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The Social Cost of Carbon on an Optimal Balanced Growth Path

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Abstract This paper derives analytically the growth rate of the social cost of carbon (SSC) on an optimal balanced growth path. More specifically, the paper examines a deterministic Ramsey model of optimal economic growth with carbon emissions. In this model, restrictions on technology and preferences are imposed that guarantee optimal balanced growth, i.e., that guarantee an optimal path with constant and positive economic growth and a constant stock of carbon in the atmosphere. The paper exploits these restrictions to show that the growth rate of the SCC on the optimal balanced growth path is negative, provided the elasticity of marginal utility of consumption with respect to consumption is larger than or equal to one. There seems to be consensus in the literature that this latter requirement is fulfilled in reality.

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1. Introduction

The social cost of carbon (SCC) is defined as the present value of the marginal damage from a small carbon emission increase. It represents an externality that is not considered by market agents in their decision making process. This externality can however be corrected with a Pigovian tax. Complete internalization of the externality requires the Pigovian tax to equal the SCC on the optimal carbon emission path. As a consequence, using Pigovian taxation or alternative climate change policies requires understanding of the determinants of the SSC on the optimal carbon emission path.

While the present paper examines an analytical Ramsey model, the SCC is usually estimated in integrated assessment models, i.e. in simulation models that integrate economic and scientific models of global warming. The first step in doing so is to specify the relative marginal damage of carbon. The marginal damage of carbon can be modeled as reduction in the productivity to produce consumption goods, as utility loss from reduced environmental amenities or as the sum of both. The rate at which the relative marginal damage of carbon grows over time depends on assumptions on firms' technology and households' preferences. Once values for the relative marginal damage of carbon for different time periods are estimated, the next step to calculate the SCC is then to employ a social discount rate to convert the stream of future relative marginal damages of carbon into a present value. The literature usually employs a constant social discount rate that is inferred from historical data of the market rate of return. Sometimes a lower social discount rate than the market rate of return is employed to incorporate the view that due to externalities and other distortions the social return to investment differs from the private return to investment.[1] Furthermore, the market rate of return can either be the high and risky rate of return on capital or the low and risk-free return on government bonds.

Employment of a constant social discount rate could be justified with a view that world temperature increases are is too unimportant to influence the social and the private return to capital investment of the world economy. Indeed, this seems to be a plausible explanation for the stylized fact of Kaldor of constancy of the market rate of return in historical long-run data. However, world temperature is predicted to increase to levels that have not been witness in the past. For this reason, most environmental scientists and economists would doubt that also future world temperature increases will be too unimportant to influence the social and the private return to capital investment. This is a problem in a Ramsey model with an endogenous social discount rate and endogenous time paths of consumption and carbon emissions, as this model does in general not imply a constant social discount rate. One can, however, impose parameter restrictions on this model that guarantee the future economy to be on an optimal balanced growth path, on which the social discount rate is constant. On such a balanced growth path there is constant and positive economic growth and a constant stock of carbon in the atmosphere. Imposing such parameter restrictions might seem restrictive. However, they are not much more restrictive than parameter restrictions that must be fulfilled to explain constancy of the market rate of return in historical long-run data.

After showing parameter restrictions that are required for optimal balanced growth in an analytical Ramsey model, the present paper exploits these restrictions to derive the growth rate of the SSC on an optimal balanced growth path. This growth rate is shown to be negative, provided the

[1] See Groom et al. (2005).

elasticity of marginal utility of consumption with respect to consumption is larger or equal to one. The latter requirement seems according to literature's consensus view to be fulfilled in reality.

Section 2 presents and solves a deterministic Ramsey growth model with a standard carbon stock accumulation equation commonly employed in environmental economics. Section 3 shows that the carbon stock accumulation equation assumed in section 2 implies convergence towards a constant steady state-level of the carbon stock.[2] Section 4 presents restrictions on technology that guarantee existence of a balanced growth path, while section 5 presents restrictions on preferences that guarantee the balanced growth path to be socially optimal.[3] Section 6 exploits these restrictions to derive the growth rate of the SSC on the optimal balanced growth path. Finally, section 7 concludes by briefly discussing the merits of this paper's assumption that the economy will converge towards an optimal balanced growth path with positive economic growth.

