

## **Chance of Revolts and Ability of Oppressions: A Comment on the Acemoglu–Robinson Model**

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**Abstract** In the original framework of Professors Acemoglu and Robinson, the government is unable to oppress the revolution once it is brought about. However, actual civil wars are unpredictable. With this notion, I introduce uncertainty depending on military expenditures of the government. Then an interesting argument follows: if the likelihood of successful oppression is sufficiently larger than a certain level of destruction rate and there are cheap-but-effective devices such as biochemicals, citizens in a dictatorial country may have a trade-off between economic prosperity and domestic military threats.

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# 1 Preliminaries

This commentary article considers a small extension of the model of institutional change considered by a series of works of Professors Acemoglu and Robinson. The Acemoglu-Robinson model (e.g. Acemoglu and Robinson 2006) considers institutional change as a result of social conflict between (political) elite and non-elite. In that model, institution evolves either by violent events such as revolutions and coups or political concessions made by redistribution of incomes from the elite (typically *rich*) to the non-elite (typically *poor*). The review of their model and two of relevant results are as follows. In the original framework, however, revolutions cannot be oppressed by the government, and the dictator considers making concessions. This paper considers the revolution succeeds contingently and its probability depends on the military expenditure.

Let  $\delta$  and  $1 - \delta$  respectively be the population shares of the elite and the non-elite. Assume  $0 < \delta < 1/2$ ; hence, the non-elite is the majority of this economy. Suppose the income levels of the elite and the non-elite are respectively given by  $y^r$  and  $y^p$  such that  $y^r > y^p > 0$ .

Suppose policy package is represented by tax rate  $\tau \in [0, 1]$ . The cost of collecting tax is non-zero and it is represented by  $C(\tau)$ : for example, opportunity losses from economic activities while tax-payments and from assignments of tax collectors who could produce other economic goods. The cost function is supposed to be at least twice differentiable,  $C'(\tau)$ ,  $C''(\tau) > 0$  and  $C'(0) = 0$ . In addition, technically, assume  $C'(1) \geq 1$  as well. Then, the *lump-sum* tax refunds are represented by  $T_r \geq 0$  to the elite and  $T_p \geq 0$  to the non-elite. In this sense, the tax refund must satisfy

$$\delta T_r + (1 - \delta)T_p = [\tau - C(\tau)] \bar{y}, \quad (1)$$

where  $\bar{y}$  is the average income level of this country defined by  $\bar{y} = \delta y^r + (1 - \delta)y^p$ .

Consider the *money metric* utility function for  $i \in \{r, p\}$  defined by

$$V^i = (1 - \tau)y^i + T_i, \quad (2)$$

where notations with subscript or superscript  $r$  indicate those of the elite and  $p$  do those of the non-elite. By a technical reason avoiding indeterminacy in collective actions, assume the government determines the tax rate at first and then the lump-sum redistributions at second. Following such decision flow, we find the first result:

**Result 1** [Acemoglu and Robinson, 2006] *If the government is dictatorial,  $T_p > T_r = 0$  followed by  $\tau > 0$  realizes as its equilibrium.*

**Proof.** If the government is dictatorial, their social planner maximizes only the utility of the elite subject to (1), so that the problem is

$$\max_{\tau, T_p} (1 - \tau)y^r + T_r \text{ s.t. } T_r = \frac{1}{\delta} \times \{[\tau - C(\tau)]\bar{y} - (1 - \delta)T_p\},$$

which is simplified to the problem

$$\max_{\tau, T_p} (1 - \tau)y^r + \frac{1}{\delta} \times \{[\tau - C(\tau)]\bar{y} - (1 - \delta)T_p\}.$$

Hence the first order condition with respect to  $\tau$  is given by  $-y^r + 1/\delta \times [1 - C'(\tau)]\bar{y} = 0$ , which is arranged to get  $C'(\tau) = 1 - \theta$ , where  $\theta = \delta y^r / \bar{y}$  is the fraction of income accrues to the elite (so that  $1 - \theta$  is that of the non-elite). By  $C'(0) = 0$  and  $C''(\tau) > 0$ ,  $\tau > 0$  must hold at the optimum. Because the first order condition with respect to  $T_p$  is negative,  $T_p = 0$  holds and then  $T_r > 0$ . ■

