's prices are symmetric across firms and thus $p_t(h) = P_{H,t}$.

The terms of trade are defined as the price of imports in terms of export prices,

$$TOT = \frac{P_{F,t}}{P_{H,t}} = \frac{\varepsilon_t P_{F,t}^*}{P_{H,t}} \tag{13}$$

An increase reflects a deterioration of the terms of trade, as import prices become relatively higher, while a decrease reflects an improvement through lower relative import prices. The terms of trade move with the nominal exchange rate but purchasing power parity (consumption-based) always hold and thus the real exchange rate is constant:

$$RER = \frac{\varepsilon_t P_t^*}{P_t} \tag{14}$$

Budget constraint and consumer optimization: Taking prices and wages as given, Home agent j maximizes (1) subject to the individual flow budget constraint,

$$M_t(j) + \sum_{s_{t+1}} B_t(s_{t+1}, j)Q(s_{t+1} \mid s_t) + \varepsilon_t \sum_{s_{t+1}} B_t^*(s_{t+1}, j)Q^*(s_{t+1} \mid s_t) \leq \tag{15}$$

$$M_{t-1}(j) + B_{t-1}(s_t, j) + \varepsilon_t B_{t-1}^*(s_t, j) + W_t l_t(j) + D_t(j) - NETT_t(j) - P_t C_t(j)$$

where $D_t(j) = \int_0^1 D_t(h)dh$ represent the dividends received by household j from the firm he owns, $NETT$ are non-distortionary (lump-sum) net taxes paid to the government denominated in Home currency and M is money accumulated. Complete markets are assumed, with households having access to a full set of Arrow-Debrew securities B. Let $Q(s_{t+1} \mid s_t)$ denote the price of one unit of Home currency delivered in period $t+1$ contingent on the state of nature at $t+1$ being s_{t+1}[7]. Let $B_t(s_{t+1}, j)$ denote the claim to $B_t(s_{t+1}, j)$ units of Home currency at time $t+1$ in the state of nature s_{t+1} that household j buys at time t and brings into time $t+1$ ($B_t^*(s_{t+1}, j)$ and $Q^*(s_{t+1} \mid s_t)$ are similarly defined in terms of Foreign currency). The first order conditions with respect to consumption, labor effort and each Arrow-Debrew security, yield,

$$\frac{W_t}{P_t} = \kappa C_t(j) \tag{16}$$

$$\frac{C_t P_t}{C_{t+1} P_{t+1}} = \frac{\varepsilon_t C_t^* P_t^*}{\varepsilon_{t+1} C_{t+1}^* P_{t+1}^*} \tag{17}$$

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[7] $Q(.)$ is the same for all individuals under complete markets.

where the first expression shows how workers equate the marginal rate of substitution between consumption and leisure $(MRS = -\kappa C_t(j))$ to the real wage.

The monetary stance at Home, defined as μ_t, may be conveniently equated to nominal aggregate demand $P_t C_t$, which by (16) is equal to wages[8], thus defining the aggregate demand relationship $\mu_t = P_t C_t$ (AD equation). Substituting this condition in expression (17) and iterating it forward the equilibrium exchange rate is obtained:

$$\varepsilon_t = \frac{\mu_t}{\mu_t^*} \tag{18}$$

Other constraints and policy: Only Home households hold M and there is no government spending. The government budget constraint implies that seigniorage revenue is rebated to Home households in a lump-sum way:

$$M_t - M_{t-1} + \int_0^1 NETT_t(j)dj = 0 \tag{19}$$

where $M_t = \int_0^1 M_t(j)dj$. The resource constraint for Home output is,

$$Y_t = C_{H,t} + C_{H,t}^* = \frac{P_t}{2}\left[\frac{1}{P_{H,t}} + \frac{1}{\varepsilon_t P_{H,t}^*}\right]C_t \tag{20}$$

as $P_t^* C_t^* \varepsilon_t = P_t C_t$. This expression can be written as an aggregate supply relationship (AS equation):

$$Z_t l_t \tau_t = C_t \tag{21}$$

$$\text{with} \quad \frac{1}{\tau_t} = \frac{P_t}{2}\left[\frac{1}{P_{H,t}} + \frac{1}{\varepsilon_t P_{H,t}^*}\right] \tag{22}$$

where τ_t reflects the change in the value of Home output that result from fluctuations in the terms of trade, either induced by flexible prices or changes in the nominal exchange rate.

The natural rate of employment: Substituting $W = \kappa PC = \kappa\mu$ in both expressions for optimal prices (11) and (12) a constant natural rate of employment is obtained, respectively, for a flexible price world and sticky price world,

$$\bar{l} = \frac{\theta-1}{\theta\kappa} \qquad E(l) = \frac{\theta-1}{\theta\kappa} = \bar{l} \tag{23}$$

This level prevails in equilibrium in the Home economy and gives rise to a third relationship - the NR equation.

