Economics
The Open-Access, Open-Assessment E-Journal

Discussion Paper
No. 2009-11 | January 22, 2009 | http://www.economics-ejournal.org/economics/discussionpapers/2009-11

Productive Base Sustainability under Climate Change: Theoretical Results and Empirical Evidence

Dimitra Vouvaki and Anastasios Xepapadeas

University of Crete; Athens University of Economics and Business

Abstract

Climate change is one of the most urgent and severe problems on the international agenda and one of the basic factors that determine sustainability conditions. This paper attempts to reveal the connection between productive base sustainability for two large groups of countries, developed and developing, and the state of the environment, which is proxied by the stock of carbon dioxide (CO2) which is mostly responsible for the creation of the global warming phenomenon. Three different policy scenaria for the evolution of global CO2 emissions empirically confirm the strong association between the state of the environment and productive base sustainability, and provide the foundations for the formulation of sustainability policy.

Paper submitted to the special issue “Discounting the Long-Run Future and Sustainable Development”

JEL: Q01

Keywords: Productive base sustainability; current change in social welfare; accounting prices; non-optimizing economies; developed countries; developing countries; policy implications

Correspondence:

Dimitra Vouvaki, University of Crete, email: dimitra@econ.soc.uoc.gr;
Anastasios Xepapadeas, Athens University of Economics and Business and Beijer Fellow, email: xepapad@aueb.gr

1 Introduction

The concept of productive base sustainability which stems from the definition of sustainable development as nondeclining social welfare (NDSW) can be regarded as an approach which is operationally and empirically useful for the development of sustainability criteria, sustainability indicators and for the design of policies promoting sustainable development. NDSW means avoiding any decline in intergenerational social welfare defined in terms of a Ramsey–Koopmans social welfare functional (R–K SWF), either from time t forever onwards, or much less demandingly, just at time t (Riley, 1980; Dasgupta and Mäler, 2001; Pemberton and Ulph, 2001; Arrow et al., 2003).[1] Arrow et al. (2003, p. 653) define a sustainable development path at t as one with NDSW at t. They show that in an autonomous nonoptimizing economy,[2] this criterion implies the maintenance of the economy's "productive base" at t. Thus NDSW at t is equivalent to non-negative genuine investment at t, with genuine investment defined as the sum of investments, valued at accounting prices, in all productive assets such as manufactured capital, human capital, natural capital and knowledge.

In a recent paper Vouvaki and Xepapadeas (2008) used the concepts of NDSW and productive base sustainability to characterize, both theoretically

---

[1] The now widely accepted Bruntland's Report definition of states that "Sustainable development is the development that meets the needs of the present without compromising the ability of future generations to meet their own needs". In an attempt to make this definition operational, a number of auxiliary definitions have been developed which identify conditions under which an economy can be regarded as following a sustainable development path. The most prevailing of these definitions, as reported by Pezzey (2004a), associate sustainability with:

- Achieving constant utility (Solow, 1974; Hartwick, 1977).
- Having the representative agent's utility (well-being) $U(t)$ not exceeding the maximum level of utility $U^m(t)$ which can be sustained forever from t onwards given the capital stocks existing at t (Pezzey, 2004b). This definition is implied by, but does not imply, the well-known condition that the agent's utility is forever non-declining from t onwards (Pearce et al., 1990; Pezzey, 1992, 1997).
- Nondeclining social welfare (NDSW), as defined above.

[2] In the context of this paper nonoptimizing means that the saving ratio in the economy is fixed and is not chosen by maximizing lifetime utility.

and empirically, sustainability conditions and changes in current social welfare for nonoptimizing economies by direct estimation of accounting prices. In this model the consumption of natural capital was proxied by the accumulation of a local pollutant which generated environmental damages. In this paper we seek to study the likely evolution of the productive base of economies, when global natural capital which is associated with the global climate is affected by economic activity.

Climate change and global warming has been a central issue both at national and international levels. Global temperature increase, due to the well-known greenhouse effect, is likely to trigger serious detrimental impacts on a global scale which have been extensively discussed in various reports.[3] Extreme weather events, destruction of ecosystems and loss of species, rise of the sea level which will endanger coastal areas and small islands, significant adverse effects on agriculture, and redistribution of world income to the detriment of lower income countries are some of the likely consequences. It is now generally accepted that most of the global warming is caused by human activities which involve the burning of fossil fuels. This causes the emissions of 'greenhouse gasses' (GHGs) which are considered to be the main factor responsible for climate change, with carbon dioxide (CO_2) regarded as by far the most important among GHGs. The emissions and the resulting excess accumulation of GHGs can be regarded as a form of depletion of natural capital which is associated with global climate. Since the depletion of natural capital means in broad terms that a smaller amount of this part of the productive base of the economy is passed to future generations, climate change is directly linked with sustainability concepts discussed above. Current reports (IPCC 2000, the Stern Report, 2006) present different possible future scenaria that include more or less pessimistic predictions for the years to come, depending on the way societies decide to handle and control the

---

[3] See for example various IPCC Reports, the European Commission Report (2006), or The Stern Report (2006). NASA reports that 2006 was the fifth warmest year on record and 2007 will likely be even warmer - possibly the warmest year in the history of instrumental measurements. Over the past 30 years Earth has warmed by about 0.6 degrees Centigrade or 1.08 degrees Fahrenheit.

global warming phenomenon today and in the immediate future.[4] If policies that have not properly addressed global warming and climate change are to be continued, this phenomenon will be intensified and will impede attempts to build and maintain a development process that could be characterized as sustainable in the productive base sense discussed above.

Based on the concepts of productive base sustainability and climate change as discussed above, the paper's main objectives are to develop first a conceptual framework capable of modelling the impact of global carbon dioxide concentration that leads to global warming and climate change, on productive base sustainability, and second, to empirically approximate this impact for two large groups of countries (developed and developing[5]). Since productive base sustainability is associated with NDSW, our approach provides a direct theoretical and empirical link between changes in current social welfare (CCSW) and global warming. The measure of CCSW is the time derivative of R-K SWF which embodies environmental damages caused by climate change. If the time derivative of an R-K SWF at time t is positive, then an economy is currently productive base sustainable and genuine investment is also positive (Arrow et al., 2003). In this sense, sustainable development is measured as the change in productive capacity. Reductions in productive capacity can be captured by negative genuine investment and imply that we leave less productive capacity to future generations to satisfy their needs. More specifically, if an economy is not currently productive base sustainable, then the time derivative of the R-K SWF at time t is negative and genuine investment is also negative. Negative genuine investment (or savings) implies that total wealth is in decline and policies that lead to persistently negative genuine savings are unsustainable. This can be considered as a *productive base approach* to sustainable development.

---

[4] "The current level or stock of greenhouse gases in the atmosphere is equivalent to around 430 parts per million (ppm) CO_2, compared with only 280ppm before the Industrial Revolution. These concentrations have already caused the world to warm by more than half a degree Celsius and will lead to at least a further half degree warming over the next few decades, because of the inertia in the climate system" (Stern, 2006, Executive Summary p. iii).

[5] The distinction between developed and developing countries is based on the OECD and non-OECD countries partitioning.

Following this methodological approach, we develop our model for the case of nonoptimizing economies[6] that we believe best fits current economic structures. We estimate CCSW conditions for two large groups of 23 developed OECD countries[7] and 21 developing non-OECD economies[8] by taking into account one of the basic environmental factors that can be held accountable for the global warming phenomenon, namely CO_2 emissions.[9] Our results suggest that under the current production structure and CO_2 emission time paths, the current estimates of CO_2 damages, and the projected time paths for emission,[10] the measured CCSW are negative. When CO_2 emission time paths are considered as a policy parameter and their future time paths are adjusted so that emissions do not increase over time, then CCSW becomes positive. Based on these results it seems that our theoretical framework might be capable of providing an assessment regarding the productive base sustainability conditions of economies, along with some policy suggestion for attaining or maintaining productive base sustainability under conditions of climate change.

