The Design of a 'Two-Pillar' Monetary Policy Strategy

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Abstract

In this paper, it is argued that money supply in a narrow sense and repo interest rate are two independent monetary policy instruments when the effect of interest rate policy cannot be efficiently transmitted to the economy through the monetary and financial markets. In this case, the control of money supply is necessary to reduce the discrepancy between the repo interest rate and the interest rates at which private agents lend and borrow. Using a simple macro-economic model, this study shows how a two-pillar monetary policy strategy as practiced by the European central bank (ECB) can be conceived to guarantee macroeconomic stability and the credibility of monetary policy. This strategy can be interpreted as a combination of inflation targeting and monetary targeting. Well conceived monetary targeting with a commitment to a long-run money growth rate corresponding to inflation target could reinforce the credibility of central bank announcements and the role of inflation target as strong and credible nominal anchor for private inflation expectations. However, an inflation-targeting regime associated with Friedman's money supply rule can generate dynamic instability in output, inflation and money demand. Three feedback monetary targeting rules, of which the design depends on economic structure and central bank preferences, are discussed relative to their capability of warranting macroeconomic stability.

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1. Introduction

Over the last decade, more and more central banks have adopted a new framework for conducting monetary policy known as inflation targeting, which is presented by Mishkin (1999) as a successor to and more efficient in controlling inflation than monetary targeting. In this context, the two-pillar strategy of the ECB appears quite singular.

Several economists advocate that if money demand is stable at European level, monetary targeting makes a suitable concept for the ECB. Neumann and von Hagen (1995) suggest that monetary targeting is an effective device for anchoring medium-term inflation expectations. At the same time, this approach permits sufficient flexibility for leaning against the wind of currency appreciation and for responding to short-term events. In contrast, inflation targeting is likely to either prevent the ECB from gaining credibility or require responding to price level shocks in an overly contractionary fashion.

For von Hagen (1999), the Bundesbank's experience suggests that a strategy of money growth targeting might help the ECB to successfully establish and assert its control over monetary conditions in the monetary union, to define its policy goals and its role in macroeconomic policy and to establish its reputation for consistently pursuing these goals over time.

Political considerations (the need to demonstrate continuity with the policies of the Bundesbank) have apparently dictated that the ECB pays attention to monetary aggregates as well in its two-pillar monetary policy strategy. Many observers have interpreted the ECB's two-pillar strategy as a bridge between the monetary targeting strategy of the old Bundesbank and the more up-to-date inflation targeting approach (Bernanke et *al.*, 1999; Svensson, 2000; Rudebusch and Svensson, 2002; Mayer, 2006). In effect, the "economic pillar" resembles an

implicit form of inflation targeting and the "monetary pillar" a weak type of monetary targeting. According to Assenmacher-Wesche and Gerlach (2007), this is a "misinterpretation" which has lead to the criticism of the framework for being inconsistent and lacking clarity.

The disagreement among economists about the true nature of the two-pillar strategy is arguably due to the fact that the ECB provides neither an explicit representation of the inflation process nor an explanation for why it necessitates a two-pillar framework. In other words, it lacks a theory justifying the simultaneous use of monetary targeting and inflation targeting.

Beck and Wieland (2007) develop a justification for including money in the interest rate rule (ECB-style monetary cross-checking) by allowing for imperfect knowledge regarding unobservable such as potential output and equilibrium interest rates. Empirical models, provided for example by Gerlach (2004) and Assenmacher-Wesche and Gerlach (2007), try to justify the two-pillar strategy by assuming that inflation can be decomposed into a 'trend', which is explained by a smoothed measure of past money growth, and a deviation from that trend, which is accounted for by the output gap.

The current debate on monetary policy strategy opposes generally monetary targeting to inflation targeting and questions over whether the ECB has to move to full-fledged inflation targeting.

Alesina et *al.* (2001) argue that it is hard to see why the growth rate of M3 should have a special role and the ECB could improve its policy by adopting inflation targeting. Evans and Honkapohja (2003) have shown that Friedman's *k*-percent money supply rule (strict monetary targeting) performs poorly in terms of welfare compared to optimal interest rate rule (flexible inflation targeting). Gersbach and Hahn (2003) suggest that inflation targeting is superior to monetary targeting as it makes it easier for central banks to commit to low inflation. Laubach

(2003) argues that monetary targeting facilitates communication of the central bank's type. However, this advantage is outweighed for most parameter values by the advantage of inflation targeting in terms of inflation control.

The empirical study by Rudebusch and Svensson (2002) has revealed that monetary targeting is quite inefficient, yielding both higher inflation and output variability and therefore, there is no support for the prominent role given to money growth in the Eurosystem's monetary policy strategy. That of Cabos et *al*. (2003) also supports that control problems involved in targeting broad or narrow money are larger than these associated to adopting direct inflation targets.

The Bundesbank's success story of monetary targeting is then explained as due to that their monetary policy is actually closer in practice to inflation targeting than it is to Friedman-like monetary targeting and thus might best be thought of as "hybrid" inflation targeting.[1] The Bundesbank's monetary targeting is quite similar to inflation targeting as it announced inflation target and transparently communicated to the public and market participants. Using real-time data, Gerberding et *al*. (2005) find that the Bundesbank took its monetary targets seriously, but also responded to deviations of expected inflation and output growth from target. In practice, the Bundesbank was a monetary targeter as well as an inflation targeter.

Central bankers (Freedman, 1996; King, 1996) have also noted the close similarity in the use of central bank instruments and the reaction of central banks to news and shocks under inflation forecast targeting and monetary targeting. That suggests that choice of one or other monetary regime does not seem to matter much for the day-to-day conduct of monetary policy. Their points of view are supported by empirical studies according to which, inflation targeting seems to have made little if any difference for inflation and interest rate dynamics

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[1] See Clarida and Gertler (1996), Bernanke and Mishkin (1997), Bernanke and Mihov (1997), Laubach and Posen (1997), Clarida et *al*. (1998), Michkin (1999, 2002) and Svensson (1999a, b, 2000).

(or conduct of interest rate policy) in countries adopting this strategy in the 1990s (Groeneveld et *al.*, 1998; Almeida and Goodhart, 1998).

Monetary targeting is generally associated with the monetarism. Even though the monetarism represents an important advance over prior conventional wisdom and the lessons learned from the monetarist controversy are not to forget, it has lost its steam in modern development of monetary theory and policy. Woodford (2008) argues that the most important of these lessons, and the ones that are of continuing relevance to the conduct of policy today, are not dependent on the thesis of the importance of monetary aggregates. The most important lessons from the monetarism, according to Woodford, are that monetary policy can do something about inflation, the central bank can reasonably be held accountable for controlling inflation and a verifiable commitment by the central bank to a non-inflationary policy is important. In other words, the ECB's continuing emphasis on the prominent role of money in its deliberations is not theoretically well justified. It is explained by the concern not to ignore the lessons of the monetarist controversies of the 1960s and 1970s.

Lessons learned from monetary targeting only indicate that the instability of the relationship between monetary aggregates and goal variables (inflation and nominal income) make monetary targeting problematic, but not necessarily the failure of monetary targeting. When this relationship is unstable, a central bank with high credibility can successfully stabilize inflation and output through monetary targeting if it is flexible, transparent and accountable (Mishkin, 2002). However, that includes too many conditions and explains why many economists argue against monetary targeting (Mishkin, 1999).

One must not be surprised that the argument used against monetary targeting can be returned against inflation targeting. In a context of financial instability, it is difficult to ignore the developments in money and financial markets and consider that they have no influence on

the monetary policy strategy and the latter can be theoretically formulated only on the base of New-Keynesian Phillips curve and IS curve.

In effect, the relationship between nominal interest rate, i.e. the repo interest rate that the central bank can directly control, and goal variables such as inflation and output gap is indirect and can be very unstable. In certain circumstances, it will be difficult for central banks to credibly and transparently communicate their monetary policy strategy in the framework of inflation targeting. Furthermore, inflation targeting regimes, focusing on inflation and output targets, could lead to very ample movements in interest rates and consequently in monetary and financial aggregates and asset prices, creating difficulties for ulterior monetary policy decisions. That is actually the case in many developed and emerging market economies where several major speculative financial and real bubbles simultaneously burst.

The theories of inflation targeting generally focuses on the imperfections on the supply-side of goods and services by using New-Keynesian Phillips and IS curves and is completely unaware of those on money and financial markets. It is assumed that the curve of endogenous money supply coincides with that of money demand and one has not to pay attention to money and financial markets in the theoretical construction of inflation targeting (Woodford, 1998; Rudebusch and Svensson, 1999). As all financial assets are implicitly assumed to be perfectly substitutable, controlling only the repo interest rate is equivalent to controlling all other interest rates. Another implicit assumption justifying the ignorance of money and financial markets is that perfect credibility and transparency of the central bank are sufficient to anchor inflation expectations.

