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Interactions in DSGE Models: The Boltzmann–Gibbs Machine and Social Networks Approach

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Abstract While DSGE models have been widely used by central banks for policy analysis, they seem to have been ineffective in calibrating the models for anticipating financial crises. To bring DSGE models closer to real situations, some of researchers have revised the traditional DSGE models. One of the modified DSGE models is the adaptive belief system model. In this framework, changes in sentiment can be expounded by a Boltzmann–Gibbs distribution, and in addition to externally caused fluctuations endogenous interactions are also considered. Methodologically, heuristic switching models are mesoscopic. For this reason, the social network structure is not described in the adaptive belief system models, even though the network structure is an important factor of interaction. The interaction behavior should ideally be based on some kind of social network structures. Today, the Boltzmann–Gibbs distribution is widely used in economic modeling. However, the question is whether the Boltzmann–Gibbs distribution can be directly applied, without considering the underlying social network structure more seriously. To this day, it seems that few scholars have discussed the relationship between social networks and the Boltzmann–Gibbs distribution. Therefore, this paper proposes a network based ant model and tries to compare the population dynamics in the Boltzmann–Gibbs model with different network structure models applied to stylized DSGE models. We find that both the Boltzmann–Gibbs model and the network-based ant model could generate herding behavior. However, it is difficult to envisage the population dynamics generated by the Boltzmann–Gibbs model and the network-based ant model having the same distribution, particularly in popular empirical network structures such as small world networks and scale-free networks. In addition, our simulation results further suggest that the population dynamics of the Boltzmann–Gibbs model and the circle network ant model can be considered with the same distribution under specific parameters settings. This finding is consistent with the study of thermodynamics, on which the Boltzmann–Gibbs distribution is based, namely, the local interaction.

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I. Introduction

While DSGE models have been widely used by central banks for policy analysis, the global financial crisis has apparently challenged the credibility of DSGE models; it may thus be risky for governments to use DSGE models as a tool for policy making. In fact, it is not easy to generate a crash or a bubble in a traditional DSGE model with incredible assumptions such as the representative agent and rational expectations. To apply DSGE models to situations closer to real world situations, many researchers have added heterogeneity, bounded rationality, learning and interaction, in the hope of calibrating the modified DSGE models to match the real world economy. For heterogeneity, Bask (2007) introduced two different types of agents (fundamentalists and chartists) to a DSGE model, and similar to Bask (2007), Chang et al. (2010) and Wen (2010) also tried to construct DSGE models with heterogeneous agents which can match some stylized facts of macroeconomics.

Besides the representative agent hypothesis, bounded rationality is another possible modification for improving DSGE models. Therefore, economists have started to model how individuals form expectations and learn and adapt their behaviors. This direction of research combines the **adaptive learning mechanisms** of Evans and Honkapojah (2001) with DSGE models. They have said that agents should not be assumed to be more clever than econometricians and, therefore, the agent should learn the underlying model as time passes. Recently, Orphanides and Williams (2007a,b.), Milani (2009), Branch and McGough (2009) and Chen and Kulthanavit (2010) have applied adaptive learning mechanisms to some new Keynesian DSGE models. However, none of the aforementioned modified models allow for an endogenous evolution of different rules. In other words, the proportion of different types of agents has to be exogenously given. Agents, therefore, keep employing the same rule and do not try to learn from past history. Nonetheless, this simplification underestimates the uncertainty faced by each agent. In the most general sense, the behavior of the agent, one can never be certain about the duration of the biased trend, since the trend can last a few weeks, months or years. Branch and McGough (2009) have also pointed out that further research should focus on the stability of equilibrium and interactions of learning behaviors.

In order to consider the adaptive behaviors of agents, economists have tried to introduce some statistical mechanics into traditional economic models. The most popular statistical mechanics which refers to the statistical basis of thermodynamics is Boltzmann-Gibbs distribution. More precisely, the Boltzmann-Gibbs distribution could be thought of as a tool for evolving the micro-structure of all agents in the economic system. In the world of thermodynamics, the system is composed of many interacting particles and different statistical mechanics are developed to deal with the relationships between the macro and micro states. Thus, through the Boltzmann-Gibbs distribution, the proportion of different behavioral rules can evolve over time. In other words, we can generate endogenous population dynamics. Brock and Hommes (1997, 1998) can be regarded as the pioneers of this kind of research, also known as the ***adaptive belief system model***. In economic research, the maximum use of the Boltzmann-Gibbs distribution has been to study financial markets' anomalies. During the last decade, the Boltzmann-Gibbs distribution has been widely used for modeling financial markets. A detailed survey of the use of the Boltzmann-Gibbs approach can be found in Chen et al. (2010). In general, the Boltzmann-Gibbs distribution is often used to deal with ***expectation behavior***; it can, therefore, be applied to models incorporating expectations, such as the cobweb model, asset pricing model and positive versus negative feedback model, etc.

So far, the Boltzmann-Gibbs distribution has been gradually entering DSGE models

(Bask, 2007; De Grauwe, 2010a, 2010b; Assenza et al., 2009; and Lengnick and Wohltmann, 2010). Bask (2007) combined a small open economic model with a Boltzmann-Gibbs distribution. He imposed technical and fundamental analyses as different rules in currency trade and found that chaotic dynamics and long swings may occur in the exchange rate. Assenza et al. (2009) combined human expectations in a standard DSGE model. They asked the subjects to provide two-period ahead forecasts of inflation rate and the output gap for 50 periods. Thus, the realized inflation and output gap could be determined by average individual expectations. In this experiment, subjects have only qualitative information about the macro economy; they do not know the underlying law of motion. Then, they separated the experimental data into four different forecasting rules: ADA (Adaptive Expectations), WTR (Weak Trend Followers), STR (Strong Trend Followers) and LAA (Learning Anchoring Adjustment). They found that the heuristic switching model could successfully calibrate the macroeconomic variables dynamics generated by the human subjects experiment. Lengnick and Wohltmann (2010) combined the Boltzmann-Gibbs distribution and the DSGE macroeconomic model with the financial market. They found that stock market developments are more realistically described by the Boltzmann-Gibbs distribution machine than rational DSGE models, and that the negative impact that speculative behavior of financial market participants exerts on the macro economy can be reduced by the introduction of a transaction tax. In addition, a closed economic DSGE model is augmented with the Boltzmann-Gibbs distribution in De Grauwe (2010a, 2010b). The author developed a stylized DSGE model in which agents use simple rules of heuristics to forecast the future inflation and output gap. The simulation results show that the dynamic behaviors of macroeconomic variables are more volatile in the Boltzmann-Gibbs distribution machine than in stylized DSGE models, and endogenous economic cycles can be generated in the Boltzmann-Gibbs distribution machine.

The number of applications combining the Boltzmann-Gibbs distribution machine and the DSGE macroeconomic model has been increasing, but the question remains as to whether statistical mechanisms (essentially metaphors) can be used to correlate conscious humans and unconscious particles in a reliable manner. Even so, methodologically, models connected with the Boltzmann-Gibbs distribution machine belong to the ***mesoscopic*** genre, i.e., individual details are considered irrelevant. Of course, the social network structure is also not described in those models. However, physicists have developed statistical mechanisms for dealing with the interaction of particles on the basis of existing structures within particles. Whether the Boltzmann-Gibbs distribution machine can be applied directly, without considering the underlying network structure more seriously, is still an open question. In particular, economists have used the Boltzmann-Gibbs distribution machine to describe interaction behaviors. In general, the network structure is an important factor of interaction, as any interaction behavior should generally be based on some kind of social network structure. In this case, we seem to know in-depth about the tool that we use. Thus, we need a deeps fundamental insight into the system's dynamics and how it can be traced back to structural properties of the underlying interaction network.

In actual fact, the impact of social networks on economic behavior has become an important issue recently. In order to describe a specific network structure, a social network is broadly understood as a collection of nodes and links between nodes. The extant literature can be roughly classified into three kinds. The first kind treats networks as endogenously determined, and studies the process of formation of networks. In this regard, agents add or delete their links for maximizing utility (or profit) according to a network formation game. In this area, the social network can be applied to free trade networks, market sharing agreements, labor markets and the co-author model. A detailed survey can be found in Jackson (2005). The second kind of literature regards networks as exogenous. In this case, network structures can be generated with different stochastic algorithms, such as random, scale-free or small world networks; these network structures have been applied to real social networks, i.e., collaborations (Vega-Redondo, 2007) and international trade and financial integration (Schiavo et al., 2010). According to the empirical results, economic networks may also reflect similar universality. Indeed, the connections of banks in an interbank network (Iori et al., 2008) show that the network structure of banks represents a scale free system where only a few banks interact with many others. In this example, banks with similar investment behaviors cluster in the network. Similar regularities can be traced in many examples, including international trade networks and financial networks (Schiavo et al., 2010). In addition to the empirical approach, applying exogenous network structures to economic models and studying their economic implications is another direction of research. In the last few years, several macroeconomic models have combined heterogeneous expectations with social network structures for modifying the setting of interaction behaviors. Alarano (2007) provided a probabilistic herding model with different network structures for agent-based final markets and found structural heterogeneity to have a crucial and non-trivial impact on the macroscopic properties of the market. Westeroff (2010) proposes a simple agent based macroeconomic model with a scale-free and lattice network structure in which firms hold heterogeneous sales expectations. Thus, each firm has fixed social relations with other firms, and they are either optimistic or pessimistic. The probability of a firm taking an optimistic view increases not only during a boom, but also with the number of its optimistic neighbors. The change in firms' sentiment causing change in national income has been observed for both a square lattice network and a scale-free network. Besides the application for studying economic implications, Alarano (2009) also constructed a hierarchical network-based ant model to overcome the N-dependence problem (of the ant model), even though a network-based ant model increases system-wide volatility. Thus, network structures become an auxiliary source of volatility except for the behavioral heterogeneity of interacting agents.

