

## **Fiscal Policy under Imperfect Competition: A Survey**

*Luís F. Costa*

*ISEG - School of Economics and Management, Technical University of Lisbon*

*Huw David Dixon*

*Cardiff Business School, University of Cardiff*

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**Abstract** This paper surveys the link between imperfect competition and the effects of fiscal policy on output, employment and welfare. We examine static and dynamic models, with and without entry under a variety of assumptions using a common analytical framework. We find that in general there is a robust relationship between the fiscal multiplier and welfare, the tantalizing possibility of Pareto improving fiscal policy is much more elusive. In general, the mechanisms are supply side, and so welfare improving policy, whilst possible, is not a general result.

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**Correspondence** Huw David Dixon, Cardiff Business School, University of Cardiff; Aberconway Building, Column Drive, Cardiff, CF10 3EU, United Kingdom; e-mail: [dixonh@Cardiff.ac.uk](mailto:dixonh@Cardiff.ac.uk)

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# 1 Introduction

In a perfectly competitive economy (sometimes referred to as "Walrasian") without market imperfections, any competitive equilibrium will be Pareto optimal. Hence there can be no efficiency motive for macroeconomic policy, whether fiscal or monetary. However, the presence of imperfect competition in the form of market power leads to an equilibrium which will in general not be Pareto optimal, with a level of output and employment below competitive equilibrium. This leads to the tantalizing possibility that fiscal policy can be used to shift the economy to a new equilibrium which will Pareto dominate the initial equilibrium. In this paper we survey and explain the literature on imperfect competition and macroeconomics in the context of fiscal policy in a "real" model without money. This was one of the key pillars of New Keynesian macroeconomics in the 1980s and 1990s, alongside the nominal models with price and wage stickiness<sup>1</sup>.

The main contribution of New Keynesian economics was to set imperfect competition at the heart of Keynesian economics and its current incarnation as the "New Keynesian/Neoclassical Synthesis". This marked a major departure from the approach of Keynes himself, especially Keynes (1936), who used a perfectly competitive market structure to give microfoundations to the supply side of the economy. Perhaps the two main reasons were (i) that the theory of imperfect competition was relatively underdeveloped at that time, (ii) Keynes's conviction that he was generalizing the existing theory with perfect competition and market clearing being a special case (hence the title of his work). Still in the 1930's, imperfect competition and macroeconomics would be mixed in Kalecki (1938) and in the Dunlop (1938) critique to the real-wage counter-cyclicity implicit in the General Theory. However, despite this promising start, four decades would pass before we can find a significant piece of work in the mainstream using imperfectly competitive microfoundations in macroeconomics. During the 1960's and the beginning of the 1970's some of the concepts and techniques that would allow the integration of imperfect competition in general-equilibrium models were developed, in particular Negishi (1961). In the second half of the 1970's we find the first attempts to integrate these concepts in macroeconomic models. Nonetheless, their success was limited due to the "subjective-demand-curve" assumption.<sup>2</sup>

Oliver Hart's model, Hart (1982), was the first to operationalise the concept of

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<sup>1</sup>See Dixon (2008) which sets this strand of literature in the context of the wider New Keynesian approach.

<sup>2</sup>A subjective demand curve is simply one that is "perceived" by the firm. It can be subject to constraint that it passes through the actual price-quantity pair that occurs in equilibrium. However, this led to endemic multiplicity of equilibria. For a short survey of the literature see Dixon and Rankin (1995).

the "objective" demand curve" in a general-equilibrium model with imperfect competition (Cournot oligopoly for each good and monopoly unions), producing some "Keynesian" outcomes, namely equilibrium with under-employment (though not involuntary unemployment) and a multiplier mechanism for autonomous demand (a non-produced good in this case) that resembles the traditional Keynesian multiplier. Oliver Hart's work gives rise to a new generation of New Keynesian models<sup>3</sup> characterised by the use of imperfect competition in general equilibrium macroeconomic models. A few notable examples are Akerlof and Yellen (1985), Bénassy (1987), Blanchard and Kyiotaki (1987), Hall (1986), Mankiw (1985), Snower (1983), and Weitzman (1982). These and other papers were analysed in surveys of the literature written at the time Dixon and Rankin (1994) or Silvestre (1993).

Despite the fact that we can find references to fiscal policy effectiveness under imperfect competition in all the above-mentioned papers, the systematic treatment of the problem, isolated from further considerations, assumptions, and results related to the general equilibrium, can only be found in the second half of the 1980's.

In this survey, we analyse the effectiveness of fiscal policy in general-equilibrium models with the following features:

1. agents are fully rational;
2. there is no uncertainty;
3. the economy is closed;
4. there is imperfect competition in goods markets;
5. labour markets are perfectly competitive;
6. prices of goods and factors are perfectly flexible;
7. public consumption has no direct effects on utilities and technologies of private agents, and we assume a benevolent government, so we can abstract from political-economy issues;
8. there is no agent heterogeneity.

These assumptions allow us to study the effect of imperfect competition in goods markets on fiscal policy, isolating it from other factors. Therefore, we can present a set of theoretical models using the same framework in order to study the effects of

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<sup>3</sup>The first generation refers to contributions such as Fischer (1977) and Taylor (1979), especially interested in price- and wage-setting rules for relatively long periods (short-run *ad hoc* nominal rigidity).

changing a particular basic assumption. We will concentrate on the effects of fiscal policy in two main objectives: aggregate output and representative-household welfare. The choice for these two objectives, especially the first one, is the usual one in the literature, but it is justified by the assumptions considered, as we will see throughout the survey. Section 2 is dedicated to simple static models and section 3 covers the dynamic models. Section 3 concludes.

## 2 Static models

In this section we develop a class of static general equilibrium models that nests most of the relevant literature on the topic<sup>4</sup>.

### 2.1 The microeconomic foundations

#### 2.1.1 Households

Let us assume there is a large number of identical households that maximise a utility function depending on the consumption of a basket of goods ( $C$ ) and leisure ( $Z$ ):

$$\max_{C,Z} U = u(C, Z). \quad (1)$$

This is a continuous twice-differentiable function with the following features<sup>5</sup>:  $u_C > 0$ ,  $u_Z > 0$ ,  $u_{CC} < 0$ ,  $u_{ZZ} > 0$ , and  $u_{CZ} = u_{ZC} > 0$ . The above-mentioned basket has a constant elasticity of substitution (CES) structure (the sub-utility function) given by

$$C = n^{\frac{1-\lambda}{1-\sigma}} \cdot \left[ \int_0^n c(j)^{\frac{\sigma-1}{\sigma}} \cdot dj \right]^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where  $c(j)$ , with  $j \in [0, n]$ , represents the consumption of variety  $j$ ,  $\sigma > 1$  stands for the (absolute value of the) elasticity of substitution between goods,  $n$  is the mass of the continuum of varieties, and  $\lambda \in [0, 1]$  controls the consumers' level of love for variety: if  $\lambda = 0$ , then there is no love for variety, when  $\lambda = 1$  we have the Dixit and Stiglitz (1977) extreme case of love for variety, and for  $\lambda \in (0, 1)$  there is some love for variety and it is larger (smaller) the closer this parameter is from one (zero).

Leisure is defined as the complement to one (by normalisation) of the time spent working ( $L$ ):

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<sup>4</sup>This model is based on the Aggregation Lecture by Luís Costa "Macroeconomics" used also for PhD courses given at York 2005 and ISEG/TULisbon.

<sup>5</sup>For sake of simplicity we use the following notation for partial derivatives:

$$f_x = \frac{\partial f}{\partial x}(x, y) \quad f_{xy} = \frac{\partial^2 f}{\partial x \partial y}(x, y) .$$

$$Z = 1 - L. \quad (3)$$

The budget constraint is given by

$$w.L + \Pi - T = \int_0^n c(j) \cdot p(j) \cdot dj, \quad (4)$$

where  $w$  represents the nominal wage,  $\Pi$  is the non-wage income (profits),  $T$  stands for the direct-tax levied on this household, and  $p(j)$ , with  $j \in [0, n]$ , is the price of good  $j$ . Households know direct taxes are a linear function of their primary income:

$$T = T_0 + t \cdot (w.L + \Pi), \quad (5)$$

where  $t \in [0, 1)$  and  $T_0 < (1 - t) \cdot (w.L + \Pi)$ .<sup>6</sup>

Considering  $C$  is a CES function, we have homothetic preferences over varieties. Thus, the representative household problem given by equations (1) to (5) can be solved in two steps:

- 1) minimising total expenditure, given the optimal choice for the quantity of private-consumption baskets ( $C$ )<sup>7</sup>;
- 2) maximising utility, given the optimal expenditure function.

From the first step we obtain the following demand function for each good

$$c(j) = \left[ \frac{p(j)}{P} \right]^{-\sigma} \cdot \frac{C}{n^{1-\lambda}}, \quad (6)$$

where  $P$  represents the relevant price (or cost-of-living) index for the household given by

$$P = \left[ \frac{1}{n^{1-\lambda}} \cdot \int_0^n p(j)^{1-\sigma} \cdot dj \right]^{\frac{1}{1-\sigma}}, \quad (7)$$

and the optimal (minimal) expenditure function is given by  $P.C$ .

Notice the demand for good  $j$  is decreasing, and with a constant price elasticity given by (in absolute value)  $\sigma$ , on the relative price of this good compared to the average ( $p(j)/P$ ), it is increasing on aggregate consumption intentions ( $C$ ), and it is not increasing on the mass of available goods ( $n$ ), with an elasticity given by  $1 - \lambda$ .

From the second step we obtain

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<sup>6</sup>Notice that it is possible to have a progressive fiscal system in this formulation, as long as  $t > 0$  and  $T_0 < 0$ .

<sup>7</sup>This problem could be solved with a general sub-utility function  $C = C(n, [c(j)]_0^n)$ , as long as it still represents homothetic preferences over goods. However, for sake of simplicity we will keep CES preferences here, as they clearly dominate the literature.

$$C = \mathfrak{C}(\omega_N, \pi_N), \quad (8)$$

$$L = \mathfrak{L}(\omega_N, \pi_N), \quad (9)$$

where  $\omega_N \equiv w.(1-t)/P$  represents the real net wage,  $\pi_N \equiv [\Pi.(1-t) - T_0]/P \equiv \pi.(1-t) - \tau_0$  stands for the net real non-wage income, equation (8) is the private consumption function where  $\mathfrak{C}_{\omega_N} > 0$  and  $\mathfrak{C}_{\pi_N} > 0$ , and equation (9) represents the labour-supply function where  $\mathfrak{L}_{\omega_N} \geq 0$  and  $\mathfrak{L}_{\pi_N} < 0$ .<sup>8</sup>

As one could expect, private consumption intentions are an increasing function of the real net wage ( $\mathfrak{C}_{\omega_N} > 0$ ) and also of the real non-wage income ( $\mathfrak{C}_{\pi_N} > 0$ ), both taken as given by households. The net real non-wage income has a negative impact on labour supply ( $\mathfrak{L}_{\pi_N} < 0$ ), but the effect of the real net wage ( $\mathfrak{L}_{\omega_N}$ ) cannot be determined ex ante, as it depends on both the substitution effect ( $> 0$ ) and on the income effect ( $< 0$ ).

### 2.1.2 Government

Firstly, let us assume government controls the real value of its public expenditure ( $G$ ) and uses it as an economic-policy decision variable. In order to avoid complicating the model due to composition effects of public expenditure, we assume the government-consumption basket has exactly the same CES composition then the private-consumption basket given by (2).

Thus, in order to minimise total expenditure in all goods for a given level of  $G$ , the demand function of each variety for public consumption,  $g(j)$  with  $j \in [0, n]$ , is given by an equation identical to (6). The relevant price index is still given by  $P$  and public consumption expenditure is  $P.G$ .

The government budget constraint is given by

$$P.G + w.\Psi = T_0 + t.(w.L + \Pi), \quad (10)$$

where  $\Psi \geq 0$  represents the quantity of unproductive labour hired by the government<sup>9</sup>, using the same labour market than firms. This is only a device used by Mankiw (1988) to simulate public-debt financing of government expenditure in a static model<sup>10</sup>.

Therefore, this equation nests several cases, corresponding to several types of financing:

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<sup>8</sup>See appendix 1.

<sup>9</sup>We call it "unproductive labour" as it does not yield non-market services, contrary to what happens in real economies.

<sup>10</sup>Assuming, of course, households exhibit non-Ricardian behaviour.

I. The case when government intends to keep the simultaneous control of the (net) autonomous tax ( $T_0 \neq 0$ ), the marginal tax rate, and also of public consumption. In this case, non-productive employment becomes the endogenous variable that adjusts to the economic situation:

$$\Psi = \frac{T_0 + t \cdot (w \cdot L + \Pi) - P \cdot G}{w}; \quad (10.I)$$

II. The case when government decides not to hire unproductive labour ( $\Psi = 0$ ) and it intends to keep the control over the marginal tax rate ( $t > 0$ ). In this case, the (net) autonomous tax becomes the endogenous variable:

$$T_0 = P \cdot G - t \cdot (w \cdot L + \Pi); \quad (10.II)$$

III. The case when government decides not to levy a (net) autonomous tax ( $T_0 = 0$ ), besides the fact it does not hire unproductive labour ( $\Psi = 0$ ). Thus, the marginal tax rate becomes endogenous:

$$t = \frac{P \cdot G}{w \cdot L + \Pi}. \quad (10.III)$$

For sake of simplicity, we will concentrate on the study of the effects of changing public consumption on the economy equilibria, ignoring the effects of changing other fiscal variables as unproductive labour ( $\Psi$ ), (net) autonomous taxes ( $T_0$ ), and the marginal tax rate ( $t$ ), when these variables are exogenous.

### 2.1.3 Industries

The productive sector is composed by a continuum of industries with mass  $n > 0$  and each industry is dedicated to producing a differentiated good. Thus, we will identify the industry that produces good  $j$  ( $\mathfrak{S}(j)$ ) as the set of firms that produce it. We assume  $m(j) \geq 1$  firms exist in this industry, so industry  $j$  is composed by firms  $i(j)$  such that:

$$\mathfrak{S}(j) = \{i(j) : i(j) = 1, \dots, m(j)\}; \quad j \in [0, n].$$

Market demand directed to industry  $j$  ( $d(j)$ ) is given by the sum of private and government demands, i.e.

$$d(j) \equiv c(j) + g(j) = \left[ \frac{p(j)}{P} \right]^{-\sigma} \cdot \frac{D}{n^{1-\lambda}}, \quad (11)$$

where  $D \equiv C + G$  represents aggregate demand, i.e. total demand for CES baskets.

In order to obtain an equilibrium in the market for good  $j$  the following condition has to hold

$$d(j) = \sum_{i(j)=1}^{m(j)} y_{i(j)}, \quad (12)$$

where  $y_{i(j)}$  represents the output (of good  $j \in [0, n]$ ) of firm  $i(j) \in \mathfrak{S}(j)$ .