Romer (2000) remark that one area in which both the traditional IS-LM approach (where the money is considered as monetary policy instrument) and IS-MP approach (where MP stands for monetary policy, i.e. interest rate rule or inflation targeting) may have simplified

too far is in their treatment of financial markets. In both approaches, the only feature of financial markets that matters for the demand for goods is 'the' real interest rate that monetary policy can powerfully and directly influence as the central bank desires. In practice, the demand for goods depends on interest rates that the central bank may not be able to control directly and tenuously as well as the level of credit which is available at those rates. An analysis, which takes more carefully account of the impacts of various developments in financial markets on the demand for goods as well as the mechanism through which the monetary policy affects these interest rates and the level of credit, would highlight many of the difficulties and uncertainties of actual policy-making.

In effect, adopting the hypothesis of imperfect money and financial markets allows understanding better the functioning of the economy and how a monetary policy is implemented. In practice, the central liquidity is not accessible at unlimited quantity because central banks limit the quantity, the quality and the types of assets accepted as collateral as well as the types of financial institutions which have direct access to the central liquidity. That implies that there could be a potential imbalance (excess of liquidity or crisis of illiquidity) on the money market.

Central bank's interventions through injection or withdrawal of liquidity become essential and have the advantage of being more flexible than the repo interest rate instrument because the latter must generally follow a well defined trend and is only modifiable (except in the event of financial crisis) with long intervals separating two interest rate decisions.

The failure of transmission mechanism which links the repo interest rate to other interest rates as well as the zero bound for nominal interest rate could greatly limit the possibility of actions through fixing the nominal interest rate for central banks adopting inflation targeting. In awful economic situations, a central bank too aggressive in reducing the repo interest rate

can quickly find itself without interest rate instrument and therefore the means of sufficiently reducing the lending interest rates and anchoring the inflation expectations.

The imperfect money and financial markets hypothesis also gives a better account of the dynamic of inflation expectations. The evolutions of the expected rate of inflation deduced from the difference of return between the indexed and un-indexed obligations show that the inflation expectations are not as static as predicts the inflation targeting literature. Some introductive teachings treat the expected rate of inflation even as fixed in the presence of serially non-correlated stochastic shocks (Romer (2000), Walsh (2002)). In my opinion, using information from money market and financial markets generally allows improving the inflation expectations of private sector compared to the case where private sector uses only information extracted from the interest rate rule, the Philips curve and the goods market equilibrium condition as it is admitted in the literature of interest rate rules and inflation targeting.

By assuming imperfect financial markets, I admit that the target of the interest rate which affects directly the decision of consumption and investment, decided by the central bank and expressed as optimal interest rate rule and function of other variables in the inflation-targeting regime, cannot be directly fixed and is not always realized due to malfunctioning of money and financial markets or shocks affecting these markets.

In effect, the central bank fixes the repo interest rate, which is determined by taking account of inflation and output targets and economic model (including money and financial markets). A modification of repo interest rate allows inducing a change in the interbank money market interest rate, affecting then the interest rates determined on the credit or debt market at which firms and consumers can borrow.

If this transmission mechanism is perturbed by exogenous shocks or endogenous instability, adopting monetary targeting under inflation-targeting regime may have many

advantages in terms of monitoring the inflation expectations and controlling the money market interest rate and other interest rates. It implies that there are good reasons for the inflation-targeting central bank, by designing an appropriate money growth rule, to flexibly monitor the level of liquidity in the monetary market and the economy (i.e. to target other interest rates) and therefore control the inflation expectations in order to ensure the dynamic stability of the economy.

The above discussion allows us to consider the monetary aggregates and repo interest rate as two independent monetary policy instruments. In this sense, I do not have the need to argue neither against the monetary targeting nor against the inflation targeting. They are complementary and can be simultaneously used to prevent macroeconomic and financial instability of dynamic nature. The combination of these two instruments can be considered a kind of two-pillar monetary policy strategy. However, we need to study how this strategy can be theoretically conceived in order to possess the property of ensuring the dynamic stability of the economy. Using this 'two-pillar' monetary policy strategy, central banks could simultaneously dispose of two policy instruments (repo interest rate and money supply) to affront an increasingly uncertain economic environment. However, it is not simply the two-pillar monetary policy strategy adopted by ECB without having developed theoretical foundation or the reintroduction of the monetarism in the inflation-targeting framework through the adoption of Friedman's *k*-percent money growth rule. In effect, as I will show in the following, the design of monetary targeting rules is crucial for the success of such monetary policy strategy.

The remainder of the paper is organized as follows. In the next section, I present a theoretical model in which money market plays a role. In the section after, I characterize the optimal reaction function of the central bank. In the fourth section, I give the reduced model and the dynamic equation of the expected inflation. In the fifth section, I analyze the dynamic

stability of the economy under Friedman's *k*-percent money growth rule. The sixth section examines three alternative feedback monetary targeting rules. The final section concludes.

2. The Model

I consider a continuous time closed economy model described by an inflation adjustment equation, an aggregate spending relationship linking output to real interest rate and an equilibrium condition in asset markets (domestic money and short-run bonds):

$$\pi_t = \pi_t^e + \alpha\,(y_t - y^*) + \varepsilon_{\pi t}, \qquad \alpha > 0, \tag{1}$$

$$y_t = -\beta\,(i_t - \pi_t^e) + \varepsilon_{dt}, \qquad \beta > 0, \tag{2}$$

$$m_t - p_t = l_1 y_t - l_2 i_t + \varepsilon_{lt}, \qquad l_1, l_2 > 0, \tag{3}$$

where π_t $(\equiv dp/dt)$ is the current inflation rate which is the time derivative of general price level p_t, π_t^e the expected inflation rate of time t conditional on information available at the moment where expectations are formed (i.e. previous to t), y_t the actual output, y^* the natural rate of output, i_t the nominal interest rate and m_t the money supply. The variables y, m and p are expressed in logarithmic terms. $\varepsilon_{\pi t}$, ε_{dt} and ε_{lt} are respectively contemporary shocks affecting the supply and the demand of goods and the demand of money.

Equation (1) stipulates that inflation is governed by an expectational Phillips curve. According to equation (2), the aggregate demand depends on expected real interest rate $(i_t - \pi_t^e)$. Equation (3) corresponds to LM curve with a real money demand depending on real income and nominal interest rate. In the following, the time index t is neglected whenever there is no confusion.

The current consensus in the inflation-targeting literature is that money market is only useful for determining money supply which endogenously responds to money demand, and so can be ignored in making monetary policy decisions (Rudebusch and Svensson, 1999, 2002; King, 2000). This is equivalent to assume that money supply is infinitely elastic.

Being aware of the importance of money and credit markets, Benjamin Friedman (2003) contests this view. For him, abandoning the role of money and the analytic of the LM curve makes it more difficult to take into account how the functioning of the banking system (and with it the credit markets more generally) matter for monetary policy and also leaves open the underlying question of how the central bank manages to fix the chosen interest rate in the first place. His concern finds some echoes in Goodhart (2007) who thinks that the central banks must still give attention to the monetary aggregates, in particular the growth rate of the bank credit allocated to the private sector, or in Christiano *et al.* (2007) showing that a monetary policy which concentrates too narrowly on inflation can, in an unintended way, to contribute to reduce the welfare via cycles of expansion and depression in the real and financial variables.

Milton Friedman (2005), using data covering three booming periods in US and Japan, shows that what happens to the quantity of money has a determinative effect on what happens to national income and to stock prices. Hafer et *al.* (2007) find that money is not redundant, notably there is a significant statistical relationship between lagged values of money and the output gap, even when lagged values of real interest rates and lagged values of the output gap are accounted for. Hafer and Jones (2008), adding money to a dynamic IS model, discover that evidence from six countries indicates that money growth usually helps predict the GDP gap and that the predictive power of a short-term real interest rate is much weaker than previous work suggests. Their results suggest that, for dynamic IS model such as that used by Rudebusch and Svensson (1999, 2002), the omission of money appears to come at a high cost.

In effect, these studies show that the empirical relationship between money and inflation must not be interpreted as simple correlation. As a causal relationship which runs from the money to inflation, it implies that controlling the quantity of money (in the narrow sense) can effectively control the inflation in the medium and long run. However, I do not exclude the possibility of inversing the causality of the relationship between these two variables if we can effectively make the money wholly endogenous as defended by the theoreticians of inflation targeting.

Considering money market as coordination device of private inflation expectations, Dai and Sidiropoulos (2003, 2005) and Dai (2006) provide some theoretical justifications of the utility of money market which has other function, i.e. the coordination device of private inflation expectations, than only endogenously determining money supply in a typical inflation-targeting framework.

The present paper gives a special attention to money market and hence it is possible to examine the adjustment dynamics of expected inflation. I assume that the central bank has not direct control over the money supply. Instead, if the central bank desires, control can be exercised over a narrow monetary aggregate such as monetary base, and variations in this aggregate are then associated with variations in broader measures of money supply.

The money supply is endogenous but it is imperfectly elastic as the banking system will increase or decrease the internal money in taking account of nominal interest rate as well as collateral and will not satisfy the money demand whenever it appears. Similarly, the central bank provides a limited quantity of central money to a limited number of banks by accepting certain categories of assets as collateral. The link between the total money supply and the monetary base, considered in the present framework as a second policy instrument beside the repo interest rate, is modeled as follows (Modigliani et al., 1970; McCallum and Hoehn, 1983; Walsh, 1999):

$$m = b + hi + \varepsilon_m. \tag{4}$$

where b is the (log) monetary base, and money multiplier ($m - b$ in log terms) is assumed to be an increasing function of nominal interest rate (i.e. $h > 0$). In addition, ε_m is a money-multiplier disturbance. The term hi represents the endogenous liquidity that the system of banks and shadow banks creates in responding to change in the market interest rate.

