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A Long Run Structural Macroeconometric Model for Germany

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Abstract:

The objective of this paper is to apply the method developed in Garratt, Lee, Pesaran, and Shin (2000) to build a structural model for Germany with a transparent and theoretically coherent foundation. The modelling strategy consists of a set of long-run structural relationships suggested by economic theory and an otherwise unrestricted VAR model. It turns out that we can rebuild the structure of the model in Garratt, Lee, Pesaran, and Shin (2003b) for German data. Five long run relations: PPP, UIP, production function, trade balance, and real money balance characterize the equilibrium state of Germany as an open economy in our structural model.

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1 Introduction

The aim of the long run structural modelling strategy by Garratt et al. (2000), Garratt, Lee, Pesaran, and Shin (2003a) and Garratt et al. (2003b) is to develop a model with transparent and theoretically coherent foundation. The authors advanced the modelling framework of King R. G. and Watson (1991), Gali (1992), Mellander, Vredin, and Warne (1991) and Crowder, Hoffman, and Rasche (1999), and developed a long-run framework suitable for modeling a small open macroeconomy. Their new strategy offers a practical approach to relationships suggested by economic theory in an otherwise unrestricted vector autoregressive (VAR) model. The core model of UK in Garratt et al. (2003b) includes transparent and theoretically founded long-run equations of the type exhibited by RBC models. The long-run relations are derived from production, arbitrage, solvency and portfolio balance conditions. The five equations of core model do not describe a closed model like for example SDGE models.

For their empirical analysis Garratt et al. (2003b) used a log-linear approximation of the five long-run equilibrium relationships. In addition they introduced a 'long-run forcing' variable as exogenous variable: the oil prices. 'Forcing' variable means that changes in oil prices have a direct influence on output, but they are not affected by the other variables in the model. Estimation of the parameters of the core model is based on a modified and generalized version of Johansen's (1991, 1995) maximum likelihood approach in the context of vector error correction models (VECM)¹.

In our paper we used the theoretical model from Garratt et al. (2003b) and estimated a long-run model for Germany. Using a VAR(2) model with unrestricted intercepts and treating the oil price as 'long-run forcing' variable we found five cointegrating relationships. With the identification restrictions derived from the economic theory we can identify the five cointegration relations as the five long run relations suggested by the theory. Tests of the overidentification restrictions can be used to test rigorously the validity of the long-run restrictions implied by economic theory. Using impulse response analysis, the effects of an exogenous oil price shock on the endogenous variables of the model can be simulated.

The plan of the paper is as follows: Section 2 describes a long-run theoretical framework for macromodelling of an open economy, and derives testable restrictions on the long-run relations. This Section also outlines, how the long-run theoretical relations are embodied in a VECM. Section 3 describes the empirical analysis underlying the construction of the model and discusses the results obtained from testing its long-run properties. The estimates of short-run dynamics and of impulse response functions for the oil price are documented. Section 4 concludes.

¹e.g. Pesaran and Shin (2002) and Garratt et al. (2000)

2 A Long Run Structural Macroeconometric Model

Similar to the core model for UK by Garratt et al. (2003b) we modelled the Germany economy as a small open economy, subject to economic developments in the rest of the world. Hence in the VAR approach both domestic and foreign variables are considered. The variables are: domestic and foreign real per capita outputs (y_t, y_t^*), domestic and foreign producer prices (p_t, p_t^*) and nominal interest rates (r_t, r_t^*), the nominal effective exchange rate (e_t), the price of oil (p_t^o) and the domestic real per capita stock of money (h_t), all data are in logarithm.

The underlying economic theory delivers five long-run relations or equilibrium conditions among these variables. They are based on production, arbitrage, solvency and portfolio balance conditions, together with stock-flow and accounting identities. The first long-run relation is a purchasing power parity relation (PPP), based on international goods market arbitrage. PPP was modified by the effect of oil prices (see Chaudhuri and Daniel (1998)). The second relation, a nominal interest rate parity relation is based on arbitrage between domestic and foreign bonds. Assuming common technological progress in production at home and overseas, a "output gap" relation is derived from the neoclassical growth model. Finally there are trade balance and real money balance relations, based on long-run solvency conditions and assumptions about the determinants of the demand for domestic and foreign assets.

The economic theory does not say anything about the statistical characteristics of the variables. If the variables are $I(1)$, the equilibrium relations become candidates for cointegrating relations in A VECM representation. We will present our model in the form of a VECM.

2.1 A Framework for Long-Run Macromodelling

The macroeconomertic modelling of a small open economy starts with a rigorous derivation of the long-run steady-state relationships expected to exist between the main variables in the core model. The analysis includes arbitrage conditions and stock-flow equilibria. These long-run relationships correspond to many of the long-run properties of RBC models and large macroeconomic models. To derive long-run relations, we make use of the arbitrage conditions, as shown below.

Production technology and output:

We assume that in the long run the aggregate output is determined by the following

production function

$$\frac{Y_t}{P_t} = F(K_t, A_t N_t) = A_t N_t F\left(\frac{K_t}{A_t N_t}, 1\right). \quad (1)$$

The constant-returns-to-scale-production-function is linear homogeneous in labour (N_t) and capital stock (K_t). We analyze a real aggregate output (Y_t/P_t), where P_t is a general price index. A_t stands for an index of labour-augmenting technological progress, assumed to be composed of a deterministic and a stochastic mean-zero components: $\ln(A_t) = a_0 + gt + u_{at}$. The fraction of the population, which is employed at time t , is assumed to follow a stationary process: $N_t/POP_t = \lambda \exp(\eta_{nt})$. Accordingly, real per capita output in logarithm: $y_t = \ln((Y_t/P_t)/POP_t)$ is given by

$$y_t = a_0 + gt + u_{at} + \ln(\lambda) + \ln(f(\kappa_t)) + \eta_{nt}, \quad (2)$$

with κ_t the capital stock per effective labour unit ($\kappa_t = K_t/(A_t N_t)$) and $f(\kappa_t)$ a well behaved function in the sense that it satisfies the Inada conditions.