#### 2.1.4 Firms

Let us now assume a firm  $i(j)$ , belonging to the industry of good  $j$ , has the following strategic behaviour<sup>11</sup>:

- it competes with other firms in its industry ( $k(j) \neq i(j) \in \mathfrak{S}(j)$ ) using quantities produced as a strategic variable - intra-industrial Cournot competition;
- it competes with firms in other industries ( $s \neq j \in [0, n]$ ) using posted prices as a strategic variable - inter-industrial Bertrand competition.

We call Cournotian Monopolistic Competition<sup>12</sup> (CMC) to this type of market structure and it allows us to nest the following as particular cases:

1. perfect competition when the number of firms in industry  $j \in [0, n]$  is very large ( $m(j) \rightarrow \infty$ ) or if varieties are close substitutes ( $\sigma \rightarrow \infty$ );
2. Dixit and Stiglitz (1977) monopolistic competition when all industries have a single producer ( $m(j) = 1, j \in [0, n]$ );
3. Cournot oligopolies in each industry when the number of firms within them ( $m(j) > 1$ ) is small.

Let us analyse the profit maximisation program for firm  $i(j)$  ( $\Pi_{i(j)}$ ), one of the producers of good  $j$ :

$$\max_{y_{i(j)}} \Pi_{i(j)} = p(j) \cdot y_{i(j)} - TC_{i(j)}, \quad (13)$$

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<sup>11</sup>Had we not considered a continuum of goods, but a finite number of varieties instead, an individual producer could be sufficiently large to consider the effects of its own actions on macroeconomic variables. In this case, we would observe a feedback effect from the macro into the microeconomic level. For a few examples of models that consider the possibility of large firms at the economy level see Costa (2001), D'Aspremont et al. (1989), or Wu and Zhang (2000), amongst other.

<sup>12</sup>See D'Aspremont et al. (1997).



where  $TC_{i(j)}$  represents total cost for this firm.

In order to keep the model simple, we assume the production technology of this good by this firm uses a single input, labour, and it is represented by

$$y_{i(j)} = \begin{cases} A_{i(j)} \cdot N_{i(j)} - \Phi & \iff N_{i(j)} > \frac{\Phi}{A_{i(j)}}; \\ 0 & \iff 0 \leq N_{i(j)} \leq \frac{\Phi}{A_{i(j)}}; \end{cases} \quad (14)$$

where  $N_{i(j)}$  represents the labour quantity hired by firm  $i(j)$ ,  $A_{i(j)} > 0$  stands for the (constant) marginal productivity of labour, and  $\Phi \geq 0$  is a technological parameter that can be interpreted in the following way: there is a minimum quantity of labour ( $\Phi/A_{i(j)}$ ) necessary for firms to work, but it does not represent production capacity in terms of good  $j$ . We can interpret this amount as administrative labour.

By looking at the first branch we can see the production function exhibits increasing returns to scale if  $\Phi > 0$  and constant returns to scale if  $\Phi = 0$ .<sup>13</sup>

Labour is acquired in perfectly competitive market and, given the fact that it is the only cost source, we can obtain the value of total costs as

$$TC_{i(j)} = w \cdot N_{i(j)}. \quad (15)$$

The firm also takes into account the effect of the market-clearing condition for good  $j$ , given by equation (12), on the residual demand it faces, i.e. it considers the effect of its sales on the price of the good:

$$p(j) = \left[ n^{1-\lambda} \cdot \frac{y_{i(j)} + \sum_{k(j) \neq i(j) \in \mathfrak{S}(j)} y_{k(j)}}{D} \right]^{-\frac{1}{\sigma}} \cdot P. \quad (16)$$

Given the market structure described above, this firm takes the quantities produced by its competitors within the same industry as given:

$$y_{k(j)} = \bar{y}_{k(j)}; \quad k(j) \neq i(j) \in \mathfrak{S}(j); \quad (17)$$

and the prices posted by its competitors in other industries:

$$p(s) = \bar{p}(s); \quad s \neq j \in [0, n]. \quad (18)$$

Finally, considering its reduced size in the economy as a whole, firm  $i(j)$  takes macroeconomic variables as given:

$$D = \bar{D}; \quad P = \bar{P}; \quad n = \bar{n}. \quad (19)$$

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<sup>13</sup>See appendix 2.

From solving the profit maximisation problem given by equations (13) to (19), we obtain the optimal price-setting rule for good  $j$  that corresponds to equalising the marginal revenue to the marginal cost ( $MC$ ):

$$p(j) \cdot \left[ 1 - \frac{S_{i(j)}}{\sigma} \right] = \frac{w}{A_{i(j)}}, \quad (20)$$

where  $S_{i(j)} \equiv y_{i(j)} / \sum_{r(j) \in \mathfrak{S}(j)} y_{r(j)}$  represents the market share of firm  $i(j)$ .

### 2.1.5 Microeconomic symmetric equilibria

Let us now assume all firms are identical, i.e. marginal productivity of labour is the same in all of them. Furthermore, we normalise it to one unit of good  $j$  per unit of labour:

$$A_{i(j)} = A = 1; \quad \forall i(j) \in \mathfrak{S}(j); \quad \forall j \in [0, n].$$

Therefore, in equilibrium there is no reason for any asymmetry to persist amongst firms of the same industry. Thus, we obtain  $S_{i(j)} = 1/m(j)$  in all industries. So we say there is a symmetric intra-industrial equilibrium.

However, if all the firms are identical in all industries, facing identical demand functions, then we also have a symmetric inter-industrial equilibrium. Consequently and for sake of simplicity, we assume the number of firms in each industry is the same ( $m(j) = m, \forall j \in [0, n]$ ).

Therefore, we can re-write the optimal price-setting rule for good  $j$ , given by (20) and the same for all the other goods in the economy due to the inter-industrial symmetry ( $p(j) = p, \forall j \in [0, n]$ ), as

$$p \cdot (1 - \mu) = \frac{w}{A}, \quad (20.a)$$

where  $\mu \equiv (p - MC)/p = 1/(\sigma \cdot m) \in [0, 1)$  is the Lerner index that represents market power of each firm in each industry. Note this index gives us the reciprocal of the (absolute value of the) price-elasticity of demand faced by each producer in a symmetric equilibrium. In the perfect competition case ( $m \rightarrow \infty$  or  $\sigma \rightarrow \infty$ )<sup>14</sup> we have  $\mu = 0$ , i.e.  $p = MC$ . In an extreme case of monopoly ( $m = 1$  and  $\sigma = 1$ ), we would have  $\mu = 1$ , i.e. each firm may post an infinitely high price relative to the marginal cost. The higher the value of  $\mu$ , the higher the representative firm's market power.

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<sup>14</sup>In order for perfect competition in goods and inputs markets may subsist in the long run, there can be no increasing returns to scale. Thus, we also have to assume that  $\Phi = 0$  in this case.

### 2.1.6 Macroeconomic constraints

Firstly, labour market has to be in equilibrium<sup>15</sup>. Thus, the (ex-post) equality between the quantities of labour supplied and demanded has to hold:

$$L = N + \Psi; \quad N = \int_0^n \sum_{i(j) \in \mathfrak{S}(j)} N_{i(j)} \cdot dj. \quad (21)$$

Notice that  $N$  gives us private-sector labour demand (both productive and "administrative"). Taking into account equilibrium symmetry, labour demand is given by  $\Psi + n.m.(y + \Phi)$ , where  $y$  stands for the equilibrium output of each firm.

We will use value added of each firm in order to define an aggregate-output concept. Considering the initial assumption that no intermediate outputs exist here, value added of firm  $i(j)$  ( $VA_{i(j)}$ ), measured in terms of consumption baskets, is given by  $[p(j)/P] \cdot y_{i(j)}$ . Thus, real aggregate output is given by

$$Y = \int_0^n \frac{p(j)}{P} \cdot \sum_{i(j) \in \mathfrak{S}(j)} y_{i(j)} \cdot dj. \quad (22)$$

In a symmetric equilibrium we have  $Y = n.m.p.y/P$ . Note that, taking into account equation (7) and equilibrium symmetry, we obtain  $P = n^{\lambda/(1-\sigma)} \cdot p$ . By substituting it in equation (16), we finally obtain the fundamental identity of national accounting  $Y = D$ .<sup>16</sup>

We have also the value of non-wage income given by the sum of the profits of all firms in the economy:

$$\Pi = \int_0^n \sum_{i(j) \in \mathfrak{S}(j)} \Pi_{i(j)} \cdot dj. \quad (23)$$

Finally, as in any other general-equilibrium model, we have to choose one good to be the *numéraire*. We choose the CES basket for that role, so that  $P = 1$ .

## 2.2 The initial general equilibrium

### 2.2.1 Some basic relations

Given the choice of the *numéraire* and given the mass of industries, we can obtain the price posted in each industry:

<sup>15</sup>Imperfect competition cannot, by itself, generate unemployment equilibria.

<sup>16</sup>This result does not depend upon microeconomic equilibrium symmetry, but it is more easily obtained under this assumption.

$$p = n^{\frac{\lambda}{1-\sigma}}. \quad (24)$$

Note this price diverges from the general level when there is some taste for variety ( $\lambda > 0$ )<sup>17</sup>.

Using equation (20.a), we can obtain the equilibrium (real) wage rate that is represented by the following expression, given the mark-up level:

$$w = (1 - \mu) . n^{\frac{\lambda}{1-\sigma}}. \quad (25)$$

Here, besides the love-for-variety effect, we can observe that a larger market power implies a smaller wage, as it contracts labour demand. The corresponding aggregate labour demand can be written as a function of aggregate output, the mass of industries, and the number of firms per industry:

$$L = Y + \Psi + n.m.\Phi, \quad (26)$$

where the first term on the right-hand side corresponds to the directly productive labour input in the private sector, the second one represents unproductive labour, and the last one is the private-sector "administrative" labour, i.e. the overhead fixed cost for the economy.

Aggregate profits can also be re-written as

$$\Pi = \mu.Y - (1 - \mu) . n.m.\Phi, \quad (23.a)$$

i.e. it is an increasing function of both the aggregate output and the mark-up level, and a decreasing function of both the mass of industries and the number of firms per industry.

### 2.2.2 A general formulation for the equilibrium

In order to deal with the various models that are nested in this general framework, we will write down the equilibrium values for the wage rate, employment, and non-wage income as functions of the government-consumption level and also of other variables and parameters. We will not have to explicitly define these functions given the fact that we are only interested on the effects of fiscal policy:

$$\begin{aligned} w^* &= w(G, \cdot); \\ L^* &= L(G, \cdot); \\ \Pi^* &= \Pi(G, \cdot); \end{aligned} \quad (27)$$

where asterisks identify the macroeconomic-equilibrium values for these variables.

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<sup>17</sup>See appendix 3.

Given the variety of fiscal-policy behaviour types considered in equations (10.I to III), have still to consider that:

$$\begin{aligned} \Psi^* &= \Psi(G, \cdot) && \text{in case I;} \\ T_0^* &= T_0(G, \cdot) && \text{in case II } (\Psi = 0); \\ t^* &= t(G, \cdot) && \text{in case III } (\Psi = 0 \text{ and } T_0 = 0). \end{aligned} \quad (28)$$

Using the fundamental identity of national accounting, the aggregate-demand definition, the consumption function, and the government budget constraint given above, we can write an equation that gives us the equilibrium value for aggregate output:

$$Y = \mathfrak{C} \{w(G, \cdot) \cdot [1 - t(G, \cdot)], \Pi(G, \cdot) \cdot [1 - t(G, \cdot)] - T_0(G, \cdot)\} + G. \quad (29)$$

From this equation we can easily see that the equilibrium value for output is given by  $Y^* = Y(G, \cdot)$ .

Once we have found the value of  $Y^*$ , we can obtain all the additional equilibrium values that depend on it, namely  $C^*$  and  $U^*$ , the latter representing the equilibrium value for households' utility (welfare).

### 2.3 Fiscal policy effectiveness

From equation (29) we can obtain the value of the output government-consumption multiplier ( $k^* = dY^*/dG$ ) using a first-order Taylor approximation and the implicit-function theorem:

$$dY = [1 + (1 - t^*) (\mathfrak{C}_{\omega_N}^* \cdot w_G^* + \mathfrak{C}_{\pi_N}^* \cdot \Pi_G^*)] \cdot dG - (w^* + \Pi^*) \cdot dt^* - \mathfrak{C}_{\pi_N}^* \cdot dT_0^*, \quad (30)$$

where we have

$$\begin{aligned} dT_0^* &= \begin{cases} 0 & \text{in case I;} \\ (1 - t^* \cdot k^*) \cdot dG & \text{in cases II and III.} \end{cases} \\ dt^* &= \begin{cases} 0 & \text{in cases I and II;} \\ \frac{1 - g^* \cdot k^*}{Y^*} \cdot dG & \text{in case III;} \end{cases} \end{aligned}$$

and  $g^* = G/Y^* \in [0, 1)$  is the weight of public consumption in aggregate expenditure.

We can expect  $k^*$  to be positive in most cases, but the main goal of this section is analysing it in specific situations, according to the various hypothesis advanced by many authors from the middle 1980's onwards. Furthermore, we are especially interested in the effect of the market power on fiscal policy effectiveness, i.e. we will analyse the sign of

$$\frac{\partial k^*}{\partial \mu}. \quad (31)$$

Finally, the analysis of fiscal policy effectiveness on households welfare can simply be done in the following way: if  $k^* > 0$ , then an expansionary fiscal policy will imply a leisure loss, as labour is the only input. Thus, welfare will only increase if i) private consumption positively reacts to an increase in public consumption and ii) if that increase is sufficiently valued by households so it more than offsets the previous leisure reduction.

In the next sub-section we will survey the main results of this strand of literature.

## 2.4 A brief survey of the literature

### 2.4.1 The initiators: Dixon and Mankiw

We may affirm the first works exclusively dedicated to this topic are Dixon (1987) and Mankiw (1988), which share the following assumptions:

1. A Cobb-Douglas utility function

$$U = C^\alpha \cdot Z^{1-\alpha} \quad \text{with } 0 < \alpha < 1. \quad (1.A)$$

2. Absence of income-dependent taxes ( $t^* = 0$ ).
3. Absence of taste for variety ( $\lambda = 0$ ).
4. A monopolistic-competition market structure ( $m = 1$ ), i.e. a constant mark-up given by  $\mu = 1/\sigma$ .
5. A fixed mass of industries ( $n$ ).

Considering these assumptions, we have a consumption function given by

$$C = \alpha \cdot \frac{w + \Pi - T_0}{P}, \quad (8.A)$$

i.e. the marginal propensity to consume is constant and identical for all types of income ( $\mathbf{e}_{\omega_N}^* = \mathbf{e}_{\pi_N}^* = \alpha$ ).

With a constant mark-up and no love for variety, the equilibrium wage rate is also constant and given by<sup>18</sup>  $w^* = 1 - \mu$ . Thus, we know this equilibrium wage will not react to fiscal policy, i.e.  $w_G^* = 0$ .

From equation (23.a) the reaction of non-wage income to fiscal policy is given by  $\Pi_G^* = \mu \cdot k^*$ .

By substituting the values above in equation (30) and solving it in order to  $k^*$  we obtain

$$k_A^* \Big|_{dT_0^*=0} = \frac{1}{1 - \alpha \cdot \mu} > 1, \quad (30.A1)$$

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<sup>18</sup>See equation (25).