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[8]For simplicity we assume hereafter $\kappa = 1$.

Optimal monetary policy: The benchmark CP-OR model considers a world integrated by two open economies and a positive productivity shock only at Home. Both countries have sticky prices and firms preset prices in their own currency (PCP), which remain fixed for one period. The Nash non-cooperative equilibrium in the CP-OR model is a first best for the world and thus no other solution for monetary policymaking among interdependent economies improve welfare in one country while not inducing any welfare loss in the other. The best each country can do is to apply inward looking monetary policies as a policy stabilization response to a productivity shock. The optimal rule, derived in Appendix, is,

$$\frac{\mu_t}{Z_t} = E_{t-1}\left(\frac{\mu_t}{Z_t}\right) \tag{24}$$

From (16), and using $\mu_t = P_t C_t$, marginal costs may be equated to optimal monetary policy as follows:

$$MC_t = \frac{\mu_t}{Z_t} = \Gamma \tag{25}$$

where Γ is a constant. Indeed, the optimal policy rule in each country is to stabilize domestic marginal costs whenever a productivity shock threatens to increase or decrease them, in accordance to,

$$\mu_t = \Gamma Z_t \tag{26}$$

Hence, in the periods after the shock producer prices are the same as before the shock, as if the productivity shock had not occurred and thus making the distortion of sticky prices seem irrelevant.

3 Money illusion

In this section we introduce money illusion in the benchmark model to contest the rational behavior of households through an assumption that is "frequently invoked and frequently resisted"[9] by economists. *Resisted* partly because it contradicts the maximizing paradigm of microeconomic theory; *invoked* since a failure to understand the importance of nominal magnitudes becomes evident in the behavior of private agents and governments through the observation of sticky prices and non-indexed contracts.

The term money illusion is attributed to John Maynard Keynes in the early 20th century. Irving Fisher wrote an important book on the subject

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[9] New Palgrave Dictionary of Economics (1987, 3: 518).

in 1928, and in the 1970s Milton Friedman distinguished between actual real wages and perceived real wages in the short term. The former, Friedman argues, is relevant for firms hiring workers while the latter is relevant for workers making labor-supply choices. In the long run the two are equal; yet in the short run workers may see their nominal wages rising and may misinterpret this increase as a rise in the perceived real wage, while the actual real wage is lower. To the extend that unanticipated changes in wages and prices generate misperceptions about relative prices, the real equilibrium of the economy is affected before expectations adjust and the natural equilibrium is reestablished.

Shafir, Diamond and Tversky [1997] describe money illusion as a bias in the assessment of the real value of economic transactions induced by a nominal evaluation. Private agents decide on nominal values and do not see properly the variation of prices, therefore being unable to infer correctly on real values. This tendency to think in nominal rather than real terms, more likely under low inflation environments, implies a lack of rationality that is alien to economists and has significant implications for economic theory. Sticky prices are usually rationalized as a result of costs of price adjustments, although it can also indicate the presence of money illusion in markets where prices are negotiated, like housing, denoting reluctance to accept that a price has decreased or to accept nominal wage cuts.

In general it is accepted a higher level of economic rationality in private agents than among governments. Yet individual decision making has revealed systematic departures from rationality that go beyond inflation considerations and happen under inflationary as well as noninflationary settings.

In this model, even though producer prices are assumed to be sticky, consumer prices vary through import prices when a productivity shock happens and the monetary authority optimally stabilizes the economy. Rational private agents equate nominal aggregate demand to nominal wages as stated in the equilibrium condition (16) derived by optimization. To introduce money illusion we will assume that agents learn nominal demand at a level $W_{H,t}^{1/\xi_H}$ lower than $W_{H,t}$ imposed from optimality, where $\xi_H > 1$ represents the degree of money illusion. In other words we are assuming that under money illusion wages are set such that private agents violate their equilibrium:

$$C_t P_t > W_{H,t}^{1/\xi_H}$$

The inability of agents to see the correct level of prices imply, therefore, a real wage loss:

$$\frac{W_{H,t}^{1/\xi_H}}{P_t} < \frac{W_{H,t}}{P_t}$$

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The intuition for the loss due to money illusion is the following: assuming a positive productivity shock and optimal stabilization policy by the monetary authority, CPI prices increase under sticky prices due to depreciation of the nominal exchange rate and consequent increase of import prices. Money illusion arises from the non awareness of private agents to depreciation and their inability to see the consequent increase of P_t that realizes following a positive productivity shock.