According to the above, both the Boltzmann-Gibbs distribution machine and the network approach have been important platforms for expressing interaction behavior, although to this day it seems that few scholars have discussed the relationship between social networks and the Boltzmann-Gibbs distribution. In order to construct a social interactive DSGE model with a network structure, we have to choose a model which can be combined with different social network structures. The ant model of Kirman (1991, 1993), inspired by the ants' foraging behavior, is one of the choices. The ant model endogenously creates swings and herding behavior in aggregate expectations through interaction and has successfully replicated stylized facts of financial markets (Chen et al., 2010). Therefore, this paper proposes a network-based ant model and tries to compare the population dynamics in the Boltzmann-Gibbs model with different network structure models which are applied to stylized New Keynesian DSGE models. In order to focus on the population dynamics generated by the Boltzmann-Gibbs model and network structures models, we follow De Grauwe (2010a, 2010b) and set the DSGE model for simplicity. Nevertheless, our model leads to a number of interesting insights. We find that both the Boltzmann-Gibbs model and network-based ant model can generate herding behavior. However, it is rather difficult to envisage the population dynamics generated by the Boltzmann-Gibbs model and the network-based ant model with the same distribution, particularly in popular empirical network structures such as small world networks and scale-free networks. In addition, our simulation results further suggest that the population dynamics of the

Boltzmann-Gibbs model and the circle network ant model can be considered with the same distribution under specific parameters settings. This finding is consistent with the study of thermodynamics for which the Boltzmann-Gibbs distribution is based on the local interaction. Although the circle network structure is not the acknowledged social network structure, according to the relative entropy between the population dynamics of the Boltzmann-Gibbs distribution and network-based ant model, the Boltzmann-Gibbs model with intensity of choice equal to 10,000 is a good approximation of the herding behavior of our network based ant model with any given network structure.

The remainder of this paper is organized as follows. In Section II, we describe the stylized DSGE model. Next, we present a version of the DSGE model with the Boltzmann-Gibbs distribution machine. In Section IV, we discuss the network-based ant model. Following that, we simulate different network structures and present the results. Section VI concludes.

II. The stylized New Keynesian DSGE model

This section describes the stylized New Keynesian DSGE model. New Keynesian DSGE models are widely used in macroeconomics because they are derived from individual optimization so that both parameters and shocks can be structural. The model consists of the following three equations:

$$y_t = a_1 E y_{t+1} + (1-a_1) y_{t-1} + a_2 (r_t - E_t \pi_{t+1}) + \varepsilon_t \tag{1}$$

$$\pi_t = b_1 E_t \pi_{t+1} + (1-b_1) \pi_{t-1} + b_2 y_t + \eta_t \tag{2}$$

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 y_t + c_3 r_{t-1} + u_t \tag{3}$$

Equation (1) is referred to as the standard aggregate demand that describes the demand side of the economy. It is derived from the Euler equation which is the result of the dynamic utility maximization of a representative household and market clearing in the goods market. The notation for aggregate demand is as follows: y_t denotes the output gap in period t, r_t is the nominal interest rate and π_t is the rate of inflation. Here, we add a logged output gap in the aggregate demand equation for describing habit formation. E_t is the expectations operator; we use it to describe how people form their expectations. In the standard New Keynesian DSGE model, the representative agent always has rational expectations. However, we focus on describing the social interaction behavior and discussing the relationship between the Boltzmann-Gibbs distribution machine and social network structure. For that reason, there are two kinds of expectations in our model, the Boltzmann-Gibbs distribution machine and the network-based ant approach.

Equation (2) is a New Keynesian Phillips curve that represents the supply side in the economic system. Under the assumption of nominal price rigidity and monopolistic competition, the New Keynesian Phillips curve can be derived from the profit maximization of a representative final goods producer and the profit maximization of intermediate goods producers which are composed of a number of heterogeneous households. To reflect the price rigidity, the intermediate goods producers can adjust their price through the Calvo pricing rule. By combining the first-order conditions of the final goods producer, the intermediate goods producer and the Calvo pricing rule, we can obtain the New Keynesian Phillips curve (Equation 2).

Equation (3) represents the Taylor rule commonly used for describing the behavior of the central bank in the standard New Keynesian DSGE model. The central bank reacts to deviations of inflation and output from targets. In Equation (3), π^* refers to the inflation target of the central bank. For convenience, π^* is set to be equal to 0. In addition, the lagged interest rate in Equation (3) represents the smoothing behavior.

Finally, as the DSGE model is the DGE (Dynamic General Equilibrium) model with stochastic terms, ε_t, η_t, and u_t are all white noise disturbance terms.

According to the aforementioned equations, we can substitute Equation (3) into Equation (1) and rewrite the matrix notation. Thus, the reduced form can be written as:

$$\begin{bmatrix} 1 & -b_2 \\ -a_2c_1 & -a_2c_2 \end{bmatrix} \times \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1-b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \times \begin{bmatrix} E_t\pi_{t+1} \\ E_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 1-b_1 & 0 \\ 0 & 1-a_1 \end{bmatrix} \times \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ a_2c_3 \end{bmatrix} \times r_{t-1} + \begin{bmatrix} \eta_t \\ a_2u_t + \varepsilon_t \end{bmatrix} \tag{4}$$

or

$$\mathbf{AZ_t} = \mathbf{BE_tZ_{t+1}} + \mathbf{CZ_{t-1}} + \mathbf{br_{t-1}} + \mathbf{V_t} \tag{5}$$

According to the above, we can have solution $\mathbf{Z_t}$ for the system.

$$\mathbf{Z_t} = \mathbf{A^{-1}}[\mathbf{BE_tZ_{t+1}} + \mathbf{CZ_{t-1}} + \mathbf{br_{t-1}} + \mathbf{V_t}] \tag{6}$$

We can derive the solution only if matrix **A** is non-singular. In other words, matrix **A** has to satisfy $(1-a_2c_2) \times a_2b_2c_1 \neq 0$. After obtaining the inflation rate (π_t) and output gap (y_t) through Equation (6), we have to substitute the solution for Equation (3) and to arrive at the interest rate (r_t).

Finally, we must emphasize that the difference between the Boltzmann-Gibbs distribution machine and the network-based ant model is the difference between the expectations of the output gap and inflation. Although agents also make forecasts of inflation, we simply assume that all agents perceive the central bank's announced inflation target π_t^* to be fully credible. In other words, we set $\mathrm{E}_t \pi_{t+1} = \pi^*$ =0 in all simulation experiments, including the Boltzmann-Gibbs distribution machine and the network-based ant model.

III. Boltzmann-Gibbs distribution machine

In discarding slick theories, to make their models more realistic, economists have started studying psychology, biology and physics, in order to rethink the operations of our economic system. The Boltzmann-Gibbs distribution machine is one example of economists borrowing from other disciplines. Actually, the Boltzmann-Gibbs distribution is developed by physicists. However, the Boltzmann-Gibbs distribution machine is used not only in economics and thermodynamics but also in psychology (Luce, 1959; Blume, 1993). The beginning of the story is that some physicists found the collision of particles to be similar to the interaction of people. As Boltzmann showed, particles are similar to many individuals, having most of the states of motion. To be more precise, the collision of constituent particles under specific structures is analogous to the interaction of people under specific social networks. For example, an individual's behavior is influenced by her/his family and friends. If one's friends use the iPhone, one has a greater willingness to buy an iPhone. In addition, although each particle (agent) is affected only by a few closed particles (agents/friends), the aggregate outcome could be a huge change. This holds in both the world of particles and human society. Since physicists have been dealing with the systems of many interacting particles for more than a century, they have developed many mature theories by using **statistical mechanics** (such as the Boltzmann-Gibbs distribution) to deal with these phenomena. Their focus has not been on the details of individual particles, but on the relationships and dynamics between particles. In other words, the Boltzmann-Gibbs distribution was developed for investigating the relationship between macroscopic and microscopic phenomena in the physical sciences.

In terms of methodology, the modeling concept is called **mesoscopic**[1] which means that individuals' details are considered irrelevant, i.e., interaction is what matters. Based on this, the setting of heterogeneity is relatively simple. Each cluster represents a behavioral rule. In other words, agents have the same behavior in the same cluster and the evolution of the Boltzmann-Gibbs distribution machine represents the microscopic structural changes. In this framework, fluctuations are not only from outside as a "given" but also from endogenous interactions. In other words, fluctuations in a macroeconomic variable due to the activities of thousands of agents need not be simply a scaled-up version of the random noise which each individual agent is subjected to. They treat the economy as a complex adaptive system and the emergence of aggregate patterns as a result of individual interactions among participants at the micro-level. Since the physical system is composed of many interacting particles, the Boltzmann-Gibbs distribution is developed to deal with relationships between the macro state and microstate. Through the Boltzmann-Gibbs distribution, the proportion of specific microstates (population dynamics) can evolve over time. In other words, the Boltzmann-Gibbs distribution can be thought of as a tool for evolving the micro structure of market participants. It can give the proportion of a particular rule of the system.

For describing the different behavioral rules of output gap expectations, we assume

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[1] In this type of study, how individual agents decide what to do may not matter very much. What happens as a result of their actions may depend much more on the interaction structure through which they act—who interacts with whom, according to what rules. Therefore, they ignore the decision details of human beings and only assume that agents follow some simple rules and care about how individual forecasting rules interact at the micro level and which aggregate outcome they co-create at the macro level.