Profit maximization on the part of the firms ensures that in the steady-state the real rate of return ρ_t will be equal to the marginal product of capital: $\rho_t = f'(\kappa_t)$. The assumption, that the steady state distribution of the real rate of return is ergodic and stationary can be written as $1 + \rho_{t+1} = (1 + \rho) \exp(\eta_{\rho,t+1})$, where $\eta_{\rho,t+1}$ is a stationary process normalized, so that $E(\exp(\eta_{\rho,t+1}|I_t)) = 1$, and I_t is the publicly available information at time t . This normalization ensures that ρ is in fact the mean of the steady state distribution of real returns, ρ_t , given by $E(f'(\kappa_\infty))$.

For the small and open economy it is reasonable to assume that, in the long-run, domestic technological process A_t is determined by the level of technological progress in the rest of the world: $A_t = \gamma A_t^* \exp(\eta_{at})$. Here γ captures the productivity differential based on fixed initial technological endowments and A_t^* represents the level of foreign technological progress. Assuming, that per capita output in the rest of the world is also determined according to a neoclassical growth model, we have:

$$y_t - y_t^* = \ln(\gamma) + \ln(\lambda/\lambda^*) + \ln(f(\kappa_t)/f^*(\kappa_t^*)) + \eta_{at} + (\eta_{nt} - \eta_{nt}^*) \quad (3)$$

Arbitrage conditions:

Following Garratt et al. (2003b), the set of arbitrage conditions which we consider in this paper are included in many macroeconomic models in one form or another. They are the (relative) Purchasing Power Parity (PPP), the Fisher Inflation Parity (FIP), and the Uncovered Interest Parity (UIP) relationships.

PPP is based on the presence of goods-market arbitrage. According to this, the price of a common basket of goods will be equal in different countries, when these

prices are measured in a common currency. The domestic price is determined by: $P_t = E_t P_t^* \exp(\eta_{PPP,t})$, where $\eta_{PPP,t}$ is assumed to follow a stationary process capturing the short-run variations in transport costs, information disparities, and the effects of tariff and non-tariff barriers. E_t is the effective exchange rate, defined as the domestic price of a unit of foreign currency at the beginning of period t . An increase in the exchange rate represents a depreciation of the home country currency. In log-linear form we have

$$p_t = e_t + p_t^* + \eta_{PPP,t} \quad (4)$$

with $p_t = \ln(P_t)$, $p_t^* = \ln(P_t^*)$ and $e_t = \ln(E_t)$.

The FIP relationship includes the equilibrium outcome of the arbitrage process between holding bonds and investing in physical assets. The nominal interest rate is determined accordingly by: $1 + R_t = (1 + E(\rho_{t+1})) \left(1 + \frac{E(P_{t+1}) - P_t}{P_t}\right) \exp(\eta_{FIP,t})$, where $(E(P_{t+1}) - P_t)/P_t$ is inflation expectation. $\eta_{FIP,t}$ is the risk premium, capturing the effects of money and goods market uncertainties on risk-averse agents. As before, we assume that $\eta_{FIP,t}$ follows a stationary process with a finite mean and variance. In log-linear form we can write²:

$$r_t = \ln(1 + \rho) + \ln\left(1 + \frac{\Delta P_{t+1}}{P_t}\right) + \eta_{FIP,t+1} + \eta_{\rho,t+1} + E(\eta_{P,t+1}) + E(\eta_{\rho,t+1}), \quad (5)$$

where $r_t = \ln(1 + R_t)$.

The third arbitrage condition is based on the UIP relationship, which captures the equilibrium outcome of the arbitrage process between holding domestic and foreign bonds. In this way any differences between interest rates across countries must be an adjustment by expected exchange rate changes to eliminate the arbitrage. For the Interest Rate Parity relationship we use the UIP equation in the form $(1 + R_t) = (1 + R_t^*) \left(1 + \frac{E(E_{t+1}) - E_t}{E_t}\right) \exp(\eta_{UIP,t+1})$. There $\eta_{UIP,t+1}$ is the risk premium associated with the effects of bonds and foreign exchange uncertainties on risk-averse agents. The IRP relationship in log-linear form can be written as³

$$r_t = r_t^* + \eta_{\Delta e,t+1} + E(\eta_{e,t+1}) + \eta_{UIP,t+1}. \quad (6)$$

Output-expenditure flow identity and long-run solvency requirements

In addition to the arbitrage condition, the economy is subject to the long-run solvency constraint obtainable from the stock-flow relationships. For the core model

²For the expectation we write $1 + E(\rho_{t+1}) = (1 + \rho_{t+1}) \exp(E(\eta_{\rho,t+1}))$ and $E(P_{t+1}) = P_{t+1} \exp(E(\eta_{P,t+1}))$. We use also here, that the steady state distribution of the real rate of return will be ergodic and stationary.

³We assume, that the expectation for the exchange rate is $E(E_{t+1}) = E_{t+1} \exp(E(\eta_{E,t+1}))$, where the expectations errors $E(\eta_{E,t+1})$ follow stationary processes.

we use the following stock identities: $D_t = H_t + B_t$, where D_t is net government debt, H_t is the stock of high-powered money, B_t is the stock of domestic bonds issued by the government. For the net foreign asset position of the economy F_t : $F_t = E_t B_t^* - (B_t - B_t^d)$, where B_t^* is the stock of foreign assets held by domestic residents, B_t^d is the stock of domestic assets held by domestic residents. And $L_t = D_t + F_t$ where L_t is the stock of financial assets held by the private sector.

For the output-expenditure identity we have $Y_t = C_t + I_t + G_t + (Ex_t - Im_t)$, where Y_t is gross domestic product, C_t consumption expenditure, I_t investment expenditure, G_t government expenditure, Ex_t and Im_t are expenditures on export and import.

In order to ensure the long-run solvency of the private sector asset/liability position, it is assumed that $L_{t+1}/Y_t = \mu \exp(\eta_{ly,t+1})$, where L_{t+1}/Y_t is a ratio of total financial assets to nominal income. The process $\eta_{ly,t+1}$ must be stationary, so that the L_{t+1}/Y_t is stationary and ergodic. This expression is based on the idea that domestic residents are neither willing nor able to accumulate claims on, or liabilities to the government and the rest of the world.