Next, consider a possibility of revolution that succeeds with some cost represented by the demolition during the violence  $\mu \in (0, 1)$ . For simplicity, the elite is supposed to be completely purged after the revolution. Then the non-elite brings about the revolt if

$$\frac{(1 - \mu)\bar{y}}{1 - \delta} > (1 - \tau)y^p + T_p. \quad (3)$$

Then, we will find further result:

**Result 2** [Acemoglu and Robinson, 2006] *Under the revolutionary pressure,  $T_r = T_p = 0$  followed by  $\tau = 0$  holds at the equilibrium. Then, the revolutionary constraint is given by  $\theta > \mu$ .*

**Proof.** The elite solves the problem to maximize their utility (2) subject to (1) and (3). Assuming no corner solution, the maximization problem is given by

$$\max_{\tau} (1 - \tau)y^r + T_r \text{ s.t. } \begin{cases} T_r = \frac{1}{\delta} \times \{[\tau - C(\tau)]\bar{y} - (1 - \delta)T_p\} \\ T_p = \frac{(1 - \mu)\bar{y}}{1 - \delta} - (1 - \tau)y^p \end{cases}.$$

By substituting all constraints into the objective function, this problem is simplified to

$$\max_{\tau} (1 - \tau)y^r + \frac{1}{\delta} \times \{[\tau - C(\tau)]\bar{y} - (1 - \mu)\bar{y} + (1 - \tau)(1 - \delta)y^p\}.$$

The first order derivative of the objective function with respect to  $\tau$  is given by  $-y^r + 1/\delta \times \{[1 - C'(\tau)]\bar{y} - (1 - \delta)y^p\}$ , which is rearranged to get

$$\frac{1}{\delta} \times \{[1 - C'(\tau)]\bar{y} - [\delta y^r + (1 - \delta)y^p]\} = -\frac{C'(\tau)}{\delta} = 0.$$

Therefore,  $\tau = 0$  is the optimum and the revolutionary constraint is given by  $\theta > \mu$ . ■

## 2 The Extension

Consider extending the Acemoglu-Robinson model with the probability for the government to oppress the rebellion,  $p \in (0, 1)$ . Assume  $p$  is influenced by the military expenditure  $M \in [0, \bar{M}]$ :  $p = \phi(M)$  with  $\phi(0) = 1$ ,  $\phi(\bar{M}) = 0$  and  $\phi'(M) > 0$ . In addition, assume  $\phi$  is at least once differentiable and it satisfies  $\phi'(0) = \infty$  and  $\phi'(\bar{M}) = 0$ . Concerning the military expenditure, the budget constraint (1) is written as

$$\delta T_r + (1 - \delta)T_p + M = [\tau - C(\tau)]\bar{y}. \quad (4)$$

Once a revolutionary incident has occurred, no one knows when the rebellion actually ends even if  $p$  is known and we need to verify the expected payoffs after the revolution in a dynamic framework (war of attrition): particularly, for the non-elite, we have

$$\begin{aligned} \sum_{t=1}^{\infty} \frac{(1 - \mu)^t (1 - p)^{t-1} p \bar{y}}{1 - \delta} &= \frac{(1 - \mu) p \bar{y}}{1 - \delta} \sum_{t=1}^{\infty} (1 - \mu)^{t-1} (1 - p)^{t-1} \\ &= \frac{(1 - \mu) \bar{y}}{1 - \delta} \times \frac{1}{(1 - \mu) + \mu/p}. \end{aligned}$$