Time derivation of equation (3) taking account of equation (4) yields:

$$\mu + h\dot{i} + \dot{\varepsilon}_m - \pi = l_1 \dot{y} - l_2 \dot{i} + \dot{\varepsilon}_l. \tag{5}$$

where $\mu = \dot{b} = db/dt$, and the dot over a variable indicates that it is a time derivative of the variable. Equation (5) implies that, in average, the money growth rate $\bar{\mu}$ must be equal to current and expected inflation rates, adjusted for the long-run growth rate of output, i.e.,

$\bar{\mu} = \bar{\pi} + l_1 \dot{y}^* = \bar{\pi}^e + l_1 \dot{y}^*$.

If the central bank desires a credible inflation-targeting policy, it could monitor the expected inflation in keeping an average long-term money growth rate consistent with its inflation target (π^T), i.e. $\bar{\mu} = \pi^T + l_1 \dot{y}^*$.[2] However, monetary targeting must not be considered as an independent strategy for achieving price stability by stabilizing inflation around a given inflation target since it faces, as shown by Svensson (1999a), an unpleasant choice between being either inefficient and transparent or efficient and non-transparent.

The way to close the model generally adopted in the inflation-targeting literature is to assume that money supply automatically adjusts to money demand so that money market can be ignored without serious consequences. In this model, I assume that money supply is partially endogenous, but not automatically equal to money demand.

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[2] This is consistent with the practice of Bundesbank. Each year, Bundesbank sets its money-growth target equal to the sum of an inflation target, a forecast of the growth of potential output, and an estimated trend in velocity (Svensson, 1999a).

The major difference which distinguishes the present model from the previous studies of inflation targeting is that the money supply obeys to its own logic and cannot be assimilated to the money demand. The maladjustment between the supply and demand of money could be due to the imperfect access of economic agents to credit, money and financial markets or to the desire of the central bank to control money growth in order to directly influence inflation expectations. Due to imperfection of the transmission mechanism of interest rate policy, the money supply becomes an independent instrument and the money market is not anymore redundant.

The central bank systematically acts to minimize fluctuations of output around the natural rate and inflation around its target. More precisely, the central bank is assumed to minimize the following loss function measured in terms of present discounted value:

$$L = \int_0^\infty \frac{1}{2} \left[\lambda(y - y^*)^2 + \kappa(\pi - \pi^T)^2 \right] \exp(-\theta t) dt, \quad \lambda, \kappa, \theta > 0, \tag{6}$$

where parameters λ and κ denote the weight that the central bank assigns to output and inflation stabilization respectively, and θ is the discount factor. This strategy of flexible inflation targeting is implemented through an optimal nominal interest rate rule, which is deduced from the optimal inflation targeting rule of the central bank.

I complete the model description by the following time sequence of events: 1) Workers form their inflation expectations and negotiate current wages. 2) Shocks realize. 3) The central bank fixes nominal interest rate following an optimal interest rate rule. 4) Firms decide their production and prices. 5) Workers revise their inflation expectations and the central bank could influence, if this is its desire, this revision with money growth rule.

3. The optimal interest rate rule

The optimal inflation targeting rule is the solution to the sequence of single period decision problems of the central bank under discretion. Since private inflation expectations are taken as given when it makes the decision of interest rate, the central bank's single period decision problems are then independent. The central bank's optimization problem simply consists of minimizing the one-period loss function in (6) subject to the economic constraint represented by equation (1). Thus, the first-order condition is given by

$$\lambda \frac{\partial y}{\partial \pi}(y - y^*) = -\kappa(\pi - \pi^T), \;\; \Rightarrow \;\; y = y^* - \frac{\kappa\alpha}{\lambda}(\pi - \pi^T), \qquad (7)$$

that, with equation (2), leads to the following nominal optimal interest rate rule:

$$i^T = \pi^e + \frac{1}{\beta}[\frac{\kappa\alpha}{\lambda}(\pi - \pi^T) - y^* + \varepsilon_d]. \qquad (8)$$

where i^T represents the target of interest rate which must be attained in order to minimize the central bank's loss function.

According to equation (8), it is optimal for the central bank to adjust the target of nominal interest rate upward to reflect expected inflation rate (to a full extend), the gap between current inflation and the inflation target, as well as increases in the output gap due to a positive demand shock.

The central bank fixes the repo interest rate, which is not explicitly modeled in this paper, in order to attain the target of interest rate. However, disturbances in financial and corporate sectors can create dislocation on financial markets and enlarge the difference between the repo interest rate and other interest rates. Furthermore, absorbing negative disturbances in goods market may require a low target of interest rate which may not be within the reach of the central bank due to negative financial market disturbances and the zero bound for the nominal repo interest rate. In this case, non-orthodox monetary policy, such as the quantitative easing policy, must be used to ease the tension on the money market or more

audaciously the credit market through strengthening banks' balance sheet and/or buying private debts on the credit market by the central bank or Treasury.

Consequently, in the process of implementing the decision of interest rate policy, the use of monetary aggregate may be very helpful particularly when the money and financial markets are perturbed and attaining the target of interest rate may needs too ample movements of the repo interest rate. Since I assume that the central bank will always attain the target of interest rate by varying the repo interest rate and the money supply, I simply admit that $i = i^T$ in the following.

4. The dynamics of expected inflation

As expected inflation rate is determined before current inflation rate and output, its dynamic trajectory can be more easily studied in a reduced dynamic system where the values of π and y are substituted by their solution in terms of expected inflation rate, exogenous variables and shocks. Once the dynamic trajectory of π^e is solved, we can determine these of π and y. Equations (1)-(2) and (8) enable us to solve inflation rate and output as follows:

$$\pi = \frac{\lambda}{\lambda + \kappa\alpha^2} \pi^e + \frac{\kappa\alpha^2}{\lambda + \kappa\alpha^2} \pi^T + \frac{\lambda}{\lambda + \kappa\alpha^2} \varepsilon_\pi , \tag{9}$$

$$y = y^* - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2} \pi^e + \frac{\kappa\alpha}{\lambda + \kappa\alpha^2} \pi^T - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2} \varepsilon_\pi . \tag{10}$$

Equations (9) and (10) are not final solutions for inflation rate and output, which can only be obtained after having solved expected inflation rate. They show that, departing from an initial equilibrium where $\pi^e = \pi^T$, an increase in inflation expectations will positively impact current inflation and negatively current output.

As I have argued before, economic agents will not blindly believe in the announced inflation target π^T in all circumstances since they cannot distinguish *a priori* between i.i.d., persistent or permanent shocks. If shocks are always i.i.d., simply using equation (9) to estimate expected inflation gives the result $\pi^e = \pi^T$. That is misleading for the central bank as well as for private agents when shocks are not perceived as random and transitory. For this reason, the revision of rational inflation expectations by market participants using more information is necessary. Since the money market (and so financial markets) conveys all information about the economy, it can serve as the co-ordination place for private agents in forming good and consensual inflation expectations. They will use a whole set of information provided by monetary and financial markets to revise their expectations. Furthermore, the inclusion of monetary targeting in the monetary strategy implies that money would have an important impact over the determination of current price level and inflation rate and consequently over that of future inflation rate.

In modern economies with developed financial markets, sophisticated financial instruments (such as inflation-indexed bonds, interest rate options, swaps and futures) are traded and implicitly convey market expectations about future inflation. These complex financial instruments are not modeled in this simple model. However, in this model, expected inflation underlying the prices of the financial assets (short term bonds) can be estimated using information about equilibrium condition on every market. Therefore, I suppose that private agents directly learn from the information conveyed by the money market to determine the expected inflation. Using equations (5), (8)-(10) and admitting $\dot{\pi}^T = 0$, we derive the following differential equation of the expected inflation (Appendix):

$$\dot{\pi}^e = \Psi(\pi^e - \mu^e) + \Psi(\dot{\varepsilon}^e_m - \dot{\varepsilon}^e_l) + \Psi l_1 \dot{y}^* - \frac{\Psi(h+l_2)}{\beta}(\dot{y}^* - \dot{\varepsilon}^e_d) - \frac{\Psi l_1 \kappa \alpha}{\lambda + \kappa \alpha^2} \dot{\varepsilon}^e_\pi. \tag{11}$$

where $\Psi = (\lambda + \kappa\alpha^2)\beta\lambda / [l_1\kappa\alpha\beta\lambda + (h + l_2)(\lambda + \kappa\alpha^2)(\beta\lambda + \kappa\alpha)] > 0$. The first term of equation (11) represents the impact of difference between the expected inflation and expected money growth rate on the adjustment of inflation expectations. The other terms represent fundamental variables influencing expected inflation: the variation of output potential and diverse shocks affecting the decisions of private agents in their choice of consumption, production and acquisition of monetary and financial assets.