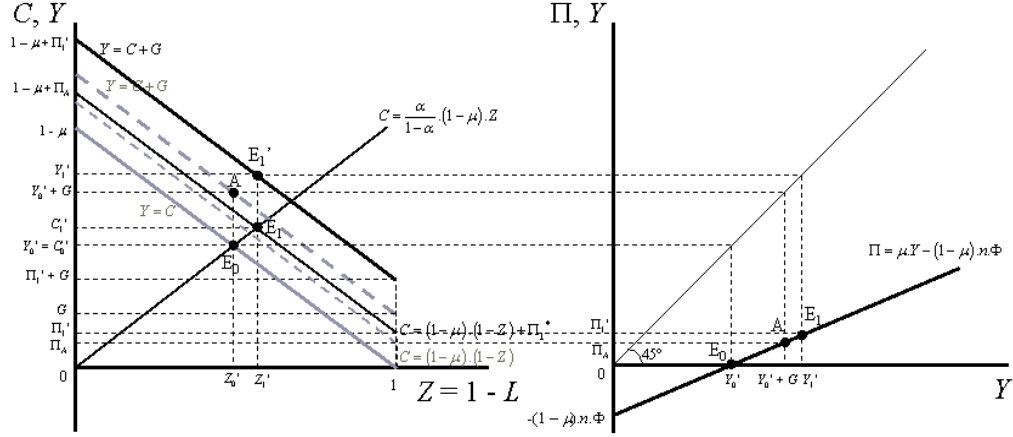


Figure 1: The Multiplier in the Dixon-Mankiw Model - Case I

in case I, where government can increase its consumption without rising taxes on households ( $dT_0^* = 0$ ). Consequently, we conclude that, in this case, a unit increase in  $G$  leads to an increase in equilibrium aggregate output of  $1/(1 - \alpha.\mu) > 1$ . Furthermore, note that this increase is larger than the one that would occur under perfect competition ( $\mu = 0$  and  $\Phi = 0$ ), where it would be equal to 1.

We can better explain what happens by using Figure 1 below.

First, consider that in the initial equilibrium government expenditure is zero ( $G = 0$ ) and profits are also zero ( $\Pi^* = 0$ ). On the left-hand panel we can depict the microeconomic decision in the leisure-consumption space using two simple graphical tools: the upward-sloping income-expansion path and the downward-sloping budget constraint. The former corresponds to equating the marginal rate of substitution between leisure and consumption ( $MRS_{Z,C} \equiv U_Z/U_C = (1 - \alpha).C/[\alpha.Z]$  in this model) to the real net wage ( $\omega_N^* = 1 - \mu$  in this model). The later is just taken from equation (4), given the equilibrium values for the wages, profits, and taxes.

Thus, the microeconomic equilibrium for the representative household is given by point  $E_0$  where it chooses an amount of leisure equal to  $Z_0^*$  and an amount of consumption given by  $C_0^*$ . Since there is government consumption, the macroeconomic equilibrium in this space is represented by a "production possibilities frontier" between output and leisure that is given by the  $Y = C$  schedule, the same as the household budget constraint. On the right-hand panel, we can represent the increasing relationship between total income and profits that corresponds to equation (23.a)

Now, let us introduce government consumption given by  $G > 0$ . The first effect on the left-hand panel is that the macroeconomic-equilibrium representation is now different from the microeconomic one, i.e. the  $Y = C + G$  curve stands above households

budget constraint. Using the same amount of labour (leisure), the macroeconomic equilibrium is now given by point A. That increase in demand leads to an increase in profits, as represented by point A in the right-hand panel. Thus, the microeconomic budget constraint would shift upwards and households would increase both leisure and consumption. But then, the macroeconomic constraint would also shift upwards, profits would increase and so on until the process ends in a new equilibrium represented by points  $E_1$  (in both panels) and  $E_1'$  (in the left-hand panel).

In a nutshell: when fiscal authorities stimulate aggregate demand through an increase in public consumption in the amount of one unit, there is an "initial" increase in output of the same amount (here, in case I). However, the mechanism does not stop there, as a larger aggregate income implies larger profits that are distributed to households. Consequently, households initiate a new "round" of the mechanism as the increase consumption, in the amount of  $\mu$  units, that will stimulate aggregate demand once more, that stimulates output...

However, for output to increase, it is necessary that private-sector employment ( $N$ ) increases. Apparently, there is a contradiction with the graphical results as leisure also increases. Nonetheless, a larger level of public consumption implies a smaller level of unproductive public employment, as we can see in equation (10.I), given by  $\Psi = (T_0 - G) / (1 - \mu)$  in this case where  $T_0$  is fixed.

Thus, despite the fact that  $L$ , total employment, decreases due to the negative effect of higher profits, there is an employment transfer from the public sector to the private sector that more than offsets the decrease in  $L$ <sup>19</sup> and leads to an increase in  $N$ .

Let us now we assume that government cannot increase its consumption without increasing taxes on households ( $dT_0^* = dG$ ), then we are in case II, where we conclude that

$$k_A^*|_{dT_0^*=dG} = \frac{1 - \alpha}{1 - \alpha \cdot \mu} > 0, \quad (30.A2)$$

i.e. a unit increase in  $G$  induces an equilibrium output increase of  $0 < 1 - \alpha < (1 - \alpha) / (1 - \alpha \cdot \mu) < 1$ .

Figure 2 pictures the multiplier mechanism in a similar way to Figure 1. However, the initial demand stimulus is now also perceived by households as a tax increase, since  $dT_0^* = dG$ . Thus, the microeconomic budget constraint shifts down by the amount of lump-sum taxes ( $G$ ). The negative income effect moves the optimal decision of households from  $E_0$  to A, reducing both consumption and leisure. Nonetheless, the macroeconomic  $Y = C + G$  curve does not move, and that means output increases

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<sup>19</sup>Remember that  $1 / (1 - \mu) > 1$ .



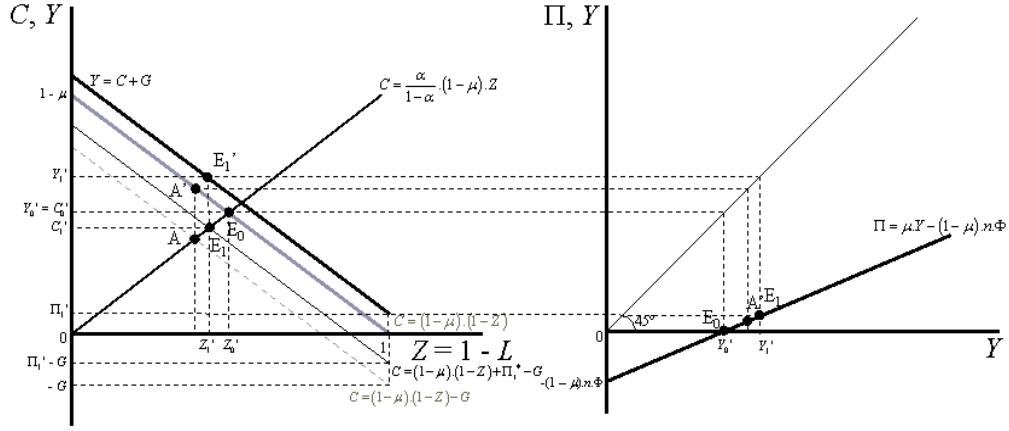


Figure 2: The Multiplier in the Dixon-Mankiw Model - Case II

to point A'. Consequently, profits increase, as shown by point A' in the right-hand panel, and another "round" of the multiplier mechanism is set in motion. At the end of the day, the new equilibrium is given by points E<sub>1</sub> (in both panels) and E<sub>1</sub>' (in the right-hand panel).

In this case, the "initial" demand stimulus of one unit of government consumption is partially crowded out, leading to a output increase of  $0 < 1 - \alpha < 1$  and then to a profits increase of  $\mu \cdot (1 - \alpha)$ , before the second "round" starts. Notice here the output increase can be easily explained by the labour-supply side: more government expenditure means more taxes and these have a positive effect on labour supply that more than offsets the negative effect of profits<sup>20</sup>. Thus, households are willing to work longer hours as their disposable income decreases, the same reason that makes them consume less.

In both these cases (I and II) we observe that fiscal policy effectiveness on output is an increasing function of the degree of monopoly that exists in the economy:

$$\frac{\partial k_A^*}{\partial \mu} \Big|_{dT_0^*=0} = \frac{\alpha}{(1-\alpha\mu)^2} > 0; \quad \frac{\partial k_A^*}{\partial \mu} \Big|_{dT_0^*=dG} = \alpha \cdot \frac{1-\alpha}{(1-\alpha\mu)^2} > 0. \quad (31.A)$$

In order to explain what happens, let us use Figure 3 that refers to case I.

This figure is very similar to Figure 1, but it assumes a larger mark-up level ( $\mu_1 > \mu_0$ ), i.e. a smaller elasticity of substitution amongst goods. In order to keep zero profits in the initial equilibrium, we also assume a larger fixed cost ( $\Phi_1 > \Phi_0$ ). As we can see in the left-hand-side panel, the larger mark-up level induces a smaller

<sup>20</sup>Remember that  $\mu < 1$ .

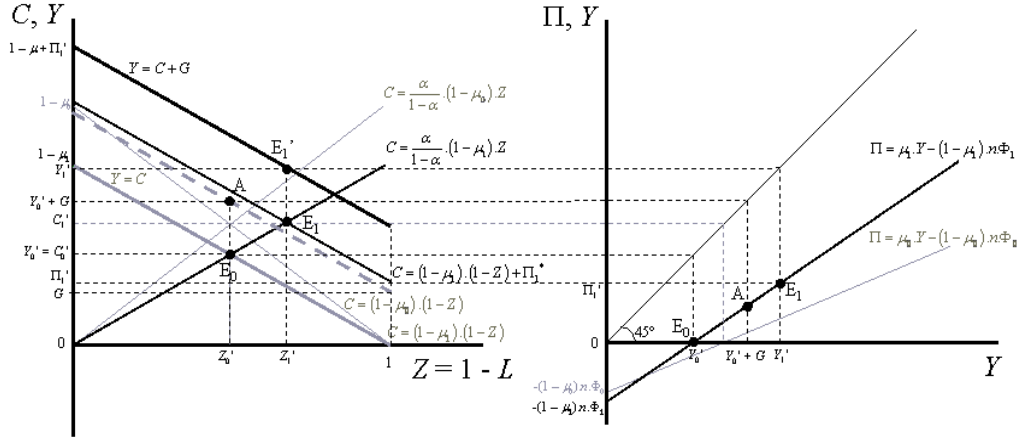


Figure 3: The Multiplier and the Mark-up in the Dixon-Mankiw Model

equilibrium wage rate, inducing a downward rotation on the income expansion path around the origin and also a downward rotation of the budget constraint about point  $(1,0)$ . On the right-hand side, a larger mark-up rotates the profit function up, but the larger fixed costs shifts it down in a parallel way.

Since the mechanism is similar to the one described in Figure 1, we can notice the output increase  $(Y_1^* - Y_0^*)$  is larger here than before, with a weaker monopoly power. So why does this happen? The answer lies on the combination of three effects: i) there is a negative substitution effect on labour supply due to the lower wage rate; ii) but the income effect of the lower wage rate is positive; and iii) there is a negative effect on labour supply due to larger profits. Thus, the crucial effect is the last one: a higher mark-up induces a larger profit windfall that will lead to a larger consumption by households, reinforcing the second-round effect of the multiplier.

In case II, where there is no partial substitution of public employment by private employment, the mechanism of profit distribution is as important as here, but the effect on labour supply is clear-cut: people would want to increase hours worked by more than in the case depicted in Figure 2. This is due to the reinforced negative effect of taxes when the rate is lower.

Given the similitude of this mechanism to the basic Keynesian model, some authors (e.g. Mankiw) identified it with the traditional Keynesian spirit. However, Dixon (1987) draws our attention to the fact that the economic mechanism that supports this outcome has much more to do with the Walrasian spirit than with the Keynesian one. In fact, the consumption-leisure choices made by households are basically the same under an expansionary fiscal policy either we face perfect or imperfect competition. The main difference has to do with non-wage income generated under

imperfect competition, not in effective demand scarcity. The effect of fiscal policy on welfare in case II, the only truly sustainable type of fiscal policy in a static model, is clear: output increases by less than public consumption. Thus, private consumption decreases due to the effect of higher taxes. Therefore, households work harder and their welfare decreases as a consequence of both effects.

## 2.4.2 Taxation

One of the critiques to the works of Dixon (1987) and Mankiw (1988) is the fact that they use lump-sum taxes to finance government expenditure. Molana and Moutos (1991) study the effect of proportional taxes in balanced-budget model without unproductive labour. Thus, the basic assumptions we have here are  $\Psi = 0$ ,  $T_0^* = 0$ , and  $0 < t^* < 1$ . All the other assumptions are identical to the previous point.

In what concerns to households, their behavioural functions are now given by

$$C = \alpha \cdot (1 - t) \cdot \frac{w + \Pi}{P}, \quad (8.B)$$

$$L = 1 - (1 - \alpha) \cdot (1 - t) \cdot \frac{w + \Pi}{w \cdot (1 - t)}. \quad (9.B)$$

Given the static equilibrium sustainability issue in a framework with a minimum of consistency, from this point onwards we assume the government always follows a balanced-budget rule without recurring to unproductive labour. Here, considering there are no (net) autonomous taxes, we are in case III, i.e. we have  $dt^* = (1 - k^* \cdot g^*) \cdot dG/Y^*$  to substitute in equation (30).

Thus, we obtain an equilibrium multiplier given by

$$k_B^* \Big|_{dt^*=(1-k^*.g^*).dG/Y^*} = \frac{Y^* - \alpha \cdot (1 - \mu + \Pi^*)}{\Delta_B}, \quad (30.B)$$

where  $\Delta_B = Y^* - \alpha \cdot (1 - \mu + \Pi^*) + \alpha \cdot (1 - g^*) \cdot (1 - \mu + \Pi^* - \mu \cdot Y^*)$ .<sup>21</sup> At first sight, the numerator, and also the denominator, appears to be either positive or negative. However, since we know that  $C^* = (1 - g^*) \cdot Y^*$  and using equation (8.B) in addition, we have  $C^* = \alpha \cdot (1 - t^*) \cdot (1 - \mu + \Pi^*)$ . If we also consider that the government budget constraint implies that  $t^* = g^*$ , it is simple to see that  $Y^* = \alpha \cdot (1 - \mu + \Pi^*)$ . Therefore,  $k_B^* \Big|_{dt^*=(1-k^*.g^*).dG/Y^*} = 0$ , i.e. fiscal policy is absolutely ineffective in this case III<sup>22</sup>.

<sup>21</sup>Since we know that, in equilibrium, we have  $-(1 - \mu) < \Pi^* - \mu \cdot Y^* = -(1 - \mu) \cdot n \cdot \Phi < 0$ , then we have  $\Delta_B = Y^* - \alpha \cdot (1 - \mu + \Pi^*) + \alpha \cdot (1 - g^*) \cdot (1 - \mu) \cdot (1 - n \cdot \Phi)$ . The constraint  $n \cdot \Phi < 1$  is a consequence of having  $1 \geq L \geq N \geq n \cdot m \cdot \Phi \geq 0$ .