2. The model

To derive the SSC analytically, we assume along the lines of the Ramsey model a social planner with perfect foresight, who maximizes in period 0 lifetime utility, $W(O)$, of an infinitely lived representative household subject to the economy's resource constraints. In a competitive economy without externalities there exists a market equilibrium equivalent to the social planner solution, while in the presence of externalities the social planner solution can be replicated in a market economy upon use of a Pigovian tax. Similar to Krautkrämer (1985), the lifetime utility function is assumed to be of the following form:[4]

$$W(0)=\int_0^{\infty} U(C,A)e^{-\rho t}dt, \qquad (1)$$

where $U(C,A)$ represents instantaneous utility and ρ denotes the constant utility discount rate. Instantaneous utility is time-separable and depends positively on consumption, C, with $U_{CC}<0$, and negatively on the stock of carbon in the atmosphere, A.[5] Note that we abstract from population growth, which seems not to be too unrealistic for the very long-run, as world population growth is predicted to slow down. For simplicity we also abstract from uncertainty, leaving its consideration to future reasearch.

Accumulation of the stock of carbon in the atmosphere is assumed to evolve according to the following differential equation:[6]

$$\dot{A}=M-\delta A, \qquad (2)$$

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[2] In this study the words steady state and balanced growth path are used interchangeable.

[3] See Bovenberg and Smulders (1995) and Pittel (2005) for a similar approach in endogenous growth models with environmental quality.

[4] The time index t is omitted. Krautkrämer (1985) assumes utility gain from environmental amenities, while eq. (1) assumes utility loss from carbon in the atmosphere.

[5] One could make the more realistic assumption that instantaneous utility depends on world temperature rather than on A and add to the model a function describing a world temperature response to changes of A (see e.g. Marten (2011). This might lead to a further optimal balanced growth restriction to be imposed on a world temperature response function, but lies outside of the scope of the present analytical paper.

[6] See Perman et al. (2003, p. 182) or Pindyck and Rubinfeld (2009, Ch. 18). A dot on a variable represents the derivative of that variable with respect to time.

where M denotes the flow of carbon emissions and δ represents a constant dissipation rate of A each period. Production of a consumption good, Y, takes place according to the following production function:

$$Y = F(K, M, \Omega, A),$$

which has constant returns to scale in capital, K, and M.[7] For simplicity, we abstract from labor input. Further, Ω denotes technology that is assumed to grow exogenously and to increase productivity in consumption goods production. Moreover, the stock of carbon in the atmosphere is assumed to reduce productivity in consumption goods production.[8] Finally, capital accumulation is assumed to evolve according to the following differential equation:

$$\dot{K} = F(K, M, \Omega, A) - C, \qquad (3)$$

where we abstract for simplicity from capital depreciation.

The Hamiltonian that the social planner maximizes is therefore:

$$H = U(C, A)e^{-\rho t} + \lambda[F(K, M, \Omega, A) - C] + \mu[M - \delta A].$$

This gives rise to the following first order conditions:

$$\frac{\partial H}{\partial C} = 0 \quad \Rightarrow \quad U_C e^{-\rho t} = \lambda, \qquad (4a)$$

$$\frac{\partial H}{\partial M} = 0 \quad \Rightarrow \quad \lambda F_M = -\mu, \qquad (4b)$$

$$\frac{\partial H}{\partial K} = -\dot{\lambda} \quad \Rightarrow \quad \lambda F_K = -\dot{\lambda}, \qquad (4c)$$

$$\frac{\partial H}{\partial A} = -\dot{\mu} \quad \Rightarrow \quad U_A e^{-\rho t} + \lambda F_A - \mu\delta = -\dot{\mu} \qquad (4d)$$

As is shown in Appendix A, if we define the social discount rate, r, as $r \equiv F_K$, then (4a) and (4c) give rise to the following modified Ramsey rule:[9]

$$r = \rho + \eta_{CC}\hat{C} + \eta_{CA}\hat{A}, \qquad (5)$$

with $\eta_{CC} \equiv -\frac{U_{CC}C}{U_C}$ and $\eta_{CA} \equiv -\frac{U_{CA}A}{U_C}$,

where η_{CC} and η_{CA} denote the elasticity of marginal utility of consumption with respect to C and A.