Wages are thus set at $W_{H,t}^{1/\xi_H} < W_{H,t}$. Firms minimize costs[10] and therefore marginal costs are defined as,

$$MC_{H,t} = \frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}}$$

Marginal costs feed into price determination: under sticky prices firms set prices in the beginning of period t, based on expected marginal costs for that period, maximizing the expected value of profits. Prices to the Home and to the Foreign market under PCP are the following:

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\begin{aligned}
P_{H,t} &= p_t(h) = \frac{\theta}{\theta - 1} E_{t-1}\left(MC_{H,t}\right) \\
P_{H,t}^* &= p_t^*(h) = \frac{\theta}{\theta - 1} \frac{1}{\varepsilon_t} E_{t-1}\left(MC_{H,t}\right)
\end{aligned}
$$

The Monetary Authority at Home under commitment announces a rule in the beginning of period t that maximize the expected utility of agents:

$$\mu_{H,t} = \underset{\mu_{H,t}}{\arg\max} E_{t-1}(\ln C_t - kl_t)$$

where $P_t C_t = \mu_{H,t}$ assuming the Monetary Authority has no money illusion and equates the monetary stance to aggregate nominal demand. The optimal policy rule is the following:

$$\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}} = \xi_H E_{t-1}\left(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}}\right)$$

Assuming constant marginal costs and equal to one as a simplification, the optimal rule may be written as,

$$\mu_{H,t} = (\xi_H Z_{H,t})^{\xi_H} \tag{27}$$

It is worth noting that, conversely to the benchmark case, under this rule the policymaker optimally responds to either a negative or a positive productivity

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[10]Derivations in Appendix.

shock with a level of monetary stance $\mu_{H,t} > Z_{H,t}$, as if he is correcting the economy for the misperception of private agents. The higher the degree of money illusion, the higher the correction induced. The Foreign country optimal rule without money illusion is, by the same token,

$$\mu_{F,t} = E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}})Z_{F,t}$$

Welfare at Home with money illusion $W_{H,t}^{MI}$ can thus be expressed as follows, where it is assumed that shocks $Z_{H,t}$ and $Z_{F,t}$ are independently lognormally distributed,

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\begin{aligned}
W_{H,t}^{MI} &= E_{t-1}(\ln C_t - l_t) \\
&= (1-\gamma)\,\xi_H E_{t-1} \ln Z_{H,t} + \gamma E_{t-1} \ln Z_{F,t} + cts.
\end{aligned}
$$

$$\Delta W_{H,t} = W_{H,t} - W_{H,t}^{MI} = (1-\gamma)\,(1-\xi_H)\,E_{t-1} \ln Z_{H,t}$$

Observe that either the absence of money illusion $\xi_H = 1$ or the absence of a productivity shock $Z_{H,t} = 1$ avoid a change in welfare. This result is intuitive since the welfare change induced by money illusion is due to the inability of private agents to see the price change driven by the exchange rate, which only realizes following a shock.

The welfare effect of a positive productivity shock in the benchmark model is a gain and the welfare effect of a negative shock a loss, with effects increasing in the magnitude of the shocks. The expression above makes clear that money illusion amplifies both effects, and thus the welfare effects of shocks are increasing in the degree of money illusion. In light of the considerations made above on the optimal rule (27), we can therefore conclude that the correction for money illusion increases welfare only if the shock is positive and the currency depreciates. Otherwise, in the case of a negative shock, money illusion and the optimal policy response of contracting the monetary stance less than it should leads to the aggravation of the welfare loss induced by the shock itself.

Furthermore, the higher the degree of openness γ (the lower the home bias) the less the welfare effect is further amplified. Indeed money illusion in this model is motivated by the misperception of agents concerning nominal exchange rates but the direct effect is on domestic producer prices. Hence, the more open the economy the less domestic prices are relevant to welfare.

4 Velocity of money

The simple version of the Quantity Theory of Money was first developed by Irving Fisher in the inter-war years and later reinforced by monetarists,

namely by Milton Friedman. The equation of exchange states that there is a constant velocity of money v and that the quantity of money M_t influences a predictable aggregate nominal demand $P_t Q_t$:

$$M_t v = P_t Q_t$$

Therefore money supply variations are fully reflected in changes of $P_t Q_t$, and this constant relation was the cornerstone of the monetarist theory. Whether both P_t and Q_t increases or only one of them depends on the aggregate supply curve. Monetarists claim that when money supply increases and the economy is at full capacity only the price level goes up proportionately.

John Maynard Keynes challenged the theory in the 1930s, arguing that increases in money supply lead to a decrease in the velocity of circulation. By the 1980s the velocity of money shifted unpredictably due to changes of people's behavior in their handling of money as well as to changes of the financial system.

To introduce uncertainty with respect to nominal aggregate demand we assume in this section that the Home economy has an aggregate demand relationship (AD equation) expressed by the following Fisher type equation:

$$\mu_{H,t} v_{H,t} = P_t C_t$$

where $v_{H,t}$ is a velocity of money exogenous shock independently lognormally distributed, and unrelated to the productivity shock $Z_{H,t}$, that arises under sudden unstable financial conditions or else technological or regulatory changes. The velocity of money, or the (average) number of times during period t that each unit of money circulates in the economy as a medium of exchange to buy goods and services, is $v_{H,t} > 1$ when the shock is positive and $0 < v_{H,t} < 1$ when the shock is negative. Hence, in the former realization each unit of money circulates more times and nominal aggregate demand is higher than in the latter realization.