<sup>22</sup>With the information obtained for the numerator, we know now that  $\Delta_B = \alpha \cdot (1 - g^*) \cdot (1 - \mu) \cdot (1 - n \cdot \Phi) > 0$ .

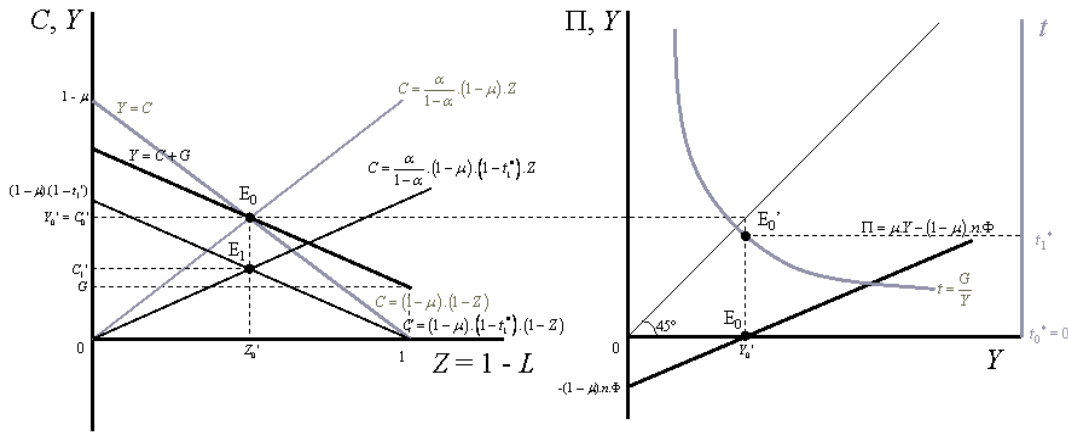


Figure 4: The Multiplier with Proportional Taxes

In Figure 4 we can observe what happens, starting from an initial equilibrium  $E_0$  with  $G = 0$ ,  $t^* = 0$ , and  $\Pi^* = 0$ . On the left-hand-side panel we now have a secondary axis to represent the tax rate, a decreasing function of output given  $G > 0$ . Thus, when positive government consumption is introduced, the tax rate increases from zero to  $t_1^* > 0$ . This implies a downward rotation of both the income expansion path and the budget constraint. In the new equilibrium  $E_1$ , private consumption was completely crowded out by government consumption and output, leisure, and profits remain unchanged, given the functionals assumed.

So, why is there such a dramatic loss of effectiveness? Contrary to case II, here an increase in public consumption only presents a potential substitution effect on labour supply, as it implies a tax-rate increase. However, this tax-rate increase has identical consequences on profits and wages, as they are both taxed at the same rate. Thus, the incentive to work more ceases to exist, unless profits decrease. But to have a decrease in profits, we would need an output fall and that is not compatible with an increase in employment in this case.

Molana and Moutos (1991) also demonstrate that, when taxes are levied only on wage income, we may even obtain a negative multiplier.

Considering there is no effect on output, here where there are only distortionary taxes, private consumption decreases and leisure remains unchanged. Once again, households welfare is smaller after implementing the expansionary fiscal policy.

### 2.4.3 Entry

Once we have explored proportional taxes, let us return to the analysis of fiscal policy finance through lump sum taxes (case II).

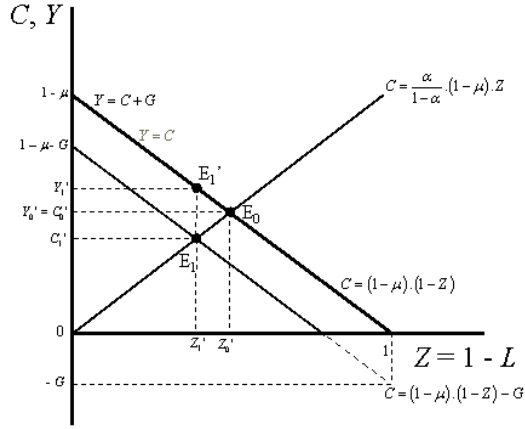


Figure 5: The Free-entry Multiplier of Startz

Dixon (1987) and Mankiw (1988) models assume the economy is in a "short-run" situation, i.e. firms are not allowed to enter or leave the productive sector. However, this situation is not sustainable in the "long run," an environment more suitable to be portrayed by a static model, corresponding to a steady state of a dynamic model.

Startz (1989) presents a "long-run" model using the basic assumptions in both Dixon (1987) and Mankiw (1988)<sup>23</sup>. Thus, the basic assumption here is that the mass of industries (or varieties),  $n$ , is not a constant, but an endogenous variable resulting from a zero-pure-profits condition. In our case, where there is no uncertainty or dynamics, since there is no opportunity cost of creating a new firm (or shutting down and existing one), the condition mentioned can be written as  $\Pi^* = 0$ .

Therefore, non-wage income ceases to respond to fiscal-policy impulses, as  $\Pi_G^* = 0$ . This feature cuts the transmission mechanism though profits into consumption and from consumption to aggregate demand again. Then, the multiplier is given by

$$k_C^* \Big|_{\substack{dT_0^*=dG \\ \Pi_G^*=0}} = 1 - \alpha > 0. \quad (30.C)$$

This multiplier is still positive, in the  $(0, 1)$  interval, but it does not depend on the existing monopoly-power level in the economy. Thus, in this model fiscal policy effectiveness would be identical in the Walrasian case ( $\mu = 0$ ) and in a highly monopolised economy ( $\mu \rightarrow 1$ ).

<sup>23</sup>In fact, Startz (1989) uses a Stone-Geary utility function instead of a Cobb-Douglas. However, the latter can be seen as a particular case of the former and the crucial property for the results obtained (i.e. constant marginal utility shares) is kept with a much simpler Cobb-Douglas function.

Figure 5 shows us what is happening in the free-entry model. There is no need for the right-hand-side panel as profits are compressed to zero by entry and exit. Thus, an increase in  $G$  shifts the microeconomic budget constraint down and the income effect of higher taxes induce an increase in labour supply and a decrease in consumption. Therefore, aggregate output increases, but there is a partial crowding out of private consumption of  $\alpha$  units for each unit of government consumption.

We can also notice that a change in  $\mu$  moves the income expansion path and the budget constraint, but it does not alter the result in terms of fiscal policy effectiveness as they both rotate in the same proportion like in the flat-rate-tax case.

Furthermore, we can observe the free-entry (or "long-run") multiplier, given by equation (30.C), is larger than the no-entry ("short-run") multiplier given by equation (30.A2):

$$\Gamma_{A(C)}^* \equiv \frac{k_C^*|_{dT_0^*=dG}}{k_A^*|_{dT_0^*=dG}} = 1 - \alpha \cdot \mu < 1.$$

As we saw when comparing both models with the same lump-sum tax financing public expenditure, the main difference between these two types of model is the way profits distribution affects private consumption. Once this mechanism is shut down, only the income effect in labour supply subsists to increase output, with the preferences assumed.

#### 2.4.4 Preferences

The main result of Startz (1989) is extremely appealing, as it eliminates the multiplier pure-profit mechanism.

However, Dixon and Lawler (1996) demonstrate that conclusion is clearly dependent on the type of preferences assumed for households<sup>24</sup>. If we keep the assumptions in Startz (1989), what some authors call the Dixon-Mankiw-Startz (DMS) framework, with the exception of the Cobb-Douglas utility function, we can see the no-entry multiplier is given by

$$k_{D1}^*|_{dT_0^*=dG} = \frac{1 - \mathfrak{C}_{\pi_N}}{1 - \mathfrak{C}_{\pi_N} \cdot \mu} > 0, \quad (30.D1)$$

which is positive and less than one if we assume the marginal propensity to consume of net non-wage income is restricted to the  $(0, 1)$  interval, as in the particular case of the DMS framework where  $\mathfrak{C}_{\pi_N} = \alpha$ .

Considering free entry, we obtain the "long-run" multiplier given by

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<sup>24</sup>In fact, that article also demonstrates Startz's result also depends upon the production technology. However, we will not analyse that side of the story here.

$$k_{D1}^* \Big|_{\substack{dT_0^*=dG \\ \Pi_G^*=0}} = 1 - \mathfrak{C}_{\pi_N} > 0, \quad (30.D2)$$

which was constant and equal to  $1 - \alpha$  in Startz (1989) particular case.

Assuming  $u(\cdot)$  still represents homothetic preferences, the graphical representations are similar to Figures 2 and 5 and the only difference is that the income expansion path is now given by  $C = \Sigma(1 - \mu) \cdot Z$ , where  $\Sigma(\cdot)$  is a general increasing function. If we assume preferences are not homothetic, the income expansion path becomes non-linear, but the outcomes are identical.

Furthermore, it is easy to observe the no-entry multiplier is larger than the free-entry one:

$$\Gamma_{D1}^* \equiv \frac{k_{D1}^* \Big|_{\substack{dT_0^*=dG \\ \Pi_G^*=0}}}{k_{D1}^* \Big|_{dT_0^*=dG}} = 1 - \mathfrak{C}_{\pi_N} \cdot \mu < 1,$$

and this result is also easily explained by the neutralisation of the profit effect<sup>25</sup>.

Thus, the previous results are similar to the DMS framework and we only have to substitute  $\alpha$  by  $\mathfrak{C}_{\pi_N}$ . However, in general, the marginal propensity to consume of profits depends upon the mark-up. Therefore, the "long-run" fiscal multiplier is the larger (smaller) the larger is the market power in the economy, when  $\mathfrak{C}_{\pi_N}$  is decreasing (increasing) with  $\mu$ .

Let us study an example using the utility function in Heijdra and van der Ploeg (1996)<sup>26</sup>, i.e. CES preferences over the consumption basket and leisure:

$$U = \left( C^{\frac{\varepsilon-1}{\varepsilon}} + a \cdot Z^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (1.D2)$$

In this case, the private consumption function is given by

$$C = \alpha \cdot \frac{w + \Pi - T_0}{1 + a^\varepsilon \cdot w^{1-\varepsilon}}, \quad (8.D2)$$

from which we can easily observe the marginal propensity to consume previously referred is not constant, but it is an endogenous variable that depends on the equilibrium value of the real wage rate:  $\mathfrak{C}_{\pi_N} = (1 + a^\varepsilon \cdot w^{1-\varepsilon})^{-1}$ . Notice also that  $\Sigma(1 - \mu) = [(1 - \mu)/a]^\varepsilon$  in this case.

Considering that we have  $w^* = 1 - \mu$ , this marginal propensity is decreasing (increasing) with the mark-up when the elasticity of substitution between consumption and leisure ( $\varepsilon$ ) is less (more) than one. Thus, fiscal policy is more (less) effective the

<sup>25</sup>Dixon and Lawler (1996) also demonstrate this is not always the case when production technology does not exhibit constant marginal returns.

<sup>26</sup>As we will see further on, this is not the only new thing introduced in this article.





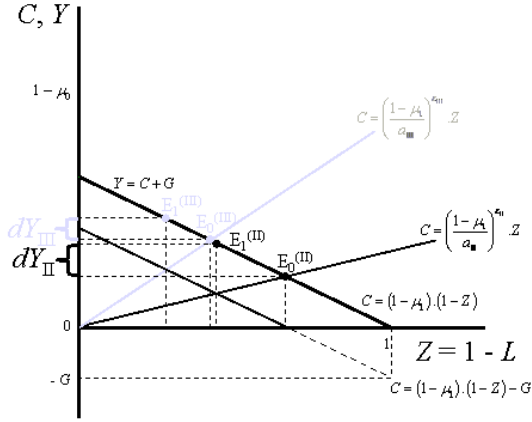


Figure 7: The Mark-up and the Multiplier with CES Preferences

path, when compared to the initial situation. In case II, a larger mark-up means a larger substitution effect. Thus, considering the effect on the budget constraint is the same in both cases, an increase in government consumption induces a higher increase in labour supply in case II than in case III. A larger elasticity of substitution means that households are willing to accept a higher reduction in leisure (and a smaller decrease in consumption) in order to respond to the corresponding tax increase.

#### 2.4.5 Increasing returns to variety

Let us now return to the functionals assumed in the DMS framework. However, we assume there is some taste for variety, i.e.  $\lambda > 0$ . In this case, equation (25) tell us that, for a given mark-up level, the real wage is an increasing function of the mass of goods existing in the economy.

This type of models, considering the love-for-variety assumption, is treated in Heijdra and van der Ploeg (1996)<sup>27</sup>. For sake of simplicity, we treat these two effects separately. Devereux et al. (1996) present a dynamic model where a similar effect arises from increase returns to specialisation, a kind of love for (intermediate-inputs) variety in the production function.

When the mass of firms and goods ( $n$ ) is fixed, i.e. when there is no entry or exit, the fiscal multiplier is still given by equation (30.A2). However, if firms are free to enter or leave the market, their mass becomes an endogenous variable given by

$$n^* = \left[ \frac{\mu \cdot Y^*}{(1-\mu) \cdot \Phi} \right]^{1-\gamma} ; \quad \gamma = \frac{\lambda}{\lambda + \sigma - 1} \in [0, \mu], \quad (32)$$

<sup>27</sup>The authors also use the CES utility function described in the previous section.

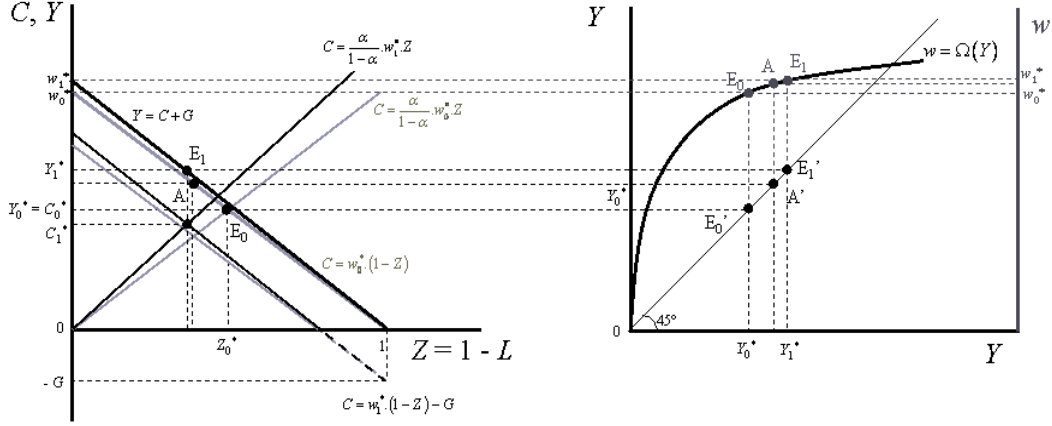


Figure 8: The Multiplier with a Varying Real Wage

a result that is obtained through the free-entry condition  $\Pi^* = 0$ .