The inflation expectations resulting from equation (11) is compatible with rational expectations hypothesis. When shocks ε_π are all transitory white noises, the solution $\pi^e = \pi^T$, resulting from mathematical expectations of equation (9), is also the steady equilibrium solution of equation (11) which turns out to be $\dot{\pi}^e = \Psi(\pi^e - \pi^T)$ with $\bar{\mu} = \pi^T + l_1\dot{y}^*$. However, equation (11) is a more realistic description of the revision mechanism of inflation expectations, since it takes account of economic, monetary and financial factors that are completely ignored in mathematical expectations of equation (9) under the assumption of i.i.d. inflationary shocks.

As equation (11) corresponds to equilibrium condition on money and financial markets, it can be considered as a condition of no arbitrage in the short-run on the financial market. It shows a direct relation between money supply and expected inflation. Indeed, the link between monetary policy and expected inflation is very complex as illustrated by equation (11). Inflation targeting (through the fixation of nominal interest rate) influences monetary supply at one hand, and real money demand directly (through nominal interest rate) and indirectly (through revenue) on the other hand. If the central bank adopts monetary targeting as another monetary policy instrument, it can, through the manipulation of the money growth rate to create excess or shortage of liquidity, influence short-run inflation expectations so that they will not significantly deviate from the inflation target over the intermediate term.

During the last decade, even though inflation-targeting central banks' principal objectives, i.e. stabilization of inflation and output gap, are relatively well achieved, too much disequilibrium on the financial markets has been accumulated, translating into bubbles in real and financial asset prices. One reason for this to repeatedly happen is that central banks do not give anymore attention to the increase in the quantity of money (or liquidity) and credits. However, they pay a particularly great attention to these aggregates when the financial system and the real economy are facing with the risk of collapsing. This asymmetrical behavior with regard to quantity of money and credits is at the origin of dramatic financial shocks that we actually live, with devastating effects on the real economy.

To avoid large self-inflicted financial shocks in the future without rejecting the recent advances in the central banking such as inflation targeting which puts accents on central bank's independence and transparency, one solution is to combine the inflation targeting with the monetary targeting through the specification of an optimal interest rate rule and an appropriate money growth rule, which are compatible with the dynamic stability of the economy.

By only manipulating the repo interest rate to indirectly affect the market interest rates, the central bank has no credible instrument of anchoring the inflation expectations besides the cheap talk about its firm intention to attain its inflation target. The real challenge appears whenever the economy is outside of equilibrium. When the rate of inflation moves away from the target announced by the central bank, verbal persuasion via the publication of the minutes, the monthly reports, the data, the procedures of decisions as well as the models used could be not enough to convince the public to adhere to the monetary policy of the central bank. Temporary but persistent shocks could make further difficult the conduct of monetary policy only based on the control of interest rate. Furthermore, speculative inflation bubbles cannot be excluded in a dynamic framework by assuming rational expectations.

In this context, private agents may find rational to lose some precious time to collect all information about the economy to form their inflation expectations instead of only using the Phillips curve, IS curve and central bank's targeting rule, not to say that using Phillips curve is also submitted to instability of the relationship in the long-run and some important pitfalls.

In this context, the central bank might be able to more effectively anchor the inflation expectations by controlling the liquidity available in the financial system. The central bank desires that the private sector believes in its objective even though shocks can deviate the realized rate of inflation from its inflation target. Knowing that the non-financial private sector scrutinizes money and financial markets to find out the market inflation expectations before determining its own ones, the central bank, concerned with ensuring its credibility, must control the growth rate of the money supply (in the narrow definition) at a level, which on average is consistent with its inflation target.

By introducing a monetary targeting rule, it is not necessary for the central bank to scrupulously make the inflation target equal to the growth rate of a chosen monetary aggregate, which may be subject to exogenous shocks or even disturbances due to speculative behaviors of financial operators.

The control of money supply implies endogenous and complex adjustment of the inflation expectations in this model. Private agents, in order to revise as fine as possible their inflation expectations, will take account of inflation dynamics not only reflected in the Phillips curve and IS curve, but also that reflected in the information conveyed by money and financial markets.

If the monetary targeting rule is well specified, the inflation target of the central bank is always realizable when the effects of shocks disappear. Although the expected and realized rates of inflation can be temporarily different from the inflation target, their difference will decrease since the dynamic stability is embedded in the economy through an appropriate

control of money growth. Without this control, an exogenous change in the inflation expectations could lead the economy to deviate far from the equilibrium corresponding to the inflation and output objectives announced by the central bank.

If the inflation target represents a potential nominal anchor of the economy, the control of money growth makes it more credible in the eyes of private agents and provides a kind of additional nominal anchor for their inflation expectations. Private agents could revise as fine as possible their inflation expectations given the state of the economy by using all available information, including that conveyed by money and financial markets.

The constant money growth rule has been considered as failure in stabilizing inflation and inflation expectations. It is hence interesting to examine its capability of dynamic stabilization in this model and then compare it with alternative feedback money growth rules.

5. The dynamic behavior of the economy under Friedman's *k*-percent rule

Equation (5) implies that, in order to stabilize current and expected inflation rates around a constant steady state level, monetary authorities are constrained to set a money growth rate consistent with their inflation target and adjusted for variation in the potential of output. One example of monetary targeting rule can be:

$$\mu = \overline{\mu} + l_1 \dot{y}^*, \quad \text{with } \overline{\mu} = \pi^T . \tag{12}$$

This is a variant of Friedman's *k*-percent rule. For von Hagen (1999), this kind of monetary targeting is a signal that the central bank is independent and fighting against price instability, and a means to define the role of monetary policy vis-à-vis other players in the macroeconomic policy game, and to structure the internal monetary policy debate.

In the absence of monetary targeting rule, inflation targeting might not be perfectly credible. The concept of imperfect credibility is used in this paper in the sense that private

agents don't automatically and uniquely use the inflation target as nominal anchor and instead, they use information extracted from current market conditions to revise their inflation expectations. In effect, to entirely believe in the inflation target is equivalent for private agents to believe that the random shocks can conceal their inflation consequences in their respective time horizon. As their time horizons are different and far from infinite and the effects of shocks cannot be mutually compensated in their respective time horizon, they might be incited to use alternative method to form their inflation expectations which correspond better to their personal time horizon of decision during which the current inflation rate could be systematically different from the expected inflation due to permanent, persistent or even stochastic shocks.[3] If this is the case, private agents could anticipate an inflation rate different from the inflation target announced by the central bank. Thus, without other warrant, inflation targeting will not necessarily offer the nominal anchor for private inflation expectations as assumed in the inflation-targeting literature.

However, the monetary targeting rule specified in equation (12) might not be considered as a warrant against major deviations of current and expected inflation rates from the inflation target and thus might not reinforce the belief of private sector on that monetary authorities will be more successful in implementing their interest rate policy consistent with their inflation target.

Taking account of equation (12) into equation (11) yields:

$$\dot{\pi}^e = \Psi(\pi^e - \overline{\mu}) + \Psi(\dot{\varepsilon}_l^e - \dot{\varepsilon}_m^e) + \frac{\Psi(h + l_2)}{\beta}(\dot{y}^* - \dot{\varepsilon}_d^e) - \frac{\Psi l_1 \kappa \alpha}{\lambda + \kappa \alpha^2}\dot{\varepsilon}_\pi^e. \tag{13}$$

If the expected inflation rate can directly jump to its equilibrium value, the issue of dynamic instability will be excluded from the model. As Buiter and Panigirtzoglou (2003), I consider inflation rate (π) and hence expected inflation rate (π^e) as predetermined. In effect,

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[3] The random nature of shocks does not exclude that the same kind of shocks occur repetitively and consecutively for several times.

it is quite reasonable to admit that π^e is a predetermined variable in a low inflation environment, where the adjustments of prices and consequently of current and expected inflation rates are quite slow due to different mechanisms causing nominal rigidities in the short-run (menu costs, partial adjustments, overlapping contracts etc.).

The dynamic behavior of the economy described by equation (13) can be summarized in the following proposition.

Proposition 1. *Under inflation targeting rule (8) combined with monetary targeting rule such as (12) (Friedman's k-percent rule), expected and realized inflation rates, real output and real money stock will follow an unstable dynamic process of adjustment.*

Proof: The eigenvalue of the dynamic equation (13) is equal to $\Psi > 0$. ∎

The solution of expected inflation is indeterminate in the sense that it will be on a divergent trajectory whenever there is a shock perturbing the economy. According to equations (9) and (10), realized inflation and output will diverge also from their average equilibrium value.

This indeterminacy appears even though Friedman's *k-percent* rule is introduced. However, as shown by equation (13), the coefficient ($\Psi > 0$) associated with π^e stays the same as in equation (11).

Since the expected inflation diverges from its equilibrium value after any shock affecting the economic system, equations (9)-(10) imply that the realized inflation and output would follow divergent trajectories. It follows that the real money demand is also unstable. This observation is interesting since instability in money demand is notably observed when, in practice, central banks use interest rate policy more intensely while keeping simple monetary targeting rule.