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the agents do not fully understand how the output gap is determined, and so the agents use simple rules, say, the optimistic rule and the pessimistic rule, to forecast the future output gap. Actually, assuming that some agents in the society are optimistic and some are pessimistic is a reasonable setting in the real economy. Therefore, in our Boltzmann-Gibbs distribution machine DSGE model, forecasts of optimistic agents systematically bias the output upwards and forecasts of pessimistic agents systematically bias the output downwards. In other words, the optimists' rule is defined by $E_{o,t}y_{t+1} = g$ and the pessimists' rule is defined by $E_{p,t}y_{t+1} = -g$, where $g > 0$ denotes the degree of bias in the estimation of the output gap.

Furthermore, the population dynamics is not static. It evolves over time in most cases. For instance, the Consumer Confidence Index (CCI) reflects the consumer sentiment of the market. After the government announces the CCI, the view of the consumer sentiment (optimistic/ pessimistic) is changed and thus the overall economic situation also changes because the optimistic/ pessimistic sentiment reflected in the index impacts the consumers' view. For this reason, the population dynamics, and the proportions of optimist and pessimist agents, can be derived from the following equations:

$$\mathrm{prob}(x(t) = o) = \alpha_{o,t} = \frac{\exp(\lambda V_{o,t})}{\exp(\lambda V_{o,t}) + \exp(\lambda V_{p,t})} \tag{7}$$

$$\mathrm{prob}(x(t) = p) = 1 - \alpha_{o,t} = \frac{\exp(\lambda V_{p,t})}{\exp(\lambda V_{o,t}) + \exp(\lambda V_{p,t})} \tag{8}$$

In this case, we can consider two alternatives o (optimist) and p (pessimist) in the Boltzmann-Gibbs distribution machine with a DSGE framework. Each will produce some gains to the agent. However, since the gain is random, the choice made by the agent is random as well. The Boltzmann-Gibbs distribution machine DSGE model assumes that the probability of the agent choosing optimism is the probability that the profits or utilities gained from choosing optimism are greater than those gained from choosing pessimism where *Vo,t* and *Vp,t* are the deterministic components of gains from alternatives *optimist* and *pessimist* at time *t*. In other words, *Vo ,t* is the temporal realized utility from being an optimist, and *Vp,t* is the temporal realized utility from being a pessimist. Equations (9) and (10) show how the agents compute the utility, *Vo,t* and *Vp,t,* for optimists' and pessimists' rules. The parameters ρ_k govern the geometrically declining weights.

$$V_{o,t} = -\sum_{k=1}^{\infty} \rho_k \left(y_{t-k} - E_{o,t-k-1} y_{t-k}\right)^2 \quad (9)$$

$$V_{p,t} = -\sum_{k=1}^{\infty} \rho_k \left(y_{t-k} - E_{p,t-k-1} y_{t-k}\right)^2 \quad (10)$$

Parameter λ is carried over from the assumed random component. In addition, there is a new interpretation for parameter λ, namely, the *intensity of choice*, because it basically measures the extent to which agents are sensitive to additional profits gained from choosing optimism instead of pessimistm. According to the above, we can obtain the aggregate expected output gap of period t+1 through Equation (11).

$$E_t y_{t+1} = \alpha_{o,t} E_{o,t} y_{t+1} + \alpha_{p,t} E_{p,t} y_{t+1} \quad (11)$$

IV. The network-based ant model

According to the above, the structure of networks is hidden in our social and economic lives and a vast amount of research has been carried out during the last few decades. For example, network analysis is not only applied to examine the transmission of information regarding about job opportunities, trade relationships, how diseases spread, how people vote and which languages they speak, but is also used in empirical works, such as the World Trade Web, the Internet, ecological networks and co-authorship networks. There is no doubt that a network structure is quite important for social interaction. Thus, we would like to introduce a network-based ant model for the New Keynesian DSGE framework. Inspired by observing the behavior of ants, "Ants, faced with two identical food sources, were observed to concentrate more on one of these, but after a period they would turn their attention to the other." (Kirman, 1993, p. 137), Kirman characterized the switching potential of each individual by two parameters, namely, a probability of self-conversion and a probability of imitation. The self-conversion probability represents the probability that the agent changes rule for personal reasons, whereas the probability of imitation refers to the agent changing the rule because of the influence of friends. Thus, the probability of agent i switching from the optimistic rule to the pessimistic rule could be represented by Equation (12):

$$\mathrm{prob}(p \to o) = s_i + m_i \omega_{ij} \sum_{j \neq i} D_o(i, j) \tag{12}$$

where s_i denotes the self-conversion (due to idiosyncratic factors) rate, and m_i refers to the imitation rate. To simplify our model, we let both the self-conversion rate and imitation rate be constant. In other words, $s_i = s_j$ and $m_i = m_j$ for each $i \neq j$, and ω_{ij} denotes the interaction strength between i and friend j. Equation (13), $D_o(i, j)$, is an indicator function that counts the number of i's friends who are optimists.

$$\mathrm{D_o(i,j)} = \begin{cases} 1, & \textit{if j is an optimistic neighbor of i} \\ 0, & \textit{otherwise} \end{cases} \tag{13}$$

Symmetrically, if the agent uses the optimistic rule in period t, the probability of agent i converting to a pessimist person could be represented by Equation (14):

$$\mathrm{prob}(o \rightarrow p) = s_i + m_i \omega_{ij} \sum_{j \neq i} D_p\,(i,j) \tag{14}$$

$$\mathrm{D_p(i,j)} = \begin{cases} 1, & \textit{if j is a pessimistic neighbor of i} \\ 0, & \textit{otherwise} \end{cases} \tag{15}$$

Finally, variable $\omega_{i,j}$ is used to describe the interaction strength between i and friend j under specific network structures such as a fully connected network, circle network, regular network, small world network and scale-free network. In order to depict the social network formation and its structure, we apply the concept of graph theory. Thus, a network G (V,E) is defined by a set of agents N and a set of links E. More specifically, V = {1, . . . , n} denotes all agents connected in some network relationship, and the number n refers to the size of the network. E denotes which pairs of agents are linked to each other so that $E = \{b_{ij}: i, j \in V\}$ encodes the relationship between any two agents in the network. Customarily, we use $b_{ij} = 1$ to indicate that there exists an edge (connection, relation) between i and j; otherwise it is zero. For this reason, we can use an N×N matrix to describe the network structure. However, we set $b_{ij} = b_{ji}$, which is known as a non-directed network in our model. Therefore, we can have a symmetric network matrix and the network formation algorithm for each specific social network structure, as follows:

(1) Fully-connected network structure

The fully-connected network has the feature that agents are completely connected with each other. In other words, each agent has (n-1) links.

(2) **Circle and regular network structures**

In a regular network structure, all agents are connected to their respective k-nearest neighbors and k is a constant number. Thus, each agent connects with k neighbors on both the left and the right. The simplest case, k=1, would be a circle network structure. In our model, the regular network structure refers to k=2, i.e., each agent makes friends with the 2-nearest neighbors from the left and the 2-nearest neighbors from the right.

(3) Small world and random network structures

Watts and Strogatz (1998) first proposed a model of small-world networks. Watts and Strogatz started with random and regular graphs. They looked at two properties of these graphs, namely, clustering and path length. Clustering is a measurement of the set of friends who all know each other. Thus they develop a clustering coefficient which provides the number of pairs of two nodes that are connected to the same node, and are also connected to each other. Path length is used to measure the average distance between two nodes, which corresponds to the degrees of separation in a social network. Their initial results showed that regular graphs have high clustering and high path lengths; random graphs of the same size tend to have low clustering and low path lengths. However, neither of these was considered to be a good model of social networks which seem to combine high clustering with short path lengths. Therefore, Watts and Strogatz tried to create a network generating algorithm to create a network which has the same property as a social network in the real world. First, they started with a regular graph with n nodes and k neighbors. Then, each agent had a rewiring probability, p, to cut off the link with each neighbor and build up a new link with one of the strangers. The probability, p, controls how random the graph is.

With p=0, the network structure is regular; with p=1 it is random. In our simulations, we consider the regular network structure and set the rewiring rate, p, equal to 0.1, 0.3, 0.5, 0.7, 0.9 and 1 to generate different random network structures.

(4) Scale-free network structure

A scale free network is a network with the power law property. Thus, the number of links originating from a given node denotes a power law distribution represented by $p(k) = k^{-\gamma}$ where k denotes the number of links. The idea of a scale-free network comes from observations of many social contexts, e.g., the citation network among scientific papers (Redner, 1998), the World Wide Web and the Internet (see, e.g., Albert et al., 1999; Faloutsos et al., 1999), telephone call and e-mail graphs (Aiello et al., 2002; Ebel et al., 2002), or the network of human sexual contacts (Liljeros et al., 2001). All of them show that only a few agents have many friends; most agents in the network have only a few friends. The most popular method to construct a scale-free network is the preferential attachment of Barabási and Albert (1999), which starts with m_0 agents and then progressively adds one new agent, i, to an existing network and builds links to existing agents with preferential attachment, according to Equation (16). That describes the rich getting richer; the probability of linking to a given agent is proportional to the number of existing links that a node has.

$$\text{prob(linking to agent i)} = \frac{k_i}{\sum_j^{N-1} k_j} \tag{16}$$

For considering the utility of different rules for each agent, we connect the interaction strength between i and friend j, ω_{ij}, and the performance of each agent. Therefore, according to Equations (9) and (10), we can assign different scores for each agent and then have the score matrix **S**, with dimensions N×N. In this case, if the agent is an optimist, it gets the score of the optimistic rule, and vice versa. By using the score matrix and the specific social network structure recorded by **N**, we can have ω_{ij} through Equation (17).

$$\omega_{ij} = \frac{N.\times S}{\sum_{i=1}^{N}(N.\times S)} \tag{17}$$

$N.\times S$ means that the element of S is multiplied by the corresponding element of N and, therefore, we can have a new matrix which contains only friends' scores. Then, each agent assigns a weight to all its friends. Thus, the agent has to sum up the scores of all friends, i.e., we have to compute $\sum_{i=1}^{N}(N.\times S)$ for each row. Finally, the friends' score matrix should be divided by $\sum_{i=1}^{N}(N.\times S)$, and after that, ω_{ij} can be generated.

