The Demand for Currency Substitution

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Abstract:

A transactions model of the demand for multiple media of exchange is developed. Some results are expected, and others are both new and surprising. There are both extensive and intensive margins to currency substitution, and inflation may affect the two margins differently, leading to subtle incentives to adopt or abandon a substitute currency. Variables not previously considered in the literature affect currency substitution in complex and somewhat unexpected ways. In particular, the level of income and the composition of consumption expenditures are important, and they interact with the other variables in the model. Independent empirical work provides support for the theory.

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I. INTRODUCTION

Currency substitution is an important phenomenon in countries with high inflation rates, complicating forecasts of money demand and making monetary policy more difficult to conduct. The basic intuition behind currency substitution is simple enough. High domestic inflation makes it costly to hold the domestic money because the currency component (and perhaps others) pays a fixed nominal rate of interest of zero and so cannot offer compensation to offset the negative return caused by inflation. Domestic residents then may find it worthwhile to use currencies from countries with low inflation rates as substitutes for the domestic currency (Cagan, 1956; Barro, 1970). This basic intuition, however, does not explain several aspects of currency substitution. Abandonment of the domestic medium of exchange is a continuous process. As inflation rates rise, people do not abruptly abandon the domestic medium for foreign substitutes. Rather, the substitution is gradual, with the amount of substitution increasing with the domestic inflation rate. Households begin buying some goods with foreign media of exchange but continue to buy other goods with the domestic medium. In addition, the evidence shows wide differences in both the extent and type of currency substitution across countries experiencing similar inflation rates. Savastano (1996), for example, shows that a country's currency substitution response to inflation depends on the country's financial institutional framework, such as the availability of liquid assets denominated in foreign currency. The theoretical literature has lagged behind the empirical evidence on these matters, offering no explanation for which goods are bought with which media, no understanding of how the factors that determine the pattern of payments interact with the factors that determine the overall degree of substitution, and little formal theoretical treatment of the role of institutional factors in determining the extent of currency substitution.

Currency substitution is clearly an aspect of money demand; people are choosing a combination of media of exchange to use in conducting market transactions. In particular, currency substitution is a type of demand for multiple media of exchange and therefore is amenable to analysis by the methods used in recent theoretical work on that subject. The present paper examines the demand for currency substitution within a money-demand framework that permits simultaneous holding of several media of exchange as well as a savings asset. The kinds of institutional factors discussed by Savastano, such as availability of liquid assets, are naturally captured by different costs and rates of return attached to the alternative assets available for carrying out transactions. The

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analysis formalizes and supports Savastano's contention that financial institutional structure is important in determining a country's currency substitution behavior. The incentives to adopt or abandon a substitute currency are more subtle than the existing empirical literature assumes, involving calculations on both an extensive and an intensive margin. The theory also shows the importance of factors not considered by Savastano or the rest of the currency substitution literature. In particular, the level of income per person and the composition of consumption expenditure for any given level of income affect the extent of currency substitution that a country experiences. The theory suggests that existing empirical investigations of currency substitution may have been misspecified by omitting relevant variables and offers guidelines on how to improve empirical specifications. It also gives guidance on what data is needed to carry out proper tests.[1] Some aspects of the theory can be tested with household survey data, as discussed below. Indeed, in an independent study Stix (2007) applies the theory developed below to a small survey data set for Croatia, Slovenia, and Slovakia, and finds support for aspects of the theory. Testing the theory with market data would be more complicated than with household survey data because it would require further theoretical development first. The theory presented here is limited to the demand side. It is not even a partial equilibrium model, so one cannot test it with market-generated data, such as aggregate money holdings. Doing that would require coupling the theory with a complementary theory of the supply side of the currency substitution market. Developing such a theory and then deriving the market equilibrium is a worthwhile endeavor, but it is far beyond the scope of the present paper, which should be regarded as a first step toward a complete theory of currency substitution.[2]

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[1]This study examines currency substitution that results from domestic inflation, i.e., a situation of "too much" money. Currency substitution also can occur when there is a shortage of the domestic money, a situation of "too little" money. See Colacelli (2005) for a nice theoretical model and an extended empirical analysis of shortage-induced currency substitution.

[2]Jovanovic (1982), Romer (1987), and Leo (2006) present general equilibrium models of the transactions demand for money. Leo's approach is the simplest and seems the most likely to be amenable to generalization to multiple media of exchange and currency substitution.

II. MODEL FUNDAMENTALS

Most models of money demand based on tight micro foundations fall into two broad categories: search or matching models on the one hand and cash-in-advance models on the other. Members of the former class typically are used to study the origins of the medium of exchange (e.g., Jones, 1976; Niehans, 1978; Kiyotaki and Wright, 1989; Craig and Waller, 2000; Howitt and Clower, 2000), although some are used to study other questions (e.g., Cavalcanti, 1999). For our purposes, these models are not useful because either they are much too stylized to address the questions to be investigated here or they are analytically intractable and require numerical solution.[3] Instead, the analysis here is based on a type of cash-in-advance model, Santomero and Seater's (1996) model of the demand for smart cards and other new means of payment. Santomero and Seater extend the Baumol-Tobin transactions model of money demand to allow consumers to use several several media of exchange simultaneously. Their approach to the demand for multiple media of exchange is much more general than their particular application to innovations in electronic means of payment and, with appropriate alteration, is well-suited to the study of currency substitution. The model simultaneously has the necessary structure and the analytical tractability to answer the questions of interest. Santomero and Seater consider the case of many media of exchange and many goods, but for our purposes, such generality merely adds tedious complications without providing any additional insights. We therefore shall restrict attention to two media of exchange that can be used to buy either or both of two goods.

A. Monies, Prices, and Interest Rates.

We will examine a country that uses two types of nominal money, m_1 and m_2. We will be interested in the case where one money is domestic and the other is foreign, but for the moment it is convenient to ignore the origins of these two monies. They could be two forms of domestic

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[3]To use some of the works already cited as examples, Jones (1976) and Niehans (1978) examine existence of equilibria and the nature of the goods that emerges as the media of exchange. Their models are not suited to studying what characteristics determine which media a particular household will use. Kiyotaki and Wright (1989) and Craig and Waller (2000) restrict the analysis to households that demand only one good, making their models inappropriate for studying how the composition of consumption affects demands for multiple media of exchange. Cavalcanti (1999) allows for two media but restricts a given household *a priori* to using one or the other of them. Howitt and Clower's (2000) model must be solved numerically.

money, such as currency and smart cards. We need to clarify the prices of goods in terms of each money and the price of one money in terms of the other. We also need to examine the real rates of return to holding each type of money.

Either type of money can be used to buy either of the two goods available in this country; consequently, we have four prices p_{ij} to consider, where i is the good and j is the type of money used to buy it. Let e denote the exchange rate between the two monies, expressed as the price of m_2 in terms of m_1 (that is, e = units of m_1 per unit of m_2). If we suppose that either kind of money can be converted costlessly into the other kind, then absence of arbitrage will guarantee that[4]

(1) $$p_{i1} = ep_{i2} \qquad i = 1,2$$

This equation is simply a Law of One Price; given any two of the three prices p_{i1}, p_{i2}, and e, it determines the remaining price.

Notice that at this point the exchange rate is a general concept; the two kinds of money may both be domestic in origin, in which case e is a domestic exchange rate, not a foreign exchange rate. This remains true even if one money is of foreign origin. What e measures is the rate of exchange between the two monies within the country in question, not between two different countries. Let e^* denote the exchange rate between m_1 and m_2 on international markets. If there were no costs of making international transactions, then absence of arbitrage would guarantee equality between e and e^*, but if there are transactions costs, then the two exchange rates can differ. We can imagine, for example, that a country sells goods in exchange for both dollars and euros and then is cut off (by war or domestic politics) from subsequent international trade and finance. The cost of international transactions has become infinite for this country. The dollars and euros already in the country can circulate as media of exchange, and an exchange rate e will be established between them, but the value of e would have no necessary relation to the exchange rate e^* prevailing on international markets. Thus equation (1) is a domestic Law of One Price, not an international one. What is

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[4]If we were to introduce a conversion cost, then we would have to consider two exchange rates, one for using m_1 to buy m_2 and one for using m_2 to buy m_1. The difference between the two exchange rates would be the bid-ask spread and would reflect the conversion cost. This bit of realism is analytically costly and seems to add nothing important to the analysis, so we simplify by supposing the cost of converting one money to another is zero.

relevant for the theory that follows is *e*, not *e**. This distinction is not especially important for the general theory, but it could be very important for empirical work. The correct measure of the exchange rate that we need is not necessarily the rate quoted on international markets.