[7] Without loss of generality, we abstract from a richer context in which fossil fuel use rather than M is an input into consumption goods production and in which fossil fuel use leads to carbon emission.

[8] Again, we abstract from the fact that in reality productivity in consumption goods production is affected by the world temperature rather than by A.

[9] A hat on a variable represents the growth rate of that variable.

Moreover, as is shown in Appendix B, combining (4a)-(4d) yields the following modified Solow-Stiglitz efficiency condition:

$$F_K = \hat{F}_M - \delta - \left(\frac{F_A + U_A / U_C}{F_M} \right), \tag{6}$$

where F_A represents marginal damage in consumption goods production from the carbon stock and the term $dC/dA = -U_A/U_C$ represents the marginal rate of substitution between consumption and the carbon stock. More generally, the left hand side of (6) represents the social return to investment in K, while the right hand side of (6) represents the social return to abatement of A. In turn, the social return to abatement of A equals the sum of the private return to abatement of A, $\hat{F}_M - \delta$, and the avoided social marginal damage from A, $-(F_A + U_A / U_C)$ multiplied with $1/F_M$.[10]

In a market economy, the social marginal damage from A represents an externality that is not considered in firms' profit maximization decisions. This externality can, however, be internalized with a Pigovian tax placed on M. As mentioned before, the externality is completely internalized if the Pigovian tax equals the SCC on the optimal carbon emission path. In turn, upon use of (6), the SSC can in the present model be derived to be:[11]

$$SCC = -\int_t^{\infty} (F_A + U_A / U_C) \frac{dA}{dM} e^{-\int_t^s r(s')ds'} ds, \tag{7}$$

that is, as mentioned before, the SCC equals the present value of the marginal damage from a small carbon emission increase. In (7), dA/dM is determined according to (2), while the social discount rate, r, evolves according to (5).

3. Convergence towards a constant level of the stock of carbon in the atmosphere

In a growth model with carbon emissions, existence of a balanced growth path with positive economic growth requires convergence of the stock of carbon in the atmosphere towards a constant steady state-level. In the last section, we assumed in (2) carbon accumulation with a constant dissipation rate of A each period. This is a common assumption in environmental economics (e.g. in Perman et. al (2003, Ch. 6.9) and in Pindyck and Rubinfeld (2009, Ch. 18)), but might be questioned. A feature of a constant dissipation rate is the fact that it guarantees fulfillment of our requirement of convergence towards a constant steady state-level of the carbon stock.[12] That this is the case can be seen from differentiation of (2) with respect to A, which gives:

$$\frac{\partial \dot{A}}{\partial A} = -\delta < 0.$$

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[10] We multiply with $1/F_M$ because both sides of (6) represent the returns of giving up one unit output. In turn, giving up one unit output for abatement implies reducing A by $1/F_M$ units (cf. Sinn (2007, p. 9).

[11] See analogously in van der Ploeg and Withagen (2011, Proposition 6).

[12] Bovenberg and Smulders (1995) assume accumulation of an environmental quality stock variable with a dissipation rate that is varying in the environmental quality stock level. Imposing some restrictions on the relation between the dissipation rate and the environmental quality stock level, there still exists in their model convergence towards a locally stable steady state-level of the environmental quality stock.