V. Collaborations and simulation results

In simulations, we follow the parameters setting of De Grauwe (2010a) for the stylized New Keynesian DSGE model. Details of parameters in the stylized New Keynesian DSGE model, Boltzmann-Gibbs machine and network-based ant model and parameters values of different network structures can be found in Table 1. In order to find out the distribution of population dynamics, we run 100 experiments for a given collaboration. For each experiment of a specific collaboration, we set the number of agents equal to 100 (1,000) and run 300 periods.

Table 1: Parameters setting of calibrated models

Parameters setting of stylized New Keynesian DSGE model		
π^*	0	the central bank's inflation target
a_1	0.5	coefficient of expected output in output equation
a_2	-0.2	the interest elasticity of output demand
b_1	0.5	coefficient of expected inflation in inflation equation
b_2	0.05	coefficient of output in inflation equation
c_1	1.5	coefficient of inflation in Taylor equation
c_2	0.5	coefficient of output in Taylor equation
c_3	0.5	interest smoothing parameter in Taylor equation
g	0.01	output forecasts optimists
ρ_k	0.5	the speed of declining weights omega in mean squared errors
$\varepsilon_t, \eta_t, u_t$	0.005	standard deviation shocks of output gap, inflation and Taylors' rule
Parameters setting of Boltzmann-Gibbs machine		
λ	100 500 1000 5000 10000 50000	intensity of choice
Parameters setting of network-based ant model		
s	0.15	self-conversion rate
m	0.7	imitation rate

Parameters setting of different network structure		
k	1	number of neighbors from the left (right) in circle network structure
k	2	number of neighbors from the left (right) in regular network structure
m_0	20	initial nodes of scale free network structure
p	0.1 0.3 0.5 0.7 0.9 1	cutting (rewriting) probability of small world network structure
Others		
N	100	number of agents
T	300	number of simulation periods for each experiment of calibrations
R	100	number of experiments for each calibration

For the Boltzmann-Gibbs machine design, we try different values of intensity of choice. In this case, if we increase the intensity of choice (λ), then the strength of social interaction is increased. Figure 1 shows the probability density function of the optimistic ratio in the Boltzmann-Gibbs machine. The first row refers to the probability density function of the optimistic ratio in the 100th period, the second row represents the probability density function of the optimistic ratio in the 200th period and the third row denotes the probability density function of the optimistic ratio in the 300th period. According to Figure 1, we can observe that if λ is low enough, say $\lambda = 100$, the fraction of optimists is very close to 0.5. As λ gets larger, the states of the probability density function of the optimistic ratio in the Boltzmann-Gibbs machine become divergent. Therefore, we can obtain bell-shaped probability density functions if λ is between 500 and 1,000. In such cases, herding behavior (animal spirit) cannot be generated. However, if the value of λ is larger than 5,000, the probability density functions of the optimistic ratio are U-shaped. In other words, to generate the herding behavior (or animal spirits),[2] the value of λ has to be set above 5,000. Then, we one can easily have a boom or bust situation easily.

One of the purposes in combining the Boltzmann-Gibbs machine and the stylized New Keynesian DSGE model is to generate booms and busts. For this reason, the focus is on self-conversion and imitation rates which can produce the herding behavior in the network-based ant model. In this case, the self-conversion rate equals 0.15 and the imitation rate equals 0.7, which meet the requirements. Figures 2 and 3 depict the probability density function of the optimistic ratio in the network-based ant model with 100 agents and 1,000 agents, respectively.

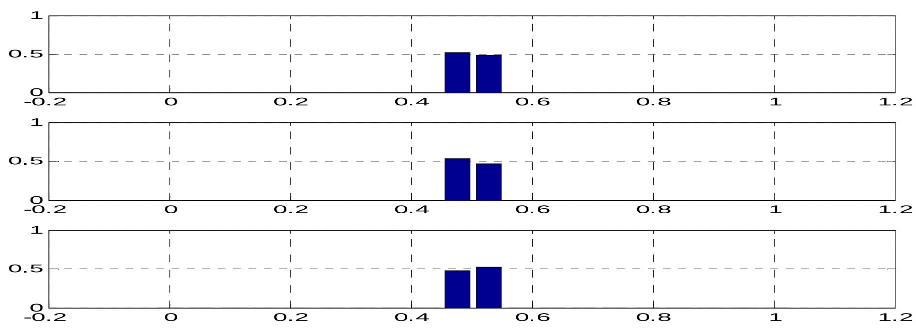
The similarity of the two population dynamics generated by the Boltzmann-Gibbs machine and the network-based ant model can be explained in three different ways. Firstly, the probability density function of the optimistic ratio for different models is sketched in order to observe the shape of the different probability density functions. Secondly, the Kolmogorov-Smirnov test is applied for all models. Finally, the relative entropy is introduced to measure the similarity between two population dynamics distributions.

[2] It means that all agents adopt the same behavior, and the phenomenon is referred to as 'animal spirits' in De Grauwe (2010a, 2010b).

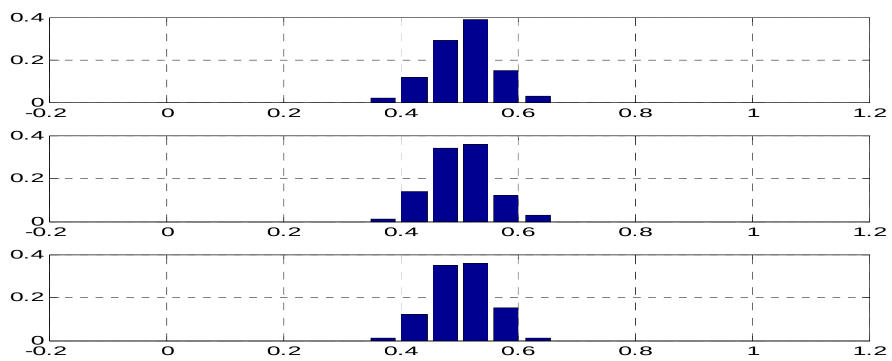
A comparison of Figures 1 and 2 shows the difference between the probability density functions of optimistic ratios for the Boltzmann-Gibbs machine and the network-based ant model. Figure 1 shows the herding behavior when the value of intensity of choice is large enough, say, larger than 5,000. However, Figure 2 shows that if the values of the self-conversion rate and imitation rate are, say, 0.15 and 0.7, respectively, it is not difficult to produce herding behavior (animal spirits) in the network-based ant model. Figure 3 (which results from a large sample) shows the same property as Figure 2. In other words, the proposed network-based ant model can generate U-shaped probability density functions of the optimistic ratio with any given network structure.

Figure 1. Probability density function of the optimistic ratio in the Boltzmann-Gibbs machine.