---

[6] A non-optimizing economy is an economy where government, whether by design or by incompetence, does not choose policies that maximize intergenerational welfare (Arrow et al., 2003).

[7] The 23 countries used in our analysis are the following: Canada, U.S.A, Austria, Belgium, Denmark, Finland, France, Greece, Italy, Portugal, Spain, Sweden, Switzerland, U.K., Japan, Iceland, Ireland, Netherlands, Norway, Australia, Mexico, Turkey, Luxembourg.

[8] The 21 developing countries used in our analysis are the following: Peru, Thailand, Paraguay, Morocco, Dominican Republic, Guatemala, Honduras, Jamaica, Bolivia, Colombia, Ecuador, Iran, Sri Lanka, Syria, Yugoslavia, India, Kenya, Madagascar, Malawi, Sierra Leone, Zimbabwe.

[9] There are two basic reasons why we use CO_2 emissions in this paper as the basic contributant to the global warming phenomenon. The first reason is that CO_2 emissions is the most important of all the other GHGs such as methane, nitrous oxides etc in terms of percentage contribution in the global warming phenomenon. The second reason is more practical and has to do with the avaliability of data on CO_2 emissions for those two large groups of countries.

[10] The Stern Report scenario corresponds to a 2.5% annual increase in CO_2 emissions.

2 Descriptive growth with emissions as an input

Starting from the concept of a non-optimizing economy in the sense that while firms maximize profits, consumers save a fixed proportion of their income, our attempt is to provide a measure of current changes in social welfare. We consider a stylized economy where the productive base includes a list of assets such as physical capital, human capital, and natural capital, along with labor augmenting (Harrod neutral) technical change and emission augmenting technical change. We consider the earth's atmosphere as a component of social overhead capital (Uzawa, 2003) which can play the role of natural capital. In this case natural capital is associated with the stock of accumulated GHGs while CO_2 emissions along with other GHGs can be thought of as a reduction of this social capital - a form of disinvestment. Thus the impact of natural capital in our model is captured by two factors: emissions of CO_2 and other GHGs which are considered as an input into the aggregate production function. Environmental damages are associated with the global stock of CO_2 and GHGs that accumulate globally and cause global warming and climate change.

Capital accumulation in our stylized economy is described by using the standard Solow model. We assume that exogenous technical change of labour augmenting type and technical change associated with emissions are present. The production function we use is of the form:

$$Y = F(K, H, AL, BZ) \tag{1}$$

where K is physical capital, H is human capital, AL is effective labour with L being labor in physical units and A reflecting labor augmenting technical change,[11] and BZ is effective input of emissions, with Z being emissions in physical units[12] and B reflecting emission saving technical change, or input

---

[11] $A(t)$: the level of labor augmented technical change is defined as $A_0 e^{gt}$.

[12] Emissions in this paper are an input in production and can be regarded as a proxy for energy. In this interpretation emissions are not treated only as a by product of output production.

augmenting technical change[13]. Using the Cobb-Douglas assumption, the production function (1) becomes:

$$Y = K^{a_1} H^{a_2} (AL)^{a_3} (BZ)^{a_4}$$

Assuming the existence of constant returns to scale: $a_1 + a_2 + a_3 + a_4 = 1$, and expressing output in per worker terms, where $y = \frac{Y}{L}, k = \frac{K}{L}$, $z = \frac{Z}{L}$ and $h = \frac{H}{L}$, we obtain:

$$\begin{aligned} \frac{Y}{L} &= \left(\frac{K}{L}\right)^{a_1} \left(\frac{H}{L}\right)^{a_2} \left(\frac{AL}{L}\right)^{a_3} \left(\frac{BZ}{L}\right)^{a_4}, \\ y &= \left(e^{gt}\right)^{a_3} \left(e^{bt} z\right)^{a_4} k^{a_1} h^{a_2}, \\ y &= e^{(ga_3 + a_4 b)t} k^{a_1} h^{a_2} z^{a_4} \ , \ ga_3 + a_4 b = \lambda \\ y &= e^{\lambda t} k^{a_1} h^{a_2} z^{a_4} \end{aligned}$$

Capital accumulation in per worker terms, assuming that the two capital goods (produced and human) depreciate at the same constant rate[14] is given by:

$$\dot{k} + \dot{h} = sy - (\eta + \delta)(k + h) \tag{2}$$

Defining $k = \hat{k} e^{\xi t}$, $h = \hat{h} e^{\xi t}$, and $z = \hat{z} e^{\xi t}$ in efficiency units we have:

$$\dot{k} = \dot{\hat{k}} e^{\xi t} + \xi \hat{k} e^{\xi t}, \ \dot{h} = \dot{\hat{h}} e^{\xi t} + \xi \hat{h} e^{\xi t} \text{ and } \dot{z} = \dot{\hat{z}} e^{\xi t} + \xi \hat{z} e^{\xi t} \tag{3}$$

Substituting $\dot{k}$ and $\dot{h}$ in (2) we obtain:

$$\dot{\hat{k}}_t e^{\xi t} + \xi \hat{k} e^{\xi t} + \dot{\hat{h}}_t e^{\xi t} + \xi \hat{h} e^{\xi t} = s e^{\lambda t} (\hat{k}_t e^{\xi t})^{a_1} (\hat{h}_t e^{\xi t})^{a_2} \left(\hat{z} e^{\xi t}\right)^{a_4} - (\eta + \delta)(\hat{k}_t e^{\xi t} + \hat{h}_t e^{\xi t}) \tag{4}$$

Solving (4) in the appendix we obtain the solution for the time path of

---

[13] $B(t) = B_0 e^{bt}$. We normalize the initial level of emission augmented technical change, by setting $B_0 = 1$ assuming that each of the groups of the countries we examine started at the beginning of our data period (1965) approximately at the same level of emissions augmenting technical change.

[14] For this assumption see Barro and Sala-i-Martin (2004).

the stock of capital:

$$\hat{k}_\tau = \left[\left(\hat{k}_t^{1-\phi} - \frac{s\Psi \left(z_t e^{\xi\tau}\right)^{a_4}}{\eta+\delta} \right) e^{-(1-\phi)(\eta+\delta+\xi)(\tau-t)} + \frac{s\Psi \left(z_t e^{\xi\tau}\right)^{a_4}}{\eta+\delta} \right]^{\frac{1}{1-\phi}}, \text{ for } \tau \geq t \tag{5}$$

Equation (5) expresses the time path of the physical capital stock in the economy as a function of the parameters of the economy and the time path of emissions per capita. In the next section we examine the way in which the path of emissions might be determined in a market economy and the impacts of these time paths on the economy's value function.

3 Value functions and policy implications under global warming

In this section we define the choice of emissions and the implied time path in a context of profit maximizing firms. Assume a representative competitive firm which solves the following profit maximization problem:

$$\begin{aligned} \max \Pi &= F(K, H, AL, BZ) - R_K K - R_H H - wL \qquad (6) \\ &\text{subject to} \quad Z \leq \bar{Z} \end{aligned}$$

Positive marginal products for the inputs and profit maximization implies that $Z = \bar{Z}$ where $\bar{Z}$ is an upper emissions limit for the representative firm. The upper bound on emissions could reflect technical constraints associated with production technologies or an emission limit determined exogenously by a regulator or an international agreement such as Kyoto. In this case aggregate emissions are constrained by the emission limit and emissions in per effective worker terms are defined as:

$$\hat{z} = \bar{Z} e^{-(\xi+\eta)t} = \frac{\bar{Z}}{L} e^{-\xi t} = \bar{z} e^{-\xi t} \tag{7}$$

where $\bar{Z}$ denotes the aggregate emission limit on CO_2 emissions and $\bar{z}$ the emission limit in per capita terms.