Hence, increases of A become gradually smaller, as A converges towards its steady state-level. In the steady state we have $\dot{A}=0$, which upon use of (2) gives the steady state-level of A, A^*, as:[13]

$$A^* = \frac{M^*}{\delta},$$

where in the steady state M must have a constant steady state-level M^* as well. Moreover, differentiation of this steady state-relationship with respect to M^* gives:

$$\frac{dA^*}{dM^*} = \frac{1}{\delta}. \qquad (8)$$

4. Restrictions on technology for existence of balanced growth

In this section we exploit the fact that our model implies stable steady-state levels for A and M to derive restrictions on technology that guarantee existence of balanced growth. We can do so because with constant A and M, our model is analogous to a standard neoclassical growth model with $Y=F(K,L,\Omega)$, where L is a possibly constant labor force and with (3) as capital accumulation equation. Uzawa (1961) has shown in a theorem that in such a model balanced growth requires technological progress to be constant and labor-augmenting, i.e. that we need $\hat{\Omega} \equiv g = \text{constant}$ and the production function to have the form $Y=F(K,\Omega L)$. Applying the Uzawa theorem to our model implies that on the balanced growth path we need as well $\hat{\Omega} \equiv g = \text{constant}$ and we need technical progress to be emission-augmenting. The latter restriction requires the production function on the balanced growth path to have the following form:

$$Y = F(K, \Omega M, A).$$

It can be shown that on a balanced growth path the capital-output ratio, K/Y, and the factor shares must be constant, where in our model the factor shares are defined as:

$$\alpha_K \equiv \frac{rK}{Y} \equiv \frac{F_K K}{Y} \text{ and } \alpha_M \equiv \frac{p_M M}{Y} \equiv \frac{F_M M}{Y} \text{ with } p_M \equiv F_M \text{ and } \alpha_K + \alpha_M = 1.$$

In what follows we first show that constancy of K/Y and of the factor shares requires technical progress to be emission-augmenting. Suppose the production function has the form $Y=F(\Omega_K K,\Omega_M M,A)$, with constant rates of capital- and emission-augmenting technical progress , Ω_K and Ω_M. In this case constant returns to scale in K and M imply that the production function can be rewritten in the following form:[14]

$$Y = F(\Omega_K K, \Omega_M M, A) = \Omega_M M F(\tilde{k}, 1, A) = \Omega_M M f(\tilde{k}, A), \qquad (9)$$

where $\tilde{k} \equiv \dfrac{\Omega_K K}{\Omega_M M}$.

[13] Cf. Perman et al. (2003, p. 183).
[14] See analogously Barro and Sala-I-Martin (2003, p. 28).

Differentiation of (9) with respect to K and M gives rise to:

$$r \equiv F_K = \Omega_K f_{\tilde{k}}(\tilde{k}, A), \tag{10}$$

$$F_M = \Omega_M [f(\tilde{k}, A) - \tilde{k} f_{\tilde{k}}(\tilde{k}, A)]. \tag{11}$$

Substituting (9)-(11) in the afore-shown factor share definitions gives the factor shares as:

$$\alpha_K = \frac{\tilde{k} f_{\tilde{k}}(\tilde{k}, A)}{f(\tilde{k}, A)} \text{ and } \alpha_M \equiv \frac{f(\tilde{k}, A) - \tilde{k} f_{\tilde{k}}(\tilde{k}, A)}{f(\tilde{k}, A)}. \tag{12}$$

Clearly, according to (12) and due to constancy of A on a balanced growth path, constancy of the factor shares requires constancy of $\tilde{k}$. In turn, since $\tilde{k} \equiv (\Omega_K K)/(\Omega_M M)$, constancy of $\tilde{k}$ requires $\Omega_K K$ and $\Omega_M M$ to grow at the same rate. If we assume this rate to be g=constant, then due to constancy of M, Ω_M must grow at the constant rate g as well. Moreover, according to (9), constancy of M, $\tilde{k}$ and A imply Y also to grow at the rate g. Therefore, constancy of K/Y implies that K must also grow at the rate g. If, however, K grows at the rate g and $\Omega_K K$ must grow at the rate g, then Ω_K must be constant. [15]

In addition, constancy of Ω_K, $\tilde{k}$ and A and the fact that Ω_M grows at the constant rate g implies, according to (10) and (11), that on the balanced growth path the social discount rate and F_K are constant, while F_M grows at the constant rate g. Finally, differentiating (9) with respect to A and applying the notation $\Omega_M \equiv \Omega$ yields:

$$F_A = \Omega_M M f_A(\tilde{k}, A) = \Omega M f_A(\tilde{k}, A). \tag{13}$$

Therefore, since M, $\tilde{k}$ and A are constant, the marginal damage in consumption goods production from the carbon stock, F_A, grows on the balanced growth path at the rate g as well.