It is easy to understand why macroeconomic instability could arise as a result of optimal nominal interest rate rule combined with rigid monetary targeting rule. For given inflation expectations, higher nominal interest rate reduces real money demand directly (through negative effect of higher nominal interest rate on demand of money for speculation or other motives) and indirectly (through its negative effect on goods demand and so on demand of money for transactions). The reduction of goods demand implies also smaller real money demand. For a given money growth rate, higher nominal interest rate implies higher monetary growth rate due to money-multiplier effect according to equation (4). With a reduced real money demand, the equilibrium condition of money market implies a higher future inflation rate that economic agents could easily anticipate if they attentively observe this market. Workers could ask higher nominal wages to compensate for the loss of purchasing power due to higher future inflation. That will effectively generate further inflationary pressures.

In effect, emerging market economies (i.e., Latin American countries during the 1980s) and transition economies (i.e., Eastern European countries in 1990s) provide numerous examples where a sharp increase in nominal interest rate is incapable of reducing expected and hence realized inflation rates.

6. Feedback monetary targeting rules

The instability result of *k*-percent money growth rule under inflation-targeting regime is due to the fact that the money growth rate is given when the interest rate policy is tightening or relaxing to answer to inflationary and demand shocks. The solution to this problem is to fine-tune monetary targeting rule so that it reacts to changing economic conditions in harmony with nominal interest rate rule.

Three feedback monetary targeting rules will be considered in the following. A component destined to counterbalance observable shocks affecting the money market can also be integrated in these rules. They can be implemented thanks to a kind of rationing or limitation of access to the central liquidity by the commercial banks and complements the interest rate rule in the 'two-pillar' monetary policy strategy. To implement well-designed 'two-pillar' strategy, it is important to choose the value of the parameters characterizing the feedback component of these rules so that the dynamic stability is imbedded in the economic system.

6.1. Money growth reacting to the variation of inflation rate

The first feedback monetary targeting rule remedying the instability due the Friedman's k-percent rule (12) negatively links the money growth rate to the variation of inflation rate:

$$\mu = \overline{\mu} + l_1 \dot{y}^* - \varphi \dot{\pi} , \qquad \text{with } \overline{\mu} = \pi^T . \tag{14}$$

Substituting $\dot{\pi}$ by $\dot{\pi}^e$ in equation (14) will not modify the results. Introducing $\dot{\pi}^e$ has even some advantages if indexed bonds are quoted on financial markets since in this case the expected inflation is directly deduced from observing the financial asset prices, while the realised inflation rate must be calculated with data which may not be available quickly.

Taking mathematical expectations of (14) leads to $\mu^e = \overline{\mu} + l_1 \dot{y}^* - \varphi \dot{\pi}^e$. Inserting the solution of μ^e into equation (11) yields:

$$\dot{\pi}^e = \frac{\Psi}{(1-\Psi\varphi)}(\pi^e - \pi^T) + \frac{\Psi}{(1-\Psi\varphi)}(\dot{\varepsilon}_m^e - \dot{\varepsilon}_l^e) - \frac{\Psi(h+l_2)}{\beta(1-\Psi\varphi)}(\dot{y}^* - \dot{\varepsilon}_d^e) - \frac{\Psi l_1 \kappa \alpha}{(\lambda + \kappa \alpha^2)(1-\Psi\varphi)}\dot{\varepsilon}_\pi^e. \tag{15}$$

Proposition 2. *i) Under inflation-targeting rule* (8) *combined with monetary targeting rule* (14), *the equilibrium solution of equation* (15) *is dynamically stable under the condition*

$1/\Psi < \varphi$. *ii) The minimal value of* φ *(*$\min \varphi = 1/\Psi$*) compatible with stable equilibrium*

decreases with λ *and* β, *increases with* κ, l_1, h, l_2. *It increases with* α *if* $\lambda > \kappa \alpha^2$.

Proof. To demonstrate the part i) of Proposition 2, it is sufficient to examine the eigenvalue

of the dynamic equation (15), which is negative when $1 - \Psi \varphi < 0$ or equivalently $1/\Psi < \varphi$.

Consequently, the dynamic system has a stable equilibrium.

To show the part ii) of Proposition 2, we take the derivatives of $\min \varphi$ with respect to

different parameters:

$$\frac{\partial \min \varphi}{\partial \lambda} = -\frac{l_1 \alpha \kappa}{(\lambda + \kappa \alpha^2)^2} - \frac{(h + l_2)\alpha \kappa}{\beta \lambda^2} < 0; \quad \frac{\partial \min \varphi}{\partial \kappa} = \frac{l_1 \alpha \lambda}{(\lambda + \kappa \alpha^2)^2} + \frac{\alpha(h + l_2)}{\beta \lambda} > 0;$$

$$\frac{\partial \min \varphi}{\partial \beta} = -\frac{(h + l_2)\kappa \alpha}{\beta^2 \lambda} < 0; \quad \frac{\partial \min \varphi}{\partial l_1} = \frac{\kappa \alpha}{(\lambda + \kappa \alpha^2)} > 0; \quad \frac{\partial \min \varphi}{\partial h} = \frac{\partial \min \varphi}{\partial l_2} = 1 + \frac{\kappa \alpha}{\lambda \beta} > 0;$$

$$\frac{\partial \min \varphi}{\partial \alpha} = \frac{l_1 \kappa (\lambda - \kappa \alpha^2)}{(\lambda + \kappa \alpha^2)^2} + \frac{+ \kappa (h + l_2)}{\lambda \beta} \geq 0 \text{ if } \lambda > \kappa \alpha^2. \quad \blacksquare$$

When $1/\Psi < \varphi$, the monetary targeting rule (14) allows sufficiently reducing the money

growth rate to equilibrate the money market following shocks that, for example, initially lead

to a rise in current and expected inflation rates and consequently to an increase in nominal

and real interest rates that implies a reduced real money demand. No further increase in

inflation rate is then justifiable by evocating the existence of excessive liquidity in the

economy.

The minimal value of φ compatible with stable equilibrium diminishes with the weight

assigned to output stabilization (greater λ) and increases if the central bank worries more

about the realization of inflation target (greater κ). It varies also with parameters (β, α, l_1, h

and l_2) reflecting the economic and financial characteristics of the economy. In particular,

more financial developments (greater β), more efficient transaction and payment system

(smaller l_1), smaller semi-interest elasticity of the money demand (smaller l_2) and supply

(smaller h) and less flexible labor market (smaller α, but under the condition $\lambda/\kappa > \alpha^2$, i.e. the central bank is a quite flexible targeter) allow the central bank to link less strongly money growth (smaller φ) to the rate of change in the inflation rate without creating macro-economic instability. When the semi-interest elasticity of money demand and supply are more important (greater substitution between money and other financial assets), the central bank must keep money supply more reactive to the rate of change in the inflation rate.

6.2. Money growth varying with the rate of change in output

The second feedback monetary targeting rule is specified to react to the variation of output:

$$\mu = \overline{\mu} + l_1 \dot{y}^* + \eta \dot{y}, \qquad \text{with } \overline{\mu} = \pi^T. \tag{16}$$

This rule is similar to the one considered by Taylor (1985), McCallum (1988a, b), Judd and Motley (1991), Hess, Small and Brayton (1993), and Feldstein and Stock (1994).

The monetary targeting rule (16) implies that the central bank accommodates to the variation of output over the current period in determining the current money growth rate. Substituting μ^e in equation (11) by its solution ($\mu^e = \overline{\mu} + l_1 \dot{y}^* + \eta \dot{y}^e$) obtained by taking the mathematical expectations of equation (16), and using the mathematical expectations of $\dot{y}$ obtained from the time derivation of equation (10), the dynamic equation of expected inflation is rewritten as:

$$\dot{\pi}^e = \Omega(\pi^e - \overline{\mu}) + \Omega(\dot{\varepsilon}_m^e - \dot{\varepsilon}_l^e) - \Omega\eta\dot{y}^* - \frac{\Omega(h+l_2)}{\beta}(\dot{y}^* - \dot{\varepsilon}_d^e) - \frac{\Omega l_1 \kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\varepsilon}_\pi^e + \frac{\Omega\eta\kappa\alpha}{(\lambda + \kappa\alpha^2)}\dot{\varepsilon}_\pi^e, \tag{17}$$

where $\Omega = \dfrac{(\lambda + \kappa\alpha^2)\Psi}{\lambda + \kappa\alpha^2 - \Psi\eta\kappa\alpha}$.

Proposition 3. *i) The dynamic equation* (17) *under inflation-targeting rule* (8) *combined with monetary targeting rule* (16) *has a stable equilibrium solution under the condition* $\eta > (\lambda + \kappa\alpha^2)/\Psi\kappa\alpha$. *ii) The minimal value of* η ($\min\eta = (\lambda + \kappa\alpha^2)/\Psi\kappa\alpha$) *compatible with*

stable equilibrium decreases with β, *increases with h,* l_1 *and* l_2. *If* $\lambda/\kappa < \sqrt{\alpha^3/\beta}$, *it decreases with* λ *and increases with* κ *and* α.