Using the standard Solow assumption, where consumption is a fixed proportion of output we have that consumption in per effective worker terms is defined as:

$$\hat{c}_\tau = (1-s)\,\hat{y} \tag{8}$$

where $\hat{y} = ye^{-\xi t}$. Thus (8) will take the form:

$$\hat{c}_\tau = (1-s)\,\Psi \hat{k}_\tau^{\phi} \hat{z}_\tau^{a_4}$$

and by replacing $\hat{k}_\tau$ by (5) in the consumption function we have:

$$\begin{aligned} \hat{c}_\tau = (1-s)\,\Psi \Bigg[\left(\hat{k}_t^{1-\phi} - \frac{s\Psi\left(\bar{z}_t e^{\xi\tau}\right)^{a_4}}{\eta+\delta} \right) e^{-(1-\phi)(\eta+\delta+\xi)(\tau-t)} + \\ \frac{s\Psi\left(\bar{z}_t e^{\xi\tau}\right)^{a_4}}{\eta+\delta}^{\frac{\phi}{1-\phi}} \left(\bar{z}e^{-\xi t}\right)^{a_4} \Bigg] \end{aligned} \tag{9}$$

The general state of the environment is introduced into the model by the variable P which is interpreted as the *stock* of CO_2 emissions which affects utility in a negative way. Then the utility function becomes a function of per capita consumption c_τ and total pollution P_τ and is assumed, as is common in this type of analysis, to have the following separable specification:

$$U(c_\tau, P_\tau) = \frac{c_\tau^{1-\sigma}}{1-\sigma} - D(P_\tau) \text{ for } 0 \leq \sigma < 1 \tag{10}$$

$$U(c_\tau, P_\tau) = \ln c_\tau - D(P_\tau) \text{ for } \sigma = 1 \tag{11}$$

In (10) σ is the elasticity of marginal utility, and P_τ is pollution stock which creates disutility. Therefore $D(P_\tau)$ can be interpreted as a damage function assumed strictly increasing and convex. We specify the damage function as $D(P_\tau) = \beta P_\tau^{\gamma}$ with $\beta > 0$ and $\gamma \geq 1$. Since the production structure is determined in per effective worker terms, we need to specify the utility function (10) in per effective worker terms. If we define consumption per effective worker as $\hat{c} = \dfrac{C}{AN}$, from the definition of per capita consumption we have:

$$\frac{C_\tau}{N_\tau} = c_\tau = \hat{c}_\tau A_t e^{g(\tau-t)}$$

then we have:

$$u\left(c_\tau\right) = \frac{1}{1-\sigma}\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)^{1-\sigma}$$

and the utility function (10) becomes:

$$U\left(c_\tau, P_\tau\right) = \frac{1}{1-\sigma}\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)^{1-\sigma} - \beta P_\tau^\gamma \tag{12}$$

We assume that the evolution of CO_2 stock, denoted by P_τ, is determined by a first-order linear differential equation:

$$\dot{P}_\tau = \sum_{j=1}^{J} Z_i - mP_\tau \ \ , P\left(t\right) = P_t \tag{13}$$

where $\sum_{j=1}^{J} Z_j = Z^T$ is the sum of aggregate emissions from $j = 1, ..., J$ countries which are possibly constrained under an international agreement, with m reflecting exponential GHGs decay.

The solution of (13) is:

$$P_\tau = (P_t - \frac{Z^T}{m})e^{-m(\tau-t)} + \frac{Z^T}{m} \tag{14}$$

Then damages from CO_2 stock for country j can be determined as:

$$D_j(P_\tau) = \beta_j\left[(P_t - \frac{Z^T}{m})e^{-m(\tau-t)} + \frac{Z^T}{m}\right]^{\gamma_j}$$

The utility flow in per effective worker terms for country j can be specified as:

$$U_j\left(\hat{k}_t, A_t, z, Z^T, P_t\right) = \frac{1}{1-\sigma}\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)_j^{1-\sigma} - \beta_j\left[(P_t - \frac{Z^T}{m})e^{-m(\tau-t)} + \frac{Z^T}{m}\right]^{\gamma_j} \tag{15}$$

The flow of total utility in the economy is $N_{j\tau}U_j(c_\tau, P_\tau)$. Therefore the value function for the economy, using (15) becomes:[15]

$$V_{jt} = \int_t^\infty e^{-\rho(\tau-t)} N_{j\tau} U_j(\hat{k}_t, A_t, \hat{z}, Z^T, P_t)dt \ , \quad N_\tau = N_{jt}e^{n_j(\tau-t)} \tag{16}$$

$$V_{jt}\left(\hat{k}_t, N_t, A_t, , z, Z^T P_t\right) = \int_t^\infty e^{-(\rho-n_j)(\tau-t)} N_{jt}\left[\frac{1}{1-\sigma}\left(\hat{c}_\tau A_t e^{g(\tau-t)}\right)_j^{1-\sigma} - \beta_j\left((P_t - \frac{Z^T}{m})e^{-m(\tau-t)} + \frac{Z^T}{m}\right)^{\gamma_j}\right]dt \tag{17}$$

It should be noted that under an effective emission limit $\hat{z}$ is defined in terms of emission limit $\bar{z}$ through (7). We do not examine how countries have reached these emissions limits. They might have been determined through an agreement such as Kyoto or limits might have been determined unilaterally. The key assumption, is however, that irrespective of how the limits have been set, they are not the outcome of an explicit optimization either at a national or at a global level, but, as is probably more realistic, they are the outcome of a non-optimizing political process. In the above formulation we could distinguish between small and large countries. A small country will consider Z^T as a fixed exogenous parameter. On the other hand, a large country might recognize its contribution in total emissions. In this case, aggregate emissions for the large country l will be defined as:

$$Z^T = \bar{Z}_l + \sum_{j \neq l} \bar{Z} = \bar{Z}_l + Z^T_{-l} \tag{18}$$

If we write $\bar{Z}_l = \bar{z}_l e^{(\xi+\eta)t}$, then accounting prices for any country l at time t can be defined as:

---

[15] A more complex structure would require additional transition equations, for say, natural resources (depletable or renewable), stocks of pollutants, human capital and so on. In this case the value function would depend on the current values of the stocks for these assets. The development of such a dynamic system, so that the value function can be defined in an operational way, is an area for future research.

$$p_{\hat{k}_{lt}} = \frac{\partial V_t}{\partial \hat{k}_{lt}},\ p_{N_{lt}} = \frac{\partial V_t}{\partial N_{lt}},\ p_{A_{lt}} = \frac{\partial V_t}{\partial A_{lt}},\ p_{P_{lt}} = \frac{\partial V_t}{\partial P_{lt}},\ p_{\bar{z}_{lt}} = \frac{\partial V_t}{\partial \bar{z}_{lt}},\ p_{\bar{Z}^T_{-lt}} = \frac{\partial V_t}{\partial \bar{Z}^T_{-lt}} \tag{19}$$

It should be noted that there is an accounting price for the emission limit $\bar{z}_l$, which is formed by two effects: the effect of the emission limit on consumption through the production function as reflected in (10), and the effect of the emission limit on environmental damages, through aggregate emissions as reflected in the second term of (15). There is also an accounting price for the aggregate emissions of all other countries since these aggregate emissions affect environmental damages.