5. Restrictions on preferences for balanced growth to be optimal

The last section has shown restrictions on technology that guarantee existence of a balanced growth path in which output grows at the constant rate g and the factor shares and the social discount rate are constant. In turn, this balanced growth path to be optimal requires restrictions on preferences that guarantee that the social planner chooses on the balanced growth path consumption to grow at the constant rate g and the stock of carbon in the atmosphere to be constant. This section presents these restrictions on preferences. Following Bovenberg and Smulders (1995) in a two-sector endogenous growth model with environmental quality, one can exploit the fact that with constant A and M our model is analogous to a Ramsey growth model with leisure time that is constant in the

[15] See Acemoglu (2009, p. 64) for a similar intuitive explanation of the Uzawa theorem within the context of the Solow model.

steady state.[16] Therefore, one can apply the two restrictions on preferences for optimal balanced growth that were derived by King et al. (1988) in a Ramsey growth model with leisure time.[17]

Possibly most simply illustrated, the first of these restrictions on preferences follows from use of the modified Ramsey rule (5).[18] The last section showed that on the balanced growth path the social discount rate and A are constant. Therefore, (5) becomes on the balanced growth path:

$$r^* = \rho + \eta_{CC}\hat{C}. \tag{14}$$

where henceforth the index (*) denotes the steady state-level or the balanced growth path of the variable. Therefore, from (14) follows that for the social planner to choose consumption to grow at the constant rate g requires the elasticity of marginal utility of consumption with respect to C, η_{CC}, to be constant on the balanced growth path. In turn, a time-separable constant-relative-risk-aversion (CRRA) instantaneous utility function does imply a constant value of η_{CC}. This is so because, as one can tell from its name, a CRRA utility function - which has in the standard case without A the form $U(C) = (1/(1-\theta))C^{1-\theta}$ - implies a constant coefficient of relative risk aversion, θ. In turn, in case of a time-separable instantaneous utility function, the elasticity of marginal utility of consumption with respect to C and the coefficient of relative risk aversion are identical.[19]

The second restrictions on preferences can be illustrated upon use of modified Solow-Stiglitz efficiency condition (6).[20] It has been shown in the last section that on the balanced growth path F_K is constant and F_M and F_A grow at the constant rate g. Since ρ is constant, the modified Solow-Stiglitz efficiency condition is only consistent with balanced growth if the marginal rate of substitution between consumption and the carbon stock, $dC/dA=-U_A/U_C$, grows at the constant rate g as well. Due to the fact that on the optimal balanced growth path C/A also grows at the constant rate g (because C grows at the rate g, while A is constant), we need to impose the restriction:

$$ES \equiv \frac{d(C/A)/(C/A)}{d(-U_A/U_C)/(-U_A/U_C)} = 1, \tag{15}$$

where ES measures the elasticity of substitution between C and A. A glance at (15) shows that ES=1 guarantees that growth of C/A at the rate g is associated with growth of $-U_A/U_C$ at the same rate g.

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[16] The fact that the model of Bovenberg and Smulders (1995) is a two-sector endogenous growth model implies different balanced growth restrictions on technology than our model. However, their household problem is similar to our model and therefore their model and our model share the same optimal balanced growth restrictions on preferences.

[17] In King et al. (1988) the balanced growth restrictions on technology that follow from application of the Uzawa theorem are the same as in our model because their model and our model are both one-sector exogenous (or neoclassical) growth models.

[18] See analogously in Acemoglu (2009, p. 307) for the "unmodified" Ramsey rule (derived in a Ramsey model without an environmental stock or a leisure time variable).

[19] The informed reader knows of course that, in addition, the elasticity of marginal utility of consumption with respect to C is the inverse of the intertemporal elasticity of substitution of C.

[20] Cf. Bovenberg and Smulders (1995, p. 378).