Thus, an aggregate demand increase induces an increase in real wages that will affect fiscal policy effectiveness as<sup>28</sup>

$$w_G^* = \gamma \cdot \frac{w^*}{Y^*} \cdot k^* = \frac{\gamma}{\alpha} \cdot [1 - (1 - \alpha) \cdot g^*] \cdot k^* > 0,$$

i.e. entry of firms, a consequence of the aggregate demand stimulus, leads to a real wage increase and consequently to a consumption increase, opening a transmission channel similar to the profit one in the no-entry model. In this case, the multiplier is given by

$$k_E^* \Big|_{\substack{dT_0^* = dG \\ \Pi_G^* = 0}} = \frac{1 - \alpha}{1 - \gamma \cdot [1 - (1 - \alpha) \cdot g^*]} \geq 0. \quad (30.E)$$

Notice that, due to  $\lambda > 0$  we have  $\gamma > 0$  and consequently a larger multiplier than in the free-entry constant-returns case  $(1 - \alpha)$ .

On the left-hand-side panel of Figure 8 we can observe that fiscal policy would change the equilibrium from point  $E_0$  to point A. That is the situation depicted in Figure 5, corresponding to a fixed-wage environment. However, point A is not an equilibrium in this model, as the real wage is a function of the aggregate output  $w^* = \Omega(Y)$  with  $\Omega'(\cdot) > 0$ . This fact can easily be observed by combining equations (25) and (32). Therefore, a higher output induce new firms to enter and that stimulates

<sup>28</sup>See appendix 4.

aggregate demand via private consumption in the case of love for variety and labour demand in the case of increasing returns to specialisation. In any case, the equilibrium wage rate goes up, as we can observe on the secondary axes of the right-hand-side panel of Figure 8. The wage increase rotates the income expansion path, the household budget constraint, and the macroeconomic constraint up in the left-hand-side panel. The new equilibrium is finally reached in point  $E_1$  with a larger output and a smaller decrease in private consumption.

Despite the fact that we are using a consumption function with constant marginal propensities to consume, this multiplier depends upon the monopoly power level in the economy through  $g^*$  and  $\gamma = \mu \cdot \lambda / (\mu \cdot \lambda + 1 - \mu)$ . It is simple to demonstrate that  $\gamma$  is increasing with the mark-up<sup>29</sup>, but it is not easy to show how  $g^*$  depends on  $\mu$ . A first glance, one could think the weight of public consumption in output should be increasing with the monopoly degree, as it means more inefficiency, thus less output. However, taking into account net profits are zero, the macroeconomic production function can be represented as

$$Y = (1 - \mu) \cdot n^{\frac{\lambda}{\sigma-1}} \cdot L.$$

In the equation above we can observe that, for the same employment level, an increase in  $\mu$  leads to a reduction in the term  $(1 - \mu)$ , but it also increases the exponent, as it corresponds to a reduction in  $\sigma$ . This means that the monopoly degree under monopolistic competition reinforces the effect of increasing returns. There is also an indirect effect that acts through  $n$ , since an increase in  $\mu$  stimulates entry.

Therefore, we can easily determine what is the effect on the multiplier when we start from a zero-government-consumption steady state

$$\frac{\partial k_E^*}{\partial \mu} \Big|_{\substack{dT_0^*=dG \\ \Pi_G^*=0 \\ g^*=0}} = \frac{1 - \alpha}{(1 - \gamma)^2} \cdot \frac{\lambda}{[1 - \mu \cdot (1 - \lambda)]^2} \geq 0.$$

In this particular case, the larger is the market power, the larger is the entry effect on the real wage, increasing the effectiveness of the initial fiscal stimulus. An identical outcome can be obtained for situations where  $g^*$  does not react dramatically to changes in the mark-up. Using numerical simulations with plausible values for the parameters, we also obtain a multiplier that is an increasing function of  $\mu$ .

Now comparing the "short-" and "long-run" multipliers, we observe that

$$\Gamma_E^* \equiv \frac{k_E^*|_{dT_0^*=dG}}{k_A^*|_{dT_0^*=dG}} = \frac{1 - \alpha \cdot \mu}{1 - \gamma \cdot [1 - (1 - \alpha) \cdot g^*]}.$$

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<sup>29</sup>  $\frac{\partial \gamma}{\partial \mu} = \frac{\gamma^2}{\lambda \cdot \mu^2} \geq 0$ .

Considering that  $\gamma$  and  $g^*$  depends upon the values of other parameters in the model, it is not possible to say *a priori* if this value is larger or smaller than one. Thus, we know that for  $\mu < \frac{\gamma \cdot [1 - (1 - \alpha) \cdot g^*]}{\alpha}$  the free-entry ("long-run") multiplier is larger than the multiplier with a fixed mass of firms, given the positive externality caused by the entry of new firms. The opposite result is obtained when the mark-up is high.

#### 2.4.6 Endogenous mark-ups

The assumption that entry of firms is done through the creation of new monopolies associated to new products hides an additional assumption that product innovation is cheaper than copying an existing good or creating a close substitute. When facing significant costs associated with creating a differentiated product, the incentive to create a new industry may be smaller than the incentive to enter an existing industry. Thus,  $m$  may be the endogenous variable in our free-entry model instead of  $n$ .<sup>30</sup>

Up to this point, we considered that  $\mu$  was an exogenous variable, as we assumed  $m$  was fixed and equal to one (a basic assumption in monopolistically competitive models). When we alter the endogenous variable in the entry process, we also endogenise  $\mu = 1/(\sigma \cdot m)$ . This value can be obtained through the zero-profit condition, assuming once again there is no love for variety ( $\lambda = 0$ ):

$$\mu^* = \frac{\phi^*}{1 + \phi^*}, \quad (33)$$

where  $\phi^* = n \cdot \Phi / Y^*$  is an increasing returns to scale indicator for the production function and it represents the weight of total fixed costs in aggregate output. We can notice its equilibrium value is a decreasing function of the equilibrium output. Note that, in this case, the market power has a negative correlation with aggregate output, which is consistent with counter-cyclical mark-ups as documented in the empirical literature<sup>31</sup>.

Despite the fact this hypothesis is considered in Dixon and Lawler (1996), the treatment of fiscal-policy effectiveness in an endogenous-mark-up framework is done in Costa (2004). However, there are other endogenous-mark-ups models, though not specifically dedicated to fiscal-policy effectiveness, that are surveyed in Rotemberg and Woodford (1999).

In the case treated here, it is the real wage that reacts to fiscal policy, as we have  $w^* = 1 - \mu^*$ . Nonetheless, considering the reduced-form macroeconomic production function with free entry  $Y = (1 - \mu) \cdot L$ , the endogenous mark-up may work as a

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<sup>30</sup>For a more detailed analysis of the underlying process and its fundamentals see Costa and Dixon (2007).

<sup>31</sup>E.g. see Martins et al. (1996) or Martins and Scarpetta (2002).

productivity shock, but it originates in the aggregate-demand side in the case of fiscal policy<sup>32</sup>.

Thus, an increase in public consumption translates into a mark-up reduction, i.e. a real-wage increase  $w_G^* = (\mu^*)^2 \cdot k^* / (n \cdot \Phi) > 0$ . Therefore, the increase in intra-industrial competition induced by an expansionary fiscal policy leads to a second stimulus in private consumption, via real wages, reinforcing the multiplier mechanism and acting as a positive externality:

$$k_F^* \Big|_{\substack{dT_0^*=dG \\ \Pi_G^*=0}} = \frac{1 - \alpha}{1 - \frac{(\mu^*)^2}{n \cdot \Phi}} \geq 0. \quad (30.F)$$

The graphical representation of this mechanism is also given by Figure 8, where  $w^* = \Omega(Y)$  is obtained from equation (33). Despite the difference in the economic mechanism, the real-wage transmission mechanism is similar to the previous model.

Considering that  $\mu$  is now an endogenous variable, it makes no sense to calculate the derivative of this multiplier in order to the mark-up. However, any change in the parameter values or exogenous variables that leads to a higher mark-up (e.g. a smaller public consumption or a higher fixed cost) induces an increase in fiscal policy effectiveness.

Finally, considering the no-entry mechanism is the same as in the previous case, we have

$$\Gamma_F^* \equiv \frac{k_F^* \Big|_{\substack{dT_0^*=dG \\ \Pi_G^*=0}}}{k_A^* \Big|_{dT_0^*=dG}} = \frac{1 - \alpha \cdot \mu}{1 - \frac{(\mu^*)^2}{n \cdot \Phi}}.$$

Thus, near the initial equilibrium where  $\mu = \mu^*$ , the "long-run" multiplier is larger than the "short-run" one, as long as the monopoly power indicator is sufficiently large, i.e. as long as  $\mu^* > \alpha \cdot n \cdot \Phi$ .

Molana and Zhang (2001) study the steady-state effects in an intertemporal model similar to Costa (2004), where they assume that  $\mu = \mu(n)$  with  $\mu'(n) < 0$ . In a way similar to Galí (1995), these authors assume that there is imperfect competition in intermediate goods markets used to produce final goods and where a larger mass of varieties increases the elasticity of substitution amongst them. Despite the different endogenous mark-up generation mechanism, the qualitative results are similar<sup>33</sup>.

In both the endogenous mark-up and the taste for variety (or increasing returns to specialisation) cases, fiscal policy (or aggregate demand management policy in

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<sup>32</sup>There is a recent interest in this subject in the business-cycle literature. For an example, see Barro and Tenreyro (2006), *inter alia*.

<sup>33</sup>Chen et al. (2005) present a model that intends to extend the DMS framework to an endogenous-mark-up situation. However, as Costa and Palma (2007) notice, their model does not hold an endogenous mark-up mechanism, only a public-consumption externality in the production function.

general) have a positive effect on the efficiency level in the economy. This allows the balanced-budget multiplier to be greater than one and simultaneously, for a given employment level, the output to be larger. Consequently, taking into account the multiplier effect of public over private consumption is given by  $k^* - 1$ , it is possible to obtain a positive final effect on households consumption. For the same reason, leisure will not decrease so much as in the previous cases.

Therefore, it is possible that fiscal policy, without any direct externalities, has a positive effect on households welfare as long as: i) the effect of the efficiency gain is large enough to guarantee that  $k^* > 1$  and ii) the increase in private consumption is sufficiently important to offset the reduction in leisure.

### 2.4.7 Extensions and generalisations

Many additional works try to analyse the relationship between market power and fiscal policy effectiveness, but we cannot go through all of them here. However, some of the most interesting results can be briefly described in this section.

Amongst static models, Molana and Montagna (2000) introduce heterogeneity in the marginal product of labour in a DMS-style framework, also keeping love for variety. There, the zero-profit condition only applies to the "marginal firm (industry)," the reason why its more efficient competitors present positive profits. In their model, the absence of taste for variety leads to the entry of less efficient firms, so it reduces the average efficiency of the economy and also fiscal policy effectiveness. Love for variety tends to oppose this effect.

Still considering static models, Torregrosa (1998) supplies a demonstration for the conjecture in Molana and Moutos (1991) stating that a negative multiplier can be obtained when there exist only proportional taxes on labour income. Reinhorn (1998) studies optimal fiscal policy in a framework where public consumption directly affects consumers utility.

Finally, Censolo and Colombo (2008) study the way fiscal policy effectiveness is influenced by differences between the composition of private and public expenditures, when different market structures (perfect and monopolistic competition) exist simultaneously in the same economy.

## 3 Intertemporal models

In the following section, we will develop a dynamic intertemporal general equilibrium model which corresponds most closely to the static models considered in the previous

section<sup>34</sup>.

### 3.1 Intertemporal household

In particular, the instantaneous household utility follows as before: 1 and 2 with  $\lambda = 0$ . The infinitely-lived household has a discount rate of  $\rho > 0$  and, instead of (1), it maximises lifetime utility:

$$\max_{C,Z} U = \int_0^{\infty} u[(C(\tau), Z(\tau))] \cdot e^{-\rho \cdot \tau} \cdot d\tau. \quad (34)$$

In the dynamic model the household owns capital  $K(\tau)$  at moment  $\tau$  which it rents out to firms at price  $r(\tau)$ : hence its total income at time  $t$  is as before, labour income  $w(\tau) \cdot L(\tau)$  and equity profits  $\Pi(\tau)$ , plus the income from capital  $R(\tau) \cdot K(\tau)$ .<sup>35</sup>

Notice that, with an infinitely-living household, Ricardian equivalence holds. Thus, since we are not interested in studying how public debt evolves overtime, nothing is lost if we assume government follows a balanced-budget rule at each moment  $\tau$ . Also, for simplicity in this section we will assume that the government finances expenditure by a lump-sum tax  $P(\tau) \cdot G(\tau) = T_0(\tau)$ , i.e. we have  $\Psi(\tau) = 0$  and  $t(\tau) = 0$ .

We still consider the preferences for varieties given by equation (2) and the resource constraint in equation (3). Therefore, the intertemporal budget constraint can be simply expressed in terms of aggregate variables. The household can choose to allocate its income between consumption, paying tax or accumulating capital. The accumulation of capital is thus:

$$\dot{K}(\tau) = \frac{w(\tau) \cdot L(\tau) + R(\tau) \cdot K(\tau) + \Pi(\tau)}{P(\tau)} - C(\tau) - G(\tau). \quad (35)$$

For simplicity we ignore time indices ( $\tau$ ) from this point onwards. Also, we continue to choose the composite good as numéraire, so  $P(\tau) = 1$ .

### 3.2 Firm and production

The representative firm's decision is inherently static, since it rents capital from the household. Each instant  $\tau$ , the firm employs labour and capital to produce output:

$$y_{i(j)} = \max \{ F(K_{i(j)}, N_{i(j)}) - \Phi \}. \quad (36)$$

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<sup>34</sup>This model is based on lecture notes by Huw Dixon "Imperfect competition and macroeconomics" used for PhD courses given at a variety of institutions, including Finnish Doctoral Programme 1996, ISEG/TULisbon 1999, Munich (CES ifo) 2000, as well as York and Cardiff.