Proof. To demonstrate the part i) of Proposition 3, it is sufficient to show that Ω is negative when $\eta > (\lambda + \kappa\alpha^2)/\Psi\kappa\alpha$. In effect, the eigenvalue of the dynamic equation (17) is equal to $\Omega = (\lambda + \kappa\alpha^2)\Psi/(\lambda + \kappa\alpha^2 - \Psi\eta\kappa\alpha)$. It is negative if $\lambda + \kappa\alpha^2 - \Psi\eta\kappa\alpha < 0$, i.e. $\eta > (\lambda + \kappa\alpha^2)/\Psi\kappa\alpha$. In this case, the expected inflation is determinate and converges to its equilibrium value after any shock.

To show the part ii) of Proposition 3, we derive $\min\eta$ with respect to different parameters:

$$\frac{\partial \min\eta}{\partial \lambda} = \frac{(h+l_2)(\beta\lambda^2 - \kappa^2\alpha^3)}{\kappa\alpha\beta\lambda^2} < 0 \text{ if } \frac{\lambda}{\kappa} < \sqrt{\frac{\alpha^3}{\beta}};$$

$$\frac{\partial \min\eta}{\partial \kappa} = \frac{(h+l_2)(\kappa^2\alpha^3 - \beta\lambda^2)}{\beta\lambda\kappa^2\alpha} > 0 \text{ if } \frac{\lambda}{\kappa} < \sqrt{\frac{\alpha^3}{\beta}}; \quad \frac{\partial \min\eta}{\partial \beta} = -\frac{(h+l_2)(\lambda + \kappa\alpha^2)}{\beta^2\lambda} < 0;$$

$$\frac{\partial \min\eta}{\partial l_1} = 1; \quad \frac{\partial \min\eta}{\partial h} = \frac{\partial \min\eta}{\partial l_2} = \frac{(\lambda + \kappa\alpha^2)(\beta\lambda + \kappa\alpha)}{\kappa\alpha\beta\lambda} > 0;$$

$$\frac{\partial \min\eta}{\partial \alpha} = \frac{(h+l_2)(\beta\lambda\kappa\alpha^2 - \beta\lambda^2 + 2\kappa^2\alpha^3)}{\beta\lambda\kappa\alpha^2} > 0, \text{ if } \frac{\lambda}{\kappa} < \sqrt{\frac{\alpha^3}{\beta}}. \quad \blacksquare$$

When $\eta > (\lambda + \kappa\alpha^2)/\Psi\kappa\alpha$, the monetary targeting rule (16), in responding to the variation of output, allows sufficiently reducing the money growth rate to equilibrate the money market following shocks that initially lead to a rise (decrease) in current and expected inflation rates. Consequently, there will not be destabilizing increase (decrease) in nominal interest rate and inflation rate in the future.

The minimal value of η compatible with stable equilibrium diminishes with the weight assigned to output stabilization (λ) and increases if the central bank worries more about the

realization of the inflation target (κ) if the initial relative weight λ/κ is small (i.e. $\lambda/\kappa < \sqrt{\alpha^3/\beta}$, which means that the central bank is a less flexible targeter). Similarly to the case of monetary targeting rule (14), more financial developments (greater β), more efficient transaction and payment system (smaller l_1), smaller semi-interest elasticity of the money demand (smaller l_2) and supply (smaller h) and less flexible labor market (smaller α, but under the condition[4] $\lambda/\kappa < \sqrt{\alpha^3/\beta}$) allow the central bank to link less strongly money growth (smaller η) to the rate of change in output without creating macro-economic instability. In contrast, financial innovations increasing the semi-interest elasticity of money demand and supply (greater substitution between money and other financial assets) require the central bank to keep money supply more reactive to the rate of change in output.

6.3. Money growth related to the variation of nominal interest rate

The third feedback monetary targeting rule takes into account the variation of nominal interest rate:

$$\mu = \overline{\mu} + l_1 \dot{y}^* - \chi \dot{i}, \qquad \text{with } \overline{\mu} = \pi^T. \qquad (18)$$

Monetary targeting rule (18) implies that the central bank answers to the variation of nominal interest rate in determining the current money growth rate. Substituting μ^e in equation (11) by $\mu^e = \overline{\mu} + l_1 \dot{y}^* - \chi \dot{i}^e$, which is the mathematical expectations of (18), and using mathematical expectations of i obtained from the time derivation of equation (8), the dynamic equation of expected inflation rate is rewritten as:

$$\dot{\pi}^e = \frac{\beta\lambda}{\beta\lambda - \Psi\chi\beta\lambda - \chi\Psi\kappa\alpha} \left[\Psi(\pi^e - \overline{\mu}) + \Psi(\dot{\varepsilon}_m^e - \dot{\varepsilon}_l^e) - \frac{\Psi(\chi + h + l_2)}{\beta}(\dot{y}^* - \dot{\varepsilon}_d^e) - \frac{\Psi l_1 \kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\varepsilon}_\pi^e \right]. \qquad (19)$$

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[4] Which is a sufficient condition for $\partial \min\eta / \partial\alpha > 0$. A less restrictive condition for $\partial \min\eta / \partial\alpha > 0$ is

Proposition 4. *i) The dynamic equation* (19) *under inflation-targeting rule* (8) *combined with monetary targeting rule* (18) *has a stable equilibrium solution under the condition* $\chi > \beta\lambda /(\Psi\beta\lambda + \Psi\kappa\alpha)$. *ii) The minimal value of* χ ($\min\chi = \beta\lambda /(\Psi\beta\lambda + \Psi\kappa\alpha)$) *compatible with stable equilibrium increases wi*th β *and decreases with* l_1, h *and* l_2. *It decreases with* λ, *increases with* κ *and* α *if* $\lambda/\kappa < \sqrt{\alpha^3/\beta}$.

Proof. The part i) of Proposition 4 is verified straightforward when $\chi > \beta\lambda /(\Psi\beta\lambda + \Psi\kappa\alpha)$. Under this condition, the eigenvalue of the dynamic equation (19) is equal to $\beta\lambda\Psi /(\beta\lambda - \Psi\chi\beta\lambda - \chi\Psi\kappa\alpha)$, which is negative if $\beta\lambda - \Psi\chi\beta\lambda - \chi\Psi\kappa\alpha < 0$, i.e. $\chi > \beta\lambda /(\Psi\beta\lambda + \Psi\kappa\alpha)$.

The part ii) of Proposition 4 can be easily demonstrated in deriving $\min\chi$ with respect to different parameters. Using the definition Ψ into the expression of $\min\chi$ leads to:

$$\min\chi = \frac{l_1\kappa\alpha\beta\lambda + (h+l_2)(\lambda + \kappa\alpha^2)(\beta\lambda + \kappa\alpha)}{(\beta\lambda + \kappa\alpha)(\lambda + \kappa\alpha^2)}. \qquad (20)$$

The derivation of $\min\chi$ given by (20) with respect to different parameters yields:

$$\frac{\partial\min\chi}{\partial\lambda} = \frac{l_1\kappa\alpha\beta(\kappa^2\alpha^3 - \beta\lambda^2)}{(\beta\lambda + \kappa\alpha)^2(\lambda + \kappa\alpha^2)^2} > 0 \text{, if } \frac{\lambda}{\kappa} < \sqrt{\frac{\alpha^3}{\beta}} \text{ ;}$$

$$\frac{\partial\min\chi}{\partial\kappa} = \frac{l_1\alpha\beta\lambda(\beta\lambda^2 - \kappa^2\alpha^3)}{(\beta\lambda + \kappa\alpha)^2(\lambda + \kappa\alpha^2)^2} < 0 \text{, if } \frac{\lambda}{\kappa} < \sqrt{\frac{\alpha^3}{\beta}} \text{ ;}$$

$$\frac{\partial\min\chi}{\partial\beta} = \frac{\lambda l_1\alpha^2\kappa^2}{(\beta\lambda + \kappa\alpha)^2(\lambda + \kappa\alpha^2)} > 0 \text{ ;}$$

$$\frac{\partial\min\chi}{\partial l_1} = \frac{\kappa\alpha\beta\lambda}{(\beta\lambda + \kappa\alpha)(\lambda + \kappa\alpha^2)} > 0 \text{; } \quad \frac{\partial\min\chi}{\partial h} = \frac{\partial\min\chi}{\partial l_2} = 1 \text{ ;}$$

$$\frac{\partial\min\chi}{\partial\alpha} = \frac{l_1\kappa\beta\lambda(\beta\lambda^2 - \beta\lambda\kappa\alpha^2 - 2\kappa^2\alpha^3)}{(\beta\lambda + \kappa\alpha)^2(\lambda + \kappa\alpha^2)^2} < 0 \text{, if } \frac{\lambda}{\kappa} < \sqrt{\frac{\alpha^3}{\beta}} \text{ .} \qquad \blacksquare$$

$\beta\lambda\kappa\alpha^2 - \beta\lambda^2 + 2\kappa^2\alpha^3 > 0$.

When $\chi > \beta\lambda / (\Psi\beta\lambda + \Psi\kappa\alpha)$, monetary targeting rule (18) allows a sufficient adjustment in the money growth rate to equilibrate the money market without creating unstable dynamics of nominal and real economic variables. In the presence of inflationary pressures and under the above condition, monetary targeting rule (18), by mopping up excessive liquidity in the economy, discards any justification of further increase in current and expected inflation rates. Comparing with monetary targeting rule (16), structural parameters l_1, h and l_2 have similar effects while λ, κ, α and β have opposite effects over the minimal value of the coefficient ($\min\chi$) linking negatively money growth to the rate of change in nominal interest rate. For parameters λ, κ and α, the results are obtained under the same condition ($\lambda/\kappa < \sqrt{\alpha^3/\beta}$) as under the feedback monetary targeting rule (16).