King et al (1988) have shown that fulfillment of these two afore-mentioned restrictions on preferences requires – replacing the carbon stock of our model for leisure time in their model – the instantaneous utility function to have the following form:[21]

$$U(C,A)=\frac{C^{1-\theta}}{1-\theta}v(A) \qquad \text{if } \theta\neq 1, \tag{16a}$$

$$\text{or} \quad U(C,A)=\ln C+v(A) \quad \text{if } \theta=1, \tag{16b}$$

where $\theta>0$ and v'<0.

It is straightforward from use of (16a) and (16b) and the definitions of η_{CC} and η_{CA} to verify that $\eta_{CC}=\theta$ and $\eta_{CA}=-(v'(A)A)/v(A)$ if $\theta\neq 1$ or $\eta_{CA}=0$ if $\theta=1$. Finally, the marginal rate of substitution between consumption and the carbon stock can be verified to equal:

$$\frac{dC}{dA}=-\frac{U_A}{U_C}=\left(\frac{1}{1-\theta}\right)\left(\frac{C}{A}\right)\left(-\frac{v'(A)A}{v(A)}\right) \quad \text{if } \theta\neq 1, \tag{17a}$$

$$\text{or} \quad \frac{dC}{dA}=-\frac{U_A}{U_C}=-Cv'(A) \qquad \text{if } \theta=1, \tag{17b}$$

where on the balanced growth path $-(v'(A)A)/v(A)$, respectively, v'(A), are constant due to constancy of A.[22]

6. The SCC on the optimal balanced growth path

As mentioned in the introduction, this section exploits the afore-shown restrictions for optimal balanced growth to derive the growth rate of the SSC on the optimal balanced growth path. In this section we first state the relevant relationships that follow from the optimal balanced growth path restrictions. Next, we combine these relationships to derive the growth rate of the SSC on the optimal balanced growth path.

In section 4 it was shown that our restrictions on technology imply that F_A grows on the balanced growth path at the constant rate g. Denoting with $\tilde{t}$ the period in which the economy has reached the balanced growth path, the balanced growth path of the marginal damage in consumption goods production from the carbon stock, F_A^*, is:

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[21] See analogously Pittel (2005, p. 449) within an AK growth model with environmental amenities and an endogenous utility discount rate and Bovenberg and Smulders (1995, footnote 4).

[22] To be more accurate, our afore-shown restrictions for optimal balanced growth only guarantee constant economic growth that must not necessarily be positive. Rearranging (14) yields: $\hat{C}=(1/\eta_{CC})(r^*-\rho)$ (#). Since we defined $r\equiv F_K$, $\hat{C}>0$ therefore requires on the balanced growth path the right hand side of the modified Solow-Stiglitz efficiency condition (6) to be larger than ρ (which can be verified from substituting (6) in (#) for $r\equiv F_K$). An analoguous argument has been made in Valente (2005) in a "capital-resource" growth model.

$$F_A^* = F_A^*(\tilde{t})e^{g(t-\tilde{t})}, \tag{18}$$

Furthermore, use of (17a) and (17b) and the fact that on a balanced growth path C must grow at the rate g, while A must be constant yields the balanced growth path of the marginal rate of substitution between consumption and the carbon stock to be:

$$\left(\frac{dC}{dA}\right)^* = \left(-\frac{U_A}{U_C}\right)^* = \left(\frac{1}{1-\theta}\right)\left(\frac{C^*(\tilde{t})}{A^*}\right)\left(-\frac{v'(A^*)A^*}{v(A^*)}\right)e^{g(t-\tilde{t})} \text{ if } \theta \neq 1, \tag{19a}$$

or $$\left(\frac{dC}{dA}\right)^* = \left(-\frac{U_A}{U_C}\right)^* = -C^*(\tilde{t})v'(A^*)e^{g(t-\tilde{t})} \qquad \text{if } \theta = 1. \tag{19b}$$