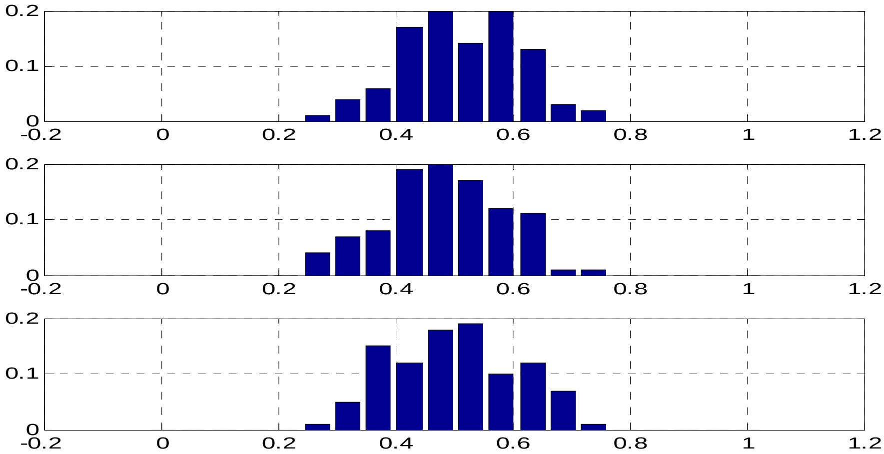
$\lambda = 100$



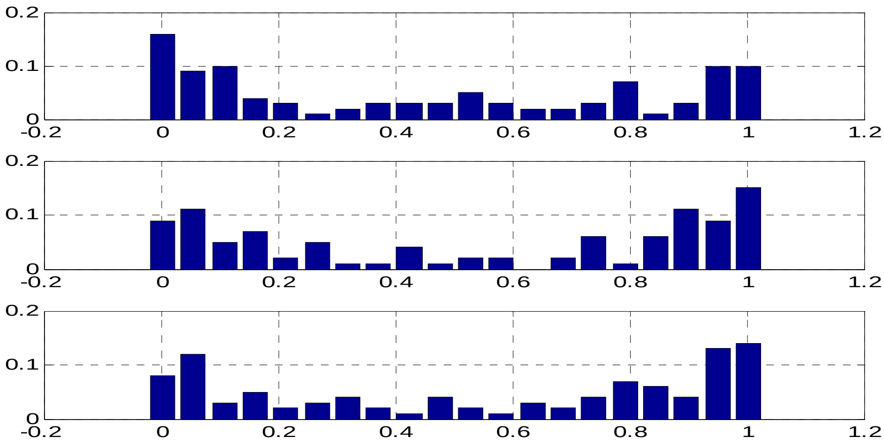
$\lambda = 500$



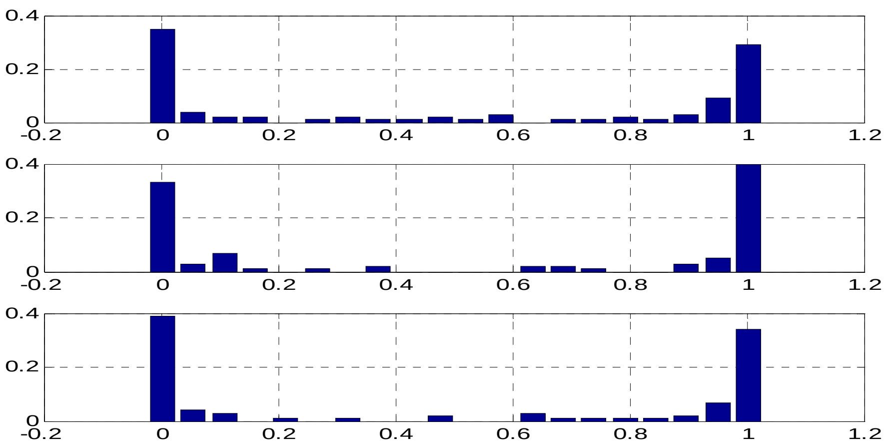
$\lambda = 1000$



$\lambda = 5000$



$\lambda = 10000$



$\lambda = 50000$

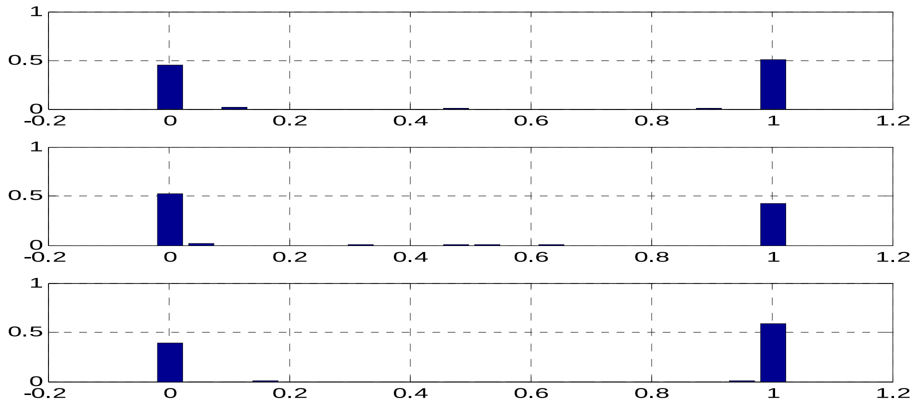
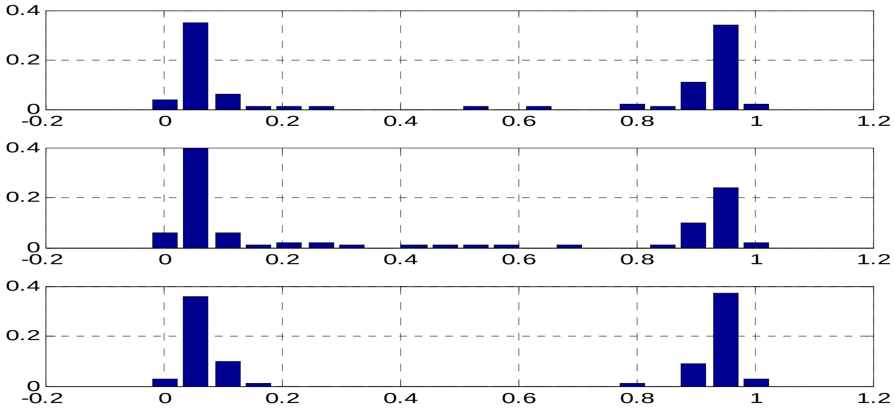
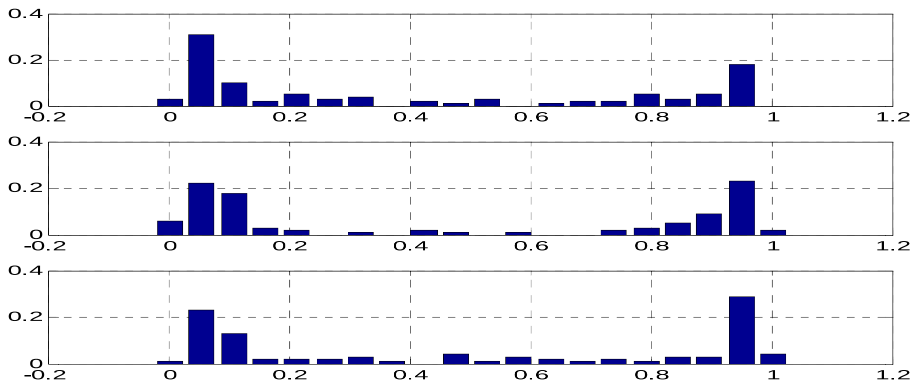


Figure 2. Probability density function of the optimistic ratio of the network-based ant model (N=100).

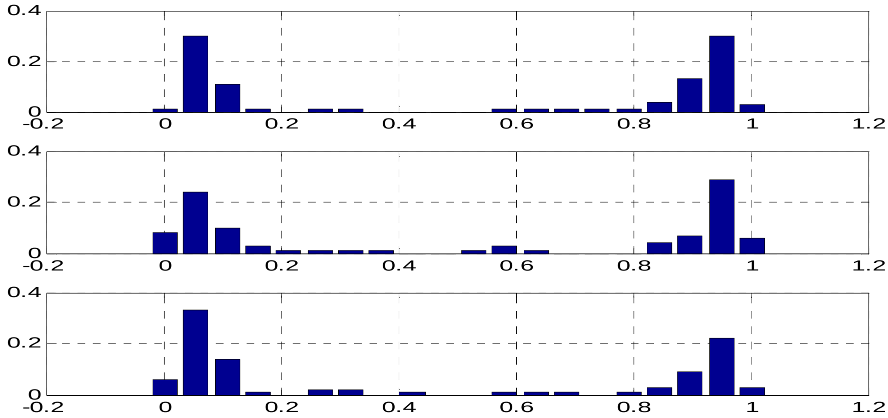
Fully-connected network



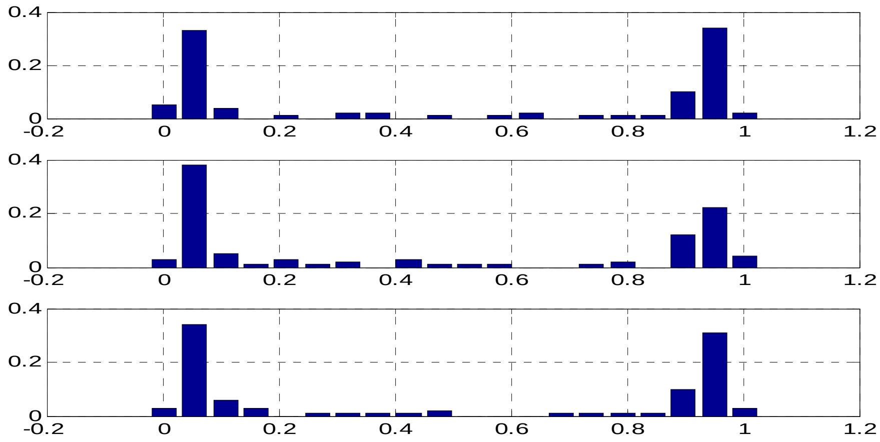
Circle network structure



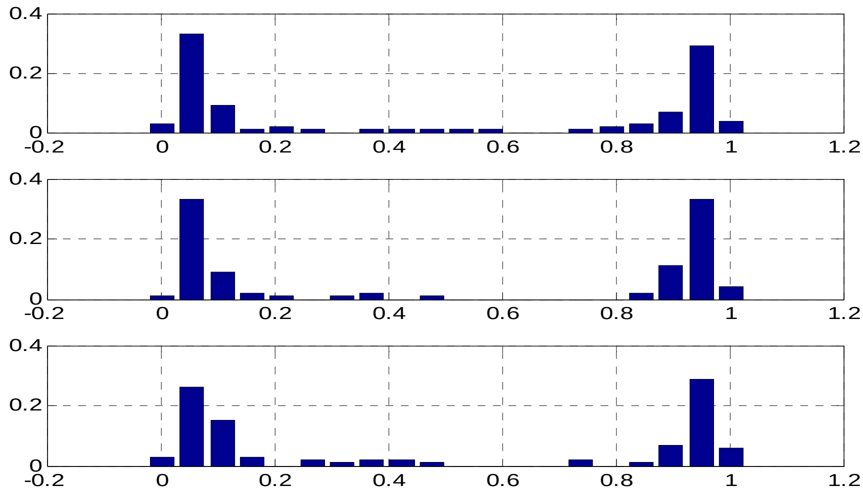
Regular network



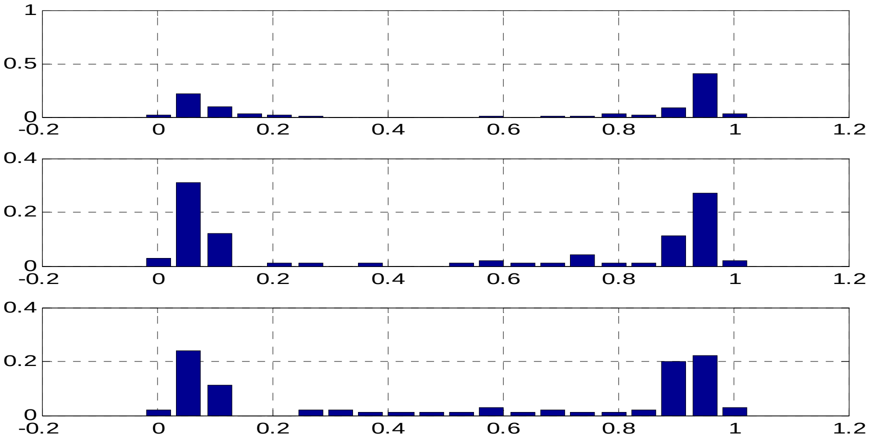
Small world network with cutting (rewriting) rate equal to 0.1