Calling into question these issues we aim at studying the consequences of uncertainty associated with different velocities of money to optimal policy-making and welfare at Home and in the Foreign country. We develop an incomplete information game between the Home and the Foreign policymakers where players play simultaneously and whereby the Home economy is subject to velocity of money shocks while the Foreign country is not. Nonetheless, the Foreign country is subject to uncertainty as the policymaker does not observe the Home velocity shock. Yet he observes the associated distribution of probabilities.

Algebraically the game specifies as follows[11]: under sticky prices and PCP firms set prices in the beginning of period t maximizing expected profits for

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[11]Derivations in Appendix.

the period:

$$P_{H,t} = p_t(h) = \frac{\theta}{\theta - 1} E_{t-1}(MC_{H,t})$$

$$P_{H,t}^* = p_t^*(h) = \frac{\theta}{\theta - 1} \frac{1}{\varepsilon_t} E_{t-1}(MC_{H,t})$$

where marginal costs are set by firms through cost minimization. Notice that without money illusion the level of marginal costs are,

$$MC_{H,t} = \frac{W_{H,t}}{Z_{H,t}}$$

The Monetary Authority at Home maximizes national welfare, indexed by the representative household's utility:

$$\mu_{H,t} = \arg\max_{\mu_{H,t}} E_{t-1}(\ln \frac{\mu_{H,t} v_{H,t}}{P_t} - k l_t)$$

where $P_t C_t = \mu_{H,t} v_{H,t} = W_{H,t}$ is used expressing that the policymaker equates the monetary stance to aggregate nominal demand and that private agents are rational. The first order condition derived defines the optimal rule announced in the beginning of period t and to which the authority commits,

$$\mu_{H,t} = E_{t-1}(\frac{\mu_{H,t} v_{H,t}}{Z_{H,t}}) \frac{Z_{H,t}}{v_{H,t}}$$

The Foreign country is assumed not to be subject to velocity shocks and thus the optimal rule is, simply,

$$\mu_{F,t} = E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) Z_{F,t}$$

To equate welfare at Home we assume constant nominal marginal costs and equal to 1 as a simplification; therefore the implemented rules in each country are,

$$\mu_{H,t} = Z_{H,t}/v_{H,t}$$
$$\mu_{F,t} = Z_{F,t}$$

National welfare with velocity shocks $W_{H,t}^{velocity}$, assuming $Z_{H,t}$, $Z_{F,t}$ and $v_{H,t}$ are independently lognormally distributed, is expressed as follows:

$$W_{H,t}^{velocity} = E_{t-1}(\ln \mu_{H,t} v_{H,t} - \ln P_t)$$
$$= (1 - \gamma) E_{t-1} \ln Z_{H,t} + \gamma E_{t-1} \ln Z_{F,t} + cts.$$

$$\Delta W_{H,t} = W_{H,t} - W_{H,t}^{velocity} = 0$$

which leads to no welfare loss since the velocity shock is taken into consideration in the policymaker's optimization problem. If the optimal policy rule implemented by the policymaker did not account for the velocity shock, there would exist a welfare loss increasing in the variance of this shock and decreasing in the degree of openness of the economy:

$$\Delta W_{H,t} = W_{H,t} - W_{H,t}^{velocity} = (1 - \gamma)0.5 Var_{t-1} \ln v_{H,t}$$

In general, it may therefore be concluded that allowing for velocity shocks reinforces the need for optimal monetary policy rules since its absence generates welfare costs. Additionally, costs from velocity shocks at Home may be reduced opening the economy.

Furthermore, allowing for uncertainty as regards the optimal rule implemented at Home when calculating Foreign welfare leads to the following result:

$$
\begin{aligned}
W_{F,t}^{uncertainty} &= E_{t-1}(\ln C_t^*) = E_{t-1}(\ln \mu_{F,t} - \ln P_t^*) \\
&= (1 - \gamma) E_{t-1} \ln Z_{F,t} + 2\gamma E_{t-1} \ln Z_{H,t} + \gamma Var_{t-1} \ln Z_{H,t} \\
&\quad -\gamma 0.5 Var_{t-1} \ln v_{H,t} + cts.
\end{aligned}
$$

$$
\begin{aligned}
\Delta W_{F,t} &= W_{F,t} - W_{F,t}^{uncertainty} \\
&= -\gamma \left[E_{t-1} \ln Z_{H,t} + Var_{t-1} \ln Z_{H,t} \right] + \gamma 0.5 Var_{t-1} \ln v_{H,t}
\end{aligned}
$$

Indeed the change in expected Foreign log consumption due to this uncertainty is a function of the Home shocks: the productivity shock generates a gain while the velocity shock generates a loss. It is also worth to stress that the welfare effect is independent of the parameter of the distribution of probabilities.