Since for any variable $\omega = \left(\hat{k}, \hat{z}\right)$ we have:

$$\hat{\omega} = \omega e^{-\xi t} = \frac{\Omega}{N} e^{-\xi t} \tag{20}$$

accounting prices in total and per capita terms are defined as:

$$p_{t\Omega_t} = \frac{\partial V_t}{\partial \hat{\omega}_t} \frac{\partial \hat{\omega}_t}{\partial \Omega_t} = \frac{e^{-\xi t}}{N_t} p_{t\hat{\omega}_t} \tag{21}$$

$$p_{t\omega_t} = \frac{\partial V_t}{\partial \hat{\omega}_t} \frac{\partial \hat{\omega}_t}{\partial \omega_t} = e^{-\xi t} p_{t\hat{\omega}_t} \tag{22}$$

4 A productive-base sustainability criterion

In our stylized economy, a positive change in current social welfare can be considered as an indicator of productive-base sustainability for the country analyzed. In other words, if:

$$\dot{V}_t = p_{K_t}\dot{K} + p_{N_t}\dot{N} + p_{A_t}\dot{A} + p_{\bar{Z}_t}\dot{\bar{Z}}_\tau + p_{P_t}\dot{P}_\tau \geq 0 \tag{23}$$

then the economy is currently productive base sustainable. More analytically, (i) if the time derivative of the social welfare function is positive, this implies that CCSW is positive and that genuine investment is also positive,[16] without however implying sustainability in individual utility terms, (ii) if the time derivative is negative, then genuine investment is negative.[17] The accounting prices for capital, population, technology, the emission limit and the pollution stock are $\left(p_{K_t}, p_{N_t}, p_{A_t}, p_{\bar{Z}_t}, p_{P_t}\right)$, while $\left(\dot{K}, \dot{N}, \dot{A}, \dot{\bar{Z}}_\tau, \dot{P}_\tau\right)$ are the rates of change of capital, population, technological change, emission limit and the pollution stock respectively.

Dividing by Nk where $k = \frac{K}{N}$, using the fact that $\dot{k} = \frac{d(K/N)}{dt} = \frac{\dot{K}}{N} - \frac{\dot{N}}{N}k$ and that the accounting price for capital in physical terms is related to the accounting price of capital in per effective worker terms, by (21) we obtain:

$$S_t = \frac{\dot{V}_t}{N_t k_t} = \frac{p_{t\hat{k}_t}}{A_t N_t}\left(\frac{\dot{k}}{k} + \frac{\dot{N}}{N}\right) + p_{tN_t}\frac{\dot{N}}{N}\frac{1}{k_t} + p_{tA_t}\frac{\dot{A}}{A}\frac{A_t}{N_t k_t} + p_{\bar{Z}_t}\frac{\dot{\bar{Z}}_\tau}{Z}\frac{Z}{N_t k_t} + p_{P_t}\frac{\dot{P}_\tau}{P}\frac{P}{N_t k_t} \tag{24}$$

where S_t measures the change in the value of the economy per unit of produced capital stock at time t and could be interpreted as the rate of return on produced capital measured in terms of social welfare. By multiplying S_t by the current stock of capital we obtain a measure of current genuine investment. Using as before $\dot{A}/A = g$; $\dot{N}/N = n$; and denoting the rate of growth of capital per worker by $\dot{k}/k = v$; by $\frac{\dot{\bar{Z}}_\tau}{Z} = \chi$; the rate of growth of the flow emission limit with $\chi < 0$ indicating that environmental policy becomes gradually more stringent and $\chi > 0$ indicating that it becomes gradually laxer; and with $\pi = \frac{\dot{P}_\tau}{P}$ the rate of change of the GHGs stock, we have that social welfare increases currently and thus development can be considered as currently sustainable in productive base terms if:

---

[16] Evidence provided by the World Bank (2006) suggests that investments in produced capital, human capital, and governance, combined with saving efforts aimed at offsetting the depletion of natural resources, can lead to future welfare increases in developing countries.

[17] As suggested by the World Bank (2006, Ch. 3), negative genuine saving rates imply that total wealth is in decline and policies leading to persistently negative genuine savings are unsustainable.

$$S_t = \frac{p_{t\hat{k}_t}}{A_t N_t}(v+n) + p_{tN_t} n \frac{1}{k_t} + p_{tA_t} g \frac{A_t}{N_t}\frac{1}{k_t} + p_{\bar{Z}_t} \chi \frac{Z}{N_t k_t} + p_{P_t} \pi \frac{P}{N_t k_t} \geq 0 \quad (25)$$

5 Results

Based on the descriptive growth model of section 2 and the methodology developed to determine whether an economy is currently productive-base sustainable, we present in this section the main empirical results. The parameters used and their numerical values,are presented in table 1 below The values correspond to the period 1965-1990.

Table 1: List of parameters

Parameters	*Values in tables*
v : Average growth of capital per worker	$(1a, 3a)$
n : Average growth of population	$(1a, 3a)$
β : Marginal damages from CO_2 stock	$(1a, 3a)$
s : Average saving rate	$(1b, 3b)$
χ : Average growth of CO_2 emissions	$(1b, 3b)$
k : Average value of capital per worker	$(1c, 3c)$
N : Average value of population per country	$(1c, 3c)$
Z : Average of CO_2 emissions per country	$(1c, 3c)$
Ψ : Constant of the production function	$(1c, 3c)$
$\phi = a_1 + a_2$	$(2, 4)$
a_3 : Production elasticity with respect to labor	$(2, 4)$
a_4 : Production elasticity with respect to emissions	$(2, 4)$
g : Rate of growth of labor augmenting technical change	$(2, 4)$
b : Rate of growth of emissions augmenting technical change	$(2, 4)$
δ : Depreciation rate	$(2, 4)$
σ : Elasticity of marginal utility	$(2, 4)$
$\lambda = ga_3 + ba_4$	$(2, 4)$
ρ : Utility discount rate	$(2, 4)$
π : Growth rate of total stock of CO_2	in (25)
γ : Parameter of the damage function	$(2, 4)$
$\Xi = \frac{\lambda}{1-a_1-a_2}$	$(2, 4)$

Parameters a_1 and a_2 are the production elasticities with respect to physical capital and with respect to human capital.[18],[19] In the context of the Barro and Sala-i-Martin (2004) assumption about the equality of marginal products of physical and human capital, we can interpret ϕ as the sum of the share of each of these two types of capital. For the case of *developed countries*: a_3 is the share of labor and a_4 is the share of emissions. For the case of *developing*

---

[18] In the competitive context all elasticities can be interpreted as the corresponding input share in output.

[19] Human capital is approximated by an index constructed from education data (Vouvaki and Xepapadeas 2008).

countries a_3 is the share of emissions and a_4 does not exist.

Tables 1a,b,c, 2, 3a,b,c and 4, present the parameter values used in our analysis of the 23 developed - OECD and the 21 developing - non-OECD countries in order to estimate the productive base sustainability criterion (25). Parameter values in tables 1a,b,c, and 3a,b,c, have been obtained using data from the Penn World tables 5.6. The estimated parameters in tables 2 and 4, are taken from Tzouvelekas et al. (2007).

Table 1a: Parameter values for the 23 developed countries

Countries	v	n	β
CANADA	0.032928687	0.021663857	0.00000000589025
U.S.A.	0.025689321	0.016860844	0.0000000170573
AUSTRIA	0.056128625	0.005204929	0.0000000112802
BELGIUM	0.033679182	0.006020976	0.0000000112802
DENMARK	0.032228406	0.009740526	0.0000000112802
FINLAND	0.038567052	0.007327035	0.0000000112802
FRANCE	0.041156021	0.008760808	0.0000000112802
GREECE	0.048409278	0.005288991	0.0000000112802
ITALY	0.038191952	0.004650733	0.0000000112802
LUXEMBOURG	0.024833593	0.00845968	0.0000000112802
PORTUGAL	0.048177233	0.009314411	0.0000000112802
SPAIN	0.059058156	0.007504434	0.0000000112802
SWEDEN	0.03556534	0.009562269	0.0000000112802
SWITZERLAND	0.033619931	0.007888075	0.00000000589025
U.K.	0.034014633	0.004932269	0.0000000112802
JAPAN	0.076563662	0.010002479	0.00000000589025
ICELAND	0.041646473	0.02103928	0.00000000589025
IRELAND	0.043979598	0.008039493	0.0000000112802
NETHERLANDS	0.030230736	0.013831165	0.0000000112802
NORWAY	0.007732509	0.014628489	0.00000000589025
AUSTRALIA	0.023857968	0.021364042	0.00000000589025
MEXICO	0.028233733	0.030730162	0.00000000589025
TURKEY	0.046517799	0.01948306	0.00000000589025