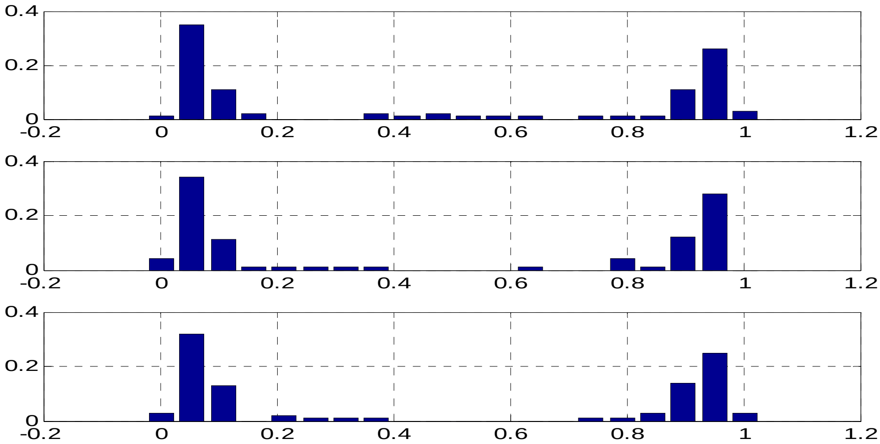
Small world network with cutting (rewriting) rate equal to 0.3



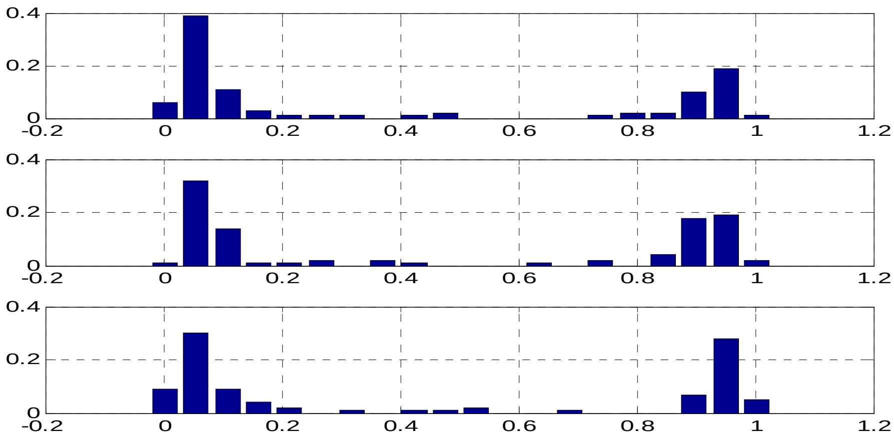
Small world network with cutting (rewriting) rate equal to 0.5



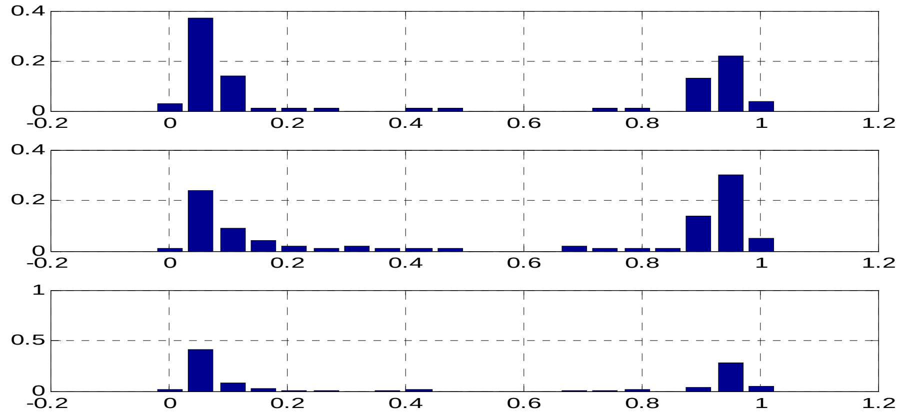
Small world network with cutting (rewriting) rate equal to 0.7



Small world network with cutting (rewriting) rate equal to 0.9



Random network



Scale-free network

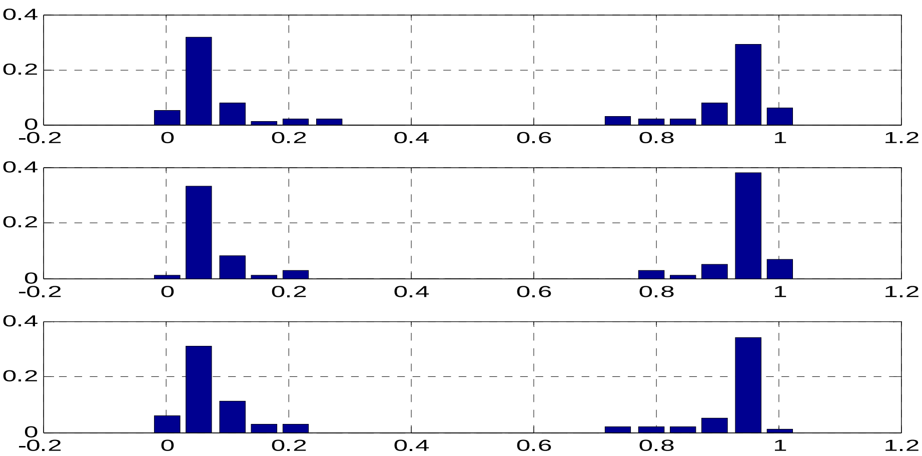
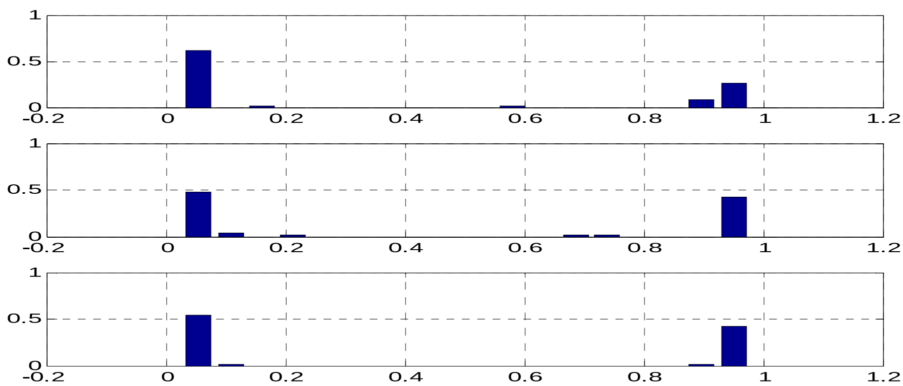
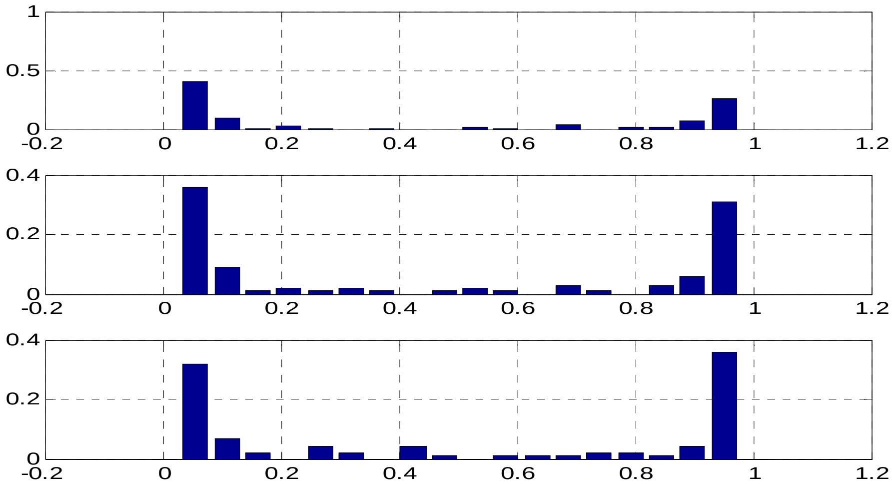


Figure 3. Probability density function of the optimistic ratio of the network-based ant model (N=1000).