Equation (1) leads directly to a parallel relation in terms of growth rates:

(2) $$\Delta p_{i1} = \Delta p_{i2} + \Delta Ee$$

where Δ denotes percentage change. Continuing with the distinctions made in the previous paragraph, the price growth rates involved here are entirely domestic and not necessarily related to the growth rates in the countries that may have issued the media of exchange. If we consider again a country cut off from rest of the world but using as media of exchange some currencies printed by other countries, it could have inflation in terms of m_1 and/or m_2 even when its supplies of m_1 and m_2 are fixed if it suffers a decline in real output. The rates of inflation would have no necessary relation to the inflation rates prevailing within the countries that originally printed m_1 and m_2. Again, the main implication of this observation is for empirical work, but we want to make sure the relevant concepts are clear.

Because there are two monies, there are two rates of inflation π_1 and π_2, one for prices in terms of m_1 and one for prices in terms of m_2. For any given real rate of return R, there are two corresponding nominal rates of return, r_1 and r_2:

(3) $$r_1 = R + \pi_1 + R\pi_1$$

(4) $$r_2 = R + \pi_2 + R\pi_2$$
$$= R + \pi_1 - \Delta e + R\pi_1 - R\Delta e$$

Solving each of these equations for R and equating the two solutions gives an expression for the relation between the two nominal rates:

(5) $$r_2 = (r_1 - \pi_1)\left(\frac{1 + \pi_2}{1 + \pi_1}\right) + \pi_2$$
$$= (r_1 - \pi_1)\left(\frac{1 + \pi_1 - \Delta e}{1 + \pi_1}\right) + \pi_1 - \Delta e$$

Finally, we can write the real rate of return in several ways

(6)
$$R = \frac{r_1 - \pi_1}{1 + \pi_1}$$

$$= \frac{r_2 - \pi_2}{1 + \pi_2}$$

$$= \frac{r_2 - \pi_1 + \Delta e}{1 + \pi_1 - \Delta e}$$

For theoretical purposes, we need the real rate of return R; equation (6) shows us how to calculate it from observable data.

B. Model Structure.

The model has the standard Baumol-Tobin structure except for the presence of more than one medium of exchange. The household receives a fixed real annual income Y^A each year, paid in J equal installments of amount $Y = Y^A/J$ at the beginning of J payments periods of equal length $1/J$. These J payments periods are the fundamental time periods of the analysis, and discussion will be couched in terms of them. Each payments period, the household exactly exhausts its income Y by buying fixed real amounts, X_g, of two different goods:

(7)
$$Y = X_1 + X_2$$

We distinguish between consumption and consumption expenditure. *Consumption* of goods occurs at a constant rate, whereas *consumption expenditures* (i.e., purchases of goods) occur at discrete sub-intervals of the payments period, chosen optimally by the household.[5] Between such "shopping trips," the household holds inventories of the two goods, which it gradually consumes until exactly exhausting them at the moment it is time to make another shopping trip. A separate shopping trip is required for each type of good. Each type of commodity inventory pays a unique real rate or

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[5]A more complete model would allow the household to determine the values of X_1 and X_2, and perhaps also Y through choice of labor supply, simultaneously with the quantities of m_1 and m_2. Jovanovic (1982) and Romer (1987) provide such a model. As their analyses show, the qualitative conclusions concerning money demand are not changed by these generalizations, but the analysis is greatly complicated. We therefore restrict attention to the simpler case where Y, X_1, and X_2 are predetermined.

return, R_{Xg}. This rate may be implicit or explicit and may be positive or negative.

Goods must be purchased with a medium of exchange that the household has on hand at the time of the purchase. That is the cash-in-advance constraint of the model. There are two media of exchange, m_i, available to the household: m_1 is the nominal domestic currency, m_2 is a nominal foreign currency. The corresponding real quantities are denoted M_1 and M_2. The household can use either or both types of money to buy each type of good. Denote the quantity of good g bought with money I by X_{gi}. The household may use medium M_1 on some shopping trips for good g and medium M_2 on others (although we will see momentarily that the household never chooses to do this but rather uses only one medium to purchase a given good). Thus

(8) $$X_g = X_{g1} + X_{g2}$$

There are Z_{gi} trips to purchase good g with money I. Each such trip has associated with it the real shopping cost B_{gi}, a lump-sum amount paid each trip but not depending on the amount spent. This cost may be explicit, such as a delivery charge or a check-cashing fee, or implicit, such as a time cost.

The household spends only a fraction of its income on any one shopping trip. Unspent income is held in a single real savings asset, S, and in money balances. Savings earn the real rate of return R_S, and the two kinds of money earn rates of return R_{Mi}. It is presumed that $R_S > R_{Mi} > R_{Xg}$.[6] When the household exhausts its holdings of a medium of exchange, the cash-in-advance constraint forces it to replenish those holdings by converting some of its saving asset to the desired medium. It does that by making periodic "trips to the bank." There are T_i conversion trips to obtain m_i, and each such trip has associated with it the real conversion cost A_i. This cost, like shopping trip costs, is a lump-sum amount paid explicitly or implicitly each time a conversion is made but does not depend on the size of the conversion. As in the simple Baumol-Tobin model, optimal conversions are evenly spaced. Shopping trips occur between conversion trips and also are evenly

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[6]It is not necessary that all money interest rates exceed all inventory rates of return, but imposing that requirement simplifies the discussion. It is trivial to show that, in the more general case, money i will not be used to purchase good g if the rate of return R_{Xg} on good g exceeds the rate of return R_{Mi} on money I.

spaced.[7] There are N_{gi} shopping trips to buy good g with money I per conversion of S into m_i. The total number of shopping trips, Z_{gi}, to buy good g with money I is thus $T_i N_{gi}$.

Finally, each of the assets, S and m_i, carries a real fixed cost F_i that must be paid if that asset is held at any time during the payments period. These fixed costs capture such things as monthly account fees. For currencies, the fixed costs may be zero; however, monies, even foreign ones, need not be currency but rather may be bank accounts which often do carry fixed fees. Consequently, fixed costs are carried through the analysis for all assets to maintain complete generality. Figure 1, anticipating the solution to follow, shows the time paths of total wealth, the savings asset, and the medium of exchange during a payments period for the simple case of one medium of exchange and one good. Total wealth is the sum of the savings asset, the holdings of the medium of exchange, and the stock of commodity inventories. Wealth starts at Y at the beginning of the payments period and falls at a constant rate (because of constant consumption), reaching zero exactly at the end of the period. The household puts half of Y into the saving asset, which then falls in a stair-step pattern (with just one step in this example), each step down corresponding to one trip to the bank to withdraw the medium of exchange. The medium of exchange also moves in a stair-step pattern, each step down corresponding to a shopping trip. In this example, the household puts one-third (i.e., two-sixths) of Y into the medium of exchange and then reduces it to zero in two withdrawals. It then replenishes its holding of the medium of exchange half way through the payments period, and the stair-step pattern repeats. Commodity inventories start at one-sixth of Y, move in a sawtooth pattern, jumping up with each shopping trip and then falling to zero at a constant rate because of constant consumption. At the end of the payments period, total wealth is just exhausted, a new income installment is received, and the process repeats.[8]

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[7]The proof that optimal trips are evenly spaced is tedious and unimportant to the issues discussed, so it is omitted. See Tobin (1956) for details.