<sup>35</sup>We ignore depreciation of capital in order to keep the presentation simple. Considering a positive depreciation rate,  $\delta > 0$ , does not change the quality of results.

where we assume that  $F_K > 0$ ,  $F_N > 0$ ,  $F_{KK} < 0$ ,  $F_{NN} < 0$ ,  $F_{KN} > 0$ , also that function  $F(\bullet)$  is homogeneous to degree 1 (HoD1), i.e. the technology would present constant returns to scale (CRtS) if  $\Phi$  was equal to zero, and the Inada conditions hold. The firm faces the demand curve (16). Given the real wage and rental on capital, the first order conditions for profit maximization imply (in a symmetric industry equilibrium):

$$(1 - \mu) \cdot F_{K_{i(j)}} = R; \quad (1 - \mu) \cdot F_{N_{i(j)}} = w. \quad (37)$$

Since the marginal products of labour and capital are the same across all firms (this is ensured by competitive factor markets), we can rewrite the household's accumulation equation using (37) as

$$\dot{K} = (1 - \mu) \cdot (F_N \cdot N + F_K \cdot K) + \Pi - C - G.$$

Since function  $F(\bullet)$  HoD1 in  $(K, N)$ , by Euler's Theorem<sup>36</sup> we have:

$$\dot{K} = (1 - \mu) \cdot F(K, N) + \Pi - C + G.$$

Furthermore, in a symmetric equilibrium where  $p(j) = P = 1$ , the profits of each firm are simply:

$$\begin{aligned} \Pi_{i(j)} &= p(j) \cdot y_{i(j)} - TC_{i(j)} = \\ &= y_{i(j)} - w \cdot N_{i(j)} - R \cdot K_{i(j)} = \\ &= [F(K_{i(j)}, N_{i(j)}) - \Phi] - (1 - \mu) \cdot F(K_{i(j)}, N_{i(j)}) = \\ &= \mu \cdot F(K_{i(j)}, N_{i(j)}) - \Phi, \end{aligned}$$

so that aggregating across all firms we have

$$\Pi = \mu \cdot F(K, N) - n \cdot m \cdot \Phi. \quad (38)$$

where  $N = \int_0^n \sum_{i(j) \in \mathfrak{S}(j)} N_{i(j)} \cdot dj$  is the total demand for labour and  $K = \int_0^n \sum_{i(j) \in \mathfrak{S}(j)} K_{i(j)} \cdot dj$  represents total demand for capital. Again, equilibrium in the labour market implies that  $N = L$ .

Under imperfect competition, a wedge is driven between the marginal product of each factor and the factor return: this leads to each additional unit of output yielding a marginal profit of  $\mu$  (since only a proportion  $(1 - \mu)$  is used to pay for labour and capital. There is also the overhead fixed cost, which may make the profit per firm negative or positive, depending upon the level of output.

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<sup>36</sup>When  $F(\bullet)$  is HoD1,  $F(K, N) = F_K \cdot K + F_N \cdot N$ .



### 3.3 The household's intertemporal optimization

The household chooses  $(C(\tau), L(\tau))$  to maximize lifetime utility (34) subject to the accumulation equation (35), in effect a dynamic budget constraint. The current-value Hamiltonian for this intertemporal optimisation problem is

$$\mathcal{H} = u(C, 1 - L) + \xi \cdot (w \cdot L + R \cdot K + \Pi - C - G),$$

The first-order conditions for this are

$$\begin{aligned} \mathcal{H}_C &\equiv u_C - \xi = 0; \\ \mathcal{H}_L &\equiv -u_Z + \xi \cdot w = 0; \\ \mathcal{H}_K &\equiv \xi \cdot R = -\dot{\xi} + \rho \cdot \xi; \\ \lim_{\tau \rightarrow \infty} [e^{-\rho \cdot \tau} \cdot \xi(\tau) \cdot K(\tau)] &= 0. \end{aligned}$$

Using (37) we can express  $(w, R)$  in terms of the marginal products. Hence, we derive two basic optimality conditions:

**Intra-temporal optimality** Once again<sup>37</sup>,  $M(C, Z)$ , the marginal rate of substitution between consumption and leisure equals the net real wage rate

$$M(C, Z) \equiv \frac{u_Z}{u_C} = (1 - \mu) \cdot F_N.$$

**Inter-temporal optimality** The *Euler condition*. Assuming that  $u_{CZ} = 0$ , i.e. assuming the felicity function is additively separable, this can be written as

$$\frac{\dot{C}}{C} = \theta \cdot [(1 - \mu) \cdot F_K - \rho],$$

where  $\theta \equiv -u_C / (C \cdot u_{CC})$  is the elasticity of intertemporal substitution in consumption.

### 3.4 Steady State

In the steady state, we have the condition that  $\dot{C} = 0$ . Hence the Euler condition implies that

$$(1 - \mu^*) \cdot F_K^* = \rho,$$

where asterisks stand for steady-state values. In the Walrasian case ( $\mu^* = 0$ ) this is just the modified golden rule. What imperfect competition does is to discourage

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<sup>37</sup>See appendix 1.

investment, since the returns on investment are depressed (there is a wedge between the marginal product and the rental on capital).

Now, under the assumption that function  $F(\bullet)$  is HoD1, we can write it in factor intensive form  $F(K, L) = L.F\left(\frac{K}{L}, 1\right) = L.f(k)$ , where  $k \equiv K/L$ . Hence the steady-state Euler condition is

$$f'(k^*) = \frac{\rho}{1 - \mu^*}, \quad (39)$$

where  $f'(k) = F_K\left(\frac{K}{L}, 1\right) > 0$  and  $f''(k) = F_{KK}\left(\frac{K}{L}, 1\right) < 0$ .

Let us now consider the special case of monopolistic competition where every industry is a monopoly, i.e.  $m(\tau) = 1$ . In this case, given the CES preferences in (2), the mark-up is also constant and given by  $\mu(\tau) = \mu = 1/\sigma$ .

With this particular market structure we can write the solution to this as  $k^* = k^*(\mu)$  with  $k^{*'} < 0$ . With  $F(\bullet)$  HoD1, the steady-state Euler condition is very powerful: not only is the marginal product of capital determined, but so is the steady-state wage rate

$$w^*(\mu) = f[k^*(\mu)] - \frac{\rho.k^*(\mu)}{1 - \mu}. \quad (40)$$

With this we have the income expansion path (IEP) for consumption and leisure, defined by the intertemporal optimality condition and the steady state wage

$$\frac{u_Z^*}{u_C^*} = (1 - \mu).F_N^* = w^*(\mu). \quad (41)$$

As in the static model, the IEP will be upward sloping in  $(Z, C)$ , since both consumption and leisure are normal; it will be linear if preferences are quasi-homothetic; it will be a linear ray through the origin if preferences are homothetic.

There is a steady-state relationship between income and consumption given by<sup>38</sup>:

$$C^* = L^*.f[k^*(\mu)] - n^*.\Phi - G^*. \quad (42)$$

We will call this the Euler frontier (EF).

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<sup>38</sup>This can be derived from the budget constraint:

$$\begin{aligned} C^* &= w^*(\mu).L^* + R^*.K^* + \Pi^* - G^* = \\ &= w^*(\mu).L^* + \frac{\rho}{1 - \mu}L.k^*(\mu) + \mu.L^*.f(k^*) - n^*.\Phi - G^* = \\ &= L^*.f(k^*(\mu)) - n^*.\Phi - G^*. \end{aligned}$$



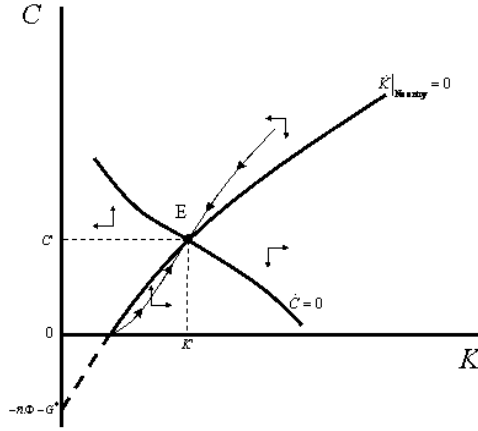


Figure 10: The Saddle-Point Stable Equilibrium

The dynamics are represented by the two isoclines:

$$\dot{C} = 0 : (1 - \mu) \cdot F_K [K, L(C, K, \mu)] - \rho = 0; \quad (43)$$

$$\dot{K} = 0 : F [K, L(C, K, \mu)] - n \cdot \Phi - G - C = 0. \quad (44)$$

The consumption isocline is *downward* sloping in  $(K, C)$ : it is defined by the equality of the marginal revenue product of capital being equal to the discount rate. To the right of the consumption isocline, consumption is falling, since  $(1 - \mu) \cdot F_K < \rho$ ; to the left it is increasing. The capital isocline has the standard upward-sloping shape<sup>41</sup>: it need not be globally concave due to the effect of  $K$  on the labour supply. The phase diagram thus has a unique saddle-path solution as depicted in Figure 10.

### 3.5 The effect of imperfect competition on the long-run equilibrium

In this section we illustrate the effect of a change in  $\mu$  on the steady-state equilibrium from both  $(1 - L, C)$  space and  $(K, C)$  space. First, let us analyse the consequences of imperfect competition in leisure-consumption space. We have two effects of an increase in the degree of imperfect competition:

<sup>41</sup>See appendix 7. Notice that with  $\delta > 0$  the capital isocline would present the usual hump shape: increasing before the modified golden-rule capital stock and decreasing afterwards.

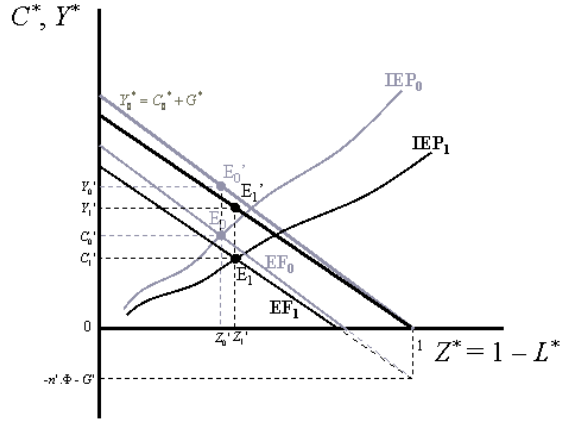


Figure 11: Market Power and the Steady-State Equilibrium (I)

- The EF curve rotates anti-clockwise. Since we have

$$f'(k^*) = \frac{\rho}{1 - \mu};$$

$$\frac{dk^*}{d\mu} = \frac{f'(k^*)}{(1 - \mu) \cdot f''(k^*)} = \frac{\rho}{(1 - \mu)^2 \cdot f''(k^*)} < 0.$$

- The real wage falls, so that the IEP moves to the right. Since from (40)

$$w^*(\mu) = f[k^*(\mu)] - \frac{\rho \cdot k^*(\mu)}{1 - \mu};$$

$$\frac{dw^*}{d\mu} = -\frac{\rho \cdot k^*(\mu)}{(1 - \mu)^2} < 0.$$

These two effects are depicted in Figure 11, where the equilibrium moves from  $E_0$  to  $E_1$  when we compare a low-markup steady-state ( $\mu = \mu_0$ ) with a large-markup one ( $\mu = \mu_1 > \mu_0$ ).

Clearly, the shift in the IEP represents a pure substitution effect. As the wage falls, the household substitutes leisure for consumption. The EF rotation, however, marks a counterbalancing income effect: income is lower for any  $L$  when  $\mu$  is higher. This operates to increase labour supply and decrease consumption. So, both income and substitution effects operate to reduce consumption: they operate in opposite ways on the labour supply. In Figure 11 leisure increases, which means that the income effect dominates for that specific example.

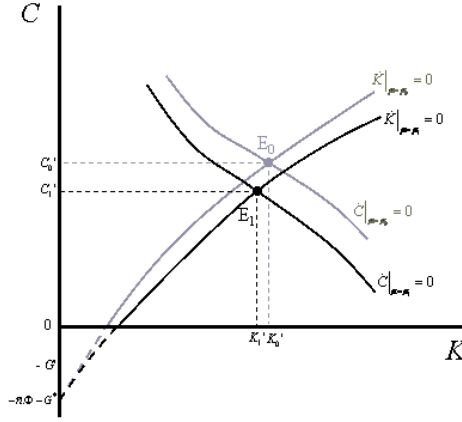


Figure 12: Market Power and the Steady-State Equilibrium (II)

Turning to capital-consumption space and the phase diagram, the way to understand the effect of  $\mu$  is via the effect on  $L$ : for given  $(K, C)$ , an increase in  $\mu$  increases the wedge between the marginal product of labour and the wage, hence leading to a reduction in the labour supply. Less labour means that both total output and the marginal product of capital fall. Hence we have two effects of an increase in  $\mu$ :

- The consumption isocline shifts to the left (since  $F_K$  falls as  $L$  decreases).
- The capital isocline shifts downwards, as there is less output given  $(K, C)$ .

The shift from equilibrium  $E_0$  to  $E_1$  in Figure 11 is represented in  $(K, C)$  in Figure 12. Note that whilst steady-state consumption falls, the effect on capital is potentially ambiguous. This is because the effect of  $\mu$  on labour supply is ambiguous. Here capital decreases, which is compatible with the reduction in employment observed in Figure 11.

### 3.6 Free Entry

Until now, we have assumed that the number of firms is fixed across time, so that  $n(\tau) = n$ . In this case, aggregate output is given by:

$$Y(\tau) = L(\tau) \cdot f[k(\tau)] - n \cdot \Phi. \quad (45)$$

If there is instantaneous free entry which drives profits to zero, from (38), for given  $(K, L)$ , profits are zero when

$$n(\tau) = \frac{\mu \cdot F[K(\tau), L(\tau)]}{\Phi} = \frac{\mu}{\Phi} \cdot L(\tau) \cdot f[k(\tau)]. \quad (46)$$

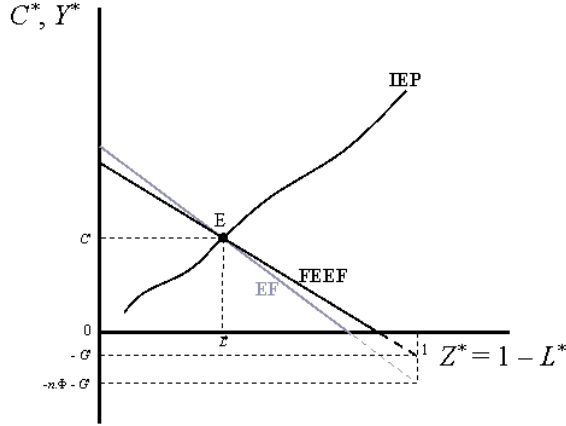


Figure 13: Steady-State Equilibrium with Free Entry (I)

In this case, aggregate output is given by

$$Y(\tau) = (1 - \mu) \cdot F [K(\tau), L(\tau)] = (1 - \mu) \cdot L(\tau) \cdot f [k(\tau)]. \quad (47)$$

Let us turn to leisure-income space. Free entry does not affect the IEP, which just depends on the real wage  $w^*(\mu)$  which is not influenced by entry. However, entry does affect the Euler frontier (42) since the level aggregate overheads  $n^* \cdot \Phi$  varies according to (46). In factor-intensive notation, we have the "Free Entry Euler Frontier", FEEF for short: In the case of free-entry, this simplifies to

$$C^* = L^* \cdot (1 - \mu) \cdot f [k^*(\mu)] - G^* \quad (48)$$

The FEEF is steeper than the EF: a higher labour supply means that there are more firms which increases the socially wasteful overhead  $n^* \cdot \Phi$  thus reducing consumption by more than if the number of firms is fixed. The two lines meet at the labour supply where the free entry number of firms happens to be equal to the exogenously given number of firms<sup>42</sup>: for labour supplies below this the FEEF lies above the EF (since there are less firms); for labour supplies above this the FEEF lies below the EF. This is depicted in Figure 13, where EF and FEEF intersect at point E.