Table 1b: Parameter values for the 23 developed countries

Countries	s	χ
CANADA	0.192667465	-0.000268545
U.S.A.	0.154015995	-0.005307595
AUSTRIA	0.224252472	0.010731307
BELGIUM	0.237979369	-0.007877097
DENMARK	0.207586337	-0.009204111
FINLAND	0.233684447	0.017010918
FRANCE	0.198030225	-0.007721138
GREECE	0.167062284	0.051160436
ITALY	0.208762513	0.021453243
LUXEMBOURG	0.208762513	-0.015120024
PORTUGAL	0.202154111	0.04375945
SPAIN	0.218688333	0.034295707
SWEDEN	0.206691146	-0.02630012
SWITZERLAND	0.319314338	0.004782584
U.K.	0.158115522	-0.008363536
JAPAN	0.300260704	0.029049305
ICELAND	0.164227689	-0.008439755
IRELAND	0.201257546	0.020256859
NETHERLANDS	0.260411589	0.001596095
NORWAY	0.285949685	0.005501036
AUSTRALIA	0.200730732	0.011726163
MEXICO	0.197188633	0.024634313
TURKEY	0.201245619	0.044337772

Table 1c: Parameter values for the 23 developed countries

Countries	k	N	Z	Ψ
CANADA	29053.44	23264538.46	34.7	0.972305717
U.S.A.	26868.12	222123115.4	43	1.110251982
AUSTRIA	22481.44	7513230.769	15.7	0.852257132
BELGIUM	28152.6	9769153.846	29.8	0.884707674
DENMARK	25440.36	5034653.846	21.9	0.801291883
FINLAND	31474.16	4758153.846	19.4	0.704514053
FRANCE	25789.96	53046269.23	17.9	0.932318299
GREECE	17145.92	9355846.154	12.3	0.610576308
ITALY	22957.64	55493192.31	15.1	0.919599527
LUXEMBOURG	37022.96	357807.6923	74.9	0.828567478
PORTUGAL	7720.64	9487769.231	5.7	0.640545578
SPAIN	16900.32	36152269.23	12.5	0.895747244
SWEDEN	27359.56	8204807.692	18	0.909050164
SWITZERLAND	53245.24	6344538.462	12.7	0.865843589
U.K.	15321.44	56133653.85	22.1	0.919075505
JAPAN	19857.68	112855269.2	11.7	0.639526001
ICELAND	13281.72	223307.6923	15.6	0.949797951
IRELAND	15612.68	3251692.308	18.5	0.723397077
NETHERLANDS	25850.72	13791192.31	24.9	0.996420422
NORWAY	41986.04	4021692.308	14.2	0.762167448
AUSTRALIA	29943.04	14228115.38	28.7	0.918414009
MEXICO	11906.36	63155307.69	9.5	0.804922725
TURKEY	5459.76	42756115.38	4	0.443093265

For tables 1a-1c - above - and 3a-3c the parameters were obtained as follows: the average of the saving rates s for the case of the developed countries was obtained from the National Accounts of the OECD database and for the case of the developing countries was obtained from the "Economics, Business, and the Environment — National Savings: Gross savings as a percent of GNI". In estimating the production function we used fixed effects estimation so Ψ was the sum of the coefficient of the production function and the fixed effects of the production function. The shares of capital, labor, emissions and the rate of growth of labor augmenting and emission augmenting technical change were obtained from Tzouvelekas et al. (2007), β which is the marginal damages from CO_2 stock was estimated for the developed countries using Fankhauser and Tol (1997) estimated damages from the doubling of CO_2 in different world regions. For the developing countries, marginal damages were obtained using Nordhaus (1998).

Table 2: Common parameter values for developed countries

$Parameter$	ϕ	a_3	a_4	g	b	ga_3	ba_4	δ
$Value$	0.325968	0.596	0.077	0.014	0.026	0.008	0.002	0.03

σ	$\lambda = ga_3 + ba_4$	ρ	γ	$\Xi = \frac{\lambda}{1-a_1-a_2}$
0.5	0.010675682	0.03	1	0.015838539

For tables 2 and 4 the parameters were obtained as follows: the depreciation rate δ was the same for the case of developed and developing countries and was obtained from Mankiw et al. (1992). The elasticity of marginal utility σ was also the same for both cases and suggests that the equal distribution of income does not have a significant weight in the utility function. The utility discount rate ρ was taken as 3%[20] and $\gamma = 1$ which implies a linear damage function.

The parameter values for the group of developing countries are summarized in table 3a,b,c that follows.

---

[20] The value of 3% has been used by a number of researchers for the estimation of marginal social costs of CO_2 emissions (see, for example, surveys by Fankhauser and Tol, 1997, Tol, 2005). The values of 1% and 2%, along with time declining rates, have also been used in these studies.

Table 3a: Parameter values for the 21 developing countries

Countries	v	n	β
PERU	0.012155219	0.026573374	0.0000000907476
THAILAND	0.064312423	0.026938467	0.0000000529394
PARAGUAY	0.0599008	0.029792926	0.0000000907476
MOROCCO	0.01180978	0.030086627	0.0000000578675
DOMINICAN REP.	0.052249044	0.029116439	0.0000000578675
GUATEMALA	0.021661835	0.025445452	0.0000000907476
HONDURAS	0.016896959	0.031984921	0.0000000907476
JAMAICA	−0.00078736	0.021162102	0.0000000907476
BOLIVIA	0.030452654	0.021997488	0.0000000907476
COLOMBIA	0.02456555	0.025270989	0.0000000907476
ECUADOR	0.039715588	0.025793195	0.0000000907476
IRAN	0.069761428	0.034655315	0.0000000692038
SRILANKA	0.030501594	0.018220663	0.0000000529394
SYRIA	0.017400356	0.0300723	0.0000000692038
YUGOSLAVIA	0.050017192	0.008301377	0.0000000692038
INDIA	0.036262979	0.019425761	0.0000000529394
KENYA	−0.007093912	0.040524848	0.0000000578675
MADAGASCAR	0.007302069	0.020755864	0.0000000578675
MALAWI	0.056975768	0.025468426	0.0000000578675
SIERRALEONE	0.048099166	0.014407023	0.0000000578675
ZIMBABWE	−0.015083099	0.036904505	0.0000000578675

Table 3b: Parameter values for the 21 developing countries

Countries	s	χ
PERU	0.1877	−0.002464966
THAILAND	0.2777297297	0.075204219
PARAGUAY	0.1564102564	0.026822542
MOROCCO	0.2085714286	0.038209517
DOMINICAN REP.	0.1932432432	0.043170423
GUATEMALA	0.11885	0.012405097
HONDURAS	0.1550263158	0.017557974
JAMAICA	0.1973	0.017935637
BOLIVIA	0.1459714286	0.029362691
COLOMBIA	0.1784285714	0.010349838
ECUADOR	0.145425	0.05290342
IRAN	0.2933793103	0.020324961
SRILANKA	0.17465	−0.003297832
SYRIA	0.1774857143	0.061044875
YUGOSLAVIA	0.1774857143	0.03099768
INDIA	0.20095	0.036739344
KENYA	0.167225	−0.006222883
MADAGASCAR	0.5808333333	0.000359423
MALAWI	0.028	−0.003835103
SIERRALEONE	0.3292	−0.007724857
ZIMBABWE	0.1395789474	0.009521261