5   Conclusions

This paper has revisited traditional issues on monetary theory in light of recent open economy macroeconomic models, stressing their welfare effects. The analysis has focused on two questions at the heart of monetary theory. The first concerns the lack of rationality represented by the observation of money illusion and its implications for economic theory under a microfounded framework. Indeed, when private agents decide on nominal values and are unable to infer correctly on real values the real wage loss changes welfare in an interdependent open economy. As the benevolent policymaker

maximizes the utility of the household, the optimal rule of policy corrects for money illusion and, following a positive shock, the change in welfare is positive. Economies with home bias benefit more from this welfare effect when productivity shocks are positive. Yet if shocks are predominantly negative a more open economy help to reduce the welfare loss.

The second question brought up in this text concerns the velocity of money. An important policy conclusion from the traditional Quantity Theory of Money is that an increase in money supply does not necessarily mean a proportional increase in output. The same conclusion is reached in the CP-OR model, where the increase of the aggregate monetary stance leads partially to the increase of consumption and partially to the increase of prices. Introducing the possibility of observing different levels of nominal aggregate demand following exogenous velocity shocks allows to reconsider the variation of those effects, as well as the consequent change of national welfare.

In this respect, the analysis sheds new light on the issue of commitment to policy rules. Namely, welfare analysis yields the conclusion that, given the velocity shock, the loss to the economy where the shock occurred may only be avoided if the policymaker implements an optimal rule taking into consideration this new source of uncertainty in the economy. Otherwise, welfare costs of velocity are inevitable. Furthermore, to the extend that openness may affect welfare, it may also be concluded that a more open economy is less prone to losses arising from unexpected velocity shocks.

6 Appendix

6.0.1 Money Illusion

Firms: The production function of each firm, that produces one variety, is represented by linear technologies in labor effort, supplied by the households,

$$Y_t(h) = Z_{H,t} l_t(h)$$

Under money illusion firms minimize costs in accordance to,

$$\min_{l_t(h)} W_{H,t}^{1/\xi_H} l_t(h)$$

$$s.t. Y_t(h) = Z_{H,t} l_t(h)$$

from where the first order condition defines the following nominal marginal costs:

$$MC_{H,t}(h) = MC_{H,t} = \frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}}$$

Price setting under sticky prices and PCP: Firms under monopolistic competition take into account the downward-sloping demand for their product (6) and set prices to maximize the present discounted value of expected profits $E_{t-1}[Q_{t-1,t} D_t(h)]$. Firms are small in the sense that they ignore the impact of their pricing and production decisions on aggregate variables and price indexes. Under **sticky prices**, firms set prices in the beginning of period t, before observing the realization of the shock, based on available info at time $t-1$.

$$\max_{p_t(h)} E_{t-1}[Q_{t-1,t} D_t(h)] =$$

$$\max_{p_t(h)} E_{t-1} \left\{ Q_{t-1,t} \left[\begin{array}{c} (p_t(h) - MC_{H,t}) \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t} \\ + (\varepsilon_t p_t^*(h) - MC_{H,t}) \left(\frac{\varepsilon_t p_t^*(h)}{\varepsilon_t P_{H,t}^*}\right)^{-\theta} C_{H,t}^* \end{array} \right] \right\}$$

$$= \max_{p_t(h)} E_{t-1} \left\{ Q_{t-1,t} \left[(p_t(h) - MC_{H,t}) \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} (C_{H,t} + C_{H,t}^*) \right] \right\}$$

where $Q_{t-1,t}$ is the households'discount rate. From the first order condition the Home-currency price $p_t(h)$ is set:

$$p_t(h) = P_{H,t} = \frac{\theta}{\theta - 1} E_{t-1}(MC_{H,t}) = \frac{\theta}{\theta - 1} E_{t-1}(\frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}})$$

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where $\frac{\theta}{\theta-1}$ is a constant markup that augments the price. The law of one price holds.