Therefore the revolutionary constraint is now rewritten as

$$\frac{(1 - \mu) \bar{y}}{1 - \delta} \times F(M) > (1 - \tau)y^p + T_p,$$

where  $F(M)$  is defined by

$$F(M) \equiv \frac{1}{(1 - \mu) + \mu/\phi(M)}.$$

Because there is no incentive for the elite to invest on military arms if the non-elite has no incentive to rebel, in order to make the discussion interesting,

suppose the non-elite has some intentions against the government. Then consider the situation such as

$$1 - \mu > (1 - \tau)(1 - \theta), \quad (5)$$

which indicates the rebellion pays for the non-elite if it succeeds in the next period (the maximum reward level). Assuming the elite gets nothing if they fails to oppress (because they are purged), the non-elite evaluate the condition for each period to get the *supergame perfect equilibrium*: the rebellion continues until it ends. If condition (5) is not satisfied, the revolutionary activities are not expected to occur; therefore,  $M = 0$  is the optimum for the government and the problem becomes identical to the original Acemoglu-Robinson model. Notice, if  $\tau = 0$ , (5) coincides with the revolutionary constraint in the original Acemoglu-Robinson model given by Result 2.

## 2.1 Military Expenditure

Assuming no corner solution with knowledge of corner values, the optimization problem subject to the constraint (5) for the dictatorial government is now given by

$$\max_{\tau} (1 - \tau)y^r + T_r \quad \text{s.t.} \quad \begin{cases} T_r = \frac{1}{\delta} \times \{[\tau - C(\tau)]\bar{y} - (1 - \delta)T_p - M\} \\ T_p = \frac{(1 - \mu)\bar{y}}{1 - \delta} \times F(M) - (1 - \tau)y^p \end{cases}. \quad (6)$$

**Remark 1** *There exists an optimal tax rate  $\tau = \tau^*$  such that  $0 < \tau^* < 1$  and it is unique.*

**Proof.** The problem is simplified to the problem as such

$$\max_{\tau} (1 - \tau)y^r + \frac{1}{\delta} \times \left\{ \begin{array}{l} [\tau - C(\tau)] \times \bar{y} - (1 - \mu)\bar{y} \times F(M) \\ + (1 - \tau)(1 - \delta)y^p - M \end{array} \right\}. \quad (7)$$

Thus the first order condition is

$$-y^r + \frac{1}{\delta} \times \left[ (1 - \mu)\bar{y} \times \left( -\frac{\partial M}{\partial \tau} \frac{\partial F}{\partial M} \right) - (1 - \delta)y^p \right] = 0,$$

which is rearranged to get the solution:

$$-\frac{\partial M}{\partial \tau} \frac{\partial F}{\partial M} = \frac{1}{1 - \mu}. \quad (8)$$

Note, we have

$$\frac{\partial F}{\partial M} = \frac{\mu\phi'(M)}{[(1-\mu)\phi(M) + \mu]^2} < 0, \quad (9)$$

$$\frac{\partial M}{\partial \tau} = [1 - C'(\tau)] \bar{y}. \quad (10)$$

In addition, by assumption, the left-hand-side of (8) is infinity at  $\tau = 0$  and attains zero at some value  $\tau = a < 1$ . For example, Figure 1 depicts the case there is a certain value  $\tau = b > a$  such that  $\phi'(b) = 0$ . Then, we can find  $\tau = \tau^*$  as the optimum tariff rate such that  $0 < \tau^* < a (< 1)$ . ■

**Proposition 1** *Under the revolutionary pressure,  $M > T_p = T_r = 0$  followed by  $\tau > 0$  realizes at its equilibrium.*

**Proof.** Let  $\mathcal{V}$  be the value function associated with (7). Apply the envelope theorem to get

$$\frac{\partial \mathcal{V}}{\partial M} = \frac{1}{\delta} \times \left[ -(1-\mu)\bar{y} \cdot \frac{\partial F}{\partial M} - 1 \right]. \quad (11)$$

At the optimum, by (9) and (10), (11) is given by

$$\frac{1}{\delta} \times \left[ \frac{(1-\mu)\bar{y}}{(1-\mu)[1-C'(\tau)]\bar{y}} - 1 \right] = \frac{1}{\delta} \times \frac{C'(\tau)}{1-C'(\tau)} > 0, \quad (12)$$

where the last inequality follows from  $C'(\tau) > 0$  and  $1 - C'(\tau) > 0$  by  $\partial M/\partial \tau > 0$  at the optimum (see Figure 2). Therefore the elite wishes to spend all tax revenue to invest on military arms; that is,  $M > 0 = T_r = T_p$  by  $\tau^* > 0$  (Remark 1). ■

As the counterpart of Result 1, Proposition 1 tells us the dictatorial government facing pressures of revolts levies tax to finance military expenditures to reinforce their regime. This insight is not discussed in the original Acemoglu-Robinson model.