Table 3c: Parameter values for the 21 developing countries

Countries	k	N	Z	Ψ
PERU	8648.615385	16312.34615	4.006080643	1.67630338
THAILAND	2866.730769	43799.96154	1.481421126	1.138801052
PARAGUAY	609.3076923	3010.769231	1.159184513	2.003815273
MOROCCO	2147.615385	18701.69231	2.280925304	1.487305361
DOMINICAN REP.	3836.615385	5408.153846	3.602092971	1.448813579
GUATEMALA	3298	6600.846154	1.701286795	2.021977514
HONDURAS	4286.192308	3492.384615	1.628043995	1.206198888
JAMAICA	4436.384615	2064.615385	7.004211969	0.984705172
BOLIVIA	5720.346154	5330.307692	2.174962904	1.303648667
COLOMBIA	10647.73077	25299.73077	4.818757212	1.438349103
ECUADOR	11560.53846	7690.538462	4.170708691	1.528243444
IRAN	8191.384615	37401.38462	11.59577103	1.966782581
SRILANKA	6924.961538	14127.73077	0.699531707	1.439297288
SYRIA	12150.84615	8287.5	7.553502822	1.993631799
YUGOSLAVIA	5422.346154	21765.53846	9.351693997	1.410042717
INDIA	1376.153846	656496.1154	1.269852887	0.740467897
KENYA	1130.076923	15803.53846	0.650252843	0.773179535
MADAGASCAR	1731.038462	8379.230769	0.258102064	1.05960691
MALAWI	365.7307692	5860.038462	0.200389063	0.735358606
SIERRALEONE	163.3076923	3162.692308	0.420699171	1.606222261
ZIMBABWE	5759.615385	6768.346154	3.451498869	0.569696794

Table 4: Common parameter values for developing countries

$Parameter$	ϕ	$a_3 = s_z$	g	b	ga_2	ba_3	δ	σ	$\lambda = ga_2 + ba_3$	ϱ	γ	$\Xi = \frac{\lambda}{1-a_1-a_3}$
$Value$	0.095117	0.330547	0.00815	0.00405	0.004684	0.001339273	0.03	0.5	0.006023273	0.03	1	0.010487368

To determine (24) and (25), we also need values regarding the growth of CO_2 emissions and the growth of CO_2 stock. Treating the future growth of CO_2 emissions as a policy variable, the evolution of the CO_2 stock can be determined using (14) as:

$$P(t) = \frac{e^{xt} Z_0}{m+x} + e^{-mt}(P_0 - \frac{Z_0}{m+x}) \tag{26}$$

where m is the exponential pollution decay on emissions, x is the rate of growth of global CO_2 emissions, P_0 is the initial stock of CO_2 and Z_0 is the initial level of total CO_2 emissions globally. Regarding the parameter values, the initial stock of pollution from CO_2 emissions P_0 was 785.3 billion tons of CO_2 obtained from Guillerminet and Tol (2005) and the initial level of global total emissions (flow) Z_0 was 6.15 billion tons of CO_2 and was obtained from Guillerminet and Tol (2005). The exponential pollution decay of emissions m, was taken at a value of 0.0083 from Reillly and Richards (1993). For the value of x we used three different scenaria regarding the evolution of CO_2 emissions. The first scenario, which was motivated by the Stern (2006) Report,[21] follows the assumption that the global CO_2 emissions increase annualy by 2.5% or $x = 0.025$ per year. The second scenario is a scenario of constant global CO_2 emissions $x = 0$, that enabled us to extract helpful results for the impact of the environmental factor on productive base sustainability. The third scenario is based on an annual increase of global emissions per 0.5% or $x = 0.005$ per year. This is a completely arbitrary scenario chosen to check whether for low rates of growth of annual global CO_2 emissions, the productive base sustainability criterion changes sign.

---

[21] "Annual emissions are still rising. Emissions of carbon dioxide grew at an average annual rate of around $2\frac{1}{2}\%$ between 1950 and 2000. In 2000, emissions of all greenhouse gases were around 42GtCO2e, increasing" (Stern, 2006, Part III: The Economics of Stabilisation, Chapter 7, pp. 169).

6 Accounting Prices and Productive Base Sustainability for Developed and Developing Countries

This section presents the results of our empirical estimations which are based on our empirical model and the parameter values described in section 5. The accounting prices for the two groups of countries and the signs of the CCSW or the productive base sustainability criterion of the economies analyzed were obtained under the three different scenaria of global CO_2 emissions described in section 5.

When we follow scenario 1 ($x = 0.025$), the results indicate that both for developed OECD and developing non-OECD economies the accounting prices of capital (APK), CO_2 emissions ($AP\mathrm{CO}_2$), and technological change (APG) are positive while the accounting prices of global emissions of CO_2 ($APGz$) and of the stock of CO_2 (APP) are negative. The signs of the accounting prices can be interpreted as follows: When capital, CO_2 emissions and technological change increase per one unit, then the social welfare also increases. On the other hand, when global emissions and the stock of CO_2 increases, this reduces social welfare and thus the sign of those accounting prices is negative. For the case of scenario 1, the sign of the current change on social welfare conditions $\left(\dot{V}\right)$ is negative which is something we expected due to the positive and high environmental degradation that the persistent increase of global annual CO_2 emissions create. Following scenario 2 ($x = 0$), we observe that the results change significantly. As far as the signs of the accounting prices are concerned, we have the same pattern, but the CCSW criterion - $\left(\dot{V}\right)$ is now positive both for the case of developed - OECD and developing - nonOECD countries. This result confirms the hypothesis that the path of global CO_2 emissions currently regarded as plausible negatively affects productive-base sustainability. Thus, our results indicate that by keeping emissions at a constant level, this environmental friendly but probably unrealistic scenario would provide positive results for the CCSW and imply current productive base sustainability. Scenario 3 ($x = 0.005$), provides the same positive values for the accounting prices of capital, CO_2 emissions, and technological change, negative values for the accounting prices of global emissions, the stock of CO_2 and the productive base sustainability criterion.[22] This pattern confirms our initial hypothesis and observation that even with a very small percentage annual increase of CO_2 emissions, the results are not optimistic for productive base sustainability. Those results are an indication of the need for a strict management of global CO_2 emissions in order to avoid the erosion of the sustainability of the productive base of the economy that the global warming phenomenon creates.

7 Policy implications

As shown from the results of our empirical analysis the accumulation of global CO_2 emissions in the atmosphere is among the main factors determining our productive base sustainability criterion (PBSC). We observe from our empirical analysis that when the annual growth of global CO_2 emissions increases from 0% to 0.5% or 2.5%, the current change in social welfare criterion changes sign and becomes negative ($\dot{V} < 0$)[23].

In particular, when we have 0% rate of growth of global and individual country emissions, then the growth of the stock of CO_2 emissions is negative. This means that the state of the environment has a positive impact on total social welfare and the PBSC is positive both for the group of the OECD and the non-OECD economies.

When we change the annual global emissions rate of growth to 0.5%, the growth of the stock of CO_2 is positive. This implies that CO_2 accumulation has a negative impact on total social welfare and the PBSC turns negative both for developed and for developing countries.

When the global emissions rate of growth increases to 2.5%, the growth of the stock of CO_2 is also positive and the change in social welfare is "more

---

[22] For the case of Mexico (developed countries) in scenario 3, the result of the Current Changes on Social Welfare Conditions $\dot{V}$ is positive in contrast to all the other countries under analysis where the sign of $\dot{V}$ is negative for 0.5% global CO_2 emissions increasement.

[23] See the results of *tables 5* and *6* in Appendix 2.

negative" relative to the 0.5% CO_2 growth scenario.

Global CO_2 emissions rate of growth can be adjusted by the use of specific policy tools such as emission limits ($\bar{Z}$) or emission taxes (τ). Those emission limits can be used at a country level so that global CO_2 emissions do not exceed a specific maximum level $Z^{global\max}$. Similar results can be obtained with the imposition of a tax as a policy tool. The Kyoto protocol can be regarded as an attempt to define $\bar{Z}$ and therefore restrict the growth of emissions of the participating countries and the growth of emissions globally. Our results suggest that in order to have productive base sustainability, the international agreements should set the limit of emissions growth very close to 0. Our results show that there is a direct relationship between the growth of emissions of each country we analyze, the growth of global emissions, the growth of CO_2 stock and PBSC. The growth of emissions of each country affects the growth of global emissions, which affects the growth of CO_2 stock. As a result, a reduction in global CO_2 emissions could have two conflicting impacts. The first one is a positive impact. Reduced emissions will produce gains in terms of reduced CO_2 stock and thus positive results for the PBSC. The second one is a negative impact. Reduced emissions in a country may imply output reduction if other cleaner ways of production are not engaged. This implies reduced consumption and capital accumulation and this can have a negative impact on productive base sustainability.