For comparison, the effects of structural and preferences parameters of the economy over the liberty of formulating these three feedback monetary targeting rules are recapitulated in the Table 1.

Table 1. The effects of parameters on the design of monetary targeting rules.

	$\min\varphi$	$\min\eta$	$\min\chi$
λ	$-$	$-$, if $\lambda/\kappa < \sqrt{\alpha^3/\beta}$	$+$, if $\lambda/\kappa < \sqrt{\alpha^3/\beta}$
κ	$+$	$+$, if $\lambda/\kappa < \sqrt{\alpha^3/\beta}$	$-$, if $\lambda/\kappa < \sqrt{\alpha^3/\beta}$
β	$-$	$-$	$+$
l_1	$+$	1	$+$
h and l_2	>1	$+$	1
α	$+$, if $\lambda > \kappa\alpha^2$	$+$, $\lambda/\kappa < \sqrt{\alpha^3/\beta}$	$-$, if $\lambda/\kappa < \sqrt{\alpha^3/\beta}$

Propositions 2, 3 and 4 summarize some results which are compatible with the view of the modern quantitative theory of money, according to which, whenever there is an inflationary pressure, the money supply must be tightened to limit the rise of prices. The implications of these propositions are also compatible with inflation-targeting framework

where shocks are assumed to be i.i.d. only. In fact, under stochastic inflation-targeting framework where expected inflation rate is set equal to the inflation target, money supply is endogenous and perfectly elastic. When the money supply is not perfectly responding to the money demand and the money and financial markets are imperfect in transmitting the effects of the repo interest rate decisions to the economy, the design of a dynamically stabilizing 'two-pillar' strategy implies that the money growth rate must not be rigidified as a *k*-percent rule according to Proposition 1. It must sufficiently react to the rate of change in current inflation rate, output or/and nominal interest rate. Therefore the money supply, which is partially endogenous and imperfectly elastic, can be well-regulated with the adoption of one of these feedback money growth rules.

Most importantly, the principal findings of this paper cast doubt on some most important clichés of the modern quantitative theory of money as well as these of the standard stochastic inflation targeting framework. In order to curb an increase in inflation rate, inflation targeting implies an increase of nominal interest rate to sufficiently raise real interest rate. But that is not sufficient to ensure economic stability when shocks are not i.i.d., when the credibility and transparency of the central bank are not perfect or/and the interest rate policy is not perfectly efficient.

Under *k*-percent money growth rule, the resulting excess of liquidity due to diminishing real money demand for transaction and speculation is translated into vicious circle of increasing expected inflation rate, increasing nominal and real interest rates and diminishing real money demand. To avoid that, the money growth rules (14), (16) and (18) suggest sufficiently diminishing the liquidity in the economy respectively when current inflation rate varies positively, current output varies negatively and nominal interest rate varies positively. These simple rules can be combined to create other stabilizing money growth rules.

7. Conclusion

Considering that money and financial markets can imperfectly transmit the effect of interest rate policy to the economy, I have argued that the cheap talk of central bankers may not be sufficient to ensure the announced inflation target as credible nominal anchor of private inflation expectations and inflation-targeting central banks have good reasons to use monetary targeting together with inflation targeting.

This strategy resembles in first view to two-pillar monetary policy strategy adopted by the ECB. But it is to notice that the ECB does not explain neither the inflation process nor give the theoretical foundation of its strategy.

Under the new 'two-pillar' monetary policy strategy suggested in this paper, the quantity of money in a narrow sense must be regulated with a rule but not in the way conceived by Milton Friedman who proposes a k percent money growth rule. In effect, the money growth rate must be flexibly adjusted to answer directly or/and indirectly to shocks affecting real as well as money and financial markets between two interest rate decisions by the central bank.

This view is strongly supported by the fact that, in recent financial and economic turmoil, many central banks massively inject liquidity in the financial system to avoid the collapse of financial and economic system because using interest rate rule is not anymore sufficient. It can also find support in the long term relationship between money and inflation revealed in empirical studies. This relationship must not be interpreted as a simple correlation. The causality between these two variables must not be misinterpreted to deny the possibility that the strict control of money supply always allows controlling the inflation rate in the medium and long term.

Using as communication and anchoring device, well conceived monetary targeting with a commitment to a long-run money growth rate identical to the inflation target, as part of this hybrid inflation-targeting regime, could effectively reinforce the credibility of the central bank and the role of inflation target as strong and credible nominal anchor for private inflation expectations.

I have shown that the inflation-targeting regime associated with Friedman's simple k-percent money growth rule can generate macro-economic instability. To guarantee the stability of economic equilibrium, feedback money growth rules must be designed to sufficiently diminish the liquidity in the economy when current inflation rate varies positively, or/and current output varies negatively or/and nominal interest rate varies positively.

Moreover, the design of these feedback money growth rules is strongly influenced by the parameters characterizing the economic structure and the preferences of the central bank. The results obtained in this paper might help stopping internal quarrels in the ECB about two-pillar strategy. They might also contribute to reanimate the debate about the monetary policy strategy where we almost only hear the enthusiast voices of these who are for abandoning the two-pillar strategy to the profit of full-fledged inflation targeting.

Appendix: Dynamics of the expected inflation rate ($\dot{\pi}^e$)

At the end of a period, private agents revise their inflation expectations for the future using the money market equilibrium condition. Taking mathematical expectation of equation (5), it yields

$$\mu^e + h i^e + \dot{\varepsilon}_m^e - \pi^e = l_1 \dot{y}^e - l_2 i^e + \dot{\varepsilon}_l^e. \tag{A.1}$$

Taking time derivation of equations (8) and (10) and the mathematical expectations of the resulting equations leads to

$$i^e = \dot{\pi}^e + \frac{1}{\beta}[\frac{\kappa\alpha}{\lambda}(\dot{\pi}^e - \dot{\pi}^T) - \dot{y}^* + \dot{\varepsilon}_d^e] \qquad (A.2)$$

$$\dot{y}^e = \dot{y}^* - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\pi}^e + \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\pi}^T - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\varepsilon}_\pi^e. \qquad (A.3)$$

Substituting (A.2)-(A.3) into (A.1) gives:

$$\mu^e + \dot{\varepsilon}_m^e - \pi^e = l_1\left[\dot{y}^* - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\pi}^e + \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\pi}^T - \frac{\kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\varepsilon}_\pi^e\right] - (h+l_2)\left[\dot{\pi}^e + \frac{1}{\beta}[\frac{\kappa\alpha}{\lambda}(\dot{\pi}^e - \dot{\pi}^T) - \dot{y}^* + \dot{\varepsilon}_d^e]\right] + \dot{\varepsilon}_l^e.$$
$$\qquad (A.4)$$

Rearranging the terms in (A.4) yields,

$$\dot{\pi}^e = \Psi(\pi^e - \mu^e - \dot{\varepsilon}_m^e + \dot{\varepsilon}_l^e) + \Psi l_1 \dot{y}^* - \frac{\Psi(h+l_2)}{\beta}(\dot{y}^* - \dot{\varepsilon}_d^e) + \frac{\Psi l_1 \kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\pi}^T + \frac{\Psi(h+l_2)\kappa\alpha}{\beta\lambda}\dot{\pi}^T - \frac{\Psi l_1 \kappa\alpha}{\lambda + \kappa\alpha^2}\dot{\varepsilon}_\pi^e.$$
$$\qquad (A.5)$$

where $\Psi = \dfrac{(\lambda + \kappa\alpha^2)\beta\lambda}{l_1\kappa\alpha\beta\lambda + (h+l_2)(\lambda + \kappa\alpha^2)(\beta\lambda + \kappa\alpha)}$. ∎

References:

Alesina, A. O. Blanchard, J. Gali, F. Giavazzi & H. Uhlig (2001), *Defining a Macroeconomic Framework for the Euro Area*. Monitoring the European Central Bank 3. London: CEPR.

Almeida, Alvaro & Charles A. E. Goodhart (1998), "Does the Adoption of Inflation Targets Affect Central Bank Behaviour?" *Banca Nazionale del Lavoro Quarterly Review* (Rome), 204 (supplement), pp. 19-107.

Assenmacher-Wesche, Katrin & Stefan Gerlach (2007), "Understanding the Link between Money Growth and Inflation in the Euro Area," in: David Cobham (Ed.) "The Travails of the Eurozone", Macmillan-Palgrave, London.

Beck, Guenter W. & Volker Wieland (2007), "Money in Monetary Policy Design under Uncertainty: The Two-Pillar Phillips Curve versus ECB-Style Cross-Checking," CFS Working Paper Series 2007/17, Center for Financial Studies.

Bernanke, B., Laubach, T., Mishkin, F., and Posen, A. (1999), *Inflation Targeting*. Princeton, New Jersey: Princeton University Press.