## 2.2 Likelihood of Oppression

Consider the likelihood ratio of the government to oppress the rebellion successfully,  $p/(1-p)$  to obtain the next proposition:

**Proposition 2** *Suppose  $p/(1-p) \geq \mu$ , then,  $\tau^*$  is decreasing in  $\mu$  and so is  $M$ . However, it is ambiguous for  $p/(1-p) < \mu$ .*

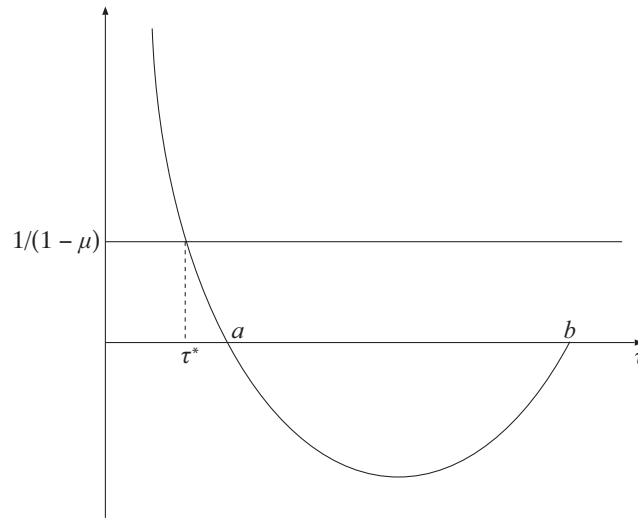


Figure 1:

**Proof.** Both sides of (8) are increasing in  $\mu$ . It is obvious for the right-hand-side,  $1/(1-\mu)$ , is increasing in  $\mu$ . For the left-hand-side given by  $-(10) \times (9)$ , because (10) is independent of  $\mu$ , consider partially differentiating (9) to get

$$\frac{\partial^2 F}{\partial M \partial \mu} = \frac{\phi'(M) \times [\phi(M) - \mu + \mu\phi(M)]}{[(1-\mu)\phi(M) + \mu]^3}. \quad (13)$$

The numerator of (13) is rewritten as  $\phi'(M) \times [p - (1-p)\mu]$ , which is negative for  $p/(1-p) < \mu$  and weakly positive for  $p/(1-p) \geq \mu$  because  $\phi'(M) > 0$ . Therefore

$$\frac{\partial}{\partial \mu} \left( -\frac{\partial M}{\partial \tau} \frac{\partial F}{\partial M} \right) \leq 0 \quad \text{if} \quad \frac{p}{1-p} \geq \mu.$$

See Figure 2. The above arguments imply the downward shift of the left-hand-side and the upward shift of the other side of (8). As an example, this figure depicts the induced reduction of the tax rate is from  $\tau^*$  to  $\tau^{**} < \tau^*$  for  $p/(1-p) \geq \mu$ . Therefore the optimum tax rate goes down in response to decline in  $\mu$ . By Proposition 1, we also find the military expenditures decline. However, the reverse is ambiguous for  $p/(1-p) < \mu$ . ■

Proposition 2 provides a comparative-statics result about the destruction rate  $\mu$  in terms of  $p/(1-p) \geq \mu$ : if the destruction rate  $\mu$  is smaller than the likelihood ratio  $p/(1-p)$ , increases in  $\mu$  induce correspondingly lower tax rates and smaller military expenses. This condition is a sufficient condition,