The foreign-currency price $p_t^*(h)$ under PCP is set maximizing profits wrt $\varepsilon_t p_t^*(h) = p_t(h)$:

$$\max_{\varepsilon_t p_t^*(h)} E_{t-1}\left[Q_{t-1,t}D_t(h)\right] =$$

$$E_{t-1}\left\{Q_{t-1,t}\left[\begin{array}{c} (p_t(h) - MC_t)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta}C_{H,t} \\ + (\varepsilon_t p_t^*(h) - MC_t)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\theta}C_{H,t}^* \end{array}\right]\right\} =$$

$$E_{t-1}\left\{Q_{t-1,t}\left[\begin{array}{c} (\varepsilon_t p_t^*(h) - MC_t)\left(\frac{\varepsilon_t p_t^*(h)}{\varepsilon_t P_{H,t}^*}\right)^{-\theta}C_{H,t} \\ + (\varepsilon_t p_t^*(h) - MC_t)\left(\frac{\varepsilon_t p_t^*(h)}{\varepsilon_t P_{H,t}^*}\right)^{-\theta}C_{H,t}^* \end{array}\right]\right\} =$$

$$E_{t-1}\left\{Q_{t-1,t}\left[(\varepsilon_t p_t^*(h) - MC_t)\left(\frac{\varepsilon_t p_t^*(h)}{\varepsilon_t P_{H,t}^*}\right)^{-\theta}\left(C_{H,t} + C_{H,t}^*\right)\right]\right\}$$

$$p_t^*(h) = P_{H,t}^* = \frac{1}{\varepsilon_t}\frac{\theta}{\theta-1}E_{t-1}\left(MC_{H,t}\right) = \frac{1}{\varepsilon_t}\frac{\theta}{\theta-1}E_{t-1}(\frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}})$$

Problem of the policymaker in country H : Under commitment the Monetary Authority announces a rule in the beginning of period t that maximizes the expected utility of agents,

$$\mu_{H,t} = \underset{\mu_{H,t}}{\arg\max} E_{t-1}(\ln C_t - kl_t)$$

where $P_t C_t = \mu_{H,t}$ assuming the Monetary Authority equates the monetary stance to aggregate nominal demand. Substituting $P_t C_t = W_{H,t} = \mu_{H,t}$ (thus assuming the Government does not know agents have money illusion) and using $E_{t-1}(l_{H,t}) = \bar{l} = \frac{\theta-1}{\kappa\theta}$ we obtain,

$$E_{t-1}(\ln\frac{\mu_{H,t}}{P_t} - kl_{H,t}) = E_{t-1}(\ln\mu_{H,t} - \ln P_t)$$

$$= E_{t-1}\ln\mu_{H,t} - E_{t-1}\ln\frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}P_{H,t}^{1-\gamma}P_{F,t}^{\gamma}$$

$$= E_{t-1}\ln\mu_{H,t}$$

$$-E_{t-1}\ln\frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\left[\frac{\theta}{\theta-1}E_{t-1}(\frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}})\right]^{1-\gamma}(\frac{\mu_{H,t}}{\mu_{F,t}})^\gamma\left[\frac{\theta}{\theta-1}E_{t-1}(\frac{W_{F,t}}{Z_{F,t}})\right]^\gamma$$

$$= E_{t-1} \ln \mu_{H,t} - E_{t-1} \ln \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left[\frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}}) \right]^{1-\gamma} \frac{\mu_{H,t}^\gamma}{\mu_{F,t}^\gamma} \left[\frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) \right]^\gamma$$

$$= (1-\gamma) E_{t-1} \ln \mu_{H,t} - (1-\gamma) \ln E_{t-1}(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}}) + \gamma E_{t-1} \ln \mu_{F,t} - \gamma \ln E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) + cts.$$

FOC wrt $\mu_{H,t}$:

$$(1-\gamma)(\ln \mu_{H,t})' - (1-\gamma) \frac{(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}})'}{E_{t-1}(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}})} = 0$$

$$\mu_{H,t}^{1/\xi_H} = \xi_H E_{t-1}(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}}) Z_{H,t}$$

Assuming constant marginal costs, equal to one,

$$MC_{H,t} = \frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}} = \frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}} = \Gamma \ cts. = 1$$

the optimal rule is,

$$\mu_{H,t} = (\xi_H Z_{H,t})^{\xi_H}$$

Problem of the policymaker in country F:

$$\mu_{F,t} = \arg \max_{\mu_{F,t}} E_{t-1}(\ln \frac{\mu_{F,t}}{P_t^*} - k l_{F,t})$$

$$E_{t-1}(\ln \mu_{F,t} - \ln P_t^*) = E_{t-1}(\ln \mu_{F,t} - \ln \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{F,t}^{*\ 1-\gamma} P_{H,t}^{*\ \gamma})$$

$$= E_{t-1} \ln \mu_{F,t} - E_{t-1} \ln \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left[\frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) \right]^{1-\gamma} \left[\frac{\mu_{F,t}}{\mu_{H,t}} \frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}}) \right]^\gamma$$

$$= (1-\gamma) \ln \mu_{F,t} - (1-\gamma) E_{t-1} \ln \left[E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) \right] + ... + cts$$