In addition, upon use of (14) and upon use of the facts that $\eta_{CC} = \theta$ and that on the balanced growth path $\hat{C} = g$, the modified Ramsey rule becomes:

$$r^* = \rho + \theta g. \tag{20}$$

Finally, we derived already in section 3 the following steady state-relationship for the carbon stock:

$$\frac{dA^*}{dM^*} = \frac{1}{\delta}. \tag{8}$$

Next, combining (18), (19a) or (19b), (20) and (8) with (7) gives the balanced growth path of the SSC as:

$$SSC^* = \Gamma(\tilde{t})e^{(g-r^*)(t-\tilde{t})} = \Gamma(\tilde{t})e^{[(1-\theta)g-\rho](t-\tilde{t})}, \tag{21}$$

with $\Gamma(\tilde{t}) \equiv F_A^*(\tilde{t}) + \left(\frac{1}{1-\theta}\right)\left(\frac{C^*(\tilde{t})}{A^*}\right)\left(-\frac{v'(A^*)A^*}{v(A^*)}\right)$ if $\theta \neq 1$,

or $\Gamma(\tilde{t}) \equiv F_A^*(\tilde{t}) - C^*(\tilde{t})v'(A^*)$ if $\theta = 1$.

Taking natural logarithms of (21) and differentiating with respect to time gives the balanced growth rate of the SSC as:

$$\widehat{SSC}^* = g - r^* = (1-\theta)g - \rho. \tag{22}$$

As a consequence, we get the following Proposition:

Proposition:

Let $g \geq 0$ and $\rho > 0$. Then the growth rate of the SCC on the optimal balanced growth path is negative if $\theta \geq 1$, i.e. if the elasticity of marginal utility of consumption with respect to consumption is larger than or equal to one.

The Proposition is a straightforward consequence of (22). Let $g \geq 0$. Then $\theta \geq 1$ guarantees the first term in *(22),* $(1-\theta)g$, to be negative or to be equal to zero. Hence, provided the second term in (22), -ρ, is negative, then the balanced growth rate of the SSC is negative. The intuitive explanation for the Proposition is as follows: According to (8), on the balanced growth path dA/dM is constant. Further, according to (7), the growth rate of the instantaneous marginal damage of carbon emissions (i.e. the sum of the growth rates of F_A^* and $(-U_A/U_C)^*$) has a positive effect on the balanced growth rate of the SCC. However, the social discount rate (i.e. the left hand side of (20)) has a negative effect on the balanced growth rate of the SCC (see (20)). The restrictions for optimal balanced growth require the instantaneous marginal damage of carbon emissions to grow at the rate g. Therefore, if $\theta \geq 1$, then (20) implies, on the balanced growth path, the negative effect of the social discount rate to unambiguously dominate the positive effect if $\rho > 0$ and $g \geq 0$.

Are the conditions of the Proposition fulfilled in reality? It is very likely that the long-run rate of technical progress, g, will be positive. Most economists would also agree that the utility discount rate is at least somewhat larger than zero (and that even a benevolent dictator should employ a somewhat positive utility discount rate). Hence, the crucial parameter is θ. It is very difficult to gauge a plausible value of the elasticity of marginal utility of consumption with respect to consumption and there is some disagreement on its true value in the literature. However, the debate is whether or not θ is larger than one or equal to one.[23] There seems to be a consensus in the literature that θ is not smaller than one, which is all what is required in the Proposition.[24] As a consequence, the SCC on the optimal balanced growth path seems to be negative. This is good news because it implies that at least in the long-run, provided an optimal balanced growth path exists, human mankind will not be burdened with ever increasing costs to internalize externalities from global warming.

7. Conclusion

The present paper examined a deterministic Ramsey model of optimal economic growth with carbon emissions. It has been shown that the sign of the growth rate of the SCC on the optimal balanced growth path depends on the value of the elasticity of marginal utility of consumption with respect to consumption. The assumption that the economy will converge towards a future optimal balanced growth path with positive economic growth can of course be questioned. Some readers might level criticism on the assumed feasibility of long-run stabilization of the carbon concentration in the atmosphere. However, if this were not feasible, then this would be a doomsday scenario. Policymakers agreed at the 2010 United Nations Climate Change Conference to the target to limit future world temperature increases to 2°C above its pre-industrial level. This target seems only feasible

[23] See e.g. Nordhaus (2007), Stern et al. (2006) and Weitzman (2007).