Our results suggest that if emissions in each country are kept constant, the gains from the reduction of the global CO_2 stock can outweigh any losses in output in individual countries, for all the countries examined and can promote support global productive base sustainability.

In our model, a parameter that can play an important role in promoting CO_2 reduction without output losses is the parameter b, the emission augmenting technological change. This parameter can be used as a potential policy tool to compensate for the negative impact that a reduction on emissions may have on growth. This can be achieved for example by international R&D cooperation.

Another issue that comes up in this context is whether and up to what point a single country is able to change the sign of global PBSC, whether unilateral policies can have results and how significant those results can be - in terms of productive base sustainability - both for the country that takes the unilateral action of reducing CO_2 emissions but also for the other countries that might benefit from this action (since global CO_2 emissions might be reduced). This is a hard issue to address because when we deal with the greenhouse effect and climate change, we refer to global magnitudes. What we measure in these cases, is the contribution of all countries in total emissions and as a result the contribution of all countries in CO_2 stock. Unilateral action can lead to the reduction of emissions in certain countries, if the policies used are effective. But can a single country with negative or zero contribution to total emissions counterbalance the positive contribution all the other countries might have on total emissions?

For instance, if the USA reduces significantly its annual emissions of CO_2 but the rest of the world keeps increasing annual CO_2 emissions, how much willthe PBSC be affected? The answer depends on several factors, such as the size of the country, the development of the country and its competitiveness, its technological progress and R&D technologies used etc. When we deal with the USA for instance, we know that this country has a large contribution to the global emissions. If the USA followed policies that could reduce its annual CO_2 emissions by say 2%, this could promote the productive base sustainability both in the USA and in the rest of the world. The final result however would mostly depend on the reaction of all the other countries to this reduction. A possible scenario could be that the USA's unilateral action could trigger more emissions by all the other countries since they might expecte that their increased emissions would be counterbalanced by the reduced emissions of USA (free riding) and the final result could be an overall negative CSW.

To explore this issue, we measured the PBSC for the case of the USA using the following assumptions: We kept the rate of growth of CO_2 emissions for the USA fixed and at the same time we assumed an annual global CO_2 emissions increase of 2.5% (for all the other countries). The results indicated that the PBSC turned out to be negative globally. This test verified our belief that even if a country with a large contribution to the global warming phenomenon followed "green" policies that lead to a reduction in the rate of growth of its own CO_2 emissions, this did not imply that the PBSC would be positive, since the reaction of other countries could counterbalance this unilateral action.

The last test we run in this paper in order to verify that our main results are robust, was to choose a logarithmic utility function[24] instead of (15) where elasticity of marginal utility was $\sigma = 0.5$. Assuming therefore that $\sigma = 1$ we obtain results for the PBSC both for developed and developing countries. The results we obtained are summarized as follows: When we follow scenario 1 ($x = 0.025$) with a global CO_2 emissions annual increase of 2.5% and a logarithmic utility function, the PBSC becomes negative, the accounting prices of capital, technical change and emissions are positive while the accounting prices of global emissions and CO_2 stock are negative. If we follow scenario 2 where global CO_2 emissions is zero ($x = 0$), we observe that the results remain the same as in the case where we used (15) with $\sigma = 0.5$, both for developed and developing countries. In particular, the productive base sustainability criterion is positive and the accounting prices of capital, technical change and emissions are positive while the accounting prices of global emissions and CO_2 stock are negative.

8 Concluding Remarks

One of the basic factors that affect the current change on social welfare conditions is CO_2 emissions along with other GHGs emissions which are considered to be the basic contributors to the global warming phenomenon. This paper attempts to formulate a theoretical model to provide empirical results for the productive base sustainability of economies under global warming. This approach can be characterized as a productive base approach to sustainable development. To achieve this, we tried to determine a criterion that measures the current change of the productive base of an economy by taking into account the environmental damage created by the global warming phe-

---

[24] Such a function has been extensively used in the Stern (2006) report, so our results about productive base sustainability could be interpreted in the context of the utility function assumptions of the Stern report.

nomenon. We considered a nonoptimizing growth framework and we derived results for the productive base sustainability of two large groups of developed OECD and developing nonOECD economies. We applied our methodology by using three different scenaria of global CO_2 emissions' growth and we obtained results for the current productive base sustainability in each one of them.

The main empirical finding of the paper under two alternative utility function specifications is that when we follow the scenaria where global CO_2 emissions increase, then the PBSC is negative for almost all the countries under analysis. When global CO_2 emissions remain constant, the PBSC is positive both for the case of developed and for the case of developing countries. Our empirical findings confirms therefore the perception that the intensification of the global warming phenomenon can erode the productive base sustainability of modern economies.

Appendix

Dividing (4) by $e^{\xi t}$ we obtain:

$$\dot{\hat{k}}_t + \dot{\hat{h}}_t = \frac{se^{\lambda t}\hat{k}_t^{a_1}e^{\xi t a_1}\hat{h}_t^{a_2}e^{a_2\xi t}z_t^{a_4 a_2}e^{a_4\xi t}}{e^{\xi t}} - (\eta+\delta+\xi)(\hat{k}_t+\hat{h}_t)$$

$$\dot{\hat{k}}_t + \dot{\hat{h}} = se^{(\lambda-\xi+a_1\xi+a_2\xi)t}\hat{k}_t^{a_1}\hat{h}_t^{a_2}\hat{z}_t^{a_4} - (\eta+\delta+\xi)(\hat{k}_t+\hat{h}_t) \tag{27}$$

Setting $\lambda - \xi + a_1\xi + a_2\xi = 0$ so that (27) becomes time autonomous we have $\xi = \frac{\lambda}{1-a_1-a_2} = \frac{ga_3+a_4b}{1-a_1-a_2}$, and

$$\dot{\hat{k}}_t + \dot{\hat{h}} = s\hat{k}_t^{a_1}\hat{h}_t^{a_2}\hat{z}^{a_4} - (\eta+\delta+\xi)(\hat{k}_t+\hat{h}_t) \tag{28}$$

Following Barro and Sala-i-Martin (2004) we assume that savings are allocated between physical and human capital so that the two marginal products of capital are equal if we use both forms of investment. For this to be achieved, the following conditions should be satisfied:

$$a_1\frac{\hat{y}_t}{\hat{k}_t} - \delta = a_2\frac{\hat{y}_t}{\hat{h}_t} - \delta$$

The equality between marginal products implies a one-to-one relationship between physical and human capital:

$$\hat{h}_t = \frac{a_2}{a_1}\hat{k}_t,\ \dot{\hat{h}}_t = \frac{a_2}{a_1}\dot{\hat{k}}_t$$

then (28) becomes:

$$\dot{\hat{k}}_t + \frac{a_2}{a_1}\dot{\hat{k}}_t = s\hat{k}^{a_1}\left(\frac{a_2}{a_1}\hat{k}_t\right)^{a_2}\hat{z}^{a_4} - (\eta+\delta+\xi)\left(\hat{k}_t\frac{a_2}{a_1}\hat{k}_t\right) \tag{29}$$

$$\left(1+\frac{a_2}{a_1}\right)\dot{\hat{k}}_t = s\hat{k}^{(a_1+a_2)}\hat{z}^{a_4}\left(\frac{a_2}{a_1}\right)^{a_2} - (\eta+\delta+\xi)\left(1+\frac{a_2}{a_1}\right)k \tag{30}$$

$$\dot{\hat{k}}_t = s\left(\frac{a_2^{a_2}a_1}{a_1^{a_2}(a_1+a_2)}\right)\hat{k}^{(a_1+a_2)}\hat{z}^{a_4} - (\eta+\delta+\xi)\,k \tag{31}$$