The modelling of the equilibrium portfolio balance of private sector assets follows Branson's (1977) Portfolio Balance Approach: The stock of financial assets held by private sector consists of the stock of high-powered money H_t plus the stock of domestic and foreign bonds, held by domestic residents. Two independent equilibrium relationships are specified relating to asset demand: namely, those relating the demand for high-power money and for foreign assets:

$$\frac{H_{t+1}}{L_t} = F_h \left(\frac{Y_t/P_t}{POP_t}, E_{t+1}(\rho_t), E_{t+1}(\rho_t^*), \frac{\Delta E_{t+1}(P_t)}{P_t}, t \right) \exp(\eta_{hl,t+1})$$

with $F_{h1} \geq 0, F_{h2} \leq 0, F_{h3} \leq 0, F_{h4} \leq 0$.

$$\frac{F_{t+1}}{L_t} = F_f \left(\frac{Y_t/P_t}{POP_t}, E_{t+1}(\rho_t), E_{t+1}(\rho_t^*), \frac{\Delta E_{t+1}(P_t)}{P_t}, t \right) \exp(\eta_{fl,t+1})$$

with $F_{f1} \leq 0, F_{f2} \leq 0, F_{f3} \geq 0, F_{f4} \geq 0$.

In view of the IRP it is clear, that in the steady state domestic and foreign bonds become perfect substitutes, and their expected rates of return are equal. Similar, given the FIP relationship the real rates of return on domestic and foreign bonds are equal to the (stationary) real rate of return on physical assets in the steady state. Hence, the asset demand relationships can be written equally as:

$$\frac{H_{t+1}}{L_t} = F_h \left(\frac{Y_t/P_t}{POP_t}, r_t, t \right) \exp(\eta_{hl,t+1})$$

$$\frac{F_{t+1}}{L_t} = F_f \left(\frac{Y_t/P_t}{POP_t}, r_t, t \right) \exp(\eta_{fl,t+1})$$

The solvency condition $L_{t+1}/Y_t = \mu \exp(\eta_{ly,t+1})$ combined with equation H_{t+1}/L_t now gives:

$$\frac{H_{t+1}}{Y_t} = \mu F_h \left(\frac{Y_t}{P_t}, r_t, t \right) \exp(\eta_{ly,t+1} + \eta_{hl,t+1}) \quad (7)$$

2.2 Econometric Formulation of the Core Model

For empirical purposes we use a log-linear approximation of the five long-run equilibrium relationships set out in the previous section in (4), (6), (3), (7) and (5).

$$(p_t - p_t^*) - e_t = a_{10} + \xi_{1,t+1} \quad (8)$$

$$r_t - r_t^* = a_{20} + \xi_{2,t+1} \quad (9)$$

$$y_t - y_t^* = a_{30} + \xi_{3,t+1} \quad (10)$$

$$h_t - y_t = a_{40} + \beta_{42}r_t + \beta_{43}y_t + \xi_{4,t+1} \quad (11)$$

$$r_t - \Delta p_t = a_{50} + \xi_{5,t+1} \quad (12)$$

where $p_t = \ln(P_t)$, $p_t^* = \ln(P_t^*)$, $e_t = \ln(E_t)$, $y_t = \ln(Y_t/P_t)$, $y_t^* = \ln(Y_t^*/P_t^*)$, $r_t = \ln(1+R_t)$, $r_t^* = \ln(1+R_t^*)$, $h_t - y_t = \ln(H_{t+1}/P_t) - \ln(Y_t/P_t)$ and $a_{30} = \ln(\gamma)$, $a_{40} = \ln(\mu)$, $a_{50} = \ln(1+\rho)$ ⁴. We have allowed only for intercept in the equations. The disturbances $\xi_{i,t+1}$ are related to the structural disturbance, $\eta_{i,t+1}$, in the following manner:

$$\xi_{1,t+1} = \eta_{PPP,t} - a_{10}$$

$$\xi_{2,t+1} = \eta_{\Delta e,t+1} + E(\eta_{e,t+1}) + \eta_{UIP,t+1} - a_{20}$$

$$\xi_{3,t+1} = \eta_{at} + (\eta_{nt} - \eta_{nt}^*)$$

$$\xi_{4,t+1} = \eta_{ly,t+1} + \eta_{hl,t+1}$$

$$\xi_{5,t+1} = \eta_{FIP,t+1} + \eta_{\rho,t+1} + E(\eta_{P,t+1}) + E(\eta_{\rho,t+1}).$$

These relationships are the same as derived in Garratt et al. (2003b). It can be difficult to identify the effects of changes in particular structural disturbances $\eta_{i,t}$ on the dynamic behavior of the macroeconomy: There are more long-run structural disturbances than there are long-run reduced form disturbances; and there is no reason to believe that the $\eta_{i,t}$ are not themselves correlated.

The five long-run relations of the core model can be written more compactly as

$$\xi_t = \beta' \mathbf{z}_{t-1} - \mathbf{a}_0 \quad (13)$$

where

$$\mathbf{z}_t = (p_t^o, r_t, y_t, \Delta p_t, p_t - p_t^*, e_t, h_t - y_t, r_t^*, y_t^*)'$$

⁴We denote $\ln(H_{t+1}/P_t)$ by h_t rather than h_{t+1} . H_{t+1} is the stock of high powered money at the beginning of period $t+1$.

$$\mathbf{a}_0 = (a_{10}, a_{20}, a_{30}, a_{40}, a_{50})', \xi_t = (\xi_{1,t}, \xi_{2,t}, \xi_{3,t}, \xi_{4,t}, \xi_{5,t})'$$

and

$$\beta' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -\beta_{42} & \beta_{43} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (14)$$

Here, p_t^o is the logarithm of the oil price. A general specification for oil prices is given by

$$\Delta p_t^o = \delta_0 + \sum_{i=1}^{s-1} \delta_i \Delta \mathbf{z}_{t-i} + \varepsilon_{o,t}$$

where $\varepsilon_{o,t}$ represents a serially uncorrelated oil price shock with zero mean and a constant variance.