Bernanke, Ben S. & Ilian Mihov (1997), "What does the Bundesbank target?" *European Economic Review*, Vol. 41, Iss. 6, pp. 1025-1053.

Bernanke, Ben S., & Frederic S. Mishkin (1997), "Inflation Targeting: A New Framework for Monetary Policy?" *Journal of Economic Perspectives* 11, no. 2 (spring), pp. 97-116.

Buiter W. H. & N. Panigirtzoglou (2003), "Overcoming the Zero Bound on Nominal Interest Rates with Negative Interest on Currency: Gesell's Solution," *Economic Journal*, 113, pp. 723-746.

Cabos, Karen, Michael Funke & Nikolaus A. Siegfried (2003), **"**Some Thoughts on Monetary Targeting vs. Inflation Targeting," *The German Economic Review*, Vol. 5, Iss. 3, pp. 219 – 238.

Christiano, Lawrence J., Roberto Motto & Massimo Rostagno (2007). Two Reasons Why Money and Credit May be Useful in Monetary Policy. *NBER Working Paper* No. W13502.

Clarida, R., Gali, J. & Gertler, M., (1998), "Monetary policy rules in practice: Some international evidence," *European Economic Review* 42, pp. 1033-1068.

Clarida, R., Gertler, M. (1996), "How the Bundesbank conducts monetary policy," *NBER Working Paper* 5581.

Dai, Meixing & Moïse Sidiropoulos (2003), "Règle du taux d'intérêt optimale, prix des actions et taux d'inflation anticipé : une étude de la stabilité macroéconomique," *Économie Appliquée*, tome LVI, n°4, p. 115-140.

Dai, Meixing & Moïse Sidiropoulos (2005), "Should Inflation-Targeting Central Banks care about dynamic instabilities in an open economy," *Ekonomia*, Vol. 8(2), pp. 1-20.

Dai, Meixing. (2006), "Inflation-targeting under a Managed Exchange Rate: the Case of Chinese Central Bank," *Journal of Chinese Economic and Business Studies*, Vol. 4, No. 3, pp. 199-219.

Evans, George W. & Seppo Honkapohja (2003) "Friedman's money supply rule vs optimal interest rate policy," *Scottish Journal of Political Economy*, Vol. 50, pp. 550-566.

Feldstein, Martin & James H. Stock (1994). The use of a monetary aggregate to target nominal GDP. In N.G. Mankiw (Ed.), *Monetary Policy*, University of Chicago Press, Chicago, 7–62.

Freedman, C. (1996), "What operating procedures should be adopted to maintain price stability? Practical issues," in: Federal Reserve Bank of Kansas City, *Achieving Price Stability*. Kansas City.

Friedman, Benjamin. M. (2003), "The LM curve: a not-so-fond farewell," *NBER Working Paper* n° 10123.

Friedman, Milton (2005), "A Natural Experiment in Monetary Policy Covering Three Episodes of Growth and Decline in the Economy and the Stock Market," *Journal of Economic Perspectives*, Vol. 19, issue 4, pp. 145-150.

Gerberding, Christina, Franz Seitz & Andreas Worms (2005), "How the Bundesbank really conducted monetary policy," *The North American Journal of Economics and Finance*, Vol. 16, Iss. 3, pp. 277-292.

Gerlach, Stefan (2004), "The Two Pillars of the European Central Bank," *Economic Policy* 40, pp. 389-439.

Gersbach, Hans & Volker Hahn (2003), "Signalling and Commitment: Monetary versus Inflation Targeting," CEPR Discussion Papers No. 4151.

Goodhart, Charles A. E. (2007), "Whatever Became of the Monetary Aggregates?" *National Institute Economic Review* 200, 56-61.

Groeneveld, H., Koedijk, K.G. & Kool, C.J.M. (1998), "Inflation Dynamics and Monetary Strategies: Evidence from Six Industrialized Countries," *Open Economies Review*, Vol. 9, No. 1, pp. 21-38.

Hafer R.W. & Garett Jones (2008). Dynamic IS Curves With and Without Money: An International Comparison. *Journal of International Money and Finance*, vol. 27(4), 609-616.

Hafer R.W., Joseph H. Haslag & Garett Jones (2007), "On money and output: Is money redundant?" *Journal of Monetary Economics*, Vol. 54, Iss. 3, pp. 945-954.

Hess, Gregory D., David H. Small & Flint Brayton (1993), "Nominal income targeting with the monetary base as instrument: an evaluation of McCallum's rule," *Proceedings*, Board of Governors of the Federal Reserve System (U.S.).

Judd, John P. & Brian Motley (1991), "Nominal Feedback Rules for Monetary Policy," *Economic Review, Federal Reserve Bank of San Francisco*, Number 3, 3-17.

King, M., (1996), "Direct inflation targets," in: Deutsche Bundesbank (Ed.), *Monetary Strategies in Europe*. Vahlen, MuKnchen.

King, Robert G. (2000), "The new IS-LM model: language, logic, and limits," *Economic Quarterly-Federal Reserve Bank of Richmond*, issue Sum, pp. 45-103.

Laubach, Thomas & Posen, Adam S. (1997), "Disciplined discretion: the German and Swiss monetary targeting frameworks in operation," Federal Reserve Bank of New York, *Research Paper* No. 9707.

Laubach, Thomas (2003), "Signalling commitment with monetary and inflation targets," *European Economic Review*, Vol. 47, Iss. 6, pp. 985-1009.

Mayer, Thomas (2006), "The ECB's monetary policy strategy: Trying to go beyond inflation targeting," *Society of Business Economists*, Vol 37, no 1.

McCallum, Bennett T. & James G. Hoehn (1983), **"**Instrument Choice for Money Stock Control with Contemporaneous and Lagged Reserve Requirements: Note," *Journal of Money, Credit and Banking*, Vol. 15, No. 1, pp. 96-101.

McCallum, Bennett T. (1988a), "Robustness Properties of a Rule for Monetary Policy," *Carnegie-Rochester Conference Series on Public Policy* 29, 173-204.

McCallum, Bennett T. (1988b), "Targets, Indicators, and Instruments of Monetary Policy," in: *Monetary Policy in an Era of Change,* Washington, D. C.: American Enterprise Institute.

Mishkin, Frederic S. (1999), "International Experiences with Different Monetary Policy Regimes," *Journal of Monetary Economics*, Vol. 43, No. 3, pp. 579-606.

Mishkin, Frederic S. (2002), "From Monetary Targeting to Inflation Targeting: Lessons from Industrialized Countries," in: *Banco de Mexico, Stabilization and Monetary Policy: The International Experience* (Bank of Mexico: Mexico City), pp. 99-139.

Modigliani, Franco, Robert Rasche & J Philip Cooper (1970), "Central Bank Policy, the Money Supply, and the Short-Term Rate of Interest," *Journal of Money, Credit and Banking*, Ohio State University Press, Vol. 2(2), pp. 166-218.

Neumann, M. J. M. & von Hagen, Jürgen (1995), "The Strategy of Monetary Targeting: Can the German Experience Provide a Model for the ECB?" University of Bonn, Germany, *Discussion Paper* No. B-324.

Romer, D. (2000), "Keynesian Macroeconomics without the LM Curve," *The Journal of Economic Perspective*, Vol. 14, n° 2, pp.149-169.

Rudebusch, G. & Svensson, Lars E. O. (1999), "Policy Rules and Inflation Targeting," in Taylor, J.B. (Ed.), *Monetary Policy Rules*. University of Chicago Press, Chicago, pp. 203-246.

Rudebusch, Glenn D. & Svensson, Lars E. O. (2002), "Eurosystem Monetary Targeting: Lessons from U.S. Data," *European Economic Review*, Vol. 46, Iss. 3, pp. 417-42.

Svensson, L.E.O. (1999a), "Inflation Targeting as a Monetary Policy Rule," *Journal of Monetary Economics* 43, pp. 607-654.

Svensson, L.E.O. (1999b), "Monetary Policy Issues for the Eurosystem," *Carnegie-Rochester Conferences Series on Public Policy* 51(1), pp. 79-136.

Svensson, L.E.O. (2000), "The First Year of the Eurosystem: Inflation Targeting or Not?" *American Economic Review: Papers and Proceedings* 90, pp. 95-99.

Taylor, John B. (1985), "What Would Nominal GNP Targeting Do to the Business Cycle?" *Carnegie-Rochester Conference Series on Public Policy* 22, pp, 61-84.

von Hagen Jürgen (1999), "Money growth targeting by the Bundesbank," *Journal of Monetary Economics, Vol.* 43, Issue 3, pp. 681-701.

Walsh, Carl E. (1999), *Monetary Theory and Policy*, the MIT Press.

Walsh, Carl E. (2002), "Teaching Inflation Targeting: An Analysis for Intermediate Macro," *Journal of Economic Education 33* (4), pp. 333-347.

Woodford, Michael (1998), "Doing without Money: Controlling Inflation in a Post-Monetary World," *Review of Economic Dynamics* 1, 173-219.

Woodford, Michael (2008), "How important is Money in the Conduct of Monetary Policy?" *Journal of Money, Credit and Banking*, Vol. 40, No. 8, 1561-1598.