Setting: $\left(\frac{a_2^{a_2} a_1}{a_1^{a_2}(a_1+a_2)}\right) = \Psi$, where Ψ is a constant, we have:

$$\dot{\hat{k}}_t = s\Psi \hat{k}_t^{a_1+a_2} \hat{z}^{a_4} - (\eta + \delta + \xi) \hat{k}_t \tag{32}$$

Setting $a_1 + a_2 = \phi$, then we have:

$$\dot{\hat{k}}_t = s\Psi \hat{k}_t^{\phi} \hat{z}^{a_4} - (\eta + \delta + \xi) \hat{k}_t \tag{33}$$

where output in efficiency units is defined as:

$$\hat{y} = \Psi \hat{k}_t^{\phi} \hat{z}^{a_4}$$

(33) is a Bernoulli equation which can be solved in the following way:

Multiplying by $\hat{k}_t^{-\phi}$ we have:

$$\dot{\hat{k}}_t \hat{k}_t^{-\phi} = s\Psi \hat{k}_t^{\phi} \hat{k}_t^{-\phi} \hat{z}^{a_4} - (\eta + \delta + \xi) \hat{k}_t \hat{k}_t^{-\phi}$$

$$\dot{\hat{k}}_t \hat{k}_t^{-\phi} = s\Psi \hat{z}^{a_4} - (\eta + \delta + \xi) \hat{k}_t \hat{k}_t^{-\phi}$$

$$\dot{\hat{k}}_t \hat{k}_t^{-\phi} + (\eta + \delta + \xi) \hat{k}_t \hat{k}_t^{-\phi} = s\Psi \hat{z}^{a_4} \tag{34}$$

$$\dot{\hat{k}}_t \hat{k}_t^{-\phi} + (\eta + \delta + \xi) \hat{k}_t^{1-\phi} = s\Psi \hat{z}^{a_4}$$

Setting $\gamma = \hat{k}_t^{1-\phi}$, we have $\dot{\gamma} = (1-\phi)\dot{\hat{k}}_t \hat{k}_t^{-\phi}$. Then:

$$\dot{\gamma} + (\eta + \delta + \xi) \gamma (1-\phi) = (1-\phi) s\Psi \hat{z}^{a_4} \tag{35}$$

which is linear in γ and the solution is the following:

$$\gamma_t = \left(\gamma_o - \frac{s\Psi \hat{z}^{a_4}}{\eta + \delta + \xi}\right) e^{-(1-\phi)(\eta+\delta+\xi)t} + \frac{s\Psi \hat{z}^{a_4}}{\eta + \delta + \xi} \tag{36}$$

replacing $\gamma_t = \hat{k}_t^{1-\phi}$, we have:

$$\hat{k}_t = \left[\left(\hat{k}_o^{1-\phi} - \frac{s\Psi \hat{z}^{a_4}}{\eta + \delta + \xi} \right) e^{-(1-\phi)(\eta+\delta+\xi)t} + \frac{s\Psi \hat{z}^{a_4}}{\eta + \delta + \xi} \right]^{\frac{1}{1-\phi}}$$

$$\hat{k}_\tau = \left[\left(\hat{k}_t^{1-\phi} - \frac{s\Psi \hat{z}^{a_4}}{\eta + \delta + \xi} \right) e^{-(1-\phi)(\eta+\delta+\xi)(\tau-t)} + \frac{s\Psi \hat{z}^{a_4}}{\eta + \delta + \xi} \right]^{\frac{1}{1-\phi}}$$

by replacing $\hat{z} = ze^{-\xi\tau}$, the solution for the time path of the stock of capital is of the form:

$$\hat{k}_\tau = \left[\left(\hat{k}_t^{1-\phi} - \frac{s\Psi \left(z_t e^{\xi\tau}\right)^{a_4}}{\eta + \delta} \right) e^{-(1-\phi)(\eta+\delta+\xi)(\tau-t)} + \frac{s\Psi \left(z_t e^{\xi\tau}\right)^{a_4}}{\eta + \delta} \right]^{\frac{1}{1-\phi}}, \text{ for } \tau \geq t \tag{37}$$

References

1. Arrow, J. K. Dasgupta, P. Mäler K.-G. (2003). Evaluating projects and assessing sustainable development in imperfect economies. *Environmental and Resource Economics* 26: 647-685.

2. Barro R., Sala-i-Martin. X. (2004). Economic Growth, 2nd Edition. McGraw-Hill: Advanced Series in Economics: 384-85.

3. Dasgupta P., Mäler, K-G., (2001), Wealth as a criterion for Sustainable Development. The Beijer International Institute of Ecological Economics, Discussion Paper 139.

4. European Commission Report. (2006),*Greenhouse gas emission trends and projections in Europe 2006,* EEA Report No 9/2006

5. Fankhauser, S. Tol, R. (1997). The social Cost of Climate Change: The IPCC Second Assessment Report and Beyond. Mitigation and Adaptation Strategies for Global Change 1: 385-403.

6. Guillerminet, M. L. Tol R. S. J. (2005). Decision Making under catastrophic risk and learning: the case of the possible collapse of the west antarctic ice sheet. Working Paper FNU79.

7. Hartwick J.M., (1977), Intergenerational equity and the investing of rents from exhaustible resources. American Economic Review 66, 972-4.

8. IPCC, (2000), *Special Report: Emissions Scenarios*, Nakicenovic, N. and Swart, R. (Eds.), Cambridge University Press, UK.

9. The Kyoto Protocol to the United Nations Framework Convention on Climate Change (1997), Kyoto, December, 1997.

10. Mankiw, N.G. Romer, D. Weil D.N. (1992). A contribution to the empirics of economic growth. Quarterly Journal of Economics: 407-437.

11. Nordhaus, W. D. (1998). Tables and Figures for Revised Estimates of the Impacts of Climate Change.
12. Pearce D., Markandya, A., Barbier, E., (1990), Sustainable Development: Economics and Environment in the third World. Edward Elgar, Worcester, UK.
13. Pemberton M., Ulph, D., (2001), Measuring income and measuring sustainability. Scandinavian Journal of Economics 103, 25-40.
14. Pezzey J., (1992), Sustainable development concepts: An economic analyses. World Bank Environmental Paper 2.
15. Pezzey J., (1997), Sustainability constraints versus "optimality" versus intertemporal concern, and axioms versus data. Land Economics 73,448–466.
16. Pezzey J., (2004a), Sustainability policy and environmental policy. Scandinavian Journal of Economics 106, 339-359.
17. Pezzey J., (2004b), One-sided sustainability tests with amenities, and changes in technology, trade and population. Journal of Environmental Economics and Management 48, 613–631.
18. Riley G., (1980), The just rate of depletion of natural resource. Journal of Environmental Economics and Management 7, 291-307.
19. Reillly, J. M. Richards, R. (1993). Climate change Damage and Trace gas, Index Issue. *Environmental and Resource Economics* 3: 41–61. Kluwer Academic Publishers.
20. Solow R., (1974), Intergenerational equity and exhaustible resources. The Review of Economic Studies. Symposium on the Economics of Exhaustible Resources 41, 29-45.
21. Stern, N. (2006). Stern Review on the Economics of Climate Change, *HM Treasury*, Cabinet Office.

22. Tol, R. S. J. (2005). The marginal damage costs of carbon dioxide emissions: an assessment of the uncertainties. *Energy Policy*. 33: 2064–2074.

23. Tzouvelekas, E. Vouvaki, D. and A. Xepapadeas. (2007). Total Factor Productivity Growth and the Environment: A Case for Green Growth Accounting. FEEM working paper 42.

24. Uzawa, H. (2003). Economic Theory and Global Warming. Cambridge University Press.

25. Vouvaki, D. and A. Xepapadeas. (2008), Total Factor Productivity Growth when Factors of Production Generate Environmental Externalities, http://mpra.ub.uni-muenchen.de/10237/, Munich RePEc Archive, MPRA Paper No. 10237.

26. World Bank (2006). Where is the Wealth of Nations? Measuring Capital for the 21st Century. Washington D.C.: World Bank.