<sup>42</sup>From (46), for given  $n$ , the critical level of labour supply is

$$\bar{L}^* = \frac{n \cdot \Phi}{\mu \cdot f [k^*(\mu)]}.$$

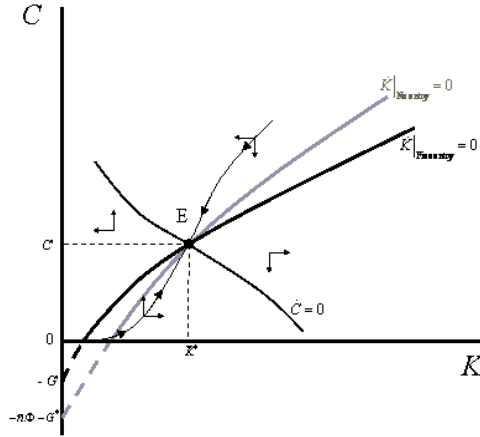


Figure 14: Steady-State Equilibrium with Free Entry (II)

If we turn to  $(K, C)$  space, free entry does not influence the consumption isocline (since overheads do not influence the marginal product of capital). The capital isocline becomes

$$\dot{K} = 0 : (1 - \mu) \cdot F [K, L(C, K, \mu)] - C - G = 0. \quad (49)$$

The capital isocline is affected: the fixed  $n$  isocline is steeper and intersects the free-entry isocline at the capital stock where the number of firms under free entry equals the fixed  $n$  (which is  $\bar{K}^*$ ). For capital stocks below that, the free entry isocline implies less overheads and lies above the fixed  $n$  isocline, and for capital above that level, it lies below the fixed  $n$  case. We depict this in figure 14.

Also, we can easily see entry does not affect the dynamics of the steady-state equilibrium<sup>43</sup>. Notice the fixed-markup monopolistically competitive model with free entry is formally equivalent to a Ramsey model with more inefficient production function given by  $(1 - \mu) \cdot F$ .

### 3.7 Fiscal Policy, entry, and imperfect competition.

We will explore the effects of an increase in government expenditure funded by a lump-sum tax. This will divide into the long-run steady-state effects and the short-run impact effects, as well as the transition towards the steady state. We will assume that in the initial position we start off with zero-profits, even in the case of a fixed number of firms. That means that the EF and FEEF both pass through the same point in steady state, i.e. point  $E_0$  in Figure 15.

<sup>43</sup>See appendix 6.



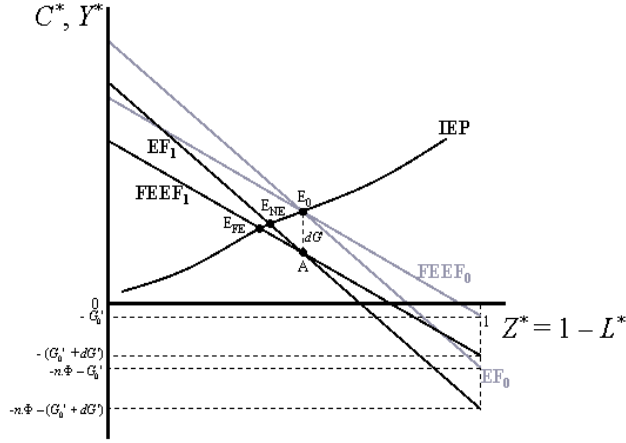


Figure 15: Long-Run Effects of Fiscal Policy

Turning first to the long-run steady-state effects of an increase in government expenditure. In leisure-consumption space, the IEP is unaffected by the change in  $G$ . The EF and FEEF are both shifted down by a vertical distance equal to the increase in government expenditure. The new steady states are  $E_{NE}$  for a fixed number of firms, and  $E_{FE}$  with free entry. As in the static case, the multiplier is "Walrasian" in the sense of being less than one and greater than zero. The drop in consumption is less than the increase in government expenditure<sup>44</sup>. How much less is determined by the slope of the EF and FEEF: a steeper slope results in more crowding out of consumption in steady state. This leads us to three simple conclusions:

- The multiplier with free-entry is smaller than the multiplier with a fixed number of firms, since FEEF is steeper than EF. This result is found in Coto-Martinez and Dixon (2003) for an open economy context.
- Employment increases (leisure decreases) as  $G$  increases, and the increase in the labour supply is greater when there is free entry.
- An increase in imperfect competition makes both the FEEF and the EF flatter, leading to less crowding out and a larger output multiplier in each case.

None of these results requires that the initial steady-state is the same (where the FEEF and EF intersect) if there are homothetic preferences (and hence a linear IEP). If the IEP is non-linear, the result will hold if the initial position is the same. The

<sup>44</sup>See the appendix.

intuition behind these results is the following. An increase in government spending financed by a lump-sum tax makes the household worse off: so, it cuts back on the good things in life, consumption and leisure. Because the economy is less efficient (at the margin) with free entry, the required effort to supply the extra output to the government is greater than with fixed  $n$ , so that consumption and leisure decline more under free-entry. An increase in imperfect competition means that whether there is a fixed number of firms or free-entry, the weight of the tax burden falls more heavily on leisure so that the crowding out of consumption is less.

If we compare the steady-states in  $(1 - L, C)$  space, there is a striking similarity between the static and dynamic models. Now let us turn to the dynamics of the model with imperfect competition. In Coto-Martinez and Dixon (2003) these results are generalised to a small open economy setting.

### 3.8 Fiscal policy: short-run dynamics.

In  $(K, C)$  we have the two accumulation equations which we assume intersect at the initial steady-state. In this case, the fixed- $n$  capital accumulation schedule is steeper than the free-entry curve, as seen above. The effect of a permanent increase in  $G$  is to shift both curves down vertically in  $(K, C)$  space. The new steady-state equilibria are  $E_{NE}$  for fixed  $n$  and  $E_{FE}$  with free entry (these two correspond exactly to the points with identical notations in Figure 15). We can see that the steady-state capital stock increases by more when there is free entry: this reflects the increase in the labour supply (decline in leisure) with the same capital/labour ratio in both cases. Since both  $E_{NE}$  and  $E_{FE}$  are saddle-point stable, consumption will drop down and follow an upward sloping path to the new steady state.

In order to compare what happens along the paths in both cases. Considering  $\beta > 0$  is the slope of the stable manifold<sup>45</sup>, we can approximate the consumption value using the first-order Taylor expansion:

$$C(\tau) = C^* + \beta \cdot [K(\tau) - K^*]. \quad (50)$$

We are especially interested in what happens at time  $\tau = 0$ , when the fiscal shock occurs. In both cases we observe a decrease in  $C(0)$  due to the combination of two effects: (i) the long-run consumption level decreases as described before and (ii) the capital stock is below its long-run optimal level (i.e.  $K(0) < K^*$ )<sup>46</sup>. However, if we want to compare the no-entry to the free-entry versions of the model, we can notice that

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<sup>45</sup>See appendix 7 for an algebraic expression.

<sup>46</sup>See the values for the long-run multipliers in the appendix.

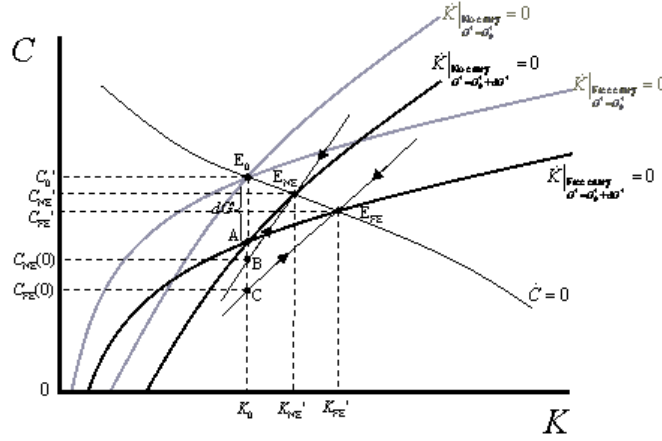


Figure 16: Short-Run Effects of Fiscal Policy

$$\Lambda C(0) = \Lambda C^* - \beta_{NE} \cdot \Lambda K^* + \Lambda \beta \cdot [K(0) - K_{NE}^*], \quad (51)$$

where  $\Lambda X \equiv X_{NE} - X_{FE}$ ,  $X_{NE} = X|_{\text{No entry}}$  and  $X_{FE} = X|_{\text{Free entry}}$ . We can see in Figure 16 that  $\Lambda C^* > 0$ , i.e. the long-run drop in consumption is larger under free entry than in the fixed- $n$  model. We can also observe that  $\Lambda K^* < 0$ , i.e. the long-run increase in the optimal capital stock is larger under free entry. Finally, we know that  $K(0) - K_{NE}^* < 0$  for the increase in government expenditure depicted in this example. Thus, we can expect a larger short-run decrease in private consumption in the free entry case ( $\Lambda C(0) < 0$ ), unless the stable manifold is much steeper in the no-entry case, i.e.  $\Lambda \beta > \frac{\Lambda C^* - \beta_{NE} \cdot \Lambda K^*}{K_{NE}^* - K(0)} > 0$ .

Let us use a numerical illustration in order to see what can happen in specific models. First, we assume the felicity function is isoelastic in both consumption and leisure, i.e.

$$u[(C(\tau), Z(\tau))] = \frac{C(\tau)^{1-\frac{1}{\theta}} - 1}{1 - \frac{1}{\theta}} + b \cdot \frac{Z(\tau)^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}},$$

where  $\theta, \psi, b > 0$ . Second, let us assume  $F(\cdot)$  is Cobb-Douglas, i.e.

$$F[K(\tau), N(\tau)] = A \cdot K(\tau)^\eta \cdot N(\tau)^{1-\eta},$$

where  $0 < \eta < 1$ . Now, we choose the following parameter values:

$\eta$	$\rho$	$\theta$	$\psi$	$\sigma$	$b$	$G_0^*$	$\Phi$
1/3	0.04	1	1	10	10/6	0.1643	0.00009

the value of  $\eta$  was chosen in order to generate a long-run capital share in total income equal to one third. The value for  $\rho$  implies a 4 per cent return on capital per period. The values for  $\theta$  and  $\psi$  imply elasticities of intertemporal substitution equal to one for both consumption and leisure. The value of  $\sigma$  gives rise to a 11 per cent price-wedge over the marginal cost in the steady state. The value for  $b$  was chosen in order to generate  $L^* = 1/3$ , the value for  $G_0^*$  is the one that leads to a 20 per cent steady-state share of government consumption in output, and the value for  $\Phi$  is such that profits are zero in the initial equilibrium ( $E_0$  in Figure 16) when  $n = 1000$ .

For this numerical illustration, a permanent one per cent increase in  $G$  leads to an immediate 1.3 per cent decrease in consumption in the no-entry case and to a 1.4 reduction in the free-entry case. Thus, in this example, despite the fact that the stable manifold is steeper in the no-entry case (i.e.  $\Lambda\beta > 0$ ), the last term on the right-hand-side of equation (51) is smaller than the sum of the positive effects. This example corresponds to Figure 16: in the no-entry case the equilibrium response of households leads to the short-run equilibrium represented by point B, whilst point C represents its no-entry counterpart.

We also varied all the parameters in their ranges and obtained similar results, i.e. for these functionals we could not numerically generate a situation where  $\Lambda C(0) < 0$ . Of course we cannot guarantee such an event would not occur with different felicity or production functions, but we can expect this result to hold in most of the real policy experiments.

### 3.8.1 Extensions and generalisations

As we saw, dynamic models allow us to study not only the long-run (steady-state) effects, but also the short-run effects that occur due to the fact that agents may use a part of their resources presently available to obtain better future outcomes, according to a discounted optimisation problem (either utility or profits). Amongst these models, Heijdra (1998) is an inevitable reference where a continuous-time dynamic model with monopolistic competition is presented, including love for variety and Ethier effects (i.e. increasing returns from diversity in the investment goods sector). Costa (2007) (the effect of capital depreciation), Devereux et al. (1996) (increasing returns to specialisation), Harms (2002) (persistence of fiscal shocks), Heijdra et al. (1998) (distortionary taxation and useful public expenditure), Linneman and Schabert (2003) (price stickiness and fiscal-monetary policies interaction), Molana (1998) (intertemporal substitution between current leisure and future consumption), or Ravn et al. (2006) (endogenous markups due to deep habits) are also examples of important references in this line of research.

On the empirical front, the recent interest on the quantitative effects of fiscal shocks, especially when markups respond counter-cyclically to them, can be observed

in Afonso and Costa (2010), Hall (2009), or Monacelli and Perotti (2009).

## 4 Concluding remarks

In this paper we studied fiscal policy effectiveness in static general equilibrium models where there is imperfect competition in goods markets. We observed this effectiveness, both over output and households welfare, and its relation with the degree of monopoly depend upon a large number of factors, namely the ones analysed here: i) the way government budget is financed; ii) the type of taxes used; iii) the possibility of free entry; iv) consumers preferences; v) the existence of increasing returns on the mass of varieties; and vi) the existence of endogenous mark-ups. Overall we find that the effectiveness of fiscal policy does indeed depend on the degree of imperfect competition. This is because the mark-up distorts the relative price of consumption and leisure (the latter becomes cheaper). For a broad range of results (with many caveats), we find that the multiplier is increasing in the degree of imperfect competition. However, the effect on welfare will still tend to be negative: the reason output increases is that households are induced to work harder by being taxed. In order to get the "Keynesian" welfare effect, you need to have some extra ingredient: for example increasing returns, love for variety, or an endogenous mark-up.

In dynamic models, many of the same issues arise, particularly if we focus on the steady-state results. However, we have additional dimension of the real-time dynamics and in particular the comparison of short- and long-run effects. In both static and dynamic models, the role of entry is crucial, as was argued by Startz (1989). With a fixed mass of varieties, extra output is produced in a marginally efficient way. With free entry, extra output sucks in additional firms and overheads. In many models this leads to a lower multiplier and lower welfare.

From the point of view of the history of economic thought it is rather strange that John Maynard Keynes, Joan Robinson the founder of monopolistic competition theory, and Richard Khan, who invented the multiplier, coexisted in the same time and place (Cambridge, England in the 1930s). Despite the space-time and intellectual proximity between them, the link was not made between imperfect competition and macroeconomics until much later<sup>47</sup>. In this survey, we have traced through general equilibrium macroeconomic models how this "tantalizing possibility" was realised in the ensuing 60 years. As we have seen, the simple fact that the imperfectly competitive equilibrium is not Pareto optimal does not imply that Pareto improving fiscal policy is generally possible. However, it does have important and more-or-less Keynesian features as regards the multiplier.

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<sup>47</sup>See Marris (1991) for more details, especially pp. 181-187.