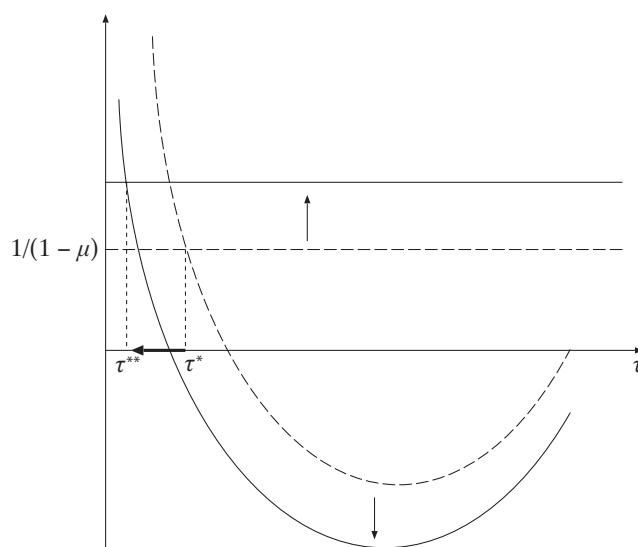


Figure 2:

however, it is significant if the political regime is viable because it is likely the government army dominates insurgent troops ( $p > 1 - p$ ). Whence, the government of the stable regime reduces the tax rate to reduce military expenditures as revolutions are getting more destructive.

### 2.3 Income Growth

Consider the influence of an increase in the average income level  $\bar{y}$  (economic growth) to examine its influence on the tariff rate and military expenditure by showing:

**Proposition 3**  $\tau^*$  is increasing in  $\bar{y}$  and so is  $M$ .

**Proof.** Consider around the equilibrium given by the first order condition (8). Because  $\partial F/\partial M$  is independent of  $\bar{y}$  and  $\partial(10)/\partial \bar{y} = 1 - C'(\tau) > 0$  around the equilibrium, we find

$$\frac{\partial}{\partial \bar{y}} \left( -\frac{\partial M}{\partial \tau} \frac{\partial F}{\partial M} \right) > 0.$$

Because the right-hand-side of (8) is independent of  $\bar{y}$ , it implies only the upward shift of the left-hand-side is induced (consider analogous picture of Figure 2). Therefore the optimum tax rate goes up and so does  $M$  by Proposition 1. ■



Above proposition asserts the economic growth causes an increase in the tariff rate and an increase in the military expenditure. This result implies the dictatorial government will prepare for revolts which likely to occur by the economic growth (or increased reward). Then consider controlling the prediction of destruction rate  $\mu$ . If  $\mu$  and  $M$  *positively* correlate, the conclusion is simple: the government keeps large military expenditures to suppress incentives of rebellions.

In order to make the argument interesting, in turn, consider the possibility  $\mu$  and  $M$  *negatively* correlate. It is practically feasible for the government: For example, biochemical weapons and demonstrations by assaulting much weaker targets such as unarmed protestant. In such cases, military expenditures can be reduced to keep larger disposable income by increased military threats of oppressing rebellions (Propositions 1 and 3). In addition, the smaller military expenditures induce resource allocations to economically productive sectors. This conclusion implies citizens in a dictatorial country may have a trade-off between economic prosperity and domestic military threats.

### 3 Concluding Comments

The issue on military expenditure and economy has long been discussed based on macroeconomic models by various researchers—recently, for example, Aizenman and Glick (2006) examined the influence of military expenditures on growth with threats of neighboring countries and Collier and Hoeffler (2007) tested the influence of international aids on the expansion of military expenditures—see Dunne et al. (2005) for a well-written survey about related works before 2004. And many works confirm that there is a negative relation or insignificance between them because the larger the amount is spent on the military arms, the less the amount is spent on the production of economic goods.

In accordance with those discussions, economic growth under dictatorial regime is unstable and it cannot be sustainable. In these studies, we will find military expansions crowd out expansions of other economic sectors and have negative impacts on economic growth. The result of the extended model, which considers endogenously strategically imposed military threats, has confirmed an analogue of the existing arguments (Proposition 3).

A new finding of this paper is the theoretical connection between the economic variables and the demolition rate in the Acemoglu-Robinson model. As an interesting argument, if the likelihood of successful oppression is sufficiently larger than a certain level of destruction rate and there are cheap

but effective devices of oppressions (e.g. biochemicals), the extended model tells the disposable income of citizens increases in exchange for larger military threats from the government (Propositions 1 and 3). Even though the imposed condition is the sufficient condition, it is likely satisfied in a stable dictatorial regime and then our discussion is viable: the likelihood ratio is likely larger than one while the destruction rate does not exceed one.

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