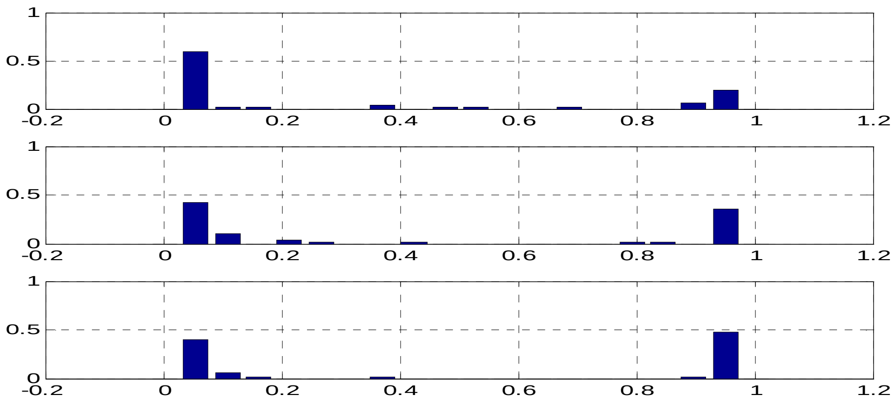
Fully-connected network



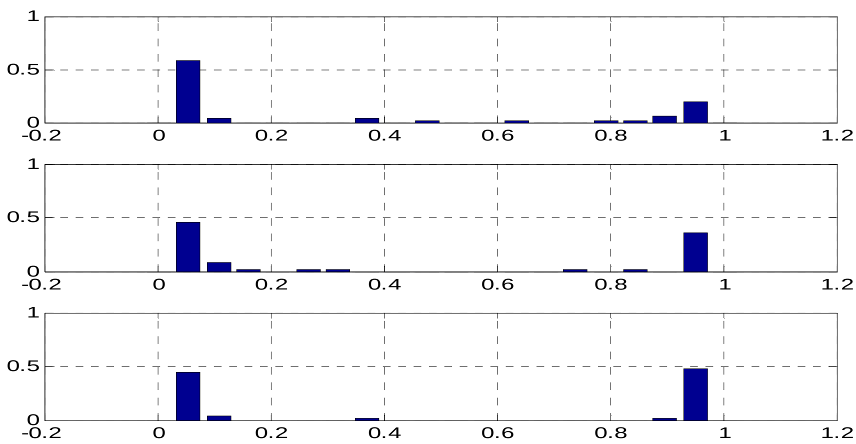
Circle network structure



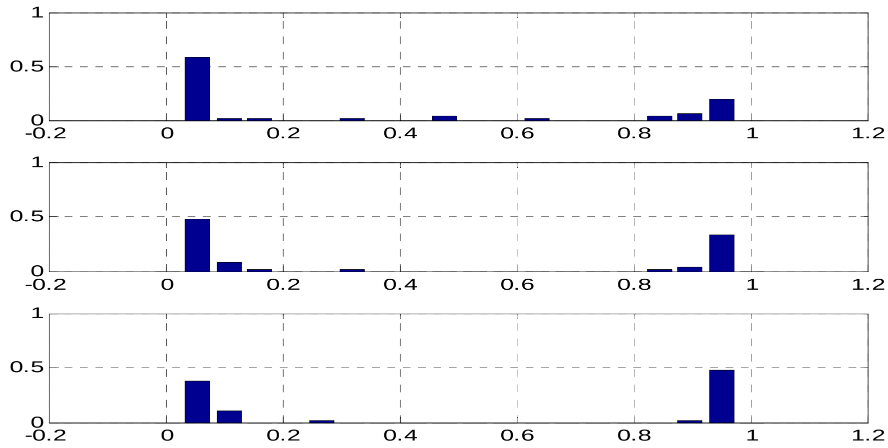
Regular network



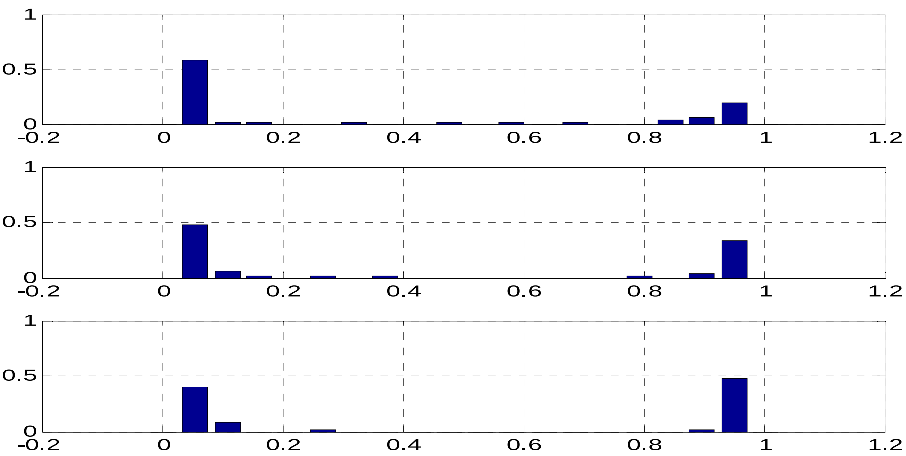
Small world network with cutting (rewriting) rate equal to 0.1



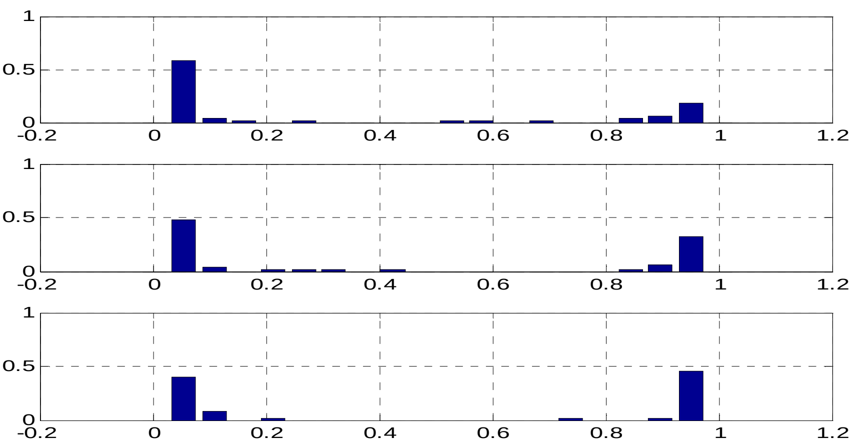
Small world network with cutting (rewriting) rate equal to 0.3



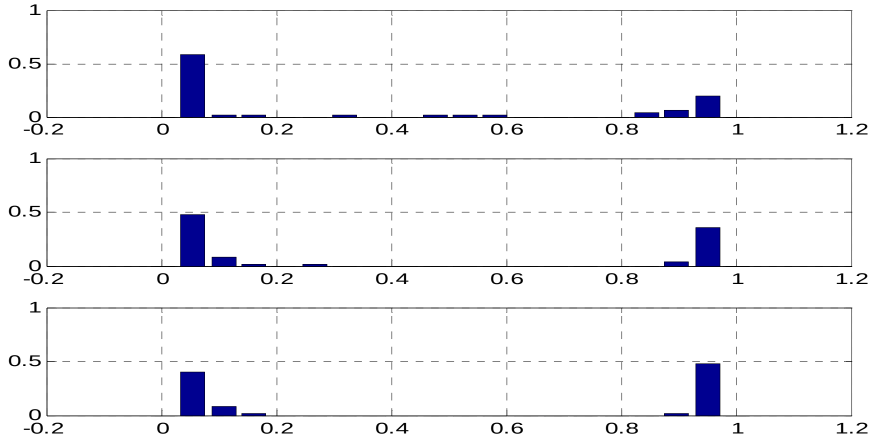
Small world network with cutting (rewriting) rate equal to 0.5



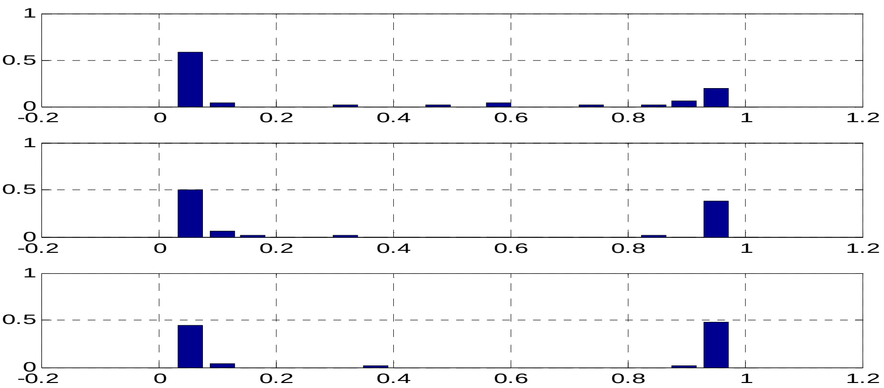
Small world network with cutting (rewriting) rate equal to 0.7



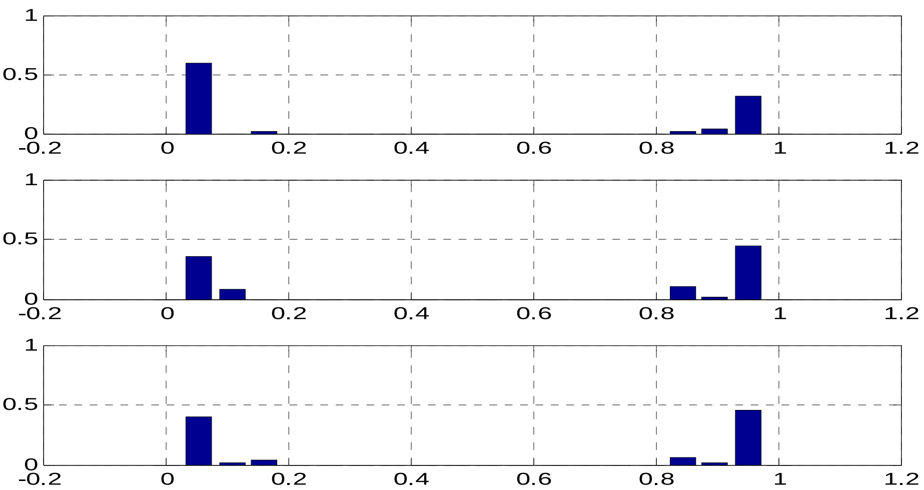
Small world network with cutting (rewriting) rate equal to 0.9



Random network



Scale-free network



However, we wonder which network structure can generate population dynamics closest to the Boltzmann-Gibbs machine. Then, we can reconsider whether the Boltzmann-Gibbs machine is a reliable tool for describing social interaction. In order to answer this question, we compare the optimists' ratios asymptotic distribution of the Boltzmann-Gibbs machine with the network-based ant model, by conducting the Kolmogorov-Smirnov test. The statistical Kolmogorov-Smirnov test can be used to compare distributions of the values in the two data vectors x1 and x2.