FOC wrt $\mu_{F,t}$:

$$\mu_{F,t} = E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) Z_{F,t}$$

Welfare with rules in country H : If $Z_{H,t}$ and $Z_{F,t}$ are independently lognormally distributed, welfare with money illusion $W_{H,t}^{MI}$ under sticky prices and full stabilization is,

$$W_{H,t}^{MI} = E_{t-1}(\ln C_t - l_t) = E_{t-1}(\ln C_t) = E_{t-1}(\ln \mu_{H,t} - \ln P_t)$$

$$= E_{t-1} \ln \mu_{H,t} - E_{t-1} \ln(\frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{H,t}^{1-\gamma} P_{F,t}^\gamma)$$

$$= E_{t-1} \ln \mu_{H,t} - E_{t-1} \ln \left[\frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left[\frac{\theta}{\theta-1} E_{t-1}(\frac{W_{H,t}^{1/\xi_H}}{Z_{H,t}}) \right]^{1-\gamma} \left[\frac{\mu_{H,t}}{\mu_{F,t}} \frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) \right]^\gamma \right]$$

$$= (1-\gamma) E_{t-1} \ln \mu_{H,t} + \gamma E_{t-1} \ln \mu_{F,t} - (1-\gamma) \ln E_{t-1}(\frac{\mu_{H,t}^{1/\xi_H}}{Z_{H,t}}) - \gamma \ln E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) + cts$$

For $\mu_{H,t} = \xi_H^{\xi_H} Z_{H,t}^{\xi_H}$ and $\mu_{F,t} = Z_{F,t}$,

$$
\begin{aligned}
W_{H,t}^{MI} &= (1-\gamma) E_{t-1} \ln \xi_H^{\xi_H} Z_{H,t}^{\xi_H} + \gamma E_{t-1} \ln Z_{F,t} - (1-\gamma) \ln E_{t-1}(\frac{\xi_H Z_{H,t}}{Z_{H,t}}) \\
&\quad -\gamma \ln E_{t-1}(\frac{Z_{F,t}}{Z_{F,t}}) + cts \\
&= (1-\gamma) \xi_H E_{t-1} \ln \xi_H Z_{H,t} + \gamma E_{t-1} \ln Z_{F,t} + cts. \\
&= (1-\gamma) \xi_H E_{t-1} \ln Z_{H,t} + \gamma E_{t-1} \ln Z_{F,t} + cts.
\end{aligned}
$$

$$
\begin{aligned}
\Delta W_{H,t} &= W_{H,t} - W_{H,t}^{MI} \\
&= (1-\gamma)(1-\xi_H) E_{t-1} \ln Z_{H,t}
\end{aligned}
$$

as welfare with sticky prices and full stabilization is,

$$W_{H,t} = (1-\gamma) E_{t-1} \ln Z_{H,t} + \gamma E_{t-1} \ln Z_{F,t} + cts$$

6.0.2   Velocity of money

Problem of the policymaker in country H :   The Monetary Authority announces a rule in the beginning of period t that maximize the expected utility of agents,

$$\mu_{H,t} = \underset{\mu_{H,t}}{\arg\max} E_{t-1}(\ln C_t - k l_t)$$

substituting $P_t C_t = \mu_{H,t} v_{H,t} = W_{H,t}$ assuming the policymaker equates the monetary stance to aggregate nominal demand and private agents are ratio-

nal, and using $E_{t-1}(l_{H,t}) = \bar{l} = \frac{\theta-1}{\kappa\theta}$,

$$E_{t-1}(\ln\frac{\mu_{H,t}v_{H,t}}{P_t} - kl_{H,t}) = E_{t-1}(\ln\mu_{H,t} - \ln P_t)$$

$$= E_{t-1}\ln\mu_{H,t}v_{H,t} - E_{t-1}\ln\frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}P_{H,t}^{1-\gamma}P_{F,t}{}^\gamma$$

$$= E_{t-1}\ln\mu_{H,t}v_{H,t}$$

$$-E_{t-1}\ln\frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}\left[\frac{\theta}{\theta-1}E_{t-1}(\frac{\mu_{H,t}v_{H,t}}{Z_{H,t}})\right]^{1-\gamma}\frac{(\mu_{H,t}v_{H,t})^\gamma}{\mu_{F,t}^\gamma}\left[\frac{\theta}{\theta-1}E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}})\right]^\gamma$$

$$= (1-\gamma)E_{t-1}\ln\mu_{H,t} + (1-\gamma)E_{t-1}\ln v_{H,t} - (1-\gamma)\ln E_{t-1}(\frac{\mu_{H,t}v_{H,t}}{Z_{H,t}})$$

$$+\gamma E_{t-1}\ln\mu_{F,t} - \gamma\ln E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) + cts.$$