[8]Note that these solution paths. The model assumes that interest income is accumulated in some unmentioned asset and not spent during the period of analysis. That assumption greatly simplifies the analysis. Relaxing it has no effect on the qualitative conclusions, as shown in the previously cited works by Jovanovic (1982) and Romer (1987).

C. Model Solution.

The household seeks to maximize the profit from managing its assets:

$$(9) \qquad V = R_S \overline{S} + \sum_{i=1}^{2} R_{Mi} \overline{M}_i + \sum_{g=1}^{2} R_{Xg} \overline{X}_g - \sum_{i=1}^{2} T_i A_i$$

$$- \sum_{i=1}^{L} \sum_{g=1}^{2} Z_{gi} B_{gi} - F_S I(S) - \sum_{i=1}^{2} F_i I(M_i)$$

where $I(Q)$ is an indicator function that is 1 if average holdings of asset Q are positive and is 0 otherwise. The terms on the right side are, in order: (1) interest earnings on average holdings of the savings asset, (2) interest earnings on average holdings of the two media of exchange, (3) interest earnings on average inventories of goods, (4) total costs paid for converting between savings on the one hand and media of exchange on the other hand, (5) total shopping trip costs, (6) the fixed cost of holding savings assets, and (7) the fixed costs of holding the two media of exchange. To maximize this profit, the household chooses optimal values of average asset holdings, trip frequencies, and the X_{gi}. This problem can be simplified in the usual way, by noting that the average asset values can be written in terms of the remaining variables (see the Appendix):

$$(10) \qquad \overline{S} = \sum_{g} \frac{X_g}{2} - \sum_{i} \sum_{g} \frac{X_{gi}}{2T_i}$$

$$(11) \qquad \overline{M}_i = \frac{\sum_{g} X_{gi}}{2T_i} - \frac{\sum_{g} X_{gi}}{2Z_{gi}}$$

$$(12) \qquad \overline{X}_{gi} = \frac{X_{gi}}{2Z_{gi}}$$

Substituting these expressions in the profit function gives

$$(13) \qquad V = R_S\left[\sum_g\left(\frac{X_g}{2} - \sum_i\frac{X_{gi}}{2T_i}\right)\right] + \sum_i R_{Mi}\left[\sum_g\left(\frac{X_{gi}}{2T_i} - \frac{X_{gi}}{2Z_{gi}}\right)\right]$$

$$+ \sum_g R_{Xg}\left(\sum_i\frac{X_{gi}}{2Z_{gi}}\right) - \sum_i T_i A_i - \sum_i\sum_g Z_{gi}B_{gi}$$

$$- F_S I(S) - \sum_i F_i I(m_i)$$

By solving for the optimal values of the T_i and Z_{gi} in terms of the X_{gi} (see the Appendix) and substituting in (9), we can write the profit function as

$$(14) \quad V = R_S\frac{X_1+X_2}{2} - [2A_1(R_S-R_{M1})(X_{11}+X_{21})]^{1/2} - [2A_2(R_S-R_{M2})\{(X_1-X_{11})+(X_2-X_{21})\}]^{1/2}$$

$$- \{[2B_{11}(R_{M1}-R_{X1})X_{11}]^{1/2} + [2B_{21}(R_{M1}-R_{X2})X_{21}]^{1/2}$$

$$- \{[2B_{12}(R_{M2}-R_{X1})(X_1-X_{11})]^{1/2} + [2B_{22}(R_{M2}-R_{X2})(X_2-X_{21})]^{1/2}\}$$

$$- F_S I(S) - F_1 I(M_1) - F_2 I(M_2)$$

All that remains is to find the optimal values of X_{11} and X_{21}. The first-order conditions are

$$(15) \qquad \frac{\partial V}{\partial X_{11}} = 0$$

$$(16) \qquad \frac{\partial V}{\partial X_{21}} = 0$$

However, the second-order conditions indicate that the profit function is convex:

$$\frac{\partial^2 V}{\partial X_{i1}\partial X_{j1}} > 0 \qquad \text{for } i,j = 1,2$$

$$(17)$$

$$\det H > 0$$

where H is the Hessian. Consequently, the interior extremum is a profit minimum, so the maximum occurs at a corner. This implies that the household always chooses to use only one medium of exchange to buy a given type of good.

There are four possibilities:

$(S > 0, X_{11} = X_1, X_{21} = X_2)$ hold S, use M_1 to buy X_1 and X_2

$(S > 0, X_{11} = X_1, X_{21} = 0)$ hold S, use M_1 to buy X_1 and M_2 to buy X_2

$(S > 0, X_{11} = 0, X_{21} = X_2)$ hold S, use M_2 to buy X_1 and M_1 to buy X_2

$(S > 0, X_{11} = 0, X_{21} = 0)$ hold S, use M_2 to buy X_1 and X_2

The foregoing possibilities all assume implicitly that the household chooses to use the savings asset. In fact, it may choose otherwise. We thus have four more possibilities:

$(S = 0, X_{11} = X_1, X_{21} = X_2)$ do not use S, use M_1 to buy X_1 and X_2

$(S = 0, X_{11} = X_1, X_{21} = 0)$ do not use S, use M_1 to buy X_1 and M_2 to buy X_2

$(S = 0, X_{11} = 0, X_{21} = X_2)$ do not use S, use M_2 to buy X_1 and M_1 to buy X_2

$(S = 0, X_{11} = 0, X_{21} = 0)$ do not use S, use M_2 to buy X_1 and X_2

The household therefore has eight possible usage patterns to consider. It chooses among them by comparing the profit functions associated with each of the eight possible patterns and picking the one with the highest profit. Stix (2007) provides evidence from household survey data that households actually do systematically use the domestic currency exclusively for some goods and a foreign currency exclusively for others, so there is good reason to explore the factors that determine how goods and currencies will be paired, that is, how the household chooses among the eight patterns just listed.

Before proceeding to a discussion of the household's choice and how that choice changes in response to various exogenous shocks, we first modify the model slightly.

D. Convenience Yields.

To this point, the discussion has treated the two types of money in a completely general way. However, because the subject of this inquiry is currency substitution, henceforth we will use M_1 to denote the domestic money and M_2 to denote a foreign money that may circulate as an alternative medium of exchange. As long as some part of M_1 and M_2 consist of currency, the nominal interest rates r_1 and r_2 on those monies cannot fully adjust to inflation. It therefore simplifies matters to assume without loss of generality that no component of nominal money (cash, checking accounts, etc.) pays nominal interest.

A minor problem arises here. Actual episodes of substantial currency substitution are driven by high rates of inflation in M_1-denominated prices, generally caused by printing excessive amounts of the nominal domestic money m_1. If the only kind of return earned on a nominal asset is the explicit nominal interest rate, then currency earns no nominal interest rate and so earns the real rate of return $R_M = -\pi/(1+\pi)$, which has a lower bound of -1 (that is, negative 100 percent). The value

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of R_M can be made arbitrarily close to this lower bound, for inflation can be made arbitrarily high, and the yield spread R_M-R_X can become negative. Once the spread becomes negative, people stop using M to buy X; it is more profitable to hold inventories of X than inventories of M. If inflation rates in both M_1-denominated prices and M_2-denominated prices, denoted π_1 and π_2, are sufficiently high, both types of money would be abandoned, and exchange would be conducted by barter. The problem with this outcome is that transactions models of the type used here are constructed under the implicit assumption that some form of money is used to buy goods; the models are incapable of handling barter.

There are two ways to address this difficulty. One approach is to build a model that accommodates barter explicitly. That is the type of model used to explore the origins of media of exchange. Although very interesting, as mentioned earlier such models either are so stylized that they cannot address the issues investigated here or are analytically intractable and can be solved only by numerical methods. The second possibility, adopted here, is to impose restrictions that prevent barter from arising. Two restrictions are needed.