For the difference-stationary variable \mathbf{z}_t the modelling strategy is now to embody ξ_t in an otherwise unrestricted VAR($s-1$) in \mathbf{z}_t . The basis of the analysis is the following conditional error correction model

$$\Delta \mathbf{z}_t = \mathbf{b} - \alpha \xi_t + \sum_{i=1}^{s-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{u}_t \quad (15)$$

where \mathbf{b} is an 9×1 vector of fixed intercepts, α is a 9×5 matrix of error-correction coefficients, Γ_i are 9×9 matrices of short-run coefficients, and \mathbf{u}_t is a 9×1 vector of disturbances assumed to be $IID(0, \Sigma)$, with Σ being a positive definite matrix. With (13) we have

$$\Delta \mathbf{z}_t = \mathbf{c} - \alpha \beta' \mathbf{z}_{t-1} + \sum_{i=1}^{s-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{u}_t \quad (16)$$

where $\mathbf{c} = \mathbf{b} + \alpha \mathbf{a}_0$ and $\beta' \mathbf{z}_{t-1}$ are the error correction terms.

For the estimation of the parameters of the core model for Germany we apply Johansen maximum likelihood approach (see Johansen (1991)). We test for the number of cointegrating relations among the 9 variables in \mathbf{z}_t . We use the Akaike- and Schwarz-Information Criterion to select the order of the underlying VAR model. We compute the LR-statistic for models with exact and over-identifying restrictions on the long-run coefficients. An exact identification in our model requires five restrictions on each of the five cointegrating vectors, or a total of twenty-five restrictions on β . The economic theory as characterized in the matrix (14) defines thirty four restrictions. Estimation of the model subject to all this restriction enables a test of the validity of the over-identifying restrictions.

3 Estimation and Testing of the Model

Data

The variables for the core model under consideration are $y_t, y_t^*, r_t, r_t^*, e_t, h_t - y_t, p_t, \tilde{p}_t, p_t^*$ and p_t^o . A detailed description of these variables is given in Table (1). The data are taken from the OECD Statistical Compendium and the IMF Data Base. They are all quarterly and seasonally adjusted data. In the model we use data after logarithm transformation. They cover the period 1991Q1-2005Q4 (56 observations). Similar to paper by Garratt et al. (2003b), we use the producer price indices to construct deviations between the domestic and foreign price levels in the PPP relationship. Instead of the retail price index we use the consumer price index to measure domestic inflation in the FIP relationship.

The Augmented DickeyFuller (ADF) test statistics for the levels and first differences of the core variables are reported in Table (2). The results suggest that it is reasonable to treat $y_t, y_t^*, r_t, r_t^*, e_t, h_t - y_t, p_t, p_t^*$ and p_t^o as I(1) variables. For these variables the unit root hypothesis is rejected when applied to their first differences for all variables at the significance level of 1% excepting y_t^* and r_t^* . For these two variables the unit root hypothesis is rejected at the significance level of 5%. When the tests are applied to levels of the variables unit root hypothesis cannot be rejected. There is, however, an exception regarding the order of integration of the price variable \tilde{p}_t . The application of the ADF test to $\Delta\tilde{p}_t$ does not reject the unit root hypothesis, but $\Delta\Delta\tilde{p}_t$ is identified as a stationary variable. This corresponds to the fact, that \tilde{p}_t is an I(2) variable. However, this variable exists according to the model description only as differences in the Fischer equation. Therefore the econometric conditions on all variables are fulfilled.

Estimation and Testing of the Long Run Relations

The first stage of our modelling sequence is to select the order of the underlying VAR in these variables. Here we find that a VAR of order two appears to be appropriate when using the AIC and SIC as the model selection criterion. Using a VAR(2) model with unrestricted intercepts and treating the oil price variable, p_t^o , as weakly exogenous for the long-run parameters, we computed Johansens λ_{trace} and λ_{max} statistics. These statistics, together with their associated 90% and 95% critical values, are reported in Table (3).

The maximal eigenvalue statistic indicates the presence of just five cointegrating relationships at the 1% significance level, which supports our a priori expectations of five cointegrating vectors. The trace test also identifies five cointegration relations at the significance level of 1%. But seven cointegration relations at 5% level. Since five

cointegration relations are in line with our a priori expectation based on the long-run theory, we proceed under the assumption that there are five cointegrating vectors.

With five cointegrating relations we require five restrictions on each relationship to exactly identify them. In view of the underlying long-run theory in the relations (8)-(12) we impose the following 25 exact-identifying restrictions on the cointegrating matrix:

$$\beta' = \begin{pmatrix} \beta_{11} & 0 & 0 & \beta_{14} & 1 & \beta_{16} & 0 & \beta_{18} & 0 \\ \beta_{21} & 1 & 0 & \beta_{24} & 0 & 0 & 0 & \beta_{28} & \beta_{29} \\ \beta_{31} & 0 & 1 & 0 & \beta_{35} & 0 & \beta_{37} & 0 & \beta_{39} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} & 0 & 0 & 1 & 0 & 0 \\ \beta_{51} & \beta_{52} & 0 & -1 & 0 & 0 & \beta_{57} & 0 & \beta_{59} \end{pmatrix} \quad (17)$$

that corresponds to $\mathbf{z}_t = (p_t^o, r_t, y_t, \Delta\tilde{p}_t, p_t - p_t^*, e_t, h_t - y_t, r_t^*, y_t^*)$. The first vector (the first row of β') relates to the PPP relationship defined by (8) and is normalised on $p_t - p_t^*$; the second relates to the IRP relationship defined by (9) and is normalised on r_t ; the third relates to the "output gap" relationship defined by (10) and is normalised on y_t ; the fourth is the money market equilibrium condition defined by (11) and is normalised on $h_t - y_t$; and the fifth is the real interest rate relationship defined by (12), normalised on $\Delta\tilde{p}_t$.