[24] As a matter of fact, negative balanced growth of the SCC is also a mathematical requirement of our Ramsey model. This is so because the social planner's mathematical problem is only well-behaved if the integral in (1) is bounded. As Smulders (2007, p. 8) explains in a model with a similar utility function as ours, this requires that instantaneous utility discounted upon employment of the utility discount rate must approach zero as time approaches infinite. This requires the balanced growth rate of this discounted instantaneous utility to be negative. However, since A is constant on the balanced growth rate, it is straightforward to verify that the balanced growth rate of this discounted instantaneous utility turns out to be equal to the balanced growth rate of the SCC (i.e. the right hand side of (22)), which therefore must be negative for mathematical reasons as well.

if it is possible to stabilize future carbon concentration in the atmosphere. Admittedly, the restrictions on technology and preferences that must be fulfilled for existence of an optimal balanced growth path are rather restrictive. However, I stressed already myself in Kögel (2009) that the common practice of using a constant social discount rate requires fulfillment of restrictive knife-edge conditions that cannot be taken for granted. Nevertheless, the 2°C world temperature increase target seems to suggest that policymakers aim for a balanced growth path as defined in this paper. Furthermore, evidence for fulfillment of the stylized facts of Kaldor in historical data suggests that sustainability might be feasible. For this reason, it might be worthwhile to accept fulfillment of the optimal balanced growth path restrictions for a moment and to explore what implications these restriction would have for the growth rate of the SCC on the optimal balanced growth path.

Appendix A: Derivation of the modified Ramsey rule (eq. (5))

Taking time derivatives of (4a) yields:

$$U_{CC}\dot{C}e^{-\rho t}+U_{CA}\dot{A}e^{-\rho t}-\rho U_{C}e^{-\rho t}=\dot{\lambda}. \tag{A1}$$

Upon substituting (4a) in (4c) for λ, multiplication with (-1) and rearranging we get:

$$\dot{\lambda}=-U_{C}e^{-\rho t}F_{K}. \tag{A2}$$

Substituting (A2) in (A1) for $\dot{\lambda}$, division by ($-e^{-\rho t}U_C$) and rearranging gives:

$$F_K=\rho+\left(-\frac{U_{CC}C}{U_C}\right)\hat{C}+\left(-\frac{U_{CA}A}{U_C}\right)\hat{A}. \tag{A3}$$

Finally, using the definition $r\equiv F_K$ and the definitions of η_{CC} and η_{CA} in the text gives rise to eq. (5) in the text.

Appendix B: Derivation of the modified Solow-Stiglitz efficiency condition (eq. (6))

Division of both sides of (4d) by μ gives:

$$\frac{U_A e^{-\rho t}}{\mu}+\frac{\lambda F_A}{\mu}-\delta=-\frac{\dot{\mu}}{\mu}. \tag{B1}$$

Taking time derivatives of (4b), multiplying with (-1) and rearranging yields:

$$\dot{\mu}=-\dot{\lambda}F_M-\lambda\dot{F}_M. \tag{B2}$$

Multiplying (4b) with (-1) and rearranging yields: $\mu=-\lambda F_M$. Substituting the latter relation for μ on both sides of (B1) gives rise to:

$$-\frac{U_A e^{-\rho t}}{\lambda F_M}-\frac{F_A}{F_M}-\delta=\frac{\dot{\mu}}{\lambda F_M}. \tag{B3}$$

Upon substituting (B2) for $\dot{\mu}$ on the right hand side of (B3) we get:

$$-\frac{U_A e^{-\rho t}}{\lambda F_M}-\frac{F_A}{F_M}-\delta=-\hat{\lambda}-\hat{F}_M. \tag{B4}$$

Rearranging (4a) and (4c) yields: $\lambda=U_C e^{-\rho t}$ and $-\hat{\lambda}=F_K$. Substituting the former relation on the left hand side of (B4) for λ and the latter relation for on the right hand side of (B4) $-\hat{\lambda}$ and rearranging terms gives eq. (6) in the text.

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