First, we suppose that all types of financial assets pay an implicit real convenience yield R_C. This convenience yield captures the great savings in transactions costs achieved by using money instead of barter to buy goods. Behavior during hyperinflations suggests that R_C is a very large number. For example, in the hyperinflations that Cagan (1956) studied, *average* inflation rates often reached hundreds of percent per *month* (far more in the case of the second Hungarian hyperinflation), but people continued using the domestic medium of exchange even though foreign media began to circulate as alternatives.[9] There clearly must be great value to using the domestic medium of exchange if people hold it in the face of such enormously negative real rates of return. We therefore assume here that R_C is sufficiently large that the real return to money

(18) $$R_M = R_C + \frac{r_M - \pi}{1 + \pi}$$

$$= R_C - \frac{\pi}{1 + \pi} \qquad \text{(because } r_M \equiv 0)$$

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[9]In one month of the second Hungarian hyperinflation, the inflation rate was just over forty quadrillion (41.9×10^{15}) percent per month, yet people still did not abandon the domestic currency completely.

is positive for any rate of inflation that will be encountered. The savings asset S also pays R_C as part of its total return. Savings assets are held only if their rate of return exceeds that on money. In the absence of inflation, nominal interest rates on short-term assets equal real interest rates and are generally are quite low (e.g., about one percent per year on one-year U.S. Treasury bills). For these assets to be held, it must be that they carry an implicit convenience yield that keeps their total return above that of money. We can suppose that S is denominated in terms of the domestic money, so the real rate of return can be written as[10]

$$(19) \qquad R_S = R_C + \frac{r_S - \pi_1}{1 + \pi_1}$$

In contrast to financial assets, physical inventories clearly do not carry a convenience yield; the convenience yield is precisely the extra value one gets by using money instead of goods to conduct transactions. The only rate of return on goods is the explicit real rate, which never is large in magnitude and usually is zero (and can be negative due to spoilage, theft, etc.). We therefore simplify the analysis by assuming that R_{Xg} is zero for all types of goods.[11]

The reader should bear in mind that the convenience yield R_C is plays no role in the subsequent analysis and is merely a convenient way to avoid having to undertake an awkward analysis of barter. Its use here is similar in spirit to imposing Inada conditions in the Solow-Swan growth model to avoid uninteresting corner solutions.

The second restriction we need to avoid barter is to assume that at least one of the profit functions associated with the eight usage patterns is positive for some choice of holdings of M_1, M_2,

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[10]Whenever necessary, we can use (2) to write R_S in terms of π_2:

$$R_S = R_C + \frac{r_S - \pi_2 - \Delta E}{1 + \pi_2 + \Delta E}$$

[11]The gain from this simplification is mathematical convenience. In some derivations below, we must manipulate expressions of the type $(R_{Mi} - R_{Xg})^x$, where x is an integer greater than one. The resulting expressions are cumbersome if R_{Xg} is non-zero because of the cross terms. Those terms are negligible in magnitude if R_{Xg} is small relative to R_{Mi}, so omitting R_{Xg} simplifies the calculations without altering the qualitative results.

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and S. The profit functions are given in Table 1. Given that we have included a convenience yield big enough to keep the R_{Mi} positive for any observed rate of inflation, this restriction is quite weak, only requiring that the rate of return on one of the monies be sufficiently high to make it profitable to make one conversion from money to goods during the payments period. In that case, the household always can choose to spend half its pay immediately to buy goods and hold the remaining half as money, run the inventories of goods down to zero, and then use the stock of money to buy goods again exactly half way through the payments period. As long as the interest earned on the money exceeds the cost of the single conversion that is made, the profit function will be positive and positive money balances will be held. *A fortiori*, it also will be positive for the optimal choice of cash management, which must yield a profit at least as high as this minimal strategy. Money rather than barter then will be used to conduct all purchases.

III. DEMANDS FOR TRANSACTIONS ASSETS

We now examine the household's choice of medium of exchange and its dependence on interest rates, transactions costs, income, and expenditure patterns. Because the solution always is in a corner (i.e., where a given money is used either exclusively or not at all to purchase a given good), we cannot use the standard first-order conditions that pertain to an interior solution. This fact complicates the analysis because, instead of just setting some first order conditions equal to zero and solving for the values of S, M_1, and M_2 that simultaneously satisfy those conditions, we must carry out a rather tedious comparison of the eight possible solutions to see which one is best, something like what one does in moving from one node to another in seeking the solution to a linear programming problem. The method of analysis is to consider how a change in a variable of interest affects the difference between various pairs of profit functions from Table 1. For example, suppose we wanted to study the effect of an increase in the real rate of return R_S on the choice of whether to use M_1 or M_2 to purchase good X_2, given that the savings asset is held and that M_1 is used to purchase good X_1. We would compute the profit difference $V_{S11}-V_{S12}$ and examine the sign of the partial derivative $\partial(V_{S11}-V_{S12})/\partial R_S$. If that sign were positive, then an increase in R_S raises the difference (i.e., makes it more positive), making it more likely that the household will choose to use M_1 to buy both goods.

A. <u>Interest Rates and Currency Substitution</u>.

For currency substitution, the most important relation is between demands for domestic and foreign money on the one hand and domestic money's real rate of return on the other. That is because currency substitution arises almost exclusively in response to high domestic inflation rates, which appear in the model as reductions in the real rate of return R_{M1} on domestic money. The nominal rate of return r_S presumably can adjust to changes in the inflation rate to satisfy the Fisher equation, leaving the real rate of return R_S unchanged; in contrast, the nominal rates of return on money r_{M1} and r_{M2} cannot make a full adjustment because part of each money stock is currency, which has a fixed nominal rate of zero. An increase in the domestic inflation rate π_1 therefore reduces R_{M1}, raises the yield spread R_S-R_{M1}, reduces the spreads $R_{M1}-R_{Xg} = R_{M1}$, and has no effect on R_{M2} or any of the spreads R_S-R_{M2} or $R_{M2}-R_{Xg} = R_{M2}$.

It may strike the reader as obvious that an increase in R_{M1}, holding constant everything else, would increase the demand for M_1 and reduce that for M_2, thus reducing the degree of any currency substitution. An increase in R_{M1} would raise the demand for M_1 through the substitution effect, and, because total levels of consumption of X_1 and X_2 are given, there is no wealth effect to work in opposite direction. The net effect therefore seems straightforward. In fact, however, the relation between R_{M1} and the demand for M_1 is not at all straightforward, as we now see.

The issues are most easily explained by considering the simpler cases where S is not held. The same results apply in the cases where S is used. Begin with the profit expression for the case where M_1 is used to buy both goods:

$$(20) \qquad V_{011} = R_{M1}\frac{X_1+X_2}{2} - [2\beta_{11}R_{M1}X_1]^{1/2} - [2\beta_{21}R_{M1}X_2]^{1/2} - F_1$$

The first derivative of this expression with respect to R_{M1} is

$$(21) \qquad \frac{\partial V_{011}}{\partial R_{M1}} = \frac{X_1+X_2}{2} - \left[\left(\frac{B_{11}X_1}{2}\right)^{1/2} + \left(\frac{B_{21}X_2}{2}\right)^{1/2}\right]R_{M1}^{-1/2}$$

which can be positive or negative. The second term of this expression goes to zero as R_{M1} becomes large, so the derivative can be negative only for "small" values of R_{M1}. Indeed, the second derivative of V_{011} with respect to R_{M1} is everywhere positive:

$$(22) \qquad \frac{\partial^2 V_{011}}{\partial R_{M1}^2} = \frac{1}{2}\left[\left(\frac{B_{11}X_1}{2}\right)^{1/2} + \left(\frac{B_{21}X_2}{2}\right)^{1/2}\right]R_{M1}^{-3/2}$$

$$> 0$$

so that V_{011} has only one turning point. That turning point occurs at the profit minimum, which can be found easily by setting the first derivative to zero and solving for R_{M1}. Plugging the resulting value of R_{M1} into V_{011} shows that V_{011} is negative there. Also, V_{011} equals $-F_1$ if $R_{M1} = 0$. Taken together, these results indicate that the profit function V_{011} has the form shown in Figure 2. All eight profit functions have the same general shape, and there is a value $R_{M1}*$ for each profit function, below which profit becomes negative. In the region where profit is positive (i.e., the region where it is relevant to the household's choice), the slope of the profit function with respect to R_{M1} is positive.