FOC wrt $\mu_{H,t}$:

$$\mu_{H,t} = E_{t-1}(\frac{\mu_{H,t}v_{H,t}}{Z_{H,t}})\frac{Z_{H,t}}{v_{H,t}}$$

If,

$$MC_{H,t} = \frac{W_{H,t}}{Z_{H,t}} = \frac{P_tC_t}{Z_{H,t}} = \frac{\mu_{H,t}v_{H,t}}{Z_{H,t}} = \Gamma \ cts. = 1 \text{ by hip.}$$

then the implemented rules are,

$$\mu_{H,t} = \frac{Z_{H,t}}{v_{H,t}}$$

Welfare with rules in country H: If $Z_{H,t}$, $Z_{F,t}$ and $v_{H,t}$ are independently lognormally distributed,

$$W_{H,t}^{velocity} = E_{t-1}(\ln\mu_{H,t}v_{H,t} - \ln P_t)$$

$$= (1-\gamma)E_{t-1}\ln\mu_{H,t} + (1-\gamma)E_{t-1}\ln v_{H,t} - (1-\gamma)\ln E_{t-1}(\frac{\mu_{H,t}v_{H,t}}{Z_{H,t}})$$

$$+\gamma E_{t-1}\ln\mu_{F,t} - \gamma\ln E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) + cts.$$

substituting $\mu_{H,t} = \frac{Z_{H,t}}{v_{H,t}}$ and $\mu_{F,t} = Z_{F,t}$,

$$= (1-\gamma)E_{t-1}\ln Z_{H,t} + \gamma E_{t-1}\ln Z_{F,t} + cts.$$

$$\Delta W_{H,t} = W_{H,t} - W_{H,t}^{velocity} = 0$$

If the policy rules ignored the velocity shock, then,

$$
\begin{aligned}
W_{H,t}^{velocity} &= E_{t-1}(\ln \mu_{H,t} v_{H,t} - \ln P_t) \\
&= (1-\gamma)E_{t-1}\ln Z_{H,t} - (1-\gamma)0.5 Var_{t-1}\ln v_{H,t} + \gamma E_{t-1}\ln Z_{F,t} + cts.
\end{aligned}
$$

$$
\Delta W_{H,t} = W_{H,t} - W_{H,t}^{velocity} = (1-\gamma)0.5 Var_{t-1}\ln v_{H,t}
$$

Welfare with rules in country F under uncertainty:

$$
W_{F,t}^{uncertainty} = E_{t-1}(\ln C_t^*) = E_{t-1}(\ln \mu_{F,t} - \ln P_t^*)
$$

$$
= E_{t-1}(\ln \mu_{F,t} - \ln \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{F,t}^{*\,1-\gamma} P_{H,t}^{*\,\gamma})
$$

$$
= E_{t-1}\ln \mu_{F,t} - E_{t-1}\ln \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \left[\frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{F,t}}{Z_{F,t}}) \right]^{1-\gamma} \left[\frac{\mu_{F,t}}{\mu_{H,t}v_{H,t}} \frac{\theta}{\theta-1} E_{t-1}(\frac{\mu_{H,t}v_{H,t}}{Z_{H,t}}) \right]^{\gamma}
$$

With probability λ there is no velocity shock and Home implements $\mu_{H,t} = Z_{H,t}$; With probability $1-\lambda$ there is a velocity shock and Home implements $\mu_{H,t} = \frac{Z_{H,t}}{v_{H,t}}$. The Foreign country implements $\mu_{F,t} = Z_{F,t}$.

$$
\begin{aligned}
&= (1-\gamma)\,E_{t-1}\ln Z_{F,t} + \gamma E_{t-1}\ln E_{t-1}\left[\lambda Z_{H,t} + (1-\lambda)\frac{Z_{H,t}}{v_{H,t}} \right] + \gamma E_{t-1}\ln v_{H,t} \\
&\quad - (1-\gamma)\ln E_{t-1}(\frac{Z_{F,t}}{Z_{F,t}}) - \gamma \ln E_{t-1}\left[\frac{\left(\lambda Z_{H,t} + (1-\lambda)\frac{Z_{H,t}}{v_{H,t}}\right)v_{H,t}}{Z_{H,t}} \right] + cts. \\
&= (1-\gamma)\,E_{t-1}\ln Z_{F,t} + 2\gamma E_{t-1}\ln Z_{H,t} + \gamma Var_{t-1}\ln Z_{H,t} - \gamma 0.5 Var_{t-1}\ln v_{H,t} + cts.
\end{aligned}
$$

Finally,

$$
\Delta W_{F,t} = W_{F,t} - W_{F,t}^{uncertainty} = -\gamma\left[E_{t-1}\ln Z_{H,t} + Var_{t-1}\ln Z_{H,t}\right] + \gamma 0.5 Var_{t-1}\ln v_{H,t}
$$

as welfare with sticky prices and full stabilization in the Foreign country is

$$
W_{F,t} = (1-\gamma)E_{t-1}\ln Z_{F,t} + \gamma E_{t-1}\ln Z_{H,t} + cts.
$$

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