We have 20 unrestricted parameters in (17), and two in the fully restricted model, yielding a total of 18 over-identifying restrictions. In addition, working with a cointegrating VAR with unrestricted intercept coefficients, there are potentially five further parameters in the five cointegrating relationships. There are just 25 parameters to be freely estimated in the cointegrating relationships and provide a total of 18 over-identifying restrictions on which the core model is based and with which the validity of the economic theory can be tested. LR statistic for testing the eighteen over-identifying restrictions in the matrix (17) results in a test statistic of 72.65 (see Tab. 4). According to Garrat et al. (2003) the relevant critical values for the joint tests of the 25 over-identifying restrictions are 67.51 at the 10% significance level and 73.19 at the 5% level. Therefore we cannot reject the over-identifying restrictions implied by the long-run theory in (4), (6), (3), (7) and (5).

The Vector Error Correction Model

The long-run relations, which incorporate all the restrictions suggested by the theory, are summarized below:

$$(p_t - p_t^*) - e_t = 0.186 + \hat{\xi}_{1,t+1} \quad (18)$$

$$r_t - r_t^* = 0.002 + \hat{\xi}_{2,t+1} \quad (19)$$

$$y_t - y_t^* = 0.051 + \hat{\xi}_{3,t+1} \quad (20)$$

$$h_t - y_t = -1.751 - \underset{(-0.34)}{20.473r_t} + \underset{(11.60)}{102.03y_t} + \hat{\xi}_{4,t+1} \quad (21)$$

$$r_t - \Delta\tilde{p}_t = 0.006 + \hat{\xi}_{5,t+1} \quad (22)$$

The first equation (18), describes the PPP relationship and is not rejected in the context of the core model. The co-movements of exchange rates and relative prices have been examined frequently in the literature. The empirical evidence on PPP appears to be sensitive to the data set used and the way in which the analysis is conducted.

The second cointegrating relation, defined by (19), is the IRP condition. This includes an intercept, which can be interpreted as the deterministic component of the risk premium associated with bonds and foreign exchange uncertainties. Its value is estimated at 0.002, implying a risk premium of approximately 0.8% per annum.

The third long-run relationship, given by (20), is the "output gap relationship" with per capita domestic and foreign output levels. This relationship indicates the moving of the output levels in tandem in the long-run. It suggests that the average long-run growth rate for Germany is the same as that in the rest of the OECD. The implied hypothesis here, that y_t and y_t^* are cointegrated, is much less restrictive than the hypothesis considered in the literature that all pairs of output variables in the OECD are cointegrated.

For the money market equilibrium (MME) condition, given by (21), we can view the left hand side as M0-velocity. This cointegration relation says that the money-income ratio increases in Germany with per capita output. There is weak evidence of a negative interest rate effect on real money balances.

Finally, the fifth equation, (22), defines the FIP relationship, where the estimated intercept implies an annual real rate of return of approximately 2.4% per annum. Our results support the FIP relationship and again highlights the important role played by the FIP relationship in a model of the macroeconomy which can incorporate interactions between variables omitted from a more partial analysis.

The short-run dynamics of the model are characterized by the error correction specifications given in Table (5). The estimates of the α -coefficients (also known as the loading coefficients) show that the long-run relations make an weak contribution in most equations. Two or maximal three coefficients are statistically relevant in the α -Matrix. The variables interest rate, income and inflation represent a statistically significant set by long-run adjustment mechanism. Similar to the estimation of the core model for UK is the Γ -Matrix (known as the short run adjustment) weakly assigned. The changes in domestic income, world interest rate and exchange rate from the previous period

$(t - 1)$ lead to the changes in the other variables of the \mathbf{z}_t vector. The rate of increase of the oil price has had a statistically significant impact on the $\Delta(p_t - p_t^*)$ and on the change of the world interest rate Δr_t^* .

Impulse Response Analysis

The impulse response functions with respect to an oil price shock are reported in Figure 1. It shows, that the oil price shock has permanent effects on the level of some series. The oil price shock has at the beginning a negative effect on the domestic output, reducing the output up to 0.5% per year. After two and half years the economy will recover from the oil price shock, and an increase in output can be expected. After 6 years the economy will finally restore the original level of output. However foreign output raises by approximately 0.024% per year, and then reduces after 2.5 years, so that the output gap returns to its equilibrium. The effect of the oil price shock on the domestic rate of inflation is permanent. After one year of up and down adjustment the domestic rate of inflation will increase by 0.16% permanently. Due to the permanent higher domestic rate of inflation, the oil price shock generates a depreciation of the nominal exchange rate in the first 8 years. But the nominal exchange rate will restore the original level after that. Further, the oil price shock is accompanied by increases in both domestic and foreign interest rates by roughly 0.2%.

4 Conclusions

This paper provides an example of macroeconomic-modelling for German data using the method developed in Garratt et al. (2003b). As basic framework for the modelling the core model for the UK by Garratt et al.(2003) is used. Following closely Garratt et al. (2003b), this paper outlines a theoretical derivation for the long-run analysis of a small open macroeconomy; introduces a practical approach to theory-based long-run relationships in an otherwise unrestricted VAR; and presents the estimates and tests of a macroeconomic model for Germany. The modelling process starts with a presentation of a set of long-run relationships between the macroeconomic variables such as interest rate, output or exchange rate. These long-run relationships are based on production, arbitrage, solvency and portfolio balance conditions, together with stock-flow and accounting identities. Further, these long-run relationships are embedded in an unrestricted VAR model with nine core variables, augmented appropriately by intercepts. The VAR model is estimated over the period 1991Q1-2005Q4, subject to the theory restrictions on the long-run coefficients using recently developed econometric techniques in Garratt et al. (2003b).

An important component of our modelling approach is the possibility for testing

formally the validity of restrictions suggested by economic theory in the context of a complete macroeconomic model. The underlying economic theory provides five long-run relations or equilibrium conditions among the nine core variables of the macro-model. The statistical tests provided little evidence to reject this view. Under the assumption that there are five long-run relationships, we obtained a model with seven freely estimated parameters. Further, the likelihood ratio tests did not reject the over-identifying restrictions suggested by economic theory, so that we conclude that the estimated model is both theory and data consistent.

In the short run analysis of the our model, we analyze the response of the levels of the model's eight endogenous variables to the oil price shocks. Among other results we found evidence that an unexpected rise in oil prices will increase domestic and foreign interest rates, will have a moderate contractionary effect on real domestic output, will increase the inflation rate and lead to a depreciation of the nominal exchange rates.