We are not really interested in how the individual profit functions V_{ijk} respond to a change in R_{M1} but rather how the profit differences respond. It is the changes in those differences that tell us whether the household switches from the domestic money to the foreign money or vice versa. For example, consider the difference

$$(23) \qquad V_{011} - V_{012} = R_{M1}\frac{X_1 + X_2}{2} - (2B_{21}R_{M1}X_2)^{1/2}$$

$$-R_{M1}\frac{X_1}{2} - R_{M2}\frac{X_2}{2} + (2B_{22}R_{M2}X_2)^{1/2} + F_2$$

which determines whether the domestic money will be used to buy good X_2 given that it is used to buy good X_1.[12] The derivative of this expression with respect to R_{M1} is

$$(24) \qquad \frac{\partial(V_{0,1,1} - V_{0,1,2})}{\partial R_{M1}} = \frac{X_2}{2} - \left(\frac{B_{21}X_2}{2}\right)^{1/2}R_{M1}^{-1/2}$$

which can be positive or negative but which is increasing in R_{M1} and positive for sufficiently large

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[12]The same qualitative results arise from other such profit differences, so we do not consider them explicitly in this study.

R_{M1}. The second derivative of the profit difference with respect to R_{M1} is everywhere positive, so there is only one turning point in the profit difference. That point occurs at the value of R_{M1} where the first derivative is zero, which is

(25) $$R_{M1} = \frac{2B_{21}}{X_2}$$

At this value of R_{M1}, the profit difference $V_{011}-V_{012}$ is

(26) $$(V_{011} - V_{012})\big|_{R_{M1} = \frac{2B_{21}}{X_2}} = -R_{M2}\frac{X_2}{2} + (2B_{22}R_{M2}X_2)^{1/2} + F_2$$

which is not necessarily negative. So although the profit difference, has the same general shape as V_{011} alone, it can be positive at its minimum (and therefore at all values of R_{M1}). Also, both the individual profit expressions V_{011} and V_{012} can be positive at the value of R_{M1} at which $V_{011}-V_{012}$ reaches its minimum.

The upshot is that it is possible for the profit difference to fall with an increase in R_{M1} even if the profit difference and the individual profit functions are positive. That means that an increase in R_{M1} could lead to a *reduction* in the use of M_1 (the domestic money), even though there are no wealth effects at work. This seemingly perverse behavior arises from the interplay of the two profit functions being compared. Each function depends on R_{M1} in a nonlinear way, positively through the direct effect of interest earnings on money balances and negatively through the indirect effect of R_{M1} on the optimal number of conversions between money and goods (and therefore on the amount of transactions costs paid). The two profit functions depend on R_{M1} in similar but not identical ways, so that the difference between them shows complicated non-linear behavior. The seeming perversity is the result. This perversity is present for sufficiently low values of R_{M1} and disappears for R_{M1} sufficiently large because (24) is increasing in R_{M1}. Consequently, the response of demand for M_1 to a change in R_{M1} (and therefore to domestic inflation π_1) is different for high initial rates of inflation (and correspondingly low values of R_{M1}) than for low initial rates.

It is important to distinguish between two different effects at work here on the demand for domestic money. The effect we have just been discussing concerns whether or not M_1 is used to buy either or both of the consumption goods. We can think of this as an extensive margin. There also

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is an intensive margin. Given that M_1 is being used to buy at least one good, the amount that is held is given by (see the Appendix)[13]

$$(27) \qquad \overline{M}_i = \left[\frac{A_i}{2(R_S - R_{Mi})} \sum_g X_{gi} \right]^{1/2} - \sum_g \left[\frac{B_{gi} X_{gi}}{2R_{Mi}} \right]^{1/2}$$

which is unambiguously increasing in R_{M1}. A change in R_{M1} can move in opposing directions along the intensive and extensive margins. An increase, for example, could reduce the number of goods purchased with M_1 but at the same time raise the amount that is held for the purchase of those goods still bought with M_1. The net change in the total demand for M_1 in this case would be ambiguous. If movement along the two margins is in complementary directions, of course, there is no ambiguity in what happens to the demand for M_1.

Note also that the precise behavior of the profit differences, their derivatives, and the quantities of each type of money held all depend on the values of the conversion costs and fixed costs. These costs capture the financial institutional structure that Savastano (1996) discusses. Savastano argues from the empirical evidence that the extent of currency substitution depends on how well developed a country's financial sector is. Countries with well-developed financial institutions can offer a wide range of transactions assets that can adjust to inflation and protect their owners from inflation, whereas countries that are financially "repressed" offer a much more restricted set of transactions assets with much less inflation protection. In terms of our model, financially developed countries will have many types of domestic money (such as demand deposits and money market mutual funds) that offer nominal interest rates, thus allowing them to adjust to inflation at least in part and help insulate the real rate of return from inflation. The fixed costs of holding those types of money will be low, as will the conversion costs associated with using them. The result will be much less incentive to substitute foreign media of exchange for domestic money in the presence of high inflation than in a financially repressed country.

These conclusions provide the fundamental results for currency substitution. Currency substitution is the abandonment of one medium of exchange for another in the face of changing

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[13]The expression obtained in the Appendix is more general than the following because here we have imposed the restriction that $R_{Xg} = 0$.

incentives. The most important such incentive historically has been changes in the domestic inflation rate, which reduces the real rate of return on domestic money. What we have just seen is that there are two dimensions to currency substitution - an extensive margin and an intensive one. Past discussions seem not to have made this distinction, measuring currency substitution solely by something like the (change in the) ratio of aggregate holdings of foreign money to holdings of domestic money holdings or the (change in the) ratio of foreign holdings to total holdings. Such a measure will be totally satisfactory only if the economy is moving along the intensive and extensive margins in a complementary way, something it need not necessarily do. Suppose, for example, that the economy is in the region where an increase in R_{M1} reduces the number of goods bought with M_1 but raises the amount of M_1 held for those goods still bought with it. We then have something similar to a "mixed market" on a stock exchange, where the number of losers, say, is larger than the number of gainers but the market as a whole gains. In the currency substitution analog, fewer goods are bought with M_1 but more M_1 is held. Should we say that currency substitution has increased or decreased? This distinction could be a useful one to explore for episodes of very high inflation, when the economy is most likely to be in the region of "perverse" behavior.

Having established the possibility of "mixed market" behavior, we now will simplify the subsequent discussion by assuming henceforth that R_{M1} initially is large enough (i.e., inflation is initially low enough) that an increase in its value makes use of M_1 more attractive. Then a reduction in R_{M1} caused by an increase in domestic inflation unambiguously will lead to a substitution of M_2 for M_1. Nonetheless, even though we now have an unambiguous qualitative response, there still is the important issue of the quantitative response, which depends on other variables we have ignored so far.

B. Conversion Costs and Fixed Costs.

The effects of the conversion costs A_j and B_{ij} and the fixed costs F_j are straightforward and do not require detailed discussion. Increases in any of these costs reduces the value of using the associated asset S, M_1, or M_2; all profit differences V_{ijk}-$V_{i'j'k'}$ change in the direction that indicates reduced value to using the asset whose associated cost has risen. Similarly, an increase in A_1, B_{11}, and B_{12} makes the derivatives of all profit differences with respect to R_{M1} move in a direction that indicates reduced likelihood to use M_1 after an increase in π_1 occurs, so that currency substitution is more likely the more costly the domestic money already is to use. These results further support

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Savastano's contentions regarding the importance of financial institutional structure to the extent of currency substitution experienced by a country.

We now turn to two variables that affect currency substitution that neither Savastano nor, apparently, the rest of the existing literature considers.

C. Level of Income.

The level of income affects currency substitution in subtle and complex ways. Income equals purchases of goods, X_1+X_2, so we can study the effects of income on currency substitution by examining the dependence of the various profit differences $V_{ijk}-V_{i'j'k'}$ on X_1+X_2. It is easiest to start with the simpler case where households choose not to hold the savings asset S. We will examine the case where S is positive shortly.