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## 5 Appendices

### 5.1 Private consumption and labour supply functions

From the second step in the optimisation process, we obtain two first-order conditions that imply equalising the marginal rate of substitution between leisure and consumption ( $M$ ) to the real wage:

$$M(C, Z) \equiv \frac{u_Z(C, Z)}{u_C(C, Z)} = \omega \cdot (1 - t) \equiv \omega_N,$$

where  $M_C = (u_{ZC} - u_{ZC} \cdot M) / u_C > 0$  and  $M_Z = (u_{ZZ} - u_{CZ} \cdot M) / u_C < 0$ . Solving this equation in order to  $L$  we obtain

$$L = 1 - H(\omega_N, C),$$

where  $H_{\omega_N} = 1/M_Z < 0$  and  $H_C = -M_C/M_Z > 0$ . Thus, taking into account that  $C = \omega_N \cdot L + \pi_N$ , we come to the following conclusion:

$$\begin{aligned} \mathfrak{C}_{\omega_N} &= \frac{-\omega_N \cdot H_{\omega_N} + L}{1 + \omega_N \cdot H_C} > 0; & \mathfrak{L}_{\omega_N} &= -H_{\omega_N} - H_C \cdot \mathfrak{C}_{\omega_N} \begin{cases} > 0 & \iff -H_{\omega_N} > H_C \cdot \mathfrak{C}_{\omega_N}; \\ = 0 & \iff -H_{\omega_N} = H_C \cdot \mathfrak{C}_{\omega_N}; \\ < 0 & \iff -H_{\omega_N} < H_C \cdot \mathfrak{C}_{\omega_N}; \end{cases} \\ \mathfrak{C}_{\pi_N} &= \frac{1}{1 + \omega_N \cdot H_C} > 0; & \mathfrak{L}_{\pi_N} &= -H_C \cdot \mathfrak{C}_{\pi_N} < 0. \end{aligned}$$

The sign of  $\mathfrak{L}_{\omega_N}$  depends on the traditional relationship between the substitution effect (given by  $-H_{\omega_N} > 0$ ) and the income effect (given by  $-H_C \cdot \mathfrak{C}_{\omega_N} < 0$ ).

### 5.2 The production function and returns to scale

Let us consider the first branch of the production function in equation (14) and let us assume we want to move from a labour utilisation of  $L_i = L_i^0 > \Phi/A_i$ , generating an output of  $y_i^0 = A_i \cdot L_i^0 - \Phi > 0$ , to another one where  $L_i = \eta \cdot L_i^0$  with  $\eta > 1$ . Thus, we obtain an output for firm  $i$  equal to  $\eta \cdot (A_i \cdot L_i^0 - \Phi) + (\eta - 1) \cdot \Phi = \eta \cdot y_i^0 + (\eta - 1) \cdot \Phi \geq \eta \cdot y_i^0$ . Therefore, if there is no need for an overhead quantity of "administrative" labour ( $\Phi = 0$ ), the production level is exactly equal to  $\eta$  times the initial one, i.e. there are constant returns to scale. However, if the production process demand an overhead quantity of this type of labour ( $\Phi > 0$ ), we have a production level greater than the initial one - in fact, greater in  $(\eta - 1) \cdot \Phi$  units of good  $j$  -, i.e. there are increasing returns to scale.

### 5.3 Love for variety and increasing returns to specialisation

In Devereux et al. (1996), instead of love for variety, a different assumption is used to model the effect of the mass of goods in the macroeconomic equilibrium: increasing returns to specialisation. There, instead of a basket of final goods we have a single final good ( $Y$ ) that is sold in a competitive market at a price  $P$ , using solely intermediate goods in its production. The production function of this final good has the same structure as equation (2):

$$Y = n^{\frac{1-\lambda}{1-\sigma}} \cdot \left[ \int_0^n d(j)^{\frac{\sigma-1}{\sigma}} \cdot dj \right]^{\frac{\sigma}{\sigma-1}},$$

where  $d(j) = \sum_{i(j) \in \mathfrak{S}(j)} y_{i(j)}$  represents the total production of intermediate good  $j$ , produced by  $m(j)$  producers. Thus, considering the final good market equilibrium ( $Y = D$ ), demand for good  $j$  by the final good sector has the same expression than equation (11). Therefore, this type of increasing returns (beside those generated by  $\Phi > 0$ ) is formally equivalent to the love for variety case explicitly analysed.

### 5.4 Wages and multipliers with increasing returns to variety

With free entry ( $\Pi^* = 0$ ), and given both the real wage in equation (25) and the equilibrium mass of firms in (32), we may substitute these values in the equilibrium equation (30) obtaining

$$w^* = \frac{Y^* - (1 - \alpha) \cdot G}{\alpha}.$$

Now, for a given value of  $n$ , we also know the equilibrium value of the non-wage income is given by  $\Pi^* = \mu \cdot Y^* - w^* \cdot n \cdot \Phi$ , so its response to the entry of new firms is given by

$$\frac{\partial \Pi^*}{\partial n} = - \frac{(1 - \mu) \cdot \Phi \cdot n^{\frac{\gamma}{1-\gamma}}}{(1 - \alpha \cdot \mu) \cdot (1 - \gamma)} \cdot \left( 1 - \gamma \cdot \frac{\mu \cdot \alpha}{\Phi \cdot n} \right).$$

Taking into account that, with free entry ( $\Pi^* = 0$ ), we have  $w^* = \mu \cdot Y^* / (\Phi \cdot n)$  and  $Y^* = \alpha \cdot w^* + (1 - \alpha) \cdot G$ , then the reaction of profits to the mass of firms is given by

$$\frac{\partial \Pi^*}{\partial n} = - \frac{(1 - \mu) \cdot \Phi \cdot n^{\frac{\gamma}{1-\gamma}}}{(1 - \alpha \cdot \mu) \cdot (1 - \gamma)} \cdot [1 - \gamma \cdot (1 - \alpha) \cdot g^*].$$

In order for the model to have economic meaning, it is necessary that this partial derivative is negative, i.e. it is necessary that  $1 - \gamma \cdot (1 - \alpha) \cdot g^* > 0$ . Considering this

assumption, an increase in the number of firms reduces total profits in the economy, as it reduces the profits of incumbents.

Heijdra and van der Ploeg (1996) advance another argument for this assumption to hold: if entry is not automatic, but it is a slow (continuous-time) process given by  $\dot{n} = \beta \cdot \Pi$ , with  $\beta > 0$ , the same assumption is necessary in order to have stability in the entry process ( $\partial \dot{n} / \partial n = \beta \cdot \partial \Pi / \partial n < 0$ ).

## 5.5 Employment as an implicit function of consumption, capital, and mark-up

The procedure to obtain the equilibrium employment in the economy as an implicit function  $L(C, K, \mu)$  is similar to the one used to derive the labour-supply function in the static model. However, we stop short of obtaining a real labour-supply function, as we only consider the substitution effect.

First, we start with the intratemporal condition equating the marginal rate of substitution of leisure by consumption to the real wage in equilibrium:

$$M(C, Z) \equiv \frac{u_Z(C, 1 - L)}{u_C(C, 1 - L)} = (1 - \mu) \cdot F_N(K, L).$$

Then, we apply the implicit-function theorem and obtain<sup>48</sup>

$$\begin{aligned} L_C(C, K, \mu) &\equiv -\frac{(1 - \mu) \cdot F_N \cdot u_{CC}}{u_{ZZ}} < 0; \\ L_K(C, K, \mu) &\equiv -\frac{(1 - \mu) \cdot F_{NK} \cdot u_C}{u_{ZZ}} > 0; \\ L_\mu(C, K, \mu) &\equiv \frac{F_N \cdot u_C}{u_{ZZ}} < 0. \end{aligned}$$

## 5.6 Dynamics of the monopolistic-competition model

Let us start with the no-entry model. Assuming all exogenous variables remain constant, we can approximate the system about its steady-state equilibrium using a first-order Taylor expansion given by

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<sup>48</sup>Remember we assumed  $u_{CZ} = 0$ . However, the signs of the following partial derivatives would not have been affected, had we assumed otherwise.

$$\begin{pmatrix} \dot{C} \\ \dot{K} \end{pmatrix} = \mathbf{J}|_{\text{No entry}} \cdot \begin{pmatrix} C - C^* \\ K - K^* \end{pmatrix},$$

$$\mathbf{J}|_{\text{No entry}} = \begin{pmatrix} C^* \cdot \theta^* \cdot F_{KN}^* \cdot L_C^* & C^* \cdot \theta^* \cdot (F_{KK}^* + F_{KN}^* \cdot L_K^*) \\ F_N^* \cdot L_C^* - 1 & F_K^* + F_N^* \cdot L_K^* \end{pmatrix}.$$

Notice that  $J_{11}|_{\text{No entry}} = \frac{F_{KN} \cdot u_Z}{u_{ZZ}} < 0$ ,  $J_{22}|_{\text{No entry}} = \frac{\rho}{1-\mu} - \frac{F_{KN} \cdot u_Z}{u_{ZZ}} > 0$ ,  $J_{21}|_{\text{No entry}} = -\frac{u_Z^2 \cdot u_{CC}}{u_C^2 \cdot (1-\mu) \cdot u_{ZZ}} - 1 < 0$ , and  $J_{12}|_{\text{No entry}} = -\frac{u_C}{u_{CC}} \cdot \left[ F_{KK} - \frac{F_{KN}^2 \cdot (1-\mu) \cdot u_C}{u_{ZZ}} \right]$ . Since we previously assumed the direct effect on output of an increase in the capital stock ( $F_K$ ) is larger than its indirect effect through the labour market ( $F_N \cdot L_K$ ), then we can unambiguously see that  $J_{12}|_{\text{No entry}} < 0$ .

Therefore, the trace, determinant, and discriminant of the Jacobian are given by

$$\begin{aligned} \text{tr} \left( \mathbf{J}|_{\text{No entry}} \right) &= (\lambda_1 + \lambda_2)|_{\text{No entry}} = \frac{\rho}{1-\mu} > 0, \\ \det \left( \mathbf{J}|_{\text{No entry}} \right) &= (\lambda_1 \cdot \lambda_2)|_{\text{No entry}} = (J_{11} \cdot J_{22} - J_{12} \cdot J_{21})|_{\text{No entry}} < 0, \\ \text{discr} \left( \mathbf{J}|_{\text{No entry}} \right) &= \left[ \text{tr} \left( \mathbf{J}|_{\text{No entry}} \right) \right]^2 - 4 \cdot \det \left( \mathbf{J}|_{\text{No entry}} \right) > 0. \end{aligned}$$

Since the discriminant of the characteristic polynomial is always positive, complex eigenvalues are ruled out, i.e. the steady-state equilibrium is non-oscillatory. Additionally, we have a negative (i.e. stable) eigenvalue ( $\lambda_1$ ) and a positive (i.e. unstable) eigenvalue ( $\lambda_2$ ), implying the steady-state equilibrium is saddle-point stable. Since we have the transversality condition that rules out explosive solutions, this no-entry equilibrium is globally stable.

In the free-entry monopolistically competitive equilibrium the Jacobian is given by

$$\mathbf{J}|_{\text{Free entry}} = \begin{pmatrix} C^* \cdot \theta^* \cdot (1-\mu) \cdot F_{KN}^* \cdot L_C^* & C^* \cdot \theta^* \cdot (1-\mu) \cdot (F_{KK}^* + F_{KN}^* \cdot L_K^*) \\ (1-\mu) \cdot F_N^* \cdot L_C^* - 1 & (1-\mu) \cdot (F_K^* + F_N^* \cdot L_K^*) \end{pmatrix}.$$

The signs of the elements in the matrix are not changed and the three measures associated with it are

$$\begin{aligned} \text{tr} \left( \mathbf{J}|_{\text{Free entry}} \right) &= (\lambda_1 + \lambda_2)|_{\text{Free entry}} = \rho > 0, \\ \det \left( \mathbf{J}|_{\text{Free entry}} \right) &= (\lambda_1 \cdot \lambda_2)|_{\text{Free entry}} = (J_{11} \cdot J_{22} - J_{12} \cdot J_{21})|_{\text{Free entry}} < 0, \\ \text{discr} \left( \mathbf{J}|_{\text{Free entry}} \right) &= \left[ \text{tr} \left( \mathbf{J}|_{\text{Free entry}} \right) \right]^2 - 4 \cdot \det \left( \mathbf{J}|_{\text{Free entry}} \right) > 0. \end{aligned}$$

Again, the usual dynamic features of an endogenous-labour Ramsey model are kept by this free-entry version: a unique globally stable non-oscillatory interior equilibrium.

In both cases analysed, the eigenvalues can be obtained as a function of the measures obtained above:

$$\begin{aligned}\lambda_1 &= \frac{1}{2} \cdot \left[ \text{tr}(\mathbf{J}) - \sqrt{\text{discr}(\mathbf{J})} \right] < 0, \\ \lambda_2 &= \frac{1}{2} \cdot \left[ \text{tr}(\mathbf{J}) + \sqrt{\text{discr}(\mathbf{J})} \right] > 0.\end{aligned}$$

## 5.7 Geometrical analysis of local dynamics in the $(K, C)$ space

The slopes of the isoclines are very simply related to the elements of the Jacobian, as they are both obtained by differentiating the two dynamic equations of the model. Therefore, we obtain

$$\left. \frac{dC}{dK} \right|_{\dot{C}=0} = -\frac{J_{12}}{J_{11}} < 0, \quad \left. \frac{dC}{dK} \right|_{\dot{K}=0} = -\frac{J_{22}}{J_{21}} > 0.$$

In order to compare the free-entry to the no-entry situation, let us consider a common initial equilibrium, i.e. one where  $n^* = n$ . It is easy to see that entry does not affect the slope of the consumption isocline as both  $J_{11}$  and  $J_{12}$  in the free-entry version are given by the values in the no-entry version simply multiplied by  $1 - \mu$ . The same happens to  $J_{22}$ , but not to  $J_{21}$ , where  $J_{21}|_{\text{Free entry}} = (1 - \mu) \cdot J_{21}|_{\text{No entry}} - \mu$ . Thus, we can see that

$$\left. \frac{dC}{dK} \right|_{\substack{\dot{K}=0 \\ \text{No entry}}} - \left. \frac{dC}{dK} \right|_{\substack{\dot{K}=0 \\ \text{Free entry}}} = \frac{\mu \cdot (F_K^* + F_N^* \cdot L_K^*)}{(F_N^* \cdot L_C^* - 1) \cdot [(1 - \mu) \cdot (F_N^* \cdot L_C^* - 1) - \mu]} > 0,$$

i.e. the capital-accumulation isocline is steeper when entry is barred than under free entry.

Now, the slope of the stable manifold is given by

$$\beta \equiv \left. \frac{dC}{dK} \right|_{\text{St. man.}} = \frac{\lambda_1 - J_{22}}{J_{21}} > 0.$$

We can easily see the stable arm is steeper than the  $\dot{K} = 0$  isocline, as  $\lambda_1 < 0$ .

## 5.8 8. The long-run multipliers

We can use the expressions obtained in the dynamic analysis to obtain the long-run fiscal multipliers. First of all, it is easy to see that

$$\frac{dC^*}{dG^*} = -\frac{J_{12}}{\det(J)} < 0, \quad \frac{dK^*}{dG^*} = \frac{J_{11}}{\det(J)} > 0.$$

Now, the long-run (net) output multiplier is given by  $1 + \frac{dC^*}{dG^*}$  and apparently cannot be signed unambiguously. However, we can use both the production and the employment functions to see that

$$\frac{dY^*}{dG^*} = J_{22} \cdot \frac{dK^*}{dG^*} + (1 + J_{21}) \cdot \frac{dC^*}{dG^*}.$$

Thus, the (net) output multiplier is unambiguously positive, i.e.  $\left| \frac{dC^*}{dG^*} \right| < 1$ .

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