Here, x1 could be regarded as the 100 optimist ratios of the 300th period in the Boltzmann-Gibbs machine and x2 is base on the network based ant model. By the definition of the Kolmogorov-Smirnov test, the null hypothesis is that x1 and x2 are from the same distribution. The alternative hypothesis is that they are from different distributions. Therefore, if the p-value of Kolmogorov-Smirnov test is larger than 0.05, x1 and x2 coming from the same distribution cannot be rejected. The results of the Kolmogorov-Smirnov test are presented in Tables 2 and 3. The simulation results show that the circle network can produce population dynamics most similar to the Kolmogorov-Smirnov test. This finding is consistent with the study of thermodynamics for which the Boltzmann-Gibbs distribution is based on the local interaction. However, it is difficult to treat the population dynamics generated by the Boltzmann-Gibbs model and network-based ant model as being from the same distribution, particularly in the popular empirical network structures such as the small world network and scale-free network. Thus, maybe we have to ponder whether the Boltzmann-Gibbs machine is the proper tool for describing social interaction under the New Keynesian DSGE framework.

According to the probability density function analysis, it seems that the Boltzmann-Gibbs machine is a robust approximation of herding behavior in a network-based ant model. However, none of the population dynamics produced by the network-based ant model with different network structures could pass the Kolmogorov-Smirnov test, besides the circle network structure. Therefore, we have to check whether the Boltzmann-Gibbs machine is a good approximation for the herding behavior for any given network structure. In order to do so, we use the relative entropy, a similar measure. Before we mention the relative entropy for measuring the similarity between two population dynamics distributions, we have to introduce the concept of **Shannon entropy** (Shannon, 1948), used to describe the uncertainty in the information theory represented by Equation (18).

$$H(p_1, p_2, \cdots, p_n) = -\sum_{i=1}^{n} p_i \log_2 p_i \tag{18}$$

Table 2: Kolmogorov-Smirnov test results (N=1000)

	Fully	Circle	Regular	SW01	SW03
λ =100	3.70E-12	2.95E-11	4.41E-15	3.70E-12	1.06E-11
λ =500	3.70E-12	8.08E-11	1.43E-14	1.06E-11	1.06E-11
λ =1000	2.95E-11	3.70E-09	1.40E-13	2.95E-11	2.17E-10
λ =5000	0.013112	0.193042	0.000322	0.008216	0.008216
λ =10000	2.75E-07	2.21E-08	1.33E-06	6.12E-07	5.22E-08
λ =50000	4.52E-14	4.41E-15	4.52E-14	4.52E-14	4.52E-14
	SW05	**SW07**	**SW09**	**Random**	**Scale-free**
λ =100	1.40E-13	3.70E-12	4.41E-15	3.96E-16	1.40E-13
λ =500	4.26E-13	3.70E-12	4.41E-15	3.96E-16	1.40E-13
λ =1000	2.95E-11	1.06E-11	4.52E-14	3.96E-16	1.40E-13
λ =5000	0.193042	0.003031	0.001029	0.000174	0.001785
λ =10000	2.85E-06	5.22E-08	1.21E-07	2.45E-05	5.96E-06
λ =50000	1.40E-13	1.34E-15	1.43E-14	1.40E-13	3.96E-16

Table 3: Kolmogorov-Smirnov test results (N=1,000)

	Fully	Circle	Regular	SW01	SW03
λ =100	1.43E-14	3.70E-12	1.06E-11	1.06E-11	1.06E-11
λ =500	1.43E-14	2.95E-11	1.06E-11	1.06E-11	1.06E-11
λ =1000	1.43E-14	1.47E-09	1.06E-11	1.06E-11	1.06E-11
λ =5000	4.81E-05	0.099376	0.008216	0.005043	0.008216
λ =10000	9.12E-09	2.21E-08	9.12E-09	9.12E-09	9.12E-09
λ =50000	3.96E-16	3.96E-16	3.96E-16	3.96E-16	3.96E-16
	SW05	**SW07**	**SW09**	**Random**	**Scale-free**
λ =100	1.06E-11	1.06E-11	1.06E-11	1.06E-11	1.40E-13
λ =500	1.06E-11	1.06E-11	1.06E-11	1.06E-11	1.40E-13
λ =1000	1.06E-11	1.06E-11	1.06E-11	1.06E-11	1.40E-13
λ =5000	0.008216	0.003031	0.001785	0.005043	0.008216
λ =10000	9.12E-09	9.12E-09	9.12E-09	9.12E-09	2.21E-08
λ =50000	3.96E-16	3.96E-16	3.96E-16	3.96E-16	1.15E-16

where $H(p_1, p_2, \cdots, p_n)$ is a continuous function, and p_i is the frequency (probability) of state i. If $p_1 = p_2 = \cdots = p_n = \frac{1}{n}$, we obtain the maximum H. It means the highest uncertainty exists in the system. However, if p_i=1 and $p_{j \neq i}$=0, H will equal zero, and in this case, state i always occurs and the degree of uncertainty in the system is 0. In our population dynamics case, we group the optimistic ratio into 10 groups and calculate the frequency for each group. The 1st group represents an optimistic ratio larger than 0 and less than 0.1, the 2nd group includes an optimistic ratio between 0.1 and 0.2,…, and so on. Therefore, we can obtain the Shannon entropy of our model through Equation (18), where n=10.

Based on the definition of Shannon entropy, Kullback and Leibler (1951) proposed **relative entropy**, which is also known as cross entropy or Kullback-Leibler divergence. Relative entropy is a measure of similarity, assuming that the baseline distribution is G and the alternative distribution is S. However, we wonder if S is a good approximation of the distribution of G. Thus, the relative entropy can be used to measure the similarity between two population dynamics distributions. The more dissimilar G and S are, the larger the relative entropy is.

Therefore, if we have two density vectors $G = (g_1, g_2, g_3, \cdots, g_n)$ (G is the frequency of the optimistic ratio derived by 100 experiments with the Boltzmann-Gibbs machine) and $S = (s_1, s_2, s_3, \cdots, s_n)$ (S is the frequency of the optimistic ratio derived by the network-based ant model of a given social network structure), the definition of relative entropy is given as Equation (19). In this case, H(G|S) will always be larger than or equal to zero; if G and S are identical, H(G|S) equals zero.

$$H(G|S) = \sum_{i=1}^{n} g_i \log_2\left(\frac{g_i}{s_i}\right) = \sum_{i=1}^{n} g_i \log_2 g_i - \sum_{i=1}^{n} g_i \log_2 s_i \qquad (19)$$

where $g_i \geq 0, s_i \geq 0$ and $\sum_{i=1}^{n} g_i = \sum_{i=1}^{n} s_i = 1$

However, relative entropy is asymmetric. In other words, H(G|S)≠H(S|G). This is why it is the Kullback-Leibler divergence rather than the Kullback-Leibler distance.

Table 4 shows the results of the relative entropy procedure. Absolute values of relative entropy are all less than 0.5 for all different social network structures if the intensity of choice equals 10,000. Therefore, the result indicates that the Boltzmann-Gibbs machine (with an intensity of choice equal to 10,000) offers a good approximation of herding behavior of our network-based ant model with any given network structure.

Table 4: Relative entropy

Intensity of choice	λ =100	λ =500	λ =1000	λ =5000	λ =10000	λ =50000
fully	0.810	0.597	-0.267	-1.160	-0.318	0.501
circle	1.448	1.235	0.372	-0.522	0.320	1.139
regular	1.066	0.852	-0.011	-0.905	-0.062	0.757
SW01	0.977	0.763	-0.100	-0.993	-0.151	0.668
SW03	1.000	0.787	-0.076	-0.970	-0.128	0.692
SW05	1.082	0.869	0.006	-0.888	-0.046	0.773
SW07	1.088	0.875	0.012	-0.882	-0.040	0.780
SW09	0.974	0.761	-0.102	-0.996	-0.154	0.666
SW10	0.985	0.772	-0.091	-0.985	-0.143	0.677
Scale free	0.797	0.583	-0.280	-1.174	-0.331	0.488

VI. Conclusion

This paper compares the population dynamics between the Boltzmann-Gibbs machine and network-based ant model under a stylized New Keynesian DSGE framework. We find that both the Boltzmann-Gibbs model and network-based ant model can generate herding behavior. However, as stated earlier, it is hard to envisage population dynamics generated by the Boltzmann-Gibbs model and the network-based ant model being from the same distribution, particularly in the popular empirical network structures such as a small world network and scale-free network. In addition, our simulation results further suggest that the population dynamics of the Boltzmann-Gibbs model and circle network ant model can be considered to be from the same distribution under specific parameter settings. The finding is consistent with the study of thermodynamics for which the Boltzmann-Gibbs distribution is based on the local interaction. Although the circle network is not the acknowledged social network structure, according to the relative entropy between the population dynamics of the Boltzmann-Gibbs distribution and the network-based ant model, the Boltzmann-Gibbs model with an intensity of choice equal to 10,000 is a good approximation of the herding behavior of our network-based ant model with any given network structure. In addition to the population dynamics, there are some other questions regarding the use of the Boltzmann-Gibbs machine to describe social interaction in the stylized New Keynesian DSGE model. For example, the frequency of herding behavior in financial markets and macroeconomic systems may be different. The opinion change could occur very rapidly in financial markets but could be slower in the macroeconomic system. In this case, we have to consider if an intensity of choice being equal to 10,000 produces too heavy an opinion change. Thus, maybe we have to further confirm if the Boltzmann-Gibbs machine is a suitable tool for calibrating social interaction under a stylized New Keynesian DSGE framework.

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