Consider the choice between using domestic money M_1 exclusively or using M_1 to buy X_1 and M_2 to buy X_2. The choice depends on the sign of the profit difference $V_{011}-V_{012}$, given in (23). We can rearrange the terms in that expression slightly to give the following more convenient form:

$$(28) \qquad V_{011} - V_{012} = (R_{M1} - R_{M2})\frac{X_2}{2} - [2B_{21}R_{M1}X_2]^{1/2} + [2B_{22}R_{M2}X_2]^{1/2} + F_2$$

The first three terms all depend positively on X_2, but the first term increases linearly in X_2 whereas the other two terms increase in the square root of X_2. Clearly, as X_2 rises, the first term dominates, and the whole expression becomes unambiguously positive. If X_2 is a normal good, then it increases when income increases, and we can conclude that a sufficiently high income will guarantee that the first term in (28) dominates everything else. The sign of that term depends directly on the sign of $R_{M1}-R_{M2}$. If $R_{M1}-R_{M2}$ is positive, then a high-income household will use only the domestic money M_1; if it is negative, the household will use M_1 to buy X_1 and M_2 to buy X_2. In the latter case, we would observe currency substitution. The intuition is straightforward. The first term in (28) captures the interest income gained by using M_1 instead of M_2 (this "gain" will be negative if $R_{M1} < R_{M2}$), the second and third terms reflect the importance of conversion costs, and the fourth term captures the extra fixed cost associated with using M_2 along with M_1.[14] As in all transactions models

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[14]Presumably this fixed cost would be zero if M_2 consisted only of foreign currency and did not include any bank accounts, such as M_2-denominated demand deposits, which might carry monthly fees or other fixed costs.

of this type, the number of conversions increases in the square root of expenditure, whereas interest income increases linearly in income. Consequently, as income rises, the interest income becomes dominant, so that high income households base their money usage decision almost exclusively on the interest income term and tend to use only the money that pays higher interest. In particular, if their government is inflating the currency, then R_{M1}-R_{M2} can become negative, and higher income households should start abandoning M_1 for M_2. The same results hold for the profit differences corresponding to other usage choices: V_{011}-V_{021}, V_{012}-V_{022}, and so forth.

Lower income households can behave quite differently. For them, the second and third terms of (28) become important because the transactions costs and fixed costs are relatively more important than interest earnings when the volume of savings on which interest is earned is small. Low income households may continue to use M_1 exclusively even if R_{M1} falls below R_{M2} if B_{22} is large relative to B_{21} or if F_2 is high. We can conclude, then, that two households facing the same interest rates, conversion costs, and fixed costs but with different income levels may make opposite choices on which type of money to use in buying a particular good.

Despite the foregoing qualifications, we do seem to have the result that currency substitution is more likely among higher-income households than lower-income households. This prediction is consistent with Dotsey's (1988) evidence that higher income reduces the fraction of household expenditures paid with currency. Dotsey's data pertain only to domestic money and explore the split between currency on the one hand and all other forms of domestic money (e.g., demand deposits) on the other. His findings therefore are not a direct test of the relations predicted by the theory developed herein, but they are suggestive. Further evidence of the income effect on currency substitution comes from a study by Stix (2007) of survey data on households in Croatia, Slovenia, and Slovakia. Stix finds that households with higher education (which seems to be positively correlated with income) are more likely to use foreign currencies in making domestic purchases.[15]

Presumably, our theoretical results carry over to entire countries. Citizens of a rich country have higher incomes on average than citizens of a poor country, so the representative agent for a rich

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[15]It also is interesting that Colacelli (2005), in studying currency substitution arising from a shortage of domestic money rather than from inflation, finds that higher income households are *less* likely to use substitute currencies, suggesting that there are important differences in the determinants of the two types of currency substitution.

country should behave more like a rich household than does the representative agent for a poor country. We thus are left with something of a mixed result for the relation between income and currency substitution. On the one hand, if R_{M1} exceeds R_{M2}, there should be less use of M_2 in rich countries than in poor countries. On the other hand, if R_{M1} is less than R_{M2}, the opposite is true.

So far, we have assumed that X_2 is a normal good. If X_2 is inferior, then its use falls with an increase in income, reversing all the conclusions concerning comparisons of rich and poor countries. Although possibly important for analyzing individual household behavior, this complication seems unlikely to be relevant for analysis of aggregate data. It seems almost certain that broad categories of goods, such as food or clothing, are normal even though individual items within those categories are inferior. We thus will ignore inferior goods hereafter.

The foregoing results were derived for the case where the savings asset S is not used. When S is used, the relation between income and currency substitution is even more complex. Consider the profit difference analogous to (28) when S is held:

$$V_{SI1} - V_{SI2} \;=\; -[2A_1(R_S - R_{MI})(X_1 + X_2)]^{1/2} - [2B_{21}R_{MI}X_2]^{1/2}$$

(29)
$$+ [2A_1(R_S - R_{MI})X_1]^{1/2} + [2A_2(R_S - R_{M2})X_2]^{1/2}$$

$$+ [2B_{22}R_{M2}X_2]^{1/2} + F_2$$

This expression is more complicated than (28) because of the presence of R_S and A_1, but the really important difference is that now all income-related terms - $X_1 + X_2$, X_1, and X_2 - enter as square roots, so that no one of them obviously becomes dominant as income rises. We therefore cannot make easy comparisons between rich and poor countries (or households). Taking the derivative of (29) with respect to $X_1 + X_2$ yields

$$(30)\quad \frac{\partial V_{S11}-V_{S12}}{\partial(X_1+X_2)} = \left\{-\left[\frac{A_1(R_S-R_{M1})}{2(X_1+X_2)}\right]^{1/2}+\left[\frac{A_1(R_S-R_{M1})}{2X_1}\right]^{1/2}\left(\frac{X_1}{X_1+X_2}\right)\right\}\left[\frac{\dfrac{dX_1}{X_1}}{\dfrac{d(X_1+X_2)}{(X_1+X_2)}}\right]$$

$$+\left\{\left[\frac{A_2(R_S-R_{M1})}{2X_2}\right]^{1/2}+\left[\frac{B_{22}R_{M2}}{2X_2}\right]^{1/2}-\left[\frac{B_{21}R_{M1}}{2X_2}\right]^{1/2}\right\}\left(\frac{X_2}{X_1+X_2}\right)\left[\frac{\dfrac{dX_2}{X_2}}{\dfrac{d(X_1+X_2)}{(X_1+X_2)}}\right]$$

$$=-\left[\frac{A_1(R_S-R_{M1})}{2(X_1+X_2)}\right]^{1/2}+\left[\frac{A_1(R_S-R_{M1})}{2X_1}\right]^{1/2}\sigma_1\varepsilon_1$$

$$+\left\{\left[\frac{A_2(R_S-R_{M1})}{2X_2}\right]^{1/2}+\left[\frac{B_{22}R_{M2}}{2X_2}\right]^{1/2}-\left[\frac{B_{21}R_{M1}}{2X_2}\right]^{1/2}\right\}\sigma_2\varepsilon_2$$

where σ_i and ε_i are the expenditure share and income elasticity of good I. This expression can be positive or negative, depending in part on the relative values of expenditure shares and income elasticities. In general, we can draw no conclusion about the sign of this expression, except that it may be different for different countries facing the same conversion costs, interest rates, and even levels of income because expenditure shares and income elasticities may well differ across countries.