References

- CHAUDHURI, K. AND DANIEL, B. (1998). Long run equilibrium real exchange rates and oil prices. *Economic Letters*, 58(2):231-238.
[Further information in IDEAS/RePEc](#)
- CROWDER, W. J., HOFFMAN, D. L., AND RASCHE, R. H. (1999). Identification, long-run relations, and fundamental innovations in a simple cointegrated system. *Review of Economics and Statistics*, 81:109-21. [Further information](#)
- GALI, J. (1992). How well does the is/lm model fit postwar us data? *Quarterly Journal of Economics*, 107:709-38. [Further information in IDEAS/RePEc](#)
- GARRATT, A., LEE, K., PESARAN, M., AND SHIN, Y. (2000). *The Structural Cointegrating VAR Approach to Macroeconometric Modelling*. S. Holly and M. Weale, Cambridge University Press. [Further information in IDEAS/RePEc](#)
- GARRATT, A., LEE, K., PESARAN, M. H., AND SHIN, Y. (2003a). Forecast uncertainties in macroeconometric modelling: an application to the uk economy. *University of Cambridge Working Paper*, available on <http://www.econ.cam.ac.uk/faculty/pesaran>.
- (2003b). A long-run structural macroeconometric model of the UK. *The Economic Journal*, 113:412-455. [Further information in IDEAS/RePEc](#)
- JOHANSEN, S. (1991). Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59:1551-1580.
[Further information in IDEAS/RePEc](#)
- (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press: Oxford. [Further information](#)
- KING R. G., PLOSSER, C. I. S. J. H. AND WATSON, M. W. (1991). Stochastic trends and economic Fluctuations. *American Economic Review*, 81:819-40.
[Further information in IDEAS/RePEc](#)
- MACKINNON, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics*, 11(6):601-619.
[Further information in IDEAS/RePEc](#)
- MELLANDER, E., VREDIN, A., AND WARNE, A. (1991). Stochastic trends and economic Fluctuations in a small open economy. *Journal of Applied Econometrics*, 7:369-94.
[Further information in IDEAS/RePEc](#)
- PESARAN, M. H. AND SHIN, Y. (2002). Long-run structural modelling. *Econometric Reviews*, 21(1):49-87. [Further information in IDEAS/RePEc](#)

Table 1: List of Variables and their Descriptions in the Core Model

y_t :	natural logarithm of the German real per capita GDP (GDP deflator) (2000=100).
p_t :	natural logarithm of the German Producer Price Index (2000=100%).
\tilde{p}_t :	natural logarithm of the German Consumer Price Index (2000=%100).
r_t :	is computed as $r_t = 0.25\ln(1 + R_t/100)$, where R_t is the 90 day Interbank discount rate per annum.
h_t :	natural logarithm of the German real per capita M1 money stock (2000=100%), Germany's share in M1 EMU (from 2003 without cash)
e_t :	natural logarithm of the nominal DM/Euro exchange rate, monthly average (2000=100%).
y_t^* :	natural logarithm of the foreign (OECD) real per capita GDP (GDP deflator) (2000=100%).
p_t^* :	natural logarithm of the foreign (OECD) Producer Price Index (2000=100%).
r_t^* :	is computed as $r_t = 0.25\ln(1 + R_t^a/100)$, where R_t^a is the weighted average 90 day interest rate per annum in the USA, UK, Japan and France
p_t^o :	natural logarithm of oil price, measured as the Average Price in US\$ per Barrel Oil.
t :	time trend, taking the values 1,2,3,... in 1991Q1, \dots , 2005Q4 respectively.

Table 2: Augmented Dickey-Fuller Unit Root Test applied to Variables in the Core Model; 1991Q1-2005Q4

variable	for the levels		for the first differences	
	t-statistic	lags	t-statistic	lags
y_t	-2.43	4	-4.03	3
y_t^*	-1.07	1	-2.97	2
r_t	-1.90	2	-3.93	1
r_t^*	-3.03	2	-3.14	1
e_t	-2.45	4	-6.96	4
$h_t - y_t$	-2.67	1	-6.89	1
p_t	-2.08	2	-4.39	1
\tilde{p}_t	-3.18	5	-1.77	7
p_t^*	-2.15	2	-4.83	1
p_t^o	-1.63	6	-4.35	5
$p_t - p_t^*$	-0.80	1	-7.57	1
$\Delta \tilde{p}_t$	-2.57	4	-11.46	3

Notes: The t-statistics are computed using ADF regressions with an intercept, a linear time trend and s lagged depended variables, when applied to the levels; and with an intercept and s lagged first-differences of dependent variable, when applied to the first difference. The order of augmentation in the Dickey-Fuller regressions is chosen using the Akaike Information Criterion, with a maximum lag order of ten. The critical values for the t-Test: -4.12 (level of significance 1%) and -3.49 (level of significance 5%) for the levels. -3.55 (level of significance 1%) and -2.91 (level of significance 5%) for the differences. The critical values are from MacKinnon (1996).

Table 3: Cointegration Rank Statistics for the Core Model

$$(y_t, y_t^*, r_t, r_t^*, e_t, h_t - y_t, p_t - p_t^*, \Delta \tilde{p}_t)$$

(A) Trace Statistic

H_0	H_1	$TestStatistic$	5% Critical values	1% Critical Values
$r \leq 0$	$r \geq 1^{**}$	352.12	192.89	204.95
$r \leq 1$	$r \geq 2^{**}$	272.01	156.00	168.36
$r \leq 2$	$r \geq 3^{**}$	198.89	124.24	133.57
$r \leq 3$	$r \geq 4^{**}$	150.45	94.15	103.18
$r \leq 4$	$r \geq 5^{**}$	107.86	68.52	76.07
$r \leq 5$	$r \geq 6^{**}$	67.01	47.21	54.46
$r \leq 6$	$r \geq 7^*$	34.57	29.68	35.65
$r \leq 7$	$r \geq 8^*$	17.38	15.41	20.04

(B) Maximum Eigenvalue Statistic

H_0	H_1	$TestStatistic$	5% Critical values	1% Critical Values
$r \leq 0$	$r = 1^{**}$	80.11	57.12	62.80
$r \leq 1$	$r = 2^{**}$	73.11	51.42	57.69
$r \leq 2$	$r = 3^*$	48.43	45.28	51.57
$r \leq 3$	$r = 4^{**}$	42.59	39.37	45.10
$r \leq 4$	$r = 5^{**}$	40.84	33.46	38.77
$r \leq 5$	$r = 6$	32.44	27.07	32.24
$r \leq 6$	$r = 7$	17.18	20.97	25.52
$r \leq 7$	$r = 8$	13.97	14.07	18.63

Notice: The underlying VECM model is of order 1 and contains unrestricted intercepts. The asymptotic critical values are taken from EVIEWS.

Table 4: Test of Overidentification

Vector Error Correction Estimates

Date: 09/02/07 Time: 11:27

Sample(adjusted): 1991:4 2005:4

Included observations: 57 after adjusting endpoints

Standard errors in () & t-statistics in []

Cointegration Restrictions:

B(1,1)=0,B(1,2)=0,B(1,3)=0,B(1,4)=0,B(1,5)=1,B(1,6)=-1,B(1,7)=0,B(1,8)=0,B(1,9)=0
B(2,1)=0,B(2,2)=1,B(2,3)=0,B(2,4)=0,B(2,5)=0,B(2,6)=0,B(2,7)=0,B(2,8)=-1,B(2,9)=0
B(3,1)=0,B(3,2)=0,B(3,3)=1,B(3,4)=0,B(3,5)=0,B(3,6)=0,B(3,7)=0,B(3,8)=0,B(3,9)=-1
B(4,1)=0,B(4,4)=0,B(4,5)=0,B(4,6)=0,B(4,7)=1,B(4,8)=0,B(4,9)=0
B(5,1)=0,B(5,2)=1,B(5,3)=0,B(5,4)=-1,B(5,5)=0,B(5,6)=0,B(5,7)=0,B(5,8)=0,B(5,9)=0

Convergence achieved after 1764 iterations.

Restrictions identify all cointegrating vectors

LR test for binding restrictions (rank = 5):

Chi-square(18) 72.65663

Cointegrating Eq:	CointEq1	CointEq2	CointEq3	CointEq4	CointEq5
P_OIL(-1)	0.000000	0.000000	0.000000	0.000000	0.000000
IR(-1)	0.000000	1.000000	0.000000	20.47302	1.000000
			(59.0727)		
			[0.34657]		
Y(-1)	0.000000	0.000000	1.000000	-102.0326	0.000000
			(8.79566)		
			[-11.6003]		
INFLAT(-1)	0.000000	0.000000	0.000000	0.000000	-1.000000
PPP(-1)	1.000000	0.000000	0.000000	0.000000	0.000000
EXR(-1)	-1.000000	0.000000	0.000000	0.000000	0.000000
H_YSA(-1)	0.000000	0.000000	0.000000	1.000000	0.000000
IR_WORLD(-1)	0.000000	-1.000000	0.000000	0.000000	0.000000
YA(-1)	0.000000	0.000000	-1.000000	0.000000	0.000000
C	-0.186435	-0.002073	-0.050758	1.751061	-0.005506

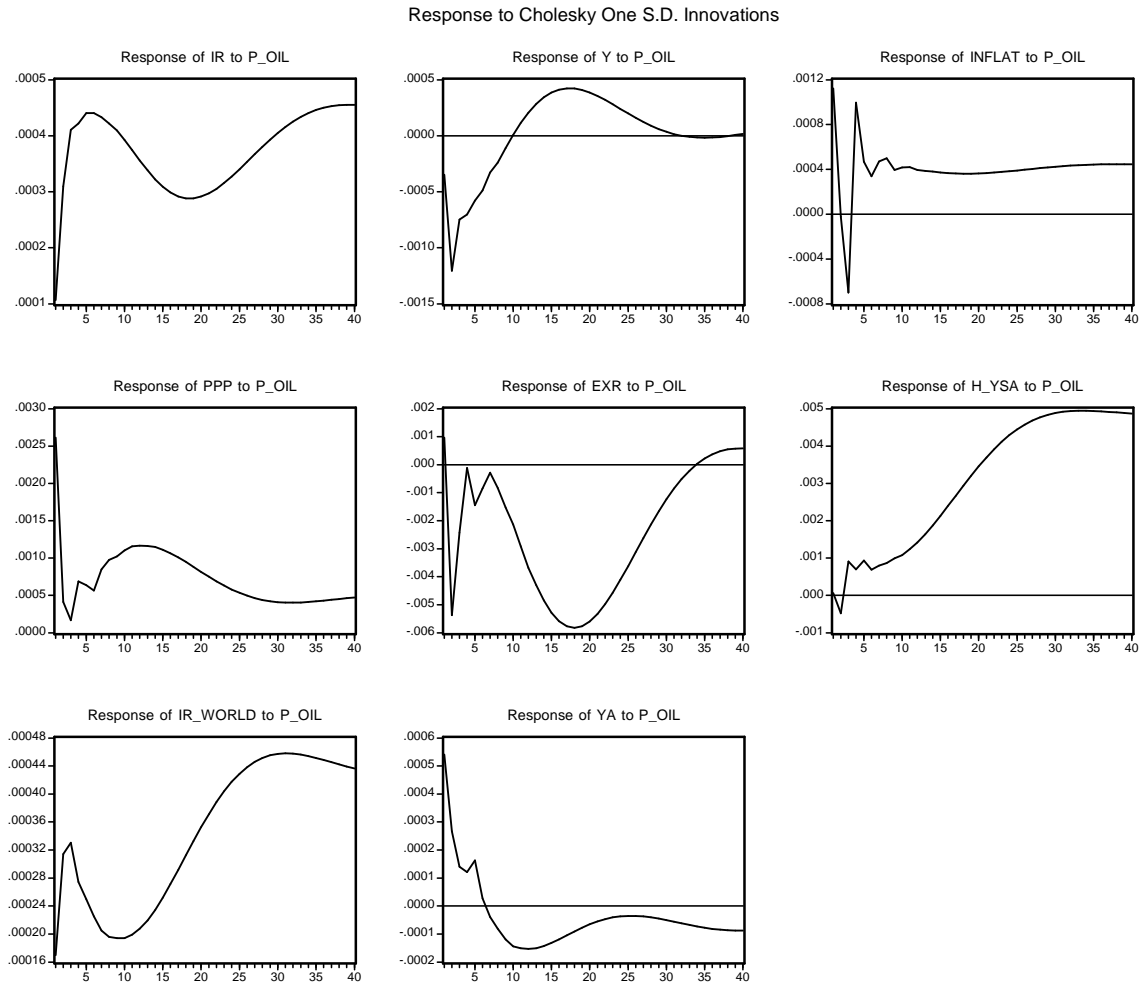


Figure 1: Impulse Responses to One SD Oil Price Shock

Table 5: Reduced Form Error Correction Specification for the Core Model of Germany

(A) α -Coefficients

Equations	Δp_t^o	Δr_t	Δy_t	$\Delta(\Delta \tilde{p}_t)$	$\Delta(p_t - p_t^*)$	Δe_t	$\Delta(h_t - y_t)$	Δr_t^*	Δy_t^*
$\hat{\xi}_{1,t}$	0.00 (0.00)	0.0006 (0.001)	0.011* (0.005)	0.029* (0.007)	0.005 (0.007)	0.059 (0.074)	0.043 (0.049)	-0.0003 (0.00008)	-0.005 (0.004)
$\hat{\xi}_{2,t}$	0.00 (0.00)	-0.133* (0.034)	0.349* (0.161)	-0.385 (0.249)	-0.180 (0.233)	-2.167 (2.55)	2.11 (1.674)	0.010 (0.026)	-0.036 (0.129)
$\hat{\xi}_{3,t}$	0.00 (0.00)	-0.014* (0.006)	-0.094* (0.028)	-0.140* (0.044)	-0.034 (0.041)	0.070 (0.449)	0.280 (0.295)	0.005 (0.005)	0.042 (0.023)
$\hat{\xi}_{4,t}$	0.00 (0.00)	0.0005* (0.0001)	0.003* (0.0006)	0.001 (0.001)	0.002 (0.0009)	-0.004 (0.010)	-0.01 (0.006)	-0.0001 (0.0001)	-0.0002 (0.0005)
$\hat{\xi}_{5,t}$	0.00 (0.00)	-0.027 (0.031)	0.128 (0.144)	1.433* (0.223)	-0.026 (0.208)	0.736 (2.283)	0.99 (1.498)	-0.054* (0.023)	0.036 (0.116)

(B) Γ -Matrix

	Δp_t^o	Δr_t	Δy_t	$\Delta(\Delta \tilde{p}_t)$	$\Delta(p_t - p_t^*)$	Δe_t	$\Delta(h_t - y_t)$	Δr_t^*	Δy_t^*
Δp_{t-1}^o	0.00 (0.00)	0.0006 (0.0008)	-0.009* (0.004)	-0.011 (0.006)	-0.015* (0.006)	0.030 (0.061)	-0.001 (0.040)	-0.0002 (0.0006)	-0.004 (0.003)
Δr_{t-1}	0.00 (0.00)	-0.106 (0.191)	-2.081* (0.892)	-2.535 (1.377)	-1.222 (1.288)	7.915 (14.1)	10.48 (9.257)	-0.108 (0.143)	0.124 (0.715)
Δy_{t-1}	0.00 (0.00)	0.022 (0.028)	-0.318* (0.131)	0.122 (0.202)	0.466* (0.189)	-1.285 (2.064)	-2.77* (1.355)	-0.006 (0.021)	0.061 (0.105)
$\Delta(\Delta \tilde{p}_{t-1})$	0.00 (0.00)	-0.0009 (0.020)	0.096 (0.094)	0.039 (0.145)	-0.111 (0.135)	-1.055 (1.482)	0.329 (0.973)	-0.015 (0.015)	0.109 (0.075)
$\Delta(p - p^*)_{t-1}$	0.00 (0.00)	-0.002 (0.017)	-0.012 (0.081)	0.286* (0.126)	-0.081 (0.117)	-1.585 (1.286)	0.58 (0.844)	0.008 (0.013)	-0.043 (0.065)
Δe_{t-1}	0.00 (0.00)	0.001 (0.002)	-0.022* (0.010)	0.031* (0.015)	0.007 (0.014)	0.234 (0.157)	-0.09 (0.103)	0.001 (0.002)	-0.009 (0.008)
$\Delta(h - y)_{t-1}$	0.00 (0.00)	-0.004 (0.004)	0.005 (0.017)	0.018 (0.026)	-0.029 (0.024)	0.18 (0.265)	0.08 (0.174)	0.001 (0.003)	0.014 (0.014)
Δr_{t-1}^*	0.00 (0.00)	0.461* (0.202)	2.187* (0.942)	2.381 (1.454)	1.179 (1.36)	-28.35 (14.89)	-11.2 (9.772)	0.619* (0.151)	0.75 (0.755)
Δy_{t-1}^*	0.00 (0.00)	-0.012 (0.042)	0.133 (0.196)	0.496 (0.302)	-1.029* (0.283)	-1.676 (3.094)	-1.44 (2.03)	-0.015 (0.031)	0.075 (0.157)

(C) ψ -Coefficients and Constant

	Δp_t^o	Δr_t	Δy_t	$\Delta(\Delta \tilde{p}_t)$	$\Delta(p_t - p_t^*)$	Δe_t	$\Delta(h_t - y_t)$	Δr_t^*	Δy_t^*
Δp_t^o	1.00* (0.00)	0.0008 (0.0007)	-0.003 (0.003)	0.009 (0.005)	0.021* (0.005)	0.008 (0.052)	0.00 (0.034)	0.001* (0.001)	0.005 (0.003)
c	0.00 (0.00)	-0.00203 (0.0003)	0.003 (0.001)	-0.003 (0.002)	0.00 (0.002)	0.00 (0.022)	0.03* (0.014)	0.00 (0.00)	0.005* (0.001)

Notice: Standard errors are given in parenthesis; * indicates significance at the 10% level.