There is still one more complication with the relation between income and currency substitution. Both (28) and (29) assume the use or non-use of S is given exogenously. In fact, however, the decision to use S is endogenous and depends on the level of income. Consider, for example, the profit difference $V_{S11}-V_{011}$:

$$(31)\quad V_{S11}-V_{011} = (R_s-R_{M1})\left(\frac{X_1+X_2}{2}\right)-[2A_1(R_S-R_{M1})(X_1+X_2)]^{1/2}-F_S$$

This equation has much the same character as (28) in terms of how income enters. The first term increases linearly in income, whereas the second increases in the square root of income. Consequently, for sufficiently large income, the first term dominates and the whole expression is positive because $R_S > R_{M1}$. The intuition is the same as before; for high income, the extra interest

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earned dominates the additional conversion costs and fixed costs from using S. We thus should expect to see more people in high income countries using S than in low income countries. The implication for currency substitution is that it is not just the value of the objective function (that is, the profit difference) that varies with income but the form of that function itself. Rich countries are likely to be using profit differences like (29) to decide which money to use, whereas poor countries are more likely to use profit differences like (28). The two types of profit differences have very different sensitivities to changes in the domestic inflation rate (which cause R_{M1} to change in the opposite direction). We can see the differences clearly by examining the derivatives of the two profit differences with respect to R_{M1}. The derivative of V_{011}-V_{012} with respect to R_{M1} is given by (24), repeated here for reference:

$$(24) \qquad \frac{\partial(V_{011} - V_{012})}{\partial R_{M1}} = \frac{X_2}{2} - \left(\frac{B_{21} X_2}{2} \right)^{1/2} R_{M1}^{-1/2}$$

whereas the same derivative for (29) is

$$(32) \qquad \frac{\partial(V_{S11} - V_{S12})}{\partial R_{M1}} = \left[\frac{A_1(X_1 + X_2)}{2(R_S - R_{M1})} \right]^{1/2} - \left[\frac{B_{21} X_2}{2 R_{M1}} \right]^{1/2} - \left[\frac{A_1 X_1}{2(R_S - R_{M1})} \right]^{1/2}$$

The latter depends on everything in the former and also on X_1, A_1, and R_S, and the signs of the two expressions can be different for several reasons.

The main implication is that there may not be a simple relation between currency substitution and inflation rates in a given data set. Our theory is not nihilistic, however. It does not say there is no relation between currency substitution and inflation or even that there is a relation that is vague or indefinite. There is a very specific relation dictated by the theory; it just is not simple.

D. Relative Expenditures.

An interesting implication of the theory presented here is that the degree of currency substitution depends on not just the level of income but also on the composition of expenditure. Two countries with the same income $X_1 + X_2$ may choose to adopt different mixes of monies for a given inflation rate and definitely will have different sensitivities to changes in the inflation rate. These implications are easily seen from (28). Holding total income $X_1 + X_2$ fixed, the first term on the right side of (28) becomes dominant, as already explained earlier. The sign of (28) then depends

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on the sign of R_{M1}-R_{M2}. Consequently, as X_2 becomes a large part of total expenditure, the country is increasingly likely to use the higher-interest money to buy it; for large enough X_2, that outcome is guaranteed. For example, if R_{M1}-R_{M2} is positive, a country with a large value of X_2 is guaranteed to have a positive value of (28) and thus to use M_1 to buy X_2, whereas a country with a lower value of X_2 (but the same value of X_1+X_2) may have a negative value of (28) and so may make the opposite choice.[16] This example is enough to illustrate the point that the choice of which monies to use depends on expenditure's composition as well as its level.[17]

It should come as no surprise by now that the amount of currency substitution a country experiences in response to a change in its domestic inflation rate also depends on its composition of expenditure. Two countries with the same income level X_1+X_2 and that had chosen the same money usage pattern (such as, for example, holding no S and using M_1 to finance all purchases) could have different marginal responses to a change in R_{M1}. For example, the right side of equation (24) is increasing in X_2; it is guaranteed to be positive for X_2 sufficiently large but can be negative for smaller values of X_2. Consequently, two countries that had equal incomes, that had chosen to hold no S, and that had chosen to use their domestic money M_1 to buy both goods (that is, they have the usage pattern [0, 1,1]) could respond in opposite ways to an increase in domestic inflation (equivalently, a decrease in R_{M1}) if they have different relative values of X_2 that cause (24) to have opposite signs for the two countries.[18]

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[16]Furthermore, because there are only two goods in this model, an increase in X_2 with X_1+X_2 held fixed necessarily entails a matching decrease in X_1. As X_1 decreases, the logic we have just used to discuss the choice of which money to use in buying X_2 works in reverse for the choice of which money to use in buying X_1. The country (or household) with a low value of X_1 is more likely to buy it with M_2 than M_1, compared to a country with high X_1. The result is a further difference from a country that has high X_1 and low X_2.

[17]Other interesting implications of expenditure composition for usage patterns are discussed by Seater (2002).

[18]It is easy to see that such a situation is possible, even if there are no fixed costs. For any given level of income, we can rule out the use of S by making A_1 and A_2 very large. We also can rule out the use of M_2 to buy X_1 by making B_{12} very large. That leaves only the usage patterns (0,1,1) and (0,1,2) to consider. We have not yet specified the values of R_{M1}, R_{M2}, B_{11}, or B_{21}, or X_2 (we are not free to choose X_1 because it is determined by choice of X_2 given that we already have fixed X_1+X_2), which gives us five degrees of freedom to make (23) barely positive

We thus see that the relation between currency substitution, inflation, and income is even more complex than it appeared in the previous section.[19]

IV. CONCLUSION

We have used an extended transactions model of the demand for money to provide a theory of the demand for currency substitution. Our theory offers a rigorous foundation for Savastano's (1996) suggestion that the extent of currency substitution in a country depends on the financial institutional framework in that country. Savastano argued that currency substitution would be less extensive in countries with well-developed financial systems that offer media of exchange that can compensate users for inflation. Our framework captures that element of institutional framework in the real rates of return, transactions costs, and fixed costs it attaches to alternative monies. Our results support Savastano's contention; financial structure does matter in determining the extent of currency substitution.

Our theory also predicts that other institutional factors not considered by Savastano or the rest of the existing currency substitution literature also are important in determining the extent of currency substitution that a country experiences. In particular, the level of income and the composition of expenditures affect the extent of currency substitution. The relation between income and currency substitution is complex and cannot be summarized as a simple relation such as "high income increases (or decreases) the extent of currency substitution," but by providing an explicit model for the demand for multiple media of exchange, the theory offers guidance in how to construct empirical specifications. The dependence of currency substitution on the composition of expenditures is perhaps the most surprising result derived. Again, the relation is complex, but the

and to make (24) either positive or negative as we choose. We thus can create a country that initially chooses the usage pattern (0,1,1) and that either retains or alters that pattern in response to a reduction in R_{M1}.

[19]It seems possible that some of the variables affecting currency substitution also would be relevant to the decision to join a currency union. The value of a currency union arises from a reduction in transactions costs compared to maintaining an independent currency. Our theory suggests that value may depend on variables, such as the composition of expenditures, not previously considered in the currency union literature (e.g., Alesina and Barro, 2001, 2002).

explicit solutions provided by the theory give guidance on how to handle that relation in empirical work. The data required to test the predictions of the theory are not readily available, but they are not impossible to assemble. The best tests probably would be based on household data, similar in nature to the data that Colacelli (2005) collected for her study of secondary currency use in Argentina. Stix (2007) using a limited survey of households in Croatia, Slovenia, and Slovakia, finds support for one of the new predictions of the theory, that currency substitution is positively related to household income. Developing more complete household survey data sets would be a valuable extension that would allow further tests.

APPENDIX

1. Average assets. Average total real assets are

$$\overline{W} = \overline{S} + \sum_{i=1}^{L} \overline{M}_i + \sum_{g=1}^{G} \overline{X}_g$$

$$= \overline{S} + \sum_{i=1}^{L} \left(\overline{M}_i + \sum_{g=1}^{G} \overline{X}_{gi} \right)$$

We also have, because trips are evenly spaced and the rate of consumption is constant,

$$\overline{W} = \sum_g \frac{X_g}{2}$$

$$\overline{M}_i + \sum_g \overline{X}_{gi} = \sum_g \frac{X_{gi}}{2T_i}$$

$$\overline{X}_{gi} = \frac{X_{gi}}{2Z_{gi}}$$

We then have

$$\overline{M}_i = \left(\overline{M}_i + \sum_g \overline{X}_{gi} \right) - \sum_g \overline{X}_{gi}$$

$$= \frac{\sum_g X_{gi}}{2T_i} - \frac{\sum_g X_{gi}}{2Z_{gi}}$$

$$\overline{S} = \overline{W} - \sum_i \left(\overline{M}_i + \sum_g \overline{X}_{gi} \right)$$

$$= \sum_g \frac{X_g}{2} - \sum_i \sum_g \frac{X_{gi}}{2T_i}$$

2. Optimal values of T_i and Z_{gi}. Substituting the foregoing expressions in the profit function gives

$$V = R_S \left[\sum_g \left(\frac{X_g}{2} - \sum_i \frac{X_{gi}}{2T_i} \right) \right] + \sum_i R_{Mi} \left[\sum_g \left(\frac{X_{gi}}{2T_i} - \frac{X_{gi}}{2Z_{gi}} \right) \right] + \sum_g R_{Xg} \left(\sum_i \frac{X_{gi}}{2Z_{gi}} \right)$$

$$- \sum_i T_i A_i - \sum_i \sum_g Z_{gi} B_{gi} - F_S I(S) - \sum_i F_i I(M_i)$$

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The first-order conditions for T_i and Z_{gi} are

$$\frac{\partial V}{\partial T_i} = 0$$

$$\frac{\partial V}{\partial Z_{gi}} = 0$$

which give the solutions

$$T_i = \left[(R_S - R_{Mi})\sum_g \frac{X_{gi}}{2A_i}\right]^{1/2}$$

$$Z_{gi} = \left[(R_{Mi} - R_{Xg})\frac{X_{gi}}{2B_{gi}}\right]^{1/2}$$

If $X_{gi}=0$ for all g, money I is not used and drops out of the maximization problem. In that case, of course, both T_i and Z_{gi} automatically are zero .

These first-order conditions assume that T_i and Z_{gi} can be treated as continuous variables, whereas in fact they must be integers. Barro (1976) shows that the correct values are found by computing the continuous solutions as above and then choosing the integers on either side of those solutions that maximize profit. He also shows that the qualitative results are the same for the continuous approximation as for the exact solution, so it is standard procedure to simplify the analysis by discussing only the continuous solution, as done in the analysis here.

3. Average assets again. Substituting these last expressions into those above for average assets gives

$$\overline{X}_{gi} = \left[\frac{X_{gi}B_{gi}}{2(R_{Mi}-R_{Xg})}\right]^{1/2} = \left[\frac{X_{gi}B_{gi}}{2R_{Mi}}\right]^{1/2}$$

$$\overline{M}_i = \left[\frac{A_i}{2(R_S-R_{Mi})}\sum_g X_{gi}\right]^{1/2} - \sum_g\left[\frac{B_{gi}X_{gi}}{2(R_{Mi}-R_{Xg})}\right]^{1/2}$$

$$\overline{S} = \sum_g \frac{X_g}{2} - \sum_i\left[\frac{A_i}{2(R_S-R_{Mi})}\sum_g X_{gi}\right]^{1/2}$$

TABLE 1
Profit Functions

$$V_{0,1,1} = R_{MI} \frac{X_1+X_2}{2} - [2B_{11}R_{MI}X_1]^{1/2} - [2B_{21}R_{MI}X_2]^{1/2} - F_1$$

$$V_{0,1,2} = R_{MI} \frac{X_1}{2} + R_{M2} \frac{X_2}{2} - [2B_{11}R_{MI}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2}$$
$$- F_1 - F_2$$

$$V_{0,2,1} = R_{MI} \frac{X_2}{2} + R_{M2} \frac{X_1}{2} - [2B_{21}R_{MI}X_2]^{1/2} - [2B_{12}R_{M2}X_1]^{1/2}$$
$$- F_1 - F_2$$

$$V_{0,2,2} = R_{M2} \frac{X_1+X_2}{2} - [2B_{12}R_{M2}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_2$$

$$V_{S,1,1} = R_S \frac{X_1+X_2}{2} - [2A_1(R_S-R_{MI})(X_1+X_2)]^{1/2}$$
$$- [2B_{11}R_{MI}X_1]^{1/2} - [2B_{21}R_{MI}X_2]^{1/2} - F_S - F_1$$

$$V_{S,1,2} = R_S \frac{X_1+X_2}{2} - [2A_1(R_S-R_{MI})X_1]^{1/2} - [2A_2(R_S-R_{M2})X_2]^{1/2}$$
$$- [2B_{11}R_{MI}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_S - F_1 - F_2$$

$$V_{S,2,1} = R_S \frac{X_1+X_2}{2} - [2A_1(R_S-R_{MI})X_2)]^{1/2} - [2A_2(R_S-R_{M2})X_1]^{1/2}$$
$$- [2B_{21}R_{MI}X_2]^{1/2} - [2B_{12}R_{M2}X_1]^{1/2} - F_S - F_1 - F_2$$

$$V_{S,2,2} = R_S \frac{X_1+X_2}{2} - [2A_2(R_S-R_{M2})(X_1+X_2)]^{1/2}$$
$$- [2B_{12}R_{M2}X_1]^{1/2} - [2B_{22}R_{M2}X_2]^{1/2} - F_S - F_2$$

NOTE: The subscripts in the profit expression V_{ijk} have the following meanings:

I = S if the saving asset is used, 0 otherwise

j = 1 or 2 as M_1 or M_2 is used to buy good 1

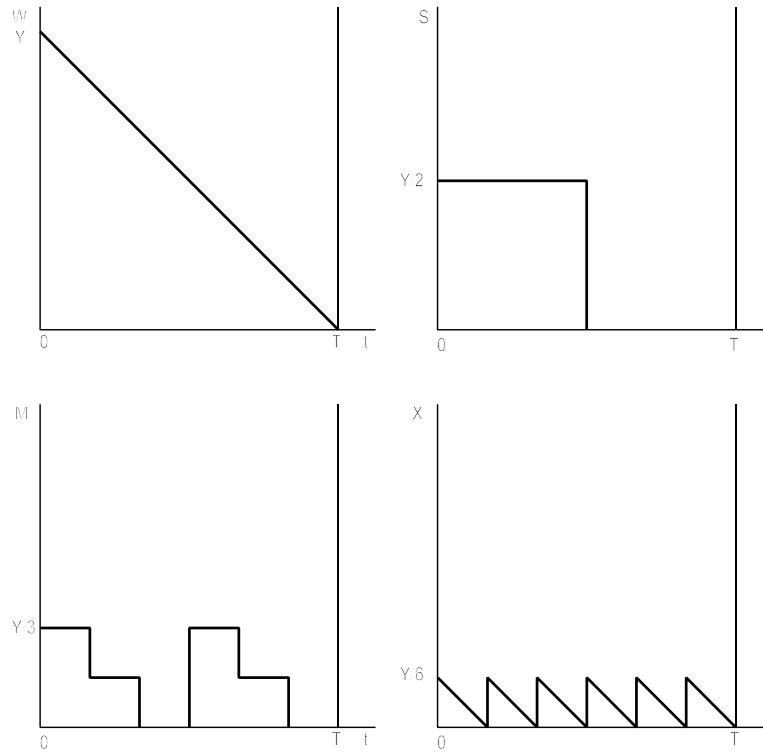k = 1 or 2 as M_1 or M_2 is used to buy good 1
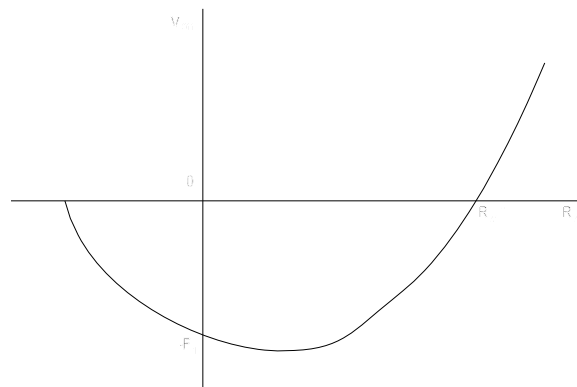
Figure 1: Illustration of asset time paths

Figure 2: Shape of the profit